Wirtschaftswissenschaftliche Fakultät
der Eberhard Karls Universität Tübingen

Neutrality-Based Effective Tax Rates

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Tübinger Diskussionsbeitrag Nr. 249
September 2002

Wirtschaftswissenschaftliches Seminar
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Abstract*

Effective tax rates (ETRs) are designed to indicate the influence of taxes on investments. Existing ETR models fail to generate ETRs that can be compared to a constant yardstick and to other ETRs. This paper develops a new ETR approach based on neutral tax systems. Integration of neutral taxation into the computation of ETRs overcomes the problem of traditional numerical concepts: Comparison of the new ETR and the statutory tax rate as a constant yardstick reveals preferential or discriminatory taxation of investments. Moreover, the comparison of different ETRs displays which investment is distorted to a higher or lower degree.

JEL Classification: H21, H25, H32.
Keywords: Effective Tax Rate, Neutral Tax System, Tax Distortion, Investment.

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* I thank Rainer Niemann for very helpful comments and suggestions.

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1 Introduction

It is well known that taxation may affect investment behavior. For this reason, capital budgeting and public finance provide many instruments to analyze tax effects, e.g. net present value (NPV), user cost of capital, and real option theory. In the last two decades, different concepts of effective tax rates (ETRs) were developed in order to provide condensed information about these tax distortions. The concepts compress the complex economic effects of the statutory tax rate, the tax base, and the time aspect of taxation into a single figure.

ETRs are designed to reveal the impact of taxation on investment behavior. In order to reach this aim, ETR concepts should reflect the individual investor's economic decision problems regarding heterogeneous investment projects. This must not be confused with using ETRs for the investment decision itself. Decision-making has to be done using the post-tax NPV or transformations like the future value or the rate of return. ETR concepts can only be used for decision-making, when the model's pre-tax assumptions are identical for all investment alternatives.

Using ETRs, taxational distortions of investment decisions can be identified. In general, this is achieved by comparing ETRs of specific investments with a neutrally taxed yardstick or with other investment's ETRs. As a limitation, not all ETR approaches discussed in the literature are able to display these distortions.

In the literature there are two kinds of ETR approaches, analytical models like the well-known King/Fullerton[1] (KF) and Devereux/Griffith[2] (DG) approaches and numerical models like the European Tax Analyzer[3] (ETA). All these approaches fail to satisfy some of the requirements on ETRs formulated in this paper.

To overcome these shortcomings, we develop a new ETR concept integrating neutral tax systems. As an improvement of existing numerical approaches, the new model consists of three main elements: An ETR is determined by the comparison of pre-tax and post-tax objective values. Further, a neutral reference tax system is integrated into the model. Every ETR is calculated based on a comparison of the actual tax burden with the neutral ideal tax burden. Although the new approach cannot fulfill all desired requirements on ETRs, it is a major improvement compared to existing models. A comparison of the new ETR and the statutory tax rate reveals preferential or discriminatory taxation. Moreover,

[1] See King/Fullerton (1984).
[2] See Devereux/Griffith (1999); Devereux/Griffith (2002).
[3] See Spengel (1995); Jacobs/Spengel (2002).

the comparison of ETRs of different investments displays the magnitude of taxational distortions.

The remainder of the article is organized as follows: Section 2 describes general requirements on ETRs. In section 3 existing ETR approaches are briefly presented and examined with respect to the defined requirements. Section 4 introduces the general idea of a meaningful ETR based on neutral taxation, followed by a short discussion of suitable objective variables for the ETR approach. The new neutrality-based model will be presented in detail in section 5; section 6 discusses its main features. Section 7 summarizes and concludes.

2 Requirements on ETRs

The following requirements have to be met by an ETR in order to indicate the impact of taxation on investments[4].

- *Reproduction of real-world investments.* At first, an instrument for examining investments must be able to consider any investment project. That is, reproduction of rising, falling, constant, and changing cash flows π_t must be possible. This comprises profitable investments with a positive NPV, zero NPV investments, and investments with a negative NPV.

- *Multi-period models.* Closely linked is the requirement of multi-period models. In general, long-term investments are not an annual repetition of a set of parameters defined in period $t = 1$. In contrast, cash flows, depreciation deductions, stock of inventories, e.g., vary over time, which leads to different tax bases in each period. Next to this tax base effect, only multi-period models take into account the time value of money and time effects of taxation. This can be achieved by linking subsequent periods, where reinvestment of cash flows is considered. ETRs depend on the assumption of the investment's time horizon. Analytical approaches assume an infinite economic life of assets, whereas numerical approaches can only consider a finite number of periods.

- *Reproduction of real tax systems.* To reveal the impact of taxation, real tax systems should be reproduced in detail by the model. This includes all relevant taxes as well

[4] They are partially taken from Niemann/Bachmann/Knirsch (2002), p. 1546.

as the legally defined tax base. The latter must be determined integrating asymmetric and state-dependent elements like loss compensation and unequal provision for unrealized gains and losses.

- *Corporate and shareholder taxes.* The significance of a tax burden comparison depends on the consideration of all taxes relevant for investment decisions. The tax burden is affected by both corporate and personal taxes, and their interaction.

  The last two requirements cause a dilemma: Neutral tax systems rely on the existence of a single tax rate for both earnings and the discount factor and a unique interest rate. These simplified rules do not reflect reality, but they are the precondition for neutral tax systems and herewith for a neutral yardstick in any ETR concept.

- *Authenticity.* Distorting tax treatment of a real investment can be identified only if it is not mixed with other types of investment. If investment projects rather than investments in single assets are analyzed, there is always a mixture of real and financial investment. If free cash flows generated by the real investment are not immediately reinvested into new assets, financial investments emerge. Since the effective tax rate for financial investment equals the statutory tax rate, mixing real and financial investment always induces a tendency towards the statutory tax rate, thereby underestimating tax distortions of real investment. A trade-off between considering financial investment as a linkage of subsequent periods and authenticity of the ETR is unavoidable. Authenticity is only possible in a one-period approach or if the disputed internal rate of return is applied as objective variable.

