EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

University of Tübingen
Working Papers in
Business and Economics

No. 145

Growth in a Circular Economy

by

Marco Compagnoni and Manfred Stadler

Faculty of Economics and Social Sciences
www.wiwi.uni-tuebingen.de

https://publikationen.uni-tuebingen.de/xmlui/handle/10900/95156

Growth in a Circular Economy

Marco Compagnoni* and Manfred Stadler**

April 2021

Abstract

We present a model of natural resources and growth that stresses the influence of an incomplete circularity of exhaustible natural resources. In particular, we analyze the recycling process and the material balance principle, two fundamental aspects of a circular economy. When market failures arise or complete recycling is not possible for technical reasons, then the equilibrium outcomes in terms of output, consumption, and prices for the material inputs are distorted compared to the socially optimal solution. However, the introduction of a market for waste and a system of subsidies/taxes on virgin and recycled resources enables an internalization of the externalities. The importance of technological progress in order to foster "circularity", i.e. both to improve resource efficiency in the production process and to enhance the backflow of materials from waste to production, is highlighted.

Keywords: Circular economy, economic growth, natural resources, recycling.

JEL Classification: O41, Q01, Q32, Q53

* PhD Sustainability: Economics, Environment, Management and Society (SUSTEEMS), University of Trento, via Verdi 26, 38122 Trento, Italy. e-mail: marco.compagnoni@unitn.it

** University of Tübingen, Department of Economics, Nauklerstr. 47, D-72074 Tübingen, Germany. e-mail: manfred.stadler@uni-tuebingen.de

1 Introduction

The sustainability of economic development is threatened by at least two environmental problems: (i) the exhaustibility of scarce natural resources that are essential for production, and (ii) the limited capacity of nature to absorb the waste of production. An appropriate theoretical framework for considering both of these problems is the concept of a circular economy. The ultimate aim of a circular economy is to minimize the extraction of virgin resources and to limit human-activity byproducts by maximizing the material and energy efficiency of the production process. Within the context of a circular economy, material resources are again employed in production after their first use. The fundamental idea is to move from the perception of waste as a problem to the perception of waste as a valuable input (EEA 2017). The advantage would be twofold: first, it offers the possibility to substitute virgin resources by a flow of semi-renewable secondary raw materials, thereby alleviating the scarcity of exhaustible resources; second, it opens the possibility to reduce the environmental burden generated by the production and consumption processes, since waste accumulation would not represent an optimal use of resources.

There exist several definitions of a circular economy (Kirchherr et al. 2017; Geissdoerfer et al. 2017) as well as some critiques to the concept that is sometimes presented too idealistically. In particular, Zotti and Bigano (2019) argued that it would be more realistic and correct to talk about "economy's circularity" instead of "circular economy", because the latter expression alludes to the idea of a perfectly closed economic system in terms of use of resources. This view is in agreement with the model analyzed in this paper. We will consider the effects of the introduction of the two most important aspects of circularity, recycling and the material balance principle. The material balance principle states that all of the materials employed in production flow into a "waste pile" after consumption; in this way, it ensures that the natural system is closed and that the evolution of the stock of waste accumulating in the environment must be taken into account when seeking to maximize the social welfare of the economy. On the other side, recycling allows a circular flux of materials from the waste pile to production again.

The major part of the research on economic growth and resource scarcity does not distinguish between the use of exhaustible virgin resources and recycled ones. However a literature discussing the effects of recycling on sustainability exists. This literature can be roughly divided into two strands: the first one considers recycling as a way to extend availability of non-renewable resources, the second one introduces recycling in the analysis of pollution abatement activities, i.e. the alleviation of waste disposal problems.

Only recently, waste production and/or reuse has been analyzed in the theory of economic growth. Pittel et al. (2010), for example, consider man-made capital, virgin and recycled resources as input factors in a growth framework. Complete circulation of matter, via a material balance constraint, is imposed. Indeed, material inputs are either bound in the stock of physical capital stock or recycled after consumption. This study assumes complete recycling and exogenous technological progress. Pittel et al. (2005) modify the model by endogenizing growth due to infinite capital accumulation or adding a human-capital sector. Both papers provide formal solutions to achieve long-run sustainability. Di Vita (2001) presents a model in which exhaustible natural resources, recyclable and non-recyclable waste are taken into account. The first type of waste is used to produce secondary raw materials and its degree of recyclability is an increasing function of R&D activity. The second kind of waste is discharged into the environment. Circulation of resources is also considered in this paper. One of the main findings is that policy is able to increase the economic growth rate by promoting research activities. Di Vita (2004) extends the results by introducing renewable resources. Furthermore, a tax and a subsidy on natural resources and on recycled materials respectively are also introduced. Finally, Di Vita (2007) considers the case in which virgin resources and reused ones are not perfect substitutes. Lafforgue and Rouge (2019) consider an endogenous growth model where the use of a non-renewable resource generates waste which can be recycled. The recycling activity can start only after the quality of the secondary raw material has reached a minimum threshold and, therefore, investment in a specific R&D sector is required to improve recycled materials quality.

None of these models, however, introduces the concept of a circular economy. Although this concept was already developed a few decades ago, it has only become popular in the last years (Boulding 1966), due to being promoted by the European Union (European Commission 2015, 2020) as well as several national governments, and firms. However, the research content of this concept is currently superficial and made up of separate ideas from several fields (Korhonen et al. 2018).

We decided to use the model of Pittel et al. (2010) as a workhorse since it is suitable for capturing the idea of a circular economy. We attempt to link this concept to a formal model of economic growth and to analyze whether this can be a way to achieve sustainable development. More specifically, we investigate whether a growth model in which the traditional linear extraction-production-consumption-dump flow of materials is replaced by a (more or less) circular one can reach sustainable long run growth. Additionally, we examine the level of economic activity and the implications for the use of resources. The effects of two important market failures will also be taken into account, leading to a solution of a decentralized economy that clearly differs from the socially optimal one. The main deviation from the basic model is the introduction of incomplete recycling due to technical reasons, in order to abandon the idealistic assumption of complete recycling.

The rest of the paper is organized as follows: Section 2 introduces the circular economy concept and discusses the consequences of incomplete recycling and of certain market failures by comparing the results of a decentralized economy to the solution of a fictitious social planner. Some policy conclusions are drawn in Section 3. Section 4 summarizes the main findings and concludes the paper.

2 The Model of a Circular Economy

For a long time it has been acknowledged that economic growth is limited by the finiteness of natural resources (Meadows et al. 1972). The exhaustible resources problem is not the only threat to development and long run sustainability of present living standards. Indeed, the more economies grow, the clearer it becomes that the flows of waste generated by human activities cannot be absorbed by nature. The consequences of these flows are climate change, that will seriously affect the everyday life of the human population all over the world in the next decades, and the depletion of environmental quality, accounting for a loss of amenity and sometimes even representing a danger for human health.

Economies are nowadays largely characterized by linear economic, material and energetic flows: resources are extracted from the natural environment, employed in the production sector or in the energy sector, consumed and eventually discarded; their life-cycle is usually singular, i.e. a second life-cycle is mostly excluded. This linear extraction-production-consumption-dump scheme is not sustainable.

Sustainable development was originally defined in the famous Bruntland report as development that meets the needs of the present without compromising the ability of future generations to meet their own needs (WCED 1987).
The most logical alternative to the linear fashion of the material and energetic flow characterizing current economies seems to be its reverse: a circular fashion. This is one of the fundamental ideas of the circular economy concept. According to Korhonen (2018, p. 39), a circular economy can be defined as "an economy constructed from societal production-consumption systems that maximizes the service produced from the linear nature-society-nature material and energy throughput flow. This is done by using cyclical material flows, renewable energy sources and cascading-type energy flows. A successful circular economy contributes to all three dimensions of sustainable development, namely social, environmental and economic dimension. A circular economy limits the throughput flow to a level that nature tolerates and utilizes ecosystem cycles in economic cycles by respecting their natural reproduction rates."

In short, a circular economic system seeks to minimize the dependency on and extraction of resources from the natural environment and to minimize the emission of pollutants and other human activities' byproducts to the environment. The ways to achieve the general objective are: design of consumption products in an ecologically efficient way and their reuse, extension of products' life cycles, efficient use of energy and material flows and, eventually, recycling of raw materials (Compagnoni 2020).

In the circular economy context, nature is no longer seen as part of the economy, but instead the economy is a subsystem of nature. This unique system is closed: on the one hand, exhaustible resources are given in a certain stock and renewable ones regenerate themselves at a natural rate that cannot be incremented while, on the other hand, human activities' byproducts cannot flow out of the system. They are hardly absorbed by nature and must be taken into account when seeking to maximize the economic subsystem welfare in the long-run.

Formal macroeconomic models considering non-renewable resources lead off with Solow's, Stiglitz's, Dasgupta and Heal's contributions, published in a symposium issue of the Review of Economic Studies in 1974. These contributions are mostly summarized in a single "Dasgupta-Heal-Solow-Stiglitz" model (D-H-S-S) (Smulders et. al 2015; Groth 2007; Scholl and Semmler 2002). This approach is characterized by a linear flow of materials. It can serve as a basis to compare to the model presented in this paper and it represents a sort of pessimistic benchmark for it.

In particular, there are two features of our model enabling it to capture a circular economic system. First, a recycling process is considered. Due to recycling, the materials are allowed to flow back from the "waste pile", where they end up after consumption, to the production process. An economic framework in which a certain share of material inputs is supplied by a recycling sector, being able to treat a large part of waste generated, would permit switching from a perception of waste as a problem to one of waste as a valuable input. Recycling represents a tool to move towards sustainability both from an income and production point of view, extending the conservation of non-renewable resources, and from an environmental one. In fact, recycling would reduce the amount of waste flowing into the environment, ending up in more pollutant treatment methods as landfilling or incineration.

Second, a material balance constraint is imposed: it states that matter can neither be created nor destroyed, but can only be transformed. This is the way to incorporate Lavoisier's law of conservation of mass into the settings of economic flows. The material balance principle constrains economic production possibilities, as all material resources extracted from nature are employed in production and eventually flow back to the environment as waste, after final products are consumed, and so it is needed to check the evolution of the waste stock. The material balance principle is present in Di Vita (2001, 2004, 2007), Lafforgue and Rouge (2019), as well as Pittel et al. (2005, 2010) in order to introduce sound material flows.

In our model, virgin resources are still essential to production and we do not take renewable resources into account. In a strict interpretation of the circular economy conception, substitution of the latter for the former is necessary to achieve complete circularity of flows. The model stresses the effects of incomplete circularity of resources flows within an economic system relying on exhaustible natural resources.

The first objective of the analysis is to verify whether and under which conditions the features of a circular economy lead to sustainability of the economic development. Secondly, the consequences of a higher or lower recycling rate, i.e. a more or less circular economy, and the effects of two important market failures are going to be investigated.

2.1 Set-up of the Model

We consider a closed economy model with infinitely lived households. The utility of households only depends on consumption C. They aim to maximize their discounted utility

$$\max(C) \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma} - 1}{1 - \sigma}\, dt \tag{1}$$

where $\rho > 0$ is the discount rate and $1/\sigma > 0$ is the constant intertemporal elasticity of substitution. The dynamic budget constraint reads

$$\dot{D} = rD - C \tag{2}$$

where D denotes assets and r the interest rate. Population is assumed to be constant and normalized to one. The households are not compensated for the waste they produce. This represents a resource in a circular economy perspective and constitutes a first market failure.

Virgin resource suppliers extract a flow of virgin, natural resources V from the stock S of non-renewing resources,

$$\dot{S} = -V \tag{3}$$

The finite upper bound on cumulative resources extraction is

$$\int_0^\infty V dt \leq S_0.$$

Costs of extraction of the natural resource are neglected. Consequently, virgin resources suppliers seek to maximize their discounted flow of profits subject to the constraint represented by finiteness and non-renewability of this kind of final good production input

$$\max(V) \int_0^\infty p_V V e^{-\int_0^t r(v)dv} dt \qquad s.t. \quad \dot{S} = -V \ , \tag{4}$$

where p_V represents the virgin resources price, while the cost of extraction of the natural resource is neglected.

The recycling sector represents one of the features of a circular economy, and this is a first clear deviation when one considers the basic D-H-S-S scenario as a benchmark for a linear economy. Recycling firms extract raw materials from the goods that are discarded after consumption, at zero cost, and they supply them to final output producers without any further processing. Accordingly, the profit maximization of these firms reads

$$\max(R) \int_0^\infty p_R R e^{-\int_0^t r(v)dv} dt \qquad s.t. \quad \dot{W} = -R + W_C \ , \tag{5}$$

where R represents recycled input and p_R its price. The constraint to profit maximization is given by the evolution of the stock of waste $\dot{W}$ that is reduced by the flow of materials recycled R and regenerated by the exogenous flow of waste coming from consumption W_C. Note that this last, continuous flow makes recycled materials a semi-renewable input. This is a first representation of the material balance constraint, introduced in its entirety in the sector of final output production.