- *Comparability of ETRs.* If an investment is analyzed under different tax regimes, the resulting ETRs must form an ordinal ranking, where a higher (lower) ETR corresponds to a lower (higher) post-tax net present value (NPV^τ). Instead of the NPV^τ, other decision criteria generating the same ranking of investment projects could be used[5]. For a given investment with NPV^τ_A and NPV^τ_B under tax systems A and B

  $$ETR_A \gtreqless ETR_B \Leftrightarrow NPV^\tau_A \lesseqgtr NPV^\tau_B \qquad \forall A, B \tag{1}$$

  must be satisfied. In this case, ETRs of investments with identical pre-tax parameters could be used as a basis for investment decisions instead of their NPV^τ. This property is lost, if the pre-tax data vary between the analyzed investment alternatives. Then, decision-making cannot rely on ETRs.

[5] A short discussion of decision criteria is presented in section 4.

- *Revelation of distortions compared to a constant yardstick.* If an investment is not taxed neutrally, decisions based on pre-tax data could be wrong when considering taxation. Under neutral taxation, the pre-tax and post-tax ranking of different investment projects coincides. Non-neutrality of tax systems causes taxational distortions, which can be measured by the deviation of the post-tax objective variable from the neutrally taxed objective variable. An ETR designed to disclose taxational distortions must refer to the investor's economic objective variable and should be comparable to a neutrally taxed yardstick that reveals preferential or discriminatory taxation of an investment[6]. Constancy of the yardstick is desirable, because it allows comparison of any ETR with the reference value. If the yardstick is variable, it must be explicitly calculated for every single investment project and cannot be generally applied.

  A possible and easy-to-understand yardstick for ETRs is the statutory tax rate τ. ETRs below (above) τ correspond to a preferential (discriminatory) taxation of the investment, respectively.

- *Comparability of distortions.* For any investment, the magnitude of tax distortion must be comparable relative to other investment's and to other tax system's distortions. If the absolute difference from the ETR of investment 1 and the constant yardstick, the statutory tax rate τ, is higher (lower) than the one of the alternative investment 2

$$|ETR_1 - \tau| \gtreqless |ETR_2 - \tau|, \tag{2}$$

  investment 1 is distorted by taxation to a higher (lower) degree. This comparison should not depend on the pre-tax parameters, i.e. initial investment I_0, NPV, cash flow structure π_t, time horizon T, and tax rules of investments 1 and 2.

3 Existing ETR approaches

3.1 The King/Fullerton approach

The well-known KF[7] approach measures the effective marginal tax rate (EMTR) of investments with a zero NPV. It relies on the concept of the user cost of capital[8] based on the following assumptions: Capital goods last forever. The investor's time horizon is

[6] See Schreiber/Spengel/Lammersen (2002), p. 3; Lammersen (2002), p. 11.
[7] See King/Fullerton (1984).
[8] See Jorgenson (1963); Hall/Jorgenson (1967).

infinite. Financial markets and capital goods markets are competitive and complete. The capital stock of the investment declines exponentially over time, because assets depreciate at a constant rate δ. Earnings amount to $\delta + r$, where r denotes the pre-tax rate of return. For assessing the impact of taxation, a simplified tax system is applied, whose tax base takes cash flows, depreciations, and inventory valuation into account. Corporate as well as shareholder taxes can be integrated into the model.

3.2 The Devereux/Griffith approach

The DG[9] approach is an extension of the KF model that analyzes mutually exclusive projects earning more than their cost of capital. This includes investments with a positive NPV[10]. Therefore, it measures the effective average tax rate (EATR). Capital invested for one period generates a cash flow that compensates for economic depreciation δ and yields an additional profit depending on the chosen rate of return $\pi = \delta + r$. The method is in fact a one-period approach[11], even though some multi-period elements like the present value of tax depreciations are taken into account. The post-tax value is derived using a very simplified tax system with few tax base elements[12].

Considering solely a uniform corporate tax, the KF-EMTR and the DG-EATR equal the statutory tax rate, if tax depreciations equal the rate of economic depreciation δ. Therefore, the statutory tax rate serves as a yardstick for revealing tax distortions. But for projects with different pre-tax rates of return, the DG-EATRs cannot be compared to each other[13].

The DG model, as the KF model, relies on quite restrictive assumptions[14]. The tax system is simplified. Moreover, most of the real-world investments, for instance investments with cash flows that do not decline exponentially, are not covered by the two models. For this reason, these models once designed for macro-econometric analyses cannot provide solutions for individual investors who need flexible models for the analysis of heterogeneous investment projects. Due to these shortcomings, analytical models will not be applied in the sequel. Instead, it will be analyzed if numerical approaches are of better use for this purpose.

9 See Devereux/Griffith (1999); Devereux/Griffith (2002).
10 In which situations the EMTR or EATR is the more suitable concept is discussed in Jacobs/Spengel (2000), pp. 338-339.
11 See Schreiber/Spengel/Lammersen (2002), p. 17.
12 See Niemann/Bachmann/Knirsch (2002), p. 1550.
13 See Niemann/Bachmann/Knirsch (2002), p. 1550.
14 See Lammersen (2002), p. 23.

3.3 The European Tax Analyzer based on numerical simulations

The ETA[15] is a numerical business simulation tool that is based on a detailed cash flow statement of the company or a specific investment. The EATR is computed by comparison of the pre-tax and post-tax rate of return $EATR^{ETA} = \frac{r-r^\tau}{r}$. The pre-tax rate r as well as the post-tax rate r^τ result from transforming the pre-tax future value of the investment (FV) or post-tax future value (FV^τ) into annualized returns

$$r = \sqrt[T]{\frac{FV}{I_0}} - 1, \qquad r^\tau = \sqrt[T]{\frac{FV^\tau}{I_0}} - 1, \tag{3}$$

where I_0 denotes the initial outlay and T the time horizon. The future values are derived from the company's cash flow statement and a valuation of the non-cash elements in period T. In each period, the balance sheet, profit and loss statement, and cash flow statement are generated in order to determine the actual tax burden and the amount of funds that can be reinvested in the next period. The tax base consists of many different elements like depreciation, stock valuation, and allowance for pension provisions. As it is a multi-period model, they can differ from year to year. Due to this detailed determination of the future value, the ETA is able to reproduce any kind of investment and any real-world tax system. If one investment is analyzed under different tax systems, the resulting EATRs build a ranking that can be used for investment decisions as explained in the requirements. But this feature gets lost if any parameter of the compared investments differs.