Final output is produced according to the Cobb-Douglas technology

$$Y = AK^{\alpha}V^{\beta}R^{\gamma}\ , \qquad \alpha, \beta, \gamma > 0,\ \ \alpha + \beta + \gamma = 1 \tag{6}$$

where K is physical capital, V is virgin resource input and R is recycled waste input. Production is enhanced by the efficiency-augmenting parameter A that grows at the exogenously given rate g_A, similarly to Kornafel and Telega (2020). Note that a double system of units is considered in this model: virgin and recycled inputs are measured in mass terms, while output, capital and consumption are measures in units terms.

Final output can be allocated either to investment in man-made capital or to consumption. Assuming a negligible rate of depreciation, capital accumulation is determined by

$$\dot{K} = Y - C \tag{7}$$

Consumption generates waste as a byproduct. The evolution of the stock of waste is defined by the material balance condition

$$\dot{W} = -R + (V + R)c(1 - x) \tag{8}$$

where $c = C/Y$ is the consumption share of output and $x \in [0, 1]$ is the share of total resources that cannot be recycled. The stock of waste W_0 is present in the environment, due to accumulation in the past. The waste pile increases by the amount of materials used for final output production, consumed and discarded, V and R. The production process is assumed not to generate any form of byproducts. The waste pile is instead reduced by the amount of materials picked up by recycling firms. Note that c does not only represent the consumption share of output, but also the “backflow rate”: the share of material inputs ending up in the waste pile. Furthermore the natural regeneration rate is zero: waste cannot be absorbed by the environment.

In this framework, the natural/economic system is completely closed. When the flow of waste coming from consumption is entirely recycled, complete circularity of material flows is achieved. At least in the sense that, although virgin resources are still essential for production, waste does not accumulate and completely flows back to production.

In the ecological economics literature, there is a lively debate, based on the laws of thermodynamics, on the possibility of complete recycling (Georgescu-Roegen 1971 and Ayres 1999). But apart from theoretical speculations, it is very clear that the fashion of material/economic flows is currently linear on a global scale and, even in the most virtuous communities, complete recycling is not observed. For this reason, we deviate from Pittel et al. (2010) in order to introduce incomplete recycling and move to a much more realistic assumption.

Consider a share x of resources that cannot be recycled after consumption. This is due to technical reasons and not to consumer behavior. In practice this share x of materials flows out of the system after consumption and W can be interpreted as the stock of recyclable waste.

Basically, after introducing mechanisms allowing for a completely circular flow of materials from the environment, through economic activities, and then back in a loop, this mechanism is partially broken allowing for a leakage of resources from the system. Therefore, we will make reference to the share x with the expression "material loss".

Two market failures characterize the considered economy. First, by introducing secondary raw materials in production, waste becomes a valuable input. Consumers, who generate waste, should be able to sell it on a market where the demand is coming from recycling firms. If no market for waste exists, consumers are not compensated for the provision of secondary materials to production, as shown by a household's budget constraint (**??**). This situation is currently largely diffused. Second, secondary raw materials suppliers and virgin materials suppliers do not take into account that a part of the inputs they provide to the final output producers, will again be available through the reflux of materials after consumption. Obviously virgin resource suppliers do not consider a possible reuse of materials in their pricing decisions as they cannot make any profit from this; recycling firms do not internalize the effect of their activity on the future availability of waste, because they operate in perfect competition.

Both of these market failures seem to be relevant. In particular, examples of markets for waste are rare and except specific cases, like junk cars, households are not remunerated for their waste. Consequently the market failures will lead to different conclusions when comparing the solution of a social planner to that of a decentralized economy.

2.2 The Socially Optimal Solution

Let us first derive the socially optimal solution. The social planner seeks to maximize the household's discounted utility (**??**), subject to the evolution paths of man-made capital (**??**), of virgin resources (**??**), and of waste stock (**??**). The present value Hamiltonian of this dynamic optimization problem is

$$\mathrm{H} = e^{-\rho t}\,\frac{C^{1-\sigma}-1}{(1-\sigma)} + \psi_1 \dot{K} + \psi_2 \dot{S} + \psi_3 \dot{W} \tag{9}$$

where ψ_1, ψ_2 and ψ_3 are the shadow prices of capital as well as virgin and recycled resources, respectively. The first-order conditions read

$$\mathrm{H_C}: \quad C^{-\sigma}e^{-\rho t} + \psi_3 m(1-x) - \psi_1 = 0 \qquad m \equiv (V+R)/Y \tag{10}$$

$$\mathrm{H_K}: \quad -\dot{\psi}_1 = \psi_1 F_K - \psi_3 F_K mc(1-x) \tag{11}$$

$$\mathrm{H_V}: \quad \psi_2 - \psi_3 c(1-x) = \psi_1 F_V - \psi_3 F_V mc(1-x) \tag{12}$$

$$\mathrm{H_R}: \quad \psi_3 - \psi_3 c(1-x) = \psi_1 F_R - \psi_3 F_R mc(1-x) \tag{13}$$

$$\mathrm{H_S}: \quad 0 = -\dot{\psi}_2 \tag{14}$$

$$\mathrm{H_W}: \quad 0 = -\dot{\psi}_3 \tag{15}$$

According to (**??**) and (**??**) it is optimal not only to exhaust virgin resources in the long-run, like in D-H-S-S model, but also recycled ones. The reason is that the waste stock is a source of valuable inputs under the circular economy features that have been introduced: not recycling part of waste and leaving it in the waste pile cannot be optimal when secondary raw materials are scarce and an essential input to production.

The consumption-savings arbitrage condition (**??**) implies that the shadow price of capital must be equal to the sum of discounted marginal utility of consumption and the "marginal recycling value of consumption byproducts (RCB)", $\psi_3 m(1-x)$, i.e. the value of the same unit of output when allocated to consumption.

The left hand side of the equations (**??**) and(**??**) represents the net marginal opportunity cost of extracting one more unit of virgin or recycled resource respectively and employing it in production. The shadow values of resource constraints are diminished by the "semi-renewability element in pricing (SRP)" of recyclable resources, the second terms of those equations' left hand side. This is due the fact that the share c of each unit of extracted materials can be used again in future production.