If a constant tax rate and a uniform capital market rate are applied under a neutral tax system, a yardstick for revealing tax distortions can be determined[16]. But if a profitable investment is regarded, this yardstick deviates from the statutory tax rate[17]. Moreover, it is not constant. It differs when parameters like the rate of return or the time horizon change: the neutrally taxed yardstick decreases while the pre-tax rate of return rises. The resulting EATRs are arbitrary and cannot be interpreted in relation to the statutory tax rate. The requirements of revelation and comparability of distortions are not satisfied, because the difference of the EATR and τ is meaningless. An EATR above the statutory tax rate can be computed even if the investment is taxed preferentially, and an EATR far below it can be computed even though a specific investment is treated discriminatory[18]. Due to these features the ETA cannot be used for analyzing neutrality of taxation[19].

[15] See Spengel (1995); Jacobs/Spengel (2002).

[16] See Niemann/Bachmann/Knirsch (2002), p. 1551.

[17] Schreiber/Spengel/Lammersen (2002), p. 5: 'The measurement of the effective tax burden on profitable investments presents a dilemma, since the statutory tax rate can no longer serve as a standard measure'.

[18] This effect is demonstrated in figure 1.

[19] See Jacobs/Spengel (2000), p. 339; Schreiber/Spengel/Lammersen (2002), p. 5.

4 Numerical EATR models based on neutral tax systems

Since existing ETR approaches suffer from diverse shortcomings and cannot be applied for disclosing tax distortions in most cases, this paper develops a new ETR concept based on neutral tax systems. The arbitrariness of EATRs of numerical models can be overcome by integrating the tax system under consideration as well as the neutral reference tax system into the EATR calculation. The new EATR is determined in the general form of

$$\begin{aligned} EATR &= \frac{\text{Actual reduction of the objective variable due to taxes}}{\text{Reduction of the objective variable under neutral taxation}} \cdot \tau \\ &= \frac{\text{Actual tax wedge}}{\text{Neutral reference tax wedge}} \cdot \tau. \end{aligned} \tag{4}$$

The fraction describes the tax wedge between the pre-tax objective variable and the actual post-tax objective variable (actual tax wedge) divided by the tax wedge that would result from neutral taxation. This is a distinction compared to existing numerical approaches, since these approaches disregard neutral reference tax systems. The new model avoids the problem of the ETA that calculates an arbitrary EATR and afterwards requires the determination of a special yardstick for interpretation. In case of the new approach, the denominator of (4) instead consists of the pre-tax value reduced by the neutrally taxed post-tax value. Thus, the actual tax burden is compared to the ideal tax burden that would result from neutral taxation. Only this reference to the neutral tax system allows determining an EATR that can be interpreted in comparison to a neutrally taxed constant yardstick. As can be easily seen, analyzing a neutral tax system results in an identical numerator and denominator, thus generating an EATR of τ, i.e. the statutory tax rate. Therefore, the statutory tax rate τ will serve as yardstick for interpreting all resulting EATRs[20].

At first sight, any investment's objective variable seems to be applicable in this general formula. Widely-used criteria for investment decisions are an investment's NPV, future value, annuity, or rate of return. Their qualification as objective variable in the new EATR approach will be discussed briefly[21].

[20] Alternatively, the EATR in (4) could be calculated without the multiplier τ. Then, the neutrally taxed yardstick would be 100%=1. All conclusions drawn in this paper would be the same using this slightly different approach. Multiplication with τ is chosen in order to obtain an easy-to-understand yardstick and because τ is common in the analytical approaches.

[21] For an introduction in calculating these objective variables, see Brealey/Myers (2000), pp. 15-114.

- The NPV is the most widespread decision criterion

$$NPV = -I_0 + \sum_{t=1}^{T} \pi_t (1+i)^{-t} \tag{5}$$

with π_t, $t = 0, ..., T$: pre-tax cash flows at time t
I_0: initial investment.

If the Johansson/Samuelson[22] (JS) tax is used as the reference tax system, $NPV^{\tau,JS} = NPV$. The neutral reference tax wedge in (4) becomes always zero and the EATR is undefined, making the NPV useless in this context.

- This problem does not occur if the future value is chosen instead. Assuming a perfect capital market, it is the present value (PV), compounded to T

$$FV = PV(1+i)^T = (I_0 + NPV)(1+i)^T. \tag{6}$$

For a given initial investment I_0, decisions based on the future value are equivalent to the NPV criterion. An ordinal ranking of post-tax future values can be generated, where a higher (lower) FV^τ corresponds to a more (less) profitable investment. In contrast to the NPV, the future value is an absolute value that is not determined relative to an alternative financial investment. Therefore, future values of investments with different initial costs or different time horizons cannot be compared for the purpose of decision-making.

- An investment's annuity An is determined by

$$An = PV \frac{(1+i)^T \cdot i}{(1+i)^T - 1}. \tag{7}$$

It can be used as a decision criterion as well. The problem of dependency of the initial investment remains.

- The internal rate of return (IRR) solves the equation

$$NPV = -I_0 + \sum_{t=1}^{T} \pi_t (1+IRR)^{-t} = 0. \tag{8}$$

The IRR concept implies that all reinvestments of positive or negative cash flows are compounded at the IRR. If another interest rate is appropriate, the calculated IRR is misleading and could result in a wrong decision that contradicts the decision

[22] See Samuelson (1964); Johansson (1969).

made on the basis of the NPV. If financial investments were always compounded at the IRR instead of the capital market rate, the problem of mixing ETRs from financial and real investment could not emerge.

Due to the well-known problems of determining the IRR[23], this concept is not applied as criterion in the new EATR approach.

- Many of the problems of the IRR can be avoided by calculating the rate as annualized return[24] $r = \sqrt[T]{\frac{FV}{I_0}} - 1$ as applied in the ETA. The interest rate for reinvestment of positive or negative cash flows can be explicitly chosen and may differ from the rate of return. The rate of return is an increasing, but nonlinear transformation of the future value. If the time horizon is constant for all investments, it generates the same ranking of investments and is therefore equivalent to the NPV as decision criterion.

Analyzed in more detail, it becomes obvious that the EATR's properties depend on the chosen objective variable. After this short discussion, the future value, the annuity, and the annualized return seem to be the most suitable objective variables for the new EATR approach. In the following, the approach based on future values and annualized returns will be demonstrated. Similarities to the annuity and differences from the annuity will be explained when necessary.

5 Neutrality-based EATRs applying different objective variables

The neutrality-based EATRs will be figured in detail using the following assumptions: Investments are made under certainty. At the perfect capital market there is a uniform and time-invariant interest rate i, which is used as the discount rate. The tax rate τ is proportional and constant over time. A complete and immediate loss offset is assumed.

5.1 EATRs based on future values

Substituting the objective variable in equation (4) by the future value, the EATR can be written as

$$EATR^{FV} = \frac{\text{Actual tax wedge}}{\text{Neutral reference tax wedge}} \cdot \tau = \frac{FV - FV^{\tau}}{FV - FV^{\tau,\, neutral}} \cdot \tau. \qquad (9)$$

[23] See Brealey/Myers (2000), pp. 98-108.

[24] See Baldwin (1959).

The pre-tax future value is computed as the sum of compounded cash flows π_t

$$FV = \sum_{t=1}^{T} \pi_t (1+i)^{T-t} = (I_0 + NPV)(1+i)^T. \tag{10}$$

The post-tax future value is obtained by compounding post-tax cash flows π_t^τ with the post-tax interest rate. Post-tax cash flows are derived from reducing cash flows π_t by annual tax payments $\tau(\pi_t - d_t)$. The statutory tax rate τ is applied to the tax base, which is composed of cash flows π_t minus depreciation deductions d_t. For reasons of simplification, d_t will be the only non-cash element of the tax base. It stands for any other revenues and expenses that differ from cash flow. Taxation of interest income will be applied, as this corresponds to tax practice in most countries.

$$FV^\tau = \sum_{t=1}^{T} (\pi_t - \tau(\pi_t - d_t))(1 + i(1-\tau))^{T-t} = (I_0 + NPV^\tau)(1 + i(1-\tau))^T. \tag{11}$$

The difference of future values in the numerator of (9) equals the investment's total tax burden. It consists of two elements. The first one is the future value of the tax payments of each period $FVTax$

$$FVTax = \sum_{t=1}^{T} \tau(\pi_t - d_t)(1 + i(1-\tau))^{T-t}. \tag{12}$$

Another reduction of the pre-tax future value is caused by the difference ΔTax of the pre-tax and post-tax discount factors i and $i(1-\tau)$

$$\Delta Tax = \sum_{t=1}^{T} \pi_t \left[(1+i)^{T-t} - (1 + i(1-\tau))^{T-t}\right]. \tag{13}$$

Therefore, the future value-based EATR^{FV} can be written as

$$EATR^{FV} = \frac{FVTax + \Delta Tax}{FV - FV^{\tau,\,neutral}} \cdot \tau. \tag{14}$$

The neutrally taxed future value $FV^{\tau,\,neutral}$ can be derived from (11) using special assumptions depending on the reference tax system. As interest income is subject to tax, the neutral reference tax system has to be the Johansson/Samuelson[25] (JS) tax, where full taxation of interest is considered. Depreciation deductions d_t in (11) are substituted by economic depreciations d_t^{JS}. They are determined as difference of present values PV_t of consecutive periods

$$d_t^{JS} = \underbrace{\sum_{j=t}^{T} \frac{\pi_j}{(1+i)^{j-t+1}}}_{PV_{t-1}} - \underbrace{\sum_{j=t+1}^{T} \frac{\pi_j}{(1+i)^{j-t}}}_{PV_t}. \tag{15}$$

[25] See Samuelson (1964); Johansson (1969).

Applying the JS tax as neutral tax system, the post-tax NPV$^{\tau,\,JS}$ equals its pre-tax value NPV as defined in equation (5), thus (11) can be simplified to

$$FV^{\tau,\,JS} = (I_0 + NPV)(1 + i(1-\tau))^T. \tag{16}$$

Inserting (10) and (16), (9) can be written as

$$EATR^{FV} = \frac{FV - FV^{\tau}}{(I_0 + NPV)\left[(1+i)^T - (1+i(1-\tau))^T\right]} \cdot \tau. \tag{17}$$

Further simplifications cannot be made without knowledge of the cash flow structure.

5.2 EATRs based on annualized returns or annuities

Substituting the objective variable in equation (4) by the annualized return instead of the future value, the EATR can be written as

$$EATR^{r} = \frac{\text{Actual tax wedge}}{\text{Neutral reference tax wedge}} \cdot \tau = \frac{r - r^{\tau}}{r - r^{\tau,\,neutral}} \cdot \tau. \tag{18}$$

Since the annualized return is a transformation of the future value, equations (10), (11), and (16) apply and are transformed using equations (3). Simplifying according to König[26] results in

$$EATR^{r} = \frac{r - r^{\tau}}{\frac{i}{1+i}(1+r)}. \tag{19}$$

Substituting the objective variable in (4) by annuities An leads to

$$EATR^{An} = \frac{An - An^{\tau}}{An - An^{\tau,\,neutral}} \cdot \tau \tag{20}$$

with

$$An = (I_0 + NPV)\frac{(1+i)^T \cdot i}{(1+i)^T - 1} \;\text{ according to (7)}, \tag{21}$$

$$An^{\tau} = (I_0 + NPV^{\tau})\frac{(1+i(1-\tau))^T \cdot i(1-\tau)}{(1+i(1-\tau))^T - 1}, \;\text{ and} \tag{22}$$

$$An^{\tau,\,JS} = (I_0 + NPV)\frac{(1+i(1-\tau))^T \cdot i(1-\tau)}{(1+i(1-\tau))^T - 1}. \tag{23}$$

[26] Transforming r in (3) to $\frac{NPV}{I_0} = \left(\frac{1+r}{1+i}\right)^T - 1$ and r^{τ} in case of JS taxation to $\frac{NPV}{I_0} = \left[\frac{1+r^{\tau,\,JS}}{1+i(1-\tau)}\right]^T - 1$, equating them and solving for r^{τ} leads to $r^{\tau,\,JS} = (1+r)\left(1 - \frac{i\cdot\tau}{1+i}\right) - 1$. Inserting in (18) results in (19). See König (1997), p. 56.

(20) can be simplified to

$$EATR^{An} = \frac{An - An^{\tau}}{i \cdot (I_0 + NPV)\left[\frac{1}{(1+i)^T - 1} - \frac{1-\tau}{(1+i(1-\tau))^T - 1} + \tau\right]} \cdot \tau. \tag{24}$$

The EATR^{FV} and EATR^r, as described above, will be illustrated in comparison to the EATR^{ETA}, as defined in section 3.3, in figure 1. The example describes an investment of $I_0 = 1{,}000$ in $t = 0$ with a time horizon of $T = 10$ years that generates uniform cash flows $\pi_t = \pi$ depending on the pre-tax rate of return r. Declining balance depreciation with a rate of 30% and transition to linear depreciation in $t = 8$ is applied for the investment's useful life of 10 years. The capital market rate is $i = 10\%$, the statutory tax rate $\tau = 40\%$. Table 1 depicts the investment for $r = 20\%$, i.e. $\pi = 389$, firstly taxed as described above, secondly using the neutral JS tax.