The right hand side of the equations (**??**), (**??**) and (**??**) shows the benefits of disposing of an additional unit of virgin resources, secondary materials or capital. The increase of any production input increases the output level not only directly, but also indirectly through the generation of valuable waste. Because of this, the last term of these equations can be interpreted as "the marginal recycling value of production inputs (RVP)".

RCB, RVP and SRP represent the effects of the features of a circular economy extending the basic D-H-S-S scenario. Note that the magnitude of all of these is reduced by the material loss.

By using the definition $z = C^{-\sigma} e^{-\rho t}$, equation (**??**) can be rewritten as

$$\psi_1 = z + \psi_3 m(1-x) \; . \tag{16}$$

By inserting this expression into equation (**??**), one obtains

$$\dot{\psi}_1 = -F_K z - \psi_3 m(1-x)(1-c)F_K$$

By differentiating (**??**) with respect to time one obtains $\dot{\psi}_1 = \dot{z} + \psi_3 \dot{m}(1-x)$ so that

$$\frac{\dot{z}}{z} = -F_K - \frac{\psi_3}{z}[m(1-c)F_K + \dot{m}](1-x)$$

Consider now equation (**??**) and rearrange it to

$$\frac{\psi_3}{z} = \frac{F_R}{[1 - c(1-x) - m(1-x)F_R + m(1-x)cF_R]}$$

Denoting the term in squared brackets by N and inserting the expression

$$\frac{\dot{z}}{z} = -F_K - \frac{F_R}{N}[m(1-c)F_K + \dot{m}](1-x)$$

together with the definition of z in terms of the growth rate $\dot{z}/z = -\sigma(\dot{C}/C) - \rho$, finally gives the extended Keynes-Ramsey rule

$$\sigma g_C = F_K - \rho + \frac{F_R\, m[(1-c)F_R + g_m]}{\frac{1}{1-x} - c - m\, F_R + m\, c\, F_R} \tag{17}$$

Compared to the original Keynes-Ramsey rule for a pure linear model (e.g. Scholl and Semmler 2002, p. 80), an additional term on the right-hand side is present. This term, due to the introduction of incomplete recycling and material balance constraint, enhances the consumption path and extends the lifetime of the economy as long as it is positive. It depends positively on the growth rate of material content of output g_m and negatively on the share of materials that cannot be recycled.

The features of our circular economy model also lead to extended Hotelling rules for recycled and virgin materials. For the derivation of the Hotelling rule for recycled material inputs, rearrange equation (**??**) for H_R as

$$F_R = -\frac{1 - c(1-x)}{\psi_1 \psi_3^{-1} - c(1-x)m}$$

Differentiating this with respect to time yields

$$\frac{\dot{F}_R}{F_R} = -\frac{\dot{c}(1-x)}{1 - c(1-x)} - \frac{\dot{\psi}_1 \psi_3^{-1} - (1-x)(\dot{m\, c})}{\psi_1 \psi_3^{-1} - c(1-x)m}$$

Rewrite (**??**) as

$$\frac{1}{\psi_1 \psi_3^{-1} - c(1-x)m} = -\frac{F_R}{1 - c(1-x)}$$

as well as equation (**??**) as $\dot{\psi}_1 = -\psi_1 F_K + \psi_3 F_K m\ c(1-x)$ and insert both equations into the expression for $\dot{F}_R/F_R$ to obtain

$$\frac{\dot{F}_R}{F_R} = -\frac{\dot{c}(1-x)}{1 - c(1-x)} - \frac{F_R}{1 - c(1-x)}[-\psi_1 \psi_3^{-1} F_K + F_K m\ c(1-x) - (1-x)(\dot{m\, c})]$$

Rearrange once again (**??**)

$$\frac{\psi_1}{\psi_3} = \frac{1 - c(1-x) + F_R m c(1-x)}{F_R}$$

and substitute this into the expression above to obtain the Hotelling rule for recycled materials

$$\frac{\dot{F}_R}{F_R} = F_K + \left[\frac{F_R}{1 - c(1-x)}(\dot{mc})(1-x) - \frac{\dot{c}(1-x)}{1 - c(1-x)}\right] \tag{18}$$

It is possible to show that, manipulating equation (**??**) for H_V and considering equations (**??**) and (**??**), the Hotelling rule for virgin resources can be derived as

$$\frac{\dot{F}_V}{F_V} = F_K + \left[\frac{F_R}{1-c(1-x)}\dot{(mc)}(1-x) - \frac{\dot{c}(1-x)}{1-c(1-x)}\frac{F_R}{F_V}\right] \tag{19}$$

The Hotelling rules, of course defining arbitrage conditions between virgin, exhaustible materials and man-made capital, are enhanced by the terms in squared brackets which reflect the semi-renewability of the secondary raw materials stock W. In both cases, if additional recycling causes a change of the share of resources flowing back after consumption, $mc(1-x)$, this influences the amount of output that can be produced in the future. A change in the reflux rate c, instead, affects the availability of materials for recycling: a positive growth rate for this flow of materials from consumption, for instance, increases the stock of recyclable waste, ceteris paribus; this implies a lower opportunity cost of extraction: the growth rate of material price is reduced. As for the Keynes-Ramsey rule, this effect has a lower magnitude, the bigger the share of resources that cannot be recycled. When a situation of no material loss is considered, $x = 0$, the solutions for the Keynes-Ramsey rule and for the Hotelling rules coincide with the ones found in Pittel et al. (2010), where complete recycling is assumed.

It remains to calculate the growth rates of consumption, output (income) and capital along the balanced growth path of the socially optimal solution.