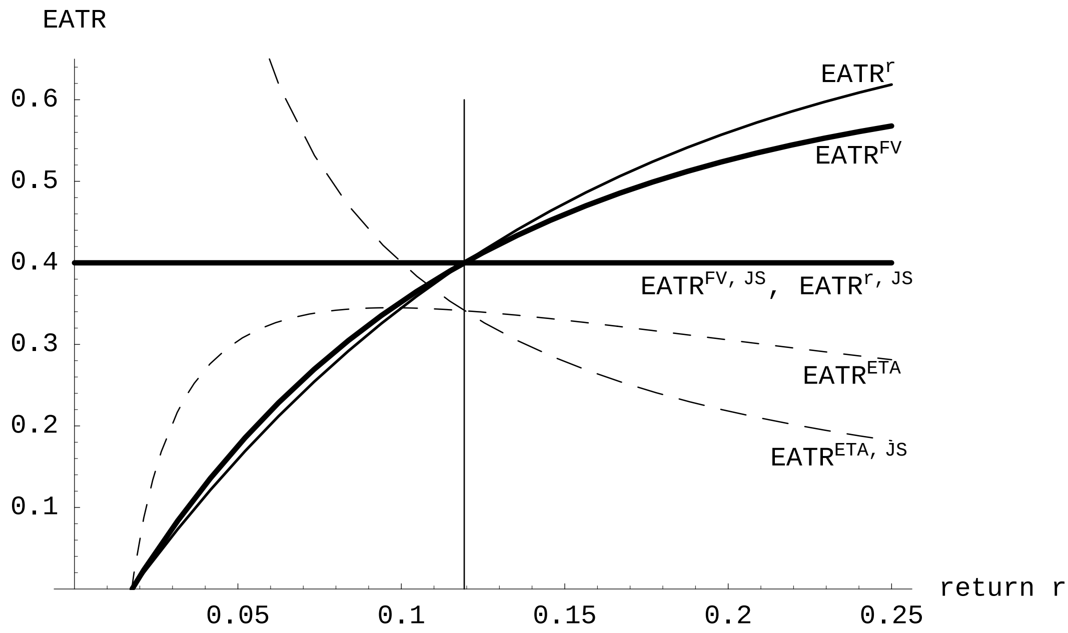
Table 1: *Determination of future values under a real and neutral tax system.*

t	0	1	2	3	4	5	6	7	8	9	10	FV$^{(\tau)}$
π_t	-1000	389	389	389	389	389	389	389	389	389	389	
d_t		300	210	147	103	72	50	35	27	27	27	
$\pi_t - d_t$		89	179	242	286	316	338	353	361	361	361	
tax_t		35	71	97	114	127	135	141	144	144	144	
π_t^{τ}		353	317	292	274	262	253	247	244	244	244	
$\pi_t(1+i)^{T-t}$		916	833	757	688	626	569	517	470	427	389	6192
$\pi_t^{\tau}(1+i(1-\tau))^{T-t}$		597	505	439	389	350	320	294	274	259	244	3672
PV_t	2387	2237	2073	1891	1692	1473	1232	966	674	353	0	
d_t^{JS}		150	165	181	199	219	241	265	292	321	353	
$\pi_t - d_t^{JS}$		239	224	207	189	169	147	123	97	67	35	
tax_t^{JS}		95	89	83	76	68	59	49	39	27	14	
$\pi_t^{\tau,JS}$		293	299	306	313	321	330	339	350	362	374	
$\pi_t^{\tau,JS}(1+i(1-\tau))^{T-t}$		495	477	460	444	429	416	404	393	383	374	4275

For the given example, the pre-tax $FV = 6{,}192$, the $FV^{\tau} = 3{,}672$, and the neutrally taxed $FV^{\tau,JS} = 4{,}275$. Transformation in annualized returns results in $r = 20\%$, as defined above, $r^{\tau} = 14\%$, and $r^{\tau,JS} = 16\%$. The pre-tax annuity is 389, the post-tax annuity is 279, and the one in case of neutral JS taxation is 324. If the newly developed approach is applied, EATR$^{FV} = 53\%$, EATR$^r = 56\%$, and the EATR^{An} is even 69%. As expected, the neutral yardstick for all of the neutrality-based EATRs is 40%, which equals the statutory tax rate. In case of the ETA, the EATR$^{ETA} = 31\%$ and the appropriate neutral yardstick EATR$^{ETA,JS} = 22\%$, which differs from the statutory tax rate[27].

[27] Using the EATR^{ETA,JS}, ETRs based on the ETA can be transformed into meaningful figures: $\frac{EATR^{ETA}}{EATR^{ETA,JS}} = \frac{r - r^{\tau}}{r - r^{\tau,JS}} = EATR^r$. But even this correction does not solve the problem of applying annualized rates of return, making this approach inferior compared to other neutrality-based concepts, as can be seen in section 6.2.

Figure 1: *Comparison of $EATR^{FV}$, $EATR^{r}$, and $EATR^{ETA}$.*



Obviously, the EATR functions differ dramatically. The EATR^{FV} and EATR^{r} are strictly increasing functions of the pre-tax rate of return. The appropriate yardstick EATR^{FV,JS} = EATR^{r,JS} is a straight line that always equals the statutory tax rate. On the contrary, the dashed line EATR^{ETA} is a concave function in the depicted interval. Its yardstick EATR^{ETA,JS} is the falling dashed line, which is strictly decreasing while the pre-tax rate of return r rises. It intersects the statutory tax rate at $r = 10\%$. At this point, $r = i$ and therefore the NPV$= 0$, which confirms that the ETA has the statutory tax rate as a yardstick, iff a marginal investment with a zero NPV is analyzed. Using the EATR^{ETA} approach for the analysis of profitable investments, each investment project requires additional calculation of the appropriate yardstick.

The three EATR lines intersect the abscissa exactly at the same rate of return. At this point the rate of return is far below i. The NPV is deeply negative and a tax reimbursement occurs. At the intersection with the abscissa the future value of the tax refund defined in (12) equals the reduction of the pre-tax future value due to taxation of interest, which is defined in (13). This is equivalent to $FV = FV^{\tau}$ and therefore the numerator of (14) and the EATR become zero. If $FV = FV^{\tau}$, the EATR^{ETA} is zero as well. Therefore, both the neutrality-based approach and the ETA yield zero EATRs at the same rate of return.

Why do EATR^{FV} and EATR^{r} rise monotonically? The reason for this distinctive feature is that the initial investment I_0, which is the basis for depreciation allowances, remains constant while the generated cash flows π_t, and therefore NPV and future value, rise with an increasing rate of return. Hence, the tax base exceeds economic profits. In case of neutral JS taxation, depreciation deductions are not restricted to I_0. In contrast, they increase simultaneously with a rising rate of return. Due to higher economic depreciations the EATR remains constant.

6 Features of the neutrality-based EATR

In this section, the features of the new EATR approach are being examined and compared to the requirements developed before. Differences and similarities between the EATR^{FV}, EATR^{r}, and EATR^{An} approaches will be emphasized. Additional comparisons will take place with the EATR^{ETA} in order to stress improvements of the newly developed approach.

6.1 The statutory tax rate as a constant yardstick

The constant yardstick of the newly developed EATR^{FV} is the statutory tax rate τ. This is valid independent of the analyzed investment, the statutory tax rate, the pre-tax rate of return, or the cash flow structure, as is evident from (9). The yardstick for assessing the taxation of any investment is the effective tax rate of a neutrally taxed investment. For $FV^{\tau} = FV^{\tau,JS}$ the EATR^{FV} is

$$EATR^{FV,JS} = \frac{FV - FV^{\tau,JS}}{FV - FV^{\tau,JS}} \cdot \tau = \tau, \tag{25}$$

which implies that deviations from the statutory tax rate can be completely explained by non-neutral taxation. If the EATR^{FV} exceeds the yardstick, the analyzed investment is taxed discriminatory; if the EATR^{FV} falls below the yardstick, its taxation is preferentially.

Regarding this feature of a constant yardstick, it makes no difference if the annualized rate, the annuity, or the future value is applied as objective variable in the newly developed EATR approach. In the three cases, the statutory tax rate serves as the constant yardstick for any investment. The functions of the EATR^{FV}, EATR^{r}, and EATR^{An} (for reason of clearness the latter is not presented in figure 1) differ, but they have exactly the same intercept point with the yardstick. Therefore, the same conclusions will be drawn concerning preferential or discriminatory tax treatment independent of the chosen objective variable.

The constant yardstick is the major distinction between the newly developed neutrality-based EATR and the EATR^{ETA}. The neutrally taxed yardstick of the ETA approach EATR^{ETA,JS}, charted as dashed line in figure 1, decreases when r rises. This cannot be explained by taxation, it just results from annualizing the rate of return in the EATR^{ETA} formula[28]. Thus, in the EATR^{ETA} approach the only chance of analyzing the tax treatment of an investment is to explicitly calculate the yardstick for each pre-tax rate of return and, moreover, for each differing structure of cash flows π_t with the same rate of return, as will be demonstrated later. The example shows an EATR^{ETA} of about 35% at a pre-tax rate of return of $r = 10\%$. At a higher rate of $r = 25\%$, the EATR^{ETA} declines to approximately 25%, even though in the first case the investment is preferentially taxed, in the second one taxation is discriminatory.

In figure 1, the intersection of the EATR lines with their appropriate yardsticks is the same for all concepts (in this example at $r \approx 12\%$). To the right of the vertical line the investment is taxed preferentially, to the left of it it is treated discriminatory.

6.2 Magnitude of tax distortions

The tax distortion μ of an investment is the deviation of the objective variable from the neutrally taxed objective variable

$$\mu = \left| \frac{\text{Neutrally taxed objective variable} - \text{Post-tax objective variable}}{\text{Neutrally taxed objective variable}} \right|. \qquad (26)$$

In case of the EATR^{FV} approach, the relative distortion μ can be measured by

$$\mu = \left| \frac{FV^{\tau,JS} - FV^{\tau}}{FV^{\tau,JS}} \right|. \qquad (27)$$

Identical $\mu_A = \mu_B$ for different investments A, B imply that the economic objective variables of A and B are distorted by taxation to the same extent. If $\mu_A \gtrless \mu_B$, investment A is distorted to a higher (lower) degree than B for all A, B. In the following, it will be analyzed if the EATR^{FV}, or more precisely $\Delta = |\text{EATR}^{FV} - \tau|$, can also provide this information, thus making the calculation of μ unnecessary.

For this purpose, four investments with identical Δ will be regarded. The investments' cash flow structures π_t and the tax rules including the depreciation systems and rates

[28] See Niemann/Bachmann/Knirsch (2002), p. 1551.

differ from each other. Investments A, B, C have the time horizon $T = 10$, the time horizon of D is $T = 5$. The capital market rate is $i = 10\%$ in all cases; the statutory tax rate and therefore the neutrally taxed yardstick is always $\tau = 40\%$. The EATR^{FV} of A is 30%, in the other cases it is 50%. Thus, $\Delta = |\text{EATR}^{FV} - \tau| = 10\%$ is identical in all cases.

Investment	A	B	C	D
T	10	10	10	5
EATR^{FV}	30%	50%	50%	50%

Inserting μ in $\Delta = |\text{EATR}^{FV} - \tau|$ applying (17) results in

$$\Delta = |EATR^{FV} - \tau| = \mu \cdot \frac{\tau(1+i(1-\tau))^T}{(1+i)^T - (1+i(1-\tau))^T}. \tag{28}$$

Equation (28) shows that $|\text{EATR}^{FV} - \tau|$ is a transformation of μ that does not depend on the rate of return of the investment. It only depends on i, T, and τ. If these parameters are held constant for all compared investments, $|\text{EATR}^{FV} - \tau|$ gives the same conclusions as the relative distortion μ, which makes the calculation of μ redundant. Even the consideration of I_0 that generally has to take place if conclusions are drawn using the future value, can be omitted here.

Using (28), $\mu = 11.2\%$ for the examples A, B, and C. Identity of EATR^{FV} assures an identical degree of relative distortion of the investments. Moreover, equivalent Δs signify an identical extent of distortion. This allows the comparison of preferentially and discriminatory taxed investments. For D, on the contrary, $\mu = 5.1\%$, because the time horizon T differs from other investments' T. Compounding interest and the taxation of interest leads to dependency of T and to the restriction of the EATR^{FV} approach with regard to comparability of distortions to investments with identical time horizons T.