Consider the first-order condition (**??**) and rearrange it to

$$\psi_3 = \frac{\gamma Y}{R\left(1-c(1-x)\right)+\gamma(V+R)c(1-x)}\psi_1 \tag{20}$$

Substituting this into equation (**??**) for H_C gives

$$-C^{-\sigma}e^{-\rho t} = \psi_1\left[-1+m(1-x)\frac{\gamma Y}{R\left(1-c(1-x)\right)+\gamma(V+R)c(1-x)}\right] \tag{21}$$

Denoting the term in squared brackets B and differentiating the whole expression with respect to time one obtains

$$C^{-\sigma}\rho e^{-\rho t} - e^{-\rho t}\left(-\sigma\frac{C^{-\sigma}}{C}\right)\dot{C} = \dot{B}\psi_1 + B\dot{\psi}_1$$

and hence

$$-\left(\sigma g_c + \rho\right) = g_{\psi_1} + g_B \tag{22}$$

The coincidence of the Hotelling rule for virgin materials and the one for recycled resources in the steady state: $\dot{F}_R/F_R = \dot{F}_V/F_V$ implies

$$\dot{R}/R = \dot{V}/V$$

This means that the two material inputs grow at the same rate. Thus, because along the balanced growth path it holds that $g_C = g_Y$ and $g_R = g_V$, one can conclude that the term B is constant over time. Insert the expression for ψ_3 from (**??**) into equation (**??**) and rearrange to obtain

$$\psi_2 - \psi_3 = \psi_1 \beta \frac{Y}{V} \left[1 - \frac{\gamma}{\beta} \frac{\beta\,(V+R)\,c\,(1-x) + V\,(1 - c\,(1-x))}{R\,(1 - c\,(1-x)) + \gamma\,(V+R)\,c\,(1-x)} \right]$$

Because in steady state $\dot{c} = 0$ and $g_R = g_V$, it follows that the term in brackets is constant. Considering now the first-order conditions (**??**) and (**??**), it is possible to conclude from the expression above that

$$g_{\psi_1} = g_V - g_Y \tag{23}$$

Due to $(\alpha + \beta + \gamma) = 1$, $g_R = g_V$ and $g_Y = g_K$, the production function (**??**) leads to

$$g_V = -\frac{1}{1-\alpha} g_A + g_Y \tag{24}$$

By inserting the expressions (**??**) and (**??**) into the equation (**??**) we obtain the growth rates induced by the social planner

$$g_C{}^{SP} = g_Y{}^{SP} = g_K{}^{SP} = \frac{1}{\sigma} \left(\frac{1}{\beta + \gamma} g_A - \rho \right) \tag{25}$$

Furthermore, by substituting the result (**??**) into the expression (**??**), we derive the rate of use of the two material inputs

$$g_R{}^{SP} = g_V{}^{SP} = \frac{1}{\sigma} \left[(1-\sigma) \frac{1}{\beta + \gamma} g_A - \rho \right] \tag{26}$$

Now we are ready to analyze the conditions under which this economy is sustainable. Let us consider equation (**??**) first. Since along the balanced growth path the growth rate of V should be negative, meaning that always less of the exhaustible virgin resource is used, it follows that

$$(1-\sigma) \frac{g_A}{\beta + \gamma} < \rho$$

This condition is generally fulfilled for $\sigma > 1$, i.e., when the intertemporal elasticity of substitution is low and a smooth consumption path is preferred by the households. As can be seen from equation (**??**), this implies a low growth rate of consumption.

Secondly, due to equation (**??**), in order to observe a positive consumption growth rate along the balanced growth path it must hold that

$$\frac{1}{\beta + \gamma} g_A > \rho \tag{27}$$

This condition depends on the material inputs' joint production elasticity, on the discount rate, and on the rate of technological progress. When this condition is satisfied, the economy is sustainable. Then the introduction of circular flows for resources, through recycling and the material balance principle, constitute a solution to overcome the collapse of the economic system.

It is worth noting that technological progress, that could easily be endogenized, is crucial for sustainability. Indeed, when the production system efficiency is not increasing, equation (**??**) reduces to ${g_C}^{SP} = -\rho/\sigma$, implying that the economy will collapse in the long run. This is due to the fact that virgin natural resources are still an essential input and, when a mechanism improving resources efficiency of production is not present, their depletion makes production impossible and recycling cannot compensate.

It is also worth noting that the growth rates along the balanced growth path are independent of the initial stocks of virgin and recyclable resources. We will come back to this point when comparing the socially optimal and the decentralized economy. Moreover, it can be seen from these conditions that incomplete recycling is not affecting the growth rates. However we will prove that it influences the material reflux rate, and consequently the production level, and the initial price of the recycled input.

Finally, let us calculate the level of consumption c along the balanced growth path. As a reminder this also accounts for the share of materials which ends up in the waste pile after consumption: the reflux rate.

Consider the expression for $\dot{z}/z$ and the first-order condition (**??**). It was shown that: $\dot{\psi}_1 = \dot{z} + \psi_3 \dot{m}(1-x)$. Hence along the balanced growth path, where $\dot{m} = 0$,

$$\dot{z}/z = \dot{\psi}_1/\psi_1$$

Equate the two expressions to find

$$\sigma g_C + \rho = \alpha \frac{Y}{K}(1 - \psi_3 \psi_1^{-1}(1-x)mc) \tag{28}$$

By inserting the expression (**??**) for ${g_C}^{SP}$ into equation (**??**) and the evolution path of capital (**??**), we obtain

$$\frac{g_A}{1-\alpha} = \left[\alpha g_K + \alpha \frac{C}{Y}\frac{Y}{K}\right]\left(1 - \psi_3 \psi_1^{-1}(1-x)mc\right)$$

Solving (**??**) for the ratio Y/K and inserting the result in the expression above we obtain

$$\frac{g_A}{1-\alpha} = \alpha g_Y \left(1 - \psi_3 \psi_1^{-1}(1-x)mc\right) + c\frac{g_A}{1-\alpha}$$

Thus the reflux rate observed in the social optimum is

$$c^{SP} = 1 - \frac{g_Y}{g_A}(1-\alpha)\alpha \left[1 - \psi_3 \psi_1^{-1} m c^{SP}(1-x)\right] \tag{29}$$

The most interesting aspect of our analysis is that the lower the material loss x is, and the more circular the economy is (considering our definition of circularity), the higher the backflow rate, i.e. the consumption level along the balanced growth path. The maximum level of consumption is reached under perfect recycling, i.e. $x = 0$.

2.3 The Solution of a Decentralized Economy

The equilibrium in each sector is analyzed separately. Utility maximization of households again leads to the Keynes-Ramsey rule

$$\sigma g_C + \rho = r \tag{30}$$

The missing market for waste changes households' consumption-savings decisions. Indeed, comparing this to the socially optimal Keynes-Ramsey rule (**??**), it is clear that the potential recycling value of consumption byproducts (RCB) is not taken into account. A first externality arises.

The final output market is characterized by perfect competition, thus firms' profit maximization leads to the well-known equalities between the price of each production input and its marginal productivity,

$$r = \alpha \frac{Y}{K} \qquad p_V = \beta \frac{Y}{Z} \qquad p_R = \gamma \frac{Y}{R} \tag{31}$$

where p_V and p_R represent virgin and recycled materials prices. Comparing the marginal revenues of the production inputs expressed by (**??**) and the first-order conditions in the social optimum makes clear that firms do not take into account the potential recycling value of each production factor (RVP): they do not internalize the effect that their input decisions have on the evolution of the waste stock. Therefore, a second externality arises.