Applying annuities instead of future values as objective variable in the newly developed neutrality-based approach generates comparable results. Identical to the future value-based approach, $|\text{EATR}^{An} - \tau|$ is a transformation of the relative distortion that depends on i, T, and τ, but is independent of the rate of return.

If the annualized return r is used as objective variable in (26) instead of the future value, relative distortion μ^r is

$$\mu^r = \left| \frac{r^{\tau,JS} - r^{\tau}}{r^{\tau,JS}} \right|. \tag{29}$$

Inserting μ^r in $\Delta^r = |\text{EATR}^r - \tau|$ applying (19) results in

$$\Delta^r = |EATR^r - \tau| = \mu^r \cdot \frac{1 + i(1-\tau) - \left[\frac{I_0}{I_0 + NPV}\right]^{\frac{1}{T}}}{i}. \tag{30}$$

$|\text{EATR}^r - \tau|$ is a transformation of μ^r that depends on i, T, τ, the initial investment I_0, and the NPV, and therefore on the rate of return of an investment. Consequently, using the EATR^r, no comparability of relative distortions is given for investments with differing pre-tax data. This identifies the EATR^{FV} and EATR^{An} as more suitable approaches for the analysis of tax distortions. If T is fixed, the EATR^{An} is a linear transformation of the future value, making the results of the EATR^{FV} and EATR^{An} comparable. Determining a rate of return includes a nonlinear transformation of the future value, which distorts some of the future value's properties. Using the simple future value instead of a nonlinear transformation seems to be a superior approach for estimating tax distortions.

6.3 Reproduction of real-world investments and tax systems

The requirement of reproduction of real-world investments and tax systems is fulfilled by the newly developed model[29] in the same way as by the ETA. As a numerical model based on a cash flow statement, the EATR^{FV} approach allows reproduction of any type of investment with any structure of initial investment I_0, NPV, and cash flows π_t. Tax base definition is totally free, as well as the determination of tax rates. Subsequent periods are linked due to the reinvestment of funds in each period. Because of this the EATR^{FV} is influenced by both the EATR of the real investment and the statutory tax rate as EATR of the financial investment. It is a mixture of both elements that does not reveal the isolated tax effect on real investment; it rather discloses the effect on the entire project that consists of real and financial investment. Due to the constant yardstick the misleading effect of mixing EATRs is smaller compared to the ETA. In case of neutral taxation, the ETA generates EATRs that are a mixture of the statutory tax rate, the EATR for financial investment, and an EATR for the real investment that deviates from it. The resulting EATR^{ETA} lies in between. In case of the neutrality-based concept, the yardstick for both parts of the investment equals the statutory tax rate. Thus, it does not depend on the proportion of financial and real investment.

Similar to the EATR^{ETA} approach, investment decisions can always be made using the EATR^{FV} if an investment is analyzed under different tax systems. In this case, a higher (lower) EATR^{FV} corresponds to a lower (higher) NPV$^\tau$ and FV^τ, respectively. If the pre-tax parameter of the analyzed investment alternatives differ, the EATR^{FV} as well as the EATR^{ETA} approach are insufficient to determine an ordinal ranking of future value-based EATRs for decision-making.

[29] In this section, properties that are described for the EATR^{FV} are the same for the EATR^r and the EATR^{An}, even if only the EATR^{FV} is explicitly mentioned.

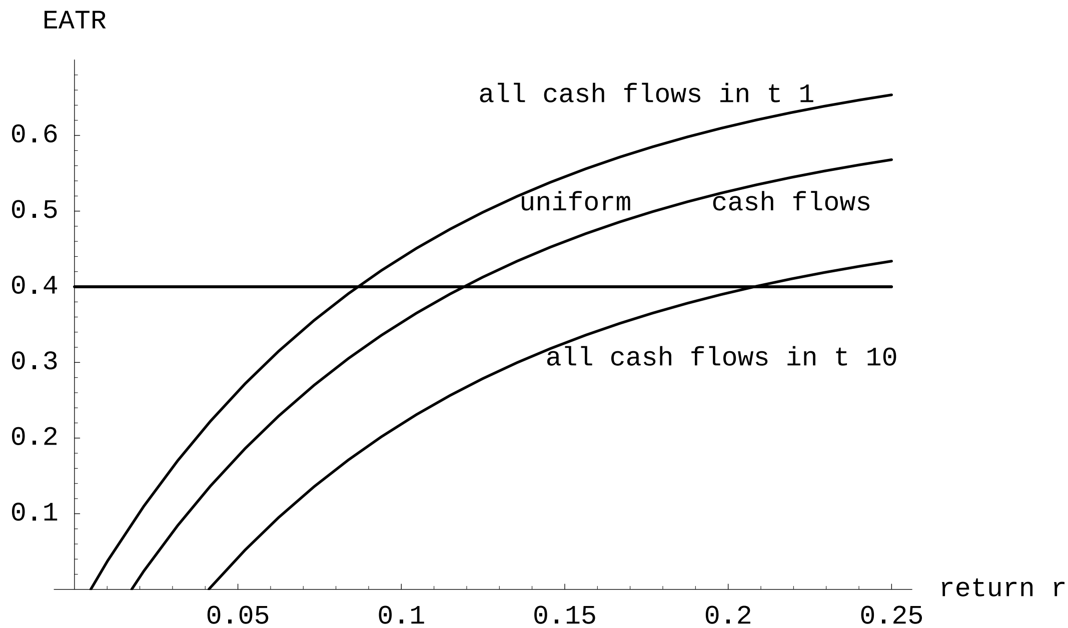
Table 2: *Effect of differing cash flow structures on economic depreciations.*

t	0	1	2	3	4	5	6	7	8	9	10
uniform π_t	-1000	389	389	389	389	389	389	389	389	389	389
d_t		300	210	147	103	72	50	35	27	27	27
tax_t		35	71	97	114	127	135	141	144	144	144
π_t^τ		353	317	292	274	262	253	247	244	244	244
PV_t	2387	2237	2073	1891	1692	1473	1232	966	674	353	0
d_t^{JS}		150	165	181	199	219	241	265	292	321	353
early π_t	-1000	2626	0	0	0	0	0	0	0	0	0
d_t		300	210	147	103	72	50	35	27	27	27
tax_t		930	-84	-59	-41	-29	-20	-14	-11	-11	-11
π_t^τ		1696	84	59	41	29	20	14	11	11	11
PV_t	2387	0	0	0	0	0	0	0	0	0	0
d_t^{JS}		2387	0	0	0	0	0	0	0	0	0
late π_t	-1000	0	0	0	0	0	0	0	0	0	6192
d_t		300	210	147	103	72	50	35	27	27	27
tax_t		-120	-84	-59	-41	-29	-20	-14	-11	-11	2466
π_t^τ		120	84	59	41	29	20	14	11	11	3726
PV_t	2387	2626	2888	3177	3495	3845	4229	4652	5117	5629	0
d_t^{JS}		-239	-263	-289	-318	-350	-384	-423	-465	-512	5629

The impact of cash flow structures on ETRs can be seen in table 2 and figure 2. The three lines in figure 2 represent three investments, whose parameters are all but one equal. For each rate of return r the investments have exactly the same pre-tax NPVs. The basis investment is the one described in section 5, but the cash flows differ according to table 2. Apart from cash flows π_t, table 2 shows depreciation deductions d_t, the resulting tax payments (reimbursements), post-tax cash flows π_t^τ, present values PV_t, and economic depreciation d_t^{JS} according to (15) of the three investments with uniform, early, and late cash flows for the example of $r = 20\%$.

Figure 2 shows EATRs based on future values. These EATRs vary dramatically if the cash flow structure is modified. The middle line shows the investment from figure 1 with $\pi = 389$ in all periods, if $r = 20\%$. The upper and lower lines represent extreme cases: The upper line shows an investment with only one positive cash flow$\pi_1 = 2,626$ in period 1 and $\pi_t = 0$ in all other periods in the case of $r = 20\%$; for the investment charted in the lower line $\pi_{10} = 6,192$ and all other cash flows are zero. Discounting the cash flows leads to identical NPVs of 1,387. The post-tax rates of return differ: for uniform cash flows it is 14%, for early cash flows it falls below 13%, and for late cash flows it is even 16%. For the described example of $r = 20\%$ the EATR^{FV} varies from 39% to 61%. Using the ETA, the EATR reacts similarly to changes of the cash flow structure. Assuming only a pre-tax rate of return without explicitly defining the cash flow structure is not sufficient

Figure 2: *EATRs of investments with different cash flow structures.*



for calculating an EATR on the basis of the ETA.

The rationale behind the differences of EATRs is that tax-deductible depreciations are equal for the charted investments; economic depreciations instead depend on the cash flow structure and vary for the three functions as can be seen in table 2. As a consequence, the present values of the differences of economic and tax-deductible depreciations are equal in the pre-tax calculation, but they differ when taxes are considered. Since taxes influence the discount factor, the post-tax present values of the investments vary depending on the cash flow structure, leading to differing EATRs.

This emphasizes that merely choosing a pre-tax rate of return, as in the DG model, is not sufficient for the assessment of real-world investments. KF- and DG-based ETRs cover only very special cases of real-world investment projects that should not be considered representative. Figure 2 depicts reasonable estimates for the range of ETRs for positive cash flows. If both positive and negative cash flows are considered, the ETR functions may even run outside the borders.

Like all other ETR approaches, neutrality-based ETRs are restricted to constant tax rates[30].

[30] This is why the authors discuss the properties of the DG-EATR in the absence of personal taxes, see Devereux/Griffith (1999), pp. 21-24.

7 Conclusion

ETRs are developed in order to provide information about distortions of investments caused by taxation. Existing ETR approaches suffer from different shortcomings. The analytical KF and DG approaches can be used for revealing tax distortions. But both of them are subject to very restricted assumptions, for instance cash flows of an investment must decline over time. Therefore, analytical ETR approaches cover only very special cases of real-world investment projects and cannot be applied in most cases.

In contrast, numerical approaches like the ETA are very flexible and allow choosing any cash flow structure, depreciation schedule, and so on. Their main shortcoming is that they lack a constant yardstick that permits revealing preferential, neutral, or discriminatory taxation. This shortcoming makes the ETA's effective tax rate arbitrary.

This paper has introduced a new EATR approach based on neutral tax systems that uses the statutory tax rate as a constant yardstick for all investments. Thus, preferential or discriminatory tax treatment can always be revealed. This feature is achieved by an extension of existing EATR concepts. Ideally, neutral taxation is integrated into calculation of the effective tax rate of every investment and compared to the actual realistic taxation. Comparison of the two tax systems leads to a yardstick for all investments that allows revelation of distortions. As interest is subject to tax, the integration of neutral taxation is realized by applying the JS tax where true economic depreciation of the investment is calculated for each period. This can be compared to the DG and KF approaches where a true economic depreciation rate is defined next to the depreciation rate of the tax system.

The neutrality-based EATR model does not only allow the comparison of effective tax rates with the statutory tax rate. Moreover, an $EATR^{FV}$ can be compared to any other $EATR^{FV}$ for investments with the identical time horizon T in order to reveal which investment is distorted to a higher degree. This is equivalent for EATRs based on annuities. This conclusion can be drawn independent of the investment's initial investment, structure of cash flows, NPV, or pre-tax rate of return. But this comparison as well as the validity of the constant yardstick are only true if one uniform tax rate is applied for profits and interest.

Existing ETR approaches as well as the newly developed model succeed in compressing the complexity of tax systems into a single figure. But this cannot change the fact that tax systems *are* complex: the resulting ETRs are appropriate for one particular investment project only. They cannot be generalized, since every little change, e.g. in the structure of cash flows or depreciation allowances, influences the resulting ETR. Due to this effect it becomes clear that the idea of determining one ETR for a broad range of investments, e.g. all investments of one country or an industry, is illusive.

Another problem with regard to the generality of ETRs has become obvious: Integrating neutral taxation in an EATR approach solves the problem of arbitrary ETRs compared to a constant yardstick. But even this improved approach cannot provide a single valid solution. The neutrality-based EATR model applying future values or annuities as objective variables generates EATRs with similar properties. They are both able to reveal tax distortions compared to a constant yardstick and other EATRs of the same approach. But they still generate *different* EATRs that cannot be compared to each other. Thus, even integrating neutrality into the model provides arbitrary EATRs, which react very sensitive with regard to the chosen objective variable.

What can we learn from this? First, EATRs can only be interpreted relative to each other; the absolute level contains no information at all. Second, it is possible to generate any EATR depending on the chosen investment project, ETR approach, and objective variable. Therefore, we should be very careful drawing any conclusions of EATRs without considering accurately under which assumptions they are determined.

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