The profit maximization of virgin and secondary raw materials producers is described by equations (**??**) and (**??**). Firms do not internalize the reflux of the resources they supply to the waste pile, so its semi-renewability is not taken into consideration in their maximizing decisions. In other words, they do not perceive that the flow of resources is partially circular and alleviates the exhaustion problem. Under these assumptions the only possible solution for the dynamics of the equilibrium prices of material inputs is represented by the standard Hotelling rule

$$g_{p_V} = g_{p_R} = r \tag{32}$$

This does not coincide with the social optimum.

Next, the growth rates of the economy under the market solution are derived. The balanced growth path (BGP) growth rates are in this case denoted by the superscript "MKT".

Express the equilibrium conditions for the material inputs prices in growth rates

$$g_{p_V} = g_Y - g_V$$

$$g_{p_R} = g_Y - g_R$$

According to the Hotelling rule (**??**), these two expressions are equal, implying ${g_V}^{MKT} = {g_R}^{MKT}$: along the BGP the rate of extraction of the two resources is the same.

In the steady state it holds that $g_Y = g_K$. Hence, it follows from the production function that

$$g_V = -1/(1-\alpha) g_A + g_Y$$

Substituting this into (**??**) yields

$$r = g_Y - g_V = 1/(1-\alpha) g_A = 1/(\beta - \gamma) g_A$$

Inserted into the Keynes-Ramsey rule (**??**) gives the BGP growth rate

$${g_C}^{MKT} = {g_Y}^{MKT} = {g_K}^{MKT} = 1/\sigma \left(\frac{1}{\beta + \gamma} g_A - \rho \right) \tag{33}$$

In addition to incomplete recycling, market failures also do not affect the long-run dynamics of the economy. Growth rates in the case of a decentralized economy coincide with the ones obtained by the social planner (compare to (**??**)).

The result that, in an exogenous growth model with a Cobb-Douglas production technology, the growth rate is not affected by market failures is well-known (e.g., Pittel et al. 2010). But this does not hold true for the level of economic activity.

Consider the Keynes-Ramsey rule (**??**) and the optimality condition (**??**) to derive

$$\sigma g_C + \rho = \alpha Y/K$$

Making use of the capital accumulation equation (**??**) and the equilibrium growth rates (**??**), we derive the value for the materials reflux rate after consumption

$$c^{MKT} = 1 - \frac{g_Y}{g_A}(1-\alpha)\alpha \tag{34}$$

The absence of the term $[1 - \psi_3 \, {\psi_1}^{-1} \, m \, c^{SP}(1-x)] < 1$ in the expression for the consumption level in the decentralized economy when comparing to the one for the social optimum (**??**) implies that the latter is higher than the former. The term in brackets is the result of the effect of the circularity of materials flow after consumption that shapes the consumption-saving decisions of the households, but only when no market failures arise and at least a part of consumed resources can be recycled. Indeed from the expression for c^{SP} it is clear that the higher the material loss is, the lower the consumption level along the balanced growth path. At the extremes, c converges to c^{MKT} for $x \to 1$ and c converges to c^* for $x \to 0$.

From now on we will denote the level of variables of an socially optimal economy and complete recycling, i.e. the (ideal) first best for the economy, with an asterisk, and the corresponding measure when $0 < x < 1$ with the superscript "SP". The final result is $c^{MKT} < c^{SP} < c^{*}$.

Whenever the reflux rate is lower than in the case of a social optimum with a perfect circularity of resources flow after consumption, this translates into a suboptimal level of recycling and this causes a suboptimal level of output at any point in time. To see this more clearly, one can integrate the whole evolution of the waste stock along the balanced growth path, i.e. the material balance condition, to obtain the initial level of recycling R_0:

$$\int_0^\infty \dot{W} dt = \int_0^\infty [-R + (V + R)c(1 - x)]dt$$

$$R_0 \equiv R_0(c) = |g_V| \frac{W_0 + c(1 - x)S_0}{1 - c(1 - x)} \tag{35}$$

We use the facts that the economy will seek to completely extract the available stocks of resources, $W_\infty = S_\infty = 0$, and that along the BGP the extraction dynamics are the same for both material inputs, $g_V = g_R$.

From equation (**??**) it is clear that the initial use of recycled materials depends positively on the consumption level and negatively on the magnitude of material loss:

- for ${R_0}^* > {R_0}^{SP} \equiv R_0(c^{SP})$ because of the incomplete circularity of materials flow after consumption;
- for ${R_0}^* > {R_0}^{MKT} \equiv R_0(c^{MKT})$ because $c^{MKT} < c^*$ due to the effect of the market failures.

Now taking into account that the growth rates are the same under the socially optimal solution and under the decentralized market solution, a lower initial recycling level determines a lower use path of waste and, because the capital accumulation dynamic coincide in the two regimes, this determines a lower level of aggregate output.

3 Policy Conclusions

In this section, some possible policy measures to restore the economy's social optimality are presented.

As shown in the previous section, consumption and output levels are suboptimal when the flux of materials from consumption cannot be completely recycled and/or when a market for waste is not present and firms do not take into account the effect of their production decisions on recyclable waste stock regeneration. It follows that two kinds of policy measures, consistent with the model, can be adopted.

The first is to improve the circularity of resources flows. Consider equation (**??**) for c^{SP} from a comparative statics perspective. It is reasonable to assume that the level of material loss x is negatively correlated to R&D efforts in "circularity".

In the first place, this specific form of technological progress is represented, in practical terms, by innovations increasing the share of recyclable materials after consumption: waste separation and materials collection technologies, like more efficient mechanical biological treatment (MBT) technologies for example.

In the second place, manufacturing technologies, producing outputs more adept to being recycled, would serve to the scope. Indeed, aligning production technologies to circular economy objectives is one of the main aims pinpointed by the European Union to promote a circular economy (European Parliament 2017; European Parliament 2009).

R&D investment in the recycling process could be implemented, for instance, through a lump-sum tax or allocating of public expenditure (note households already invest in government bonds as one can see from the optimization problem (1), but the allocation of revenues by the government is not specified). If we assume that the investment will lead for sure to an innovation that reduces the material loss at a certain point in time t, the lump-sum tax or the temporary allocation of government expenditure on research would lead to a higher output level from t on. It can be seen from equation (**??**) that for $x \to 0$, ${R_0}^{SP}$ converges to ${R_0}^{*}$. If the investment in R&D would be financed by the government issuing new debt, then, after the higher income level is reached, an income tax could be levied to restore the balance of state budget.

The suitability of a package of policies to improve the circularity of resource flow is supported by data on the share of recovered materials after their first use cycle. Even in the most successful cases for the recycling of a certain material, steel and plastics for example, a large share of the value of the resource is lost after the first use cycle (Allen MacArthur 2015). This is due to both the quality of the material after it has been used – production technologies, adapting more to the scopes and flows of a circular economy, could play a role here – and to the quantity of materials that can be recycled after the first use. Of course, when considering real data, one should also take into account a more or less virtuous recycling behavior of consumers. The OECD (2015) assessment of technological innovations in the waste management sector highlights a stagnation of the number of filed patents that was lower in 2013 than in 1997. This reinforces the argument of suitability of investment in "R&D for economy circularity".

Secondly, a set of market-oriented policies could be adopted. In order to reach the first-best economy production level, the externalities due to the two market failures must be corrected. The aim here is to close the gap between the consumption level observed in the decentralized economy, c^{MKT}, and the first-best idealistic case where the reflux of materials after consumption is complete, c^{*}, and not the one between c^{MKT} and c^{SP}, that is observed in case of positive material loss and that represents only a second-best situation.

The first market failure is represented by the absence of a market for waste in the sense that households are not remunerated for the waste they produce. Introducing this mechanism, the income constraint of households is modified. Indeed, an additional source of income dependency on the price of the recycled materials p_R, by the flow of consumption and by the material intensity of output

m is observed:

$$\dot{D} = rD - (1 - p_R m)C$$

The Hamiltonian

$$\mathrm{H} = e^{-\rho t}\ \frac{C^{1-\sigma} - 1}{1 - \sigma} + \omega(rD - C + p_R mC)$$

now leads to the first-order conditions

$$\mathrm{H}_C : C^{-\sigma} e^{-\rho t} = \omega(1 - p_R m)$$

$$\mathrm{H}_D : \dot{\omega} = -r\omega$$

where ω is the shadow price of households' wealth. The term $1 - p_R m$ represents the net cost of consumption. It accounts for the difference between the price of one unit of the consumption good, normalized to one, and the revenue from selling the waste generated per unit of consumption to recycling firms. H_C is analogous to equation (**??**) showing the internalization of RCB.

The introduction of a market for waste also corrects the externality due to the missing consideration of RVP by final output producers. It can be shown that expressing profits in terms of the net costs of consumption and maximizing, one obtains first-order conditions analogous to the ones for production inputs derived in the social planner solution, except for the "semi-renewability element in pricing (SRP)" of recyclable resources[1].

To correct for this last externality, due to the fact that material input producers do not take into account the semi-renewability of the waste stock, a system of corrections for material inputs prices can be introduced.

In order to calculate the optimal subsidy for the two material inputs, their initial prices must be calculated. Consider equation (**??**) for p_R, insert equation (**??**) for R_0 and recall that $Y = [g_Y/(1 - c)]K$ to derive

$$p_{R_0}(c) = \gamma \frac{g_Y}{|g_V|} \frac{K_0}{W_0 + (1 - x)c\, S_0} \frac{1 - c(1 - x)}{1 - c} \tag{36}$$

which is the general expression for the initial price of recycled resources. This proves to be useful to show that the higher the material loss is, the higher the initial price for the recycled input, due to the lower initial level of secondary raw materials use, R_0.

Because the aim is to achieve the first best situation, complete recycling is assumed, so that

$$p_{R_0}(c) = \gamma \frac{g_Y}{|g_V|} \frac{K_0}{W_0 + cS_0}$$

By considering equation (**??**) for p_V and inserting equation (**??**) and $Y = [g_Y/(1 - c)]K$, one obtains the initial price for virgin resources as a function of the reflux rate

$$p_{V_0}(c) = \gamma \frac{g_Y}{|g_V|} \frac{K_0}{(1 - c)S_0}$$

---

[1] Express final output producers' profit function as $\pi = (1 - p_R m)Y - p_K K - p_V V - p_R R$. Deriving with respect to R and considering $p_R = \psi_3 \psi_1^{-1}$, one obtains $\psi_1 F_R - \psi_3 F_R m\, c - \psi_3 = 0$, which equals (**??**) without SRP.

The difference, that arises between the initial market prices $p_{R_0}(c^{MKT})$ and $p_{V_0}(c^{MKT})$ and initial socially optimal prices $p_{R_0}(c^*)$ and $p_{V_0}(c^*)$ respectively, determines the optimal system of price corrections:

$$\tau_R = \gamma \frac{g_Y}{|g_V|} K_0 \left[\frac{1}{W_0 + c^* S_0} - \frac{1}{W_0 + c^{MKT} S_0} \right]$$

$$\tau_V = \gamma \frac{g_Y}{|g_V|} K_0 \left[\frac{1}{1 - c^*} - \frac{1}{1 - c^{MKT}} \right]$$

Because $c^{MKT} < c^*$ and the initial stocks of both resources are strictly positive, the optimal correction for recycled resources price τ_R is negative, meaning that, to achieve the first best situation, the initial market price for recycled waste has to be lowered by τ_R. On the opposite, the correction for the initial market price for virgin resources makes them more expensive, as $\tau_V > 0$.

To summarize: the introduction of a market for waste and the introduction of a subsidy on recycled resources and of a tax on virgin resources represent two possible devices to restore the social optimality of the economy. By increasing the level of recycling, that was shown to be the cause of the suboptimal level of output, these measures also shift the composition of final output towards a more recycled-resources intensive one, making it more sustainable.

4 Summary and Conclusion

The paper attempted to integrate the concept of a circular economy in the well-known framework of (traditional) growth theory. The transition to (more) circular economic systems could represent a device to alleviate both problems of natural resources depletion as well as waste accumulation in the environment. Since the concept is often discussed in a quite idealistic way, we concentrated on two of its most important aspects.

Indeed, we showed that a basic economic growth model with exogenous technological progress, characterized by exhaustible natural resources as an essential input, may be sustainable in the long run with the introduction of a recycling sector and of the material balance principle.

Our theoretical analysis builds on the Pittel et al. (2010) model and extends it by the introduction of the variable “material loss”, i.e. the share of materials that cannot be recycled for technical reasons. In particular, the introduction of the matetial loss concept enables a switch, depending on the magnitude of that variable, from a totally linear model to a completely closed and circular system, at least with regard to natural resources flows outgoing the consumption phase. The material loss and two specific market failures included in the model reduce the initial level of recycling and, consequently, increase the initial price of secondary raw materials. We demonstrated that the consumption level, accounting for the reflux of materials to production after consumption, as well as the output level are not optimal when recycling is incomplete and those market failures are taken into account.

The first-best outcome for the economy in terms of output and consumption levels can be achieved: by reducing the share of materials that are not flowing back to the production process through investment in "R&D for economy circularity", i.e. in forms of innovation aiming to improve the circularity of resource flow, and by introducing a market for waste and a system of corrections for initial prices of material inputs.

The perception of waste as a valuable input into such an economic system implies a complete exhaustion of the waste stock in the long-run (see also Pittel 2006). This is a clue for the environmental sustainability of circular economies.

Nowadays, European economies are far from being significantly circular. For example, the Eurostat circular material use index[2], representing the share of secondary raw materials on overall material input for domestic use at the EU level, accounts for a mere 11.9% (2019). This leaves much room for improvements, but, as stressed in this paper, recycling must be accompanied with an enhanced resource efficiency of the production processes in order to substantially overcome the problem of natural resource depletion.

References

Allen MacArthur Foundation (2015) "Growth within: a circular economy vision for a competitive Europe".

Ayres R.U. (1999) "The second law, the fourth law, recycling and limits to growth". Ecological Economics 3, 473-483.

Boulding, K. E., (1966) "The Economics of the Coming Spaceship Earth". In H. Jarrett (ed.) 1966. Environmental Quality in a Growing Economy, pp. 3-14. Baltimore, MD: Resources for the Future/Johns Hopkins University Press.

Compagnoni, M. (2020) "Regional policies for circular economy and an empirical analysis of the effects of PAYT tax in Emilia Romagna". Environmental Engineering Management Journal 19, 1711-1718.
`http://www.eemj.icpm.tuiasi.ro/pdfs/vol19/no10/Full/8_90_Compagnoni_20.pdf`

Dasgupta, P., Heal, G. (1974), "The optimal depletion of exhaustible resources". Review of Economic Studies 41, 3-28.

Di Vita, G. (2007) "Exhaustible resources and secondary materials: A macroeconomic analysis". Ecological Economics 63, 138-148.

Di Vita, G. (2004) "Renewable resources and waste recycling". Fondazione Eni Enrico Mattei, Milan, pp. 159-167.

Di Vita, G. (2001) "Technological change, growth and waste recycling". Energy Economics 23, 549-567.

European Commission (2015) "Closing the loop – An EU action plan for the Circular Economy". COM/2015/0614.

---

[2] code env_ac_cur

European Commission (2020) "New Circular Economy Action Plan" `https://ec.europa.eu/environment/circular-economy/pdf/new_circular_economy_action_plan.pdf`

European Commission (2010) "Europe 2020: a strategy for smart, sustainable and inclusive growth".

European Commission (2008) "Waste Framework Directive": Directive 2008/98/EC

EEA European Environmental Agency (2017) "Circular by design. Products in the circular economy", EEA Report, 6, Copenhagen.

European Parliament (2017) "Waste management in EU – Towards a circular economy". European Parliament Research Service.

European Parliament (2009) Directive 2009/125/EC of the European Parliament and of the Council of 21 October 2009 establishing a framework for the setting of ecodesign requirements for energy-related products, OJ L 285, p. 10–35.

Eurostat, circular material use rate `https://ec.europa.eu/eurostat/web/products-eurostat-news/-/DDN-20200312-1`

Geissdoerfer, M., Savaget, S., Bocken, N., Hultink, E. J. (2017) "The Circular Economy – A new sustainability paradigm?" Journal of Cleaner Production 143, 757-768.

Georgescu-Roegen, N. (1971) "The entropy law and the economic process". Harvard University Press, Cambridge (MA).

Groth, C. (2007) "A New-Growth Perspective on Non-Renewable Resources". In: Bretschger L., Smulders S. (eds) Sustainable Resource Use and Economic Dynamics. The Economics Of Non-Market Goods And Resources, vol. 10, Springer, Dordrecht.

Kirchherr, J., Reike, D., Hekkert, M. (2017) "Conceptualizing the circular economy: An analysis of 114 definitions". Resources, Conservation and Recycling 127, 221-232.

Korhonen, J., Honkasalo, A., Seppaila, J. (2018) "Circular Economy: the concept and its limitations". Ecological Economics 143, 37-46.

Kornafel, M., Telega, I. (2020) "Dynamics of natural capital in neoclassical growth model". International Journal of Sustainable Economy, Inderscience Enterprises Ltd, 12, 1-24.

Lafforgue, G., Rouge, L. (2019) "A dynamic model of recycling with endogenous technological breakthrough". Resource and Energy Economics 57, 101-118.

OECD (2015) "Education at a Glance" OECD publishing.

Meadows, D. H., Meadows, D. L., Randers, J., Behrens III, W. W. (1972) "The Limits to Growth; A Report for the Club of Rome's Project on the Predicament of Mankind", New York: Universe Books.

Pittel, K. (2006). "A Kuznets Curve for Recycling" SSRN Electronic Journal.

Pittel, K., Amigues J., Kuhn, T. (2005) "Endogenous growth and recycling: a material balance approach" CER-ETH Economics working paper 05/37, ETH Zurich.

Pittel, K., Amigues, J., Kuhn, T. (2010) "Recycling under a material balance constraint". Resource and Energy Economics 32, 379-394.

Scholl, A., Semmler, W., (2002) “Sustainable economic growth and exhaustible resources: A model and estimation for the US” Discrete Dynamics in Nature and Society 7, 79-92.

Smulders, S., Toman, M., Withagen C., (2015) “Growth theory and ‘green growth’”. Oxford Review of Economic Policy 30, 423–446.

Stiglitz, J. E. (1974) “Growth with exhaustible natural resources: efficient and optimal growth paths”. Review of Economic Studies, Symposium Issue 41, 123-137.

World commission on environment and development WCED (1987) “Report of the World Commission on Environment and Development: Our Common Future” Oxford University Press, New York.

Zotti, J., Bigano, A., (2019) “Write circular economy, read economy’s circularity. How to avoid going in circles”. Economia Politica 36, 629-652.