Significant Abstracts and Extended Summaries 1983-2012

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Including Four Psychological Abstracts by
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Preface

This online publication makes available some abstracts and extended summaries, which contain unpublished ideas that have occasionally been referenced. This gives them a regular DOI to facilitate access and citation. Not included are abstracts of conferences of the Association for Symbolic Logic, which are published in the Journal/Bulletin of Symbolic Logic.

In addition, this collection contains four abstracts from the years 1986–1989 representing joint psychological work by Walter H. Ehrenstein, Gabriele Heister and myself.

Tübingen, August 2022 Peter Schroeder-Heister

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Abstracts of the 7th International Congress of Logic, Methodology and Philosophy of Science, Salzburg, July 11th-16th, 1983, Vol. 2, pp. 150-153.

Section 5: Philosophical Logic

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INVERSION PRINCIPLES AND THE COMPLETENESS OF INTUITIONISTIC NATURAL DEDUCTION SYSTEMS

Some influential semantical conceptions for intuitionistic logic, in particular those of M. Dummett and D. Prawitz, consider the introduction rules (I-rules) for logical operators to be 'canonical' rules which give a meaning to these operators; the elimination rules (E-rules) are then justified with respect to a semantics depending on I-rules. The often stated harmony between I- and E-rules suggests that one might reverse this procedure, i.e. choose the E-rules to be canonical and justify the I-rules with respect to a semantics depending on E-rules. In the following, which represents an attempt in this direction, we shall define a concept of validity based on E-rules. It can then be shown that all I-rules are valid, and conversely that all valid rules are derivable in intuitionistic logic. In this sense intuitionistic logic is complete. This approach is dual to that proposed in [7] where the inversion principle, as formulated by Prawitz [3,4], is generalized to a notion of validity based on I-rules, with respect to which the completeness of intuitionistic logic could be established. So the present approach formulates an 'inverted' inversion principle.

<u>The Inversion Principle</u> In Lorenzen [2], the inversion principle is treated as a principle to establish the admissibility of rules. A rule R is called admissible in a calculus K, if its addition to the inference rules of K, yielding an extended calculus K+R, does not enlarge the set of formulas derivable in K, i.e. for each formula D, if $\vdash_{K+R} D$, then $\vdash_{K} D$. Here the 'if...then' is understood constructively, i.e. there must be an effective procedure eliminating each application of R in a derivation of D. The inversion principle is applied in such cases where the premises of R can be derived in K only by application of certain inference rules $R_1,\ldots,R_n$ of K: then we know that a derivation of the premises of R in K contains a derivation of at least some of the premises of $R_1,\ldots,R_n$; if we know furthermore that for each i $(1 \leq i \leq n)$ the step from the premises of R_i to the conclusion of R is admissible, we can infer the admissibility of R. (For a precise description see [1]). The main application of the inversion principle within formal logic is the justification of the $\wedge$-, $\vee$-, and $\exists$-E-rules as admissible rules in every calculus K having the $\wedge$-, $\vee$-, $\exists$-I-rules as the only inference rules making it possible to infer conjunctions, disjunctions and existential quantifications. As can easily be seen, the admissibility concept and thus the inversion principle in Lorenzen's version does not work for derivations from assumptions. If we defined R to be admissible in K if for all finite sets of assumptions Γ and all formulas D: if $\Gamma \vdash_{K+R} D$ then $\Gamma \vdash_{K} D$, then each admissible rule R would be derivable: Taking Γ to be the set of premises of R and D to be its conclusion, $\Gamma \vdash_{K+R} D$ would be trivially fulfilled, thus $\Gamma \vdash_{K} D$ would hold.

Following some remarks of Gentzen, Prawitz used in [3,4] a somewhat different inversion principle to describe the relation between I- and E-rules of natural deduction systems: if the major premise of an E-rule is derived using an I-rule in the last step, this derivation already 'contains', together with derivations of the minor premises of the E-rule, a derivation of the conclusion of the E-rule. This relation is made explicit in the reduction steps and normalization procedures stated by Prawitz. Such an inversion principle obviously does not allow the elimination of an E-rule R from all derivations in C+R, where C is the (canonical) part of an intuitionistic natural deduction system having only I-rules as inference rules. But we can formulate it in a way that makes it closely related to Lorenzen's inversion principle: Define for an E-rule R a derivation in C+R to be a derivation which applies R only if its major premise is the conclusion of an application of an I-rule. Then it holds in fact for all E-rules R that for all sets of assumptions Γ and all formulas D: if $\Gamma \vdash_{C+R} D$, then $\Gamma \vdash_{C} D$. The difference to Lorenzen's inversion principle is that in calculi without assumptions the major premise of an E-rule can be derived only by using an I-rule in the last step where this fact must be required in the case of calculi with assumptions. On the one hand this weakens the inversion principle, but on the other hand it makes it possible to treat → in this framework (which was not possible in Lorenzen [2]).

Prawitz' inversion principle is defined for the standard E-rules with one major premise. In Schroeder-Heister [7] it is generalized to a principle that may be used for the justification of arbitrary rules R (the E-rules being special cases thereof). The general schema for an arbitrary rule in a natural deduction system can be stated as

$$(1)\qquad \frac{\begin{array}{c}\Gamma_1\\ \vdots\,\underline{x}_1\\ A_1\end{array}\quad \cdots\quad \begin{array}{c}\Gamma_n\\ \vdots\,\underline{x}_n\\ A_n\end{array}}{A}$$

where the Γ's are (possibly empty) sets of formulas, indicating the assumptions which may be discharged by application of that rule, and the $\underline{x}$'s are sets of eigenvariables to be respected. In order to formulate an inversion principle, we assume a (possibly empty) set of non-atomic assumption- and eigenvariable-free premises to be distinguished by a star, thus arriving at the schema

$$(2)\qquad \frac{\begin{array}{c}\vdots\\ {}^*A_1\end{array}\ \cdots\ \begin{array}{c}\vdots\\ {}^*A_n\end{array}\quad \begin{array}{c}\Gamma_1\\ \vdots\,\underline{x}_1\\ B_1\end{array}\ \cdots\ \begin{array}{c}\Gamma_m\\ \vdots\,\underline{x}_m\\ B_m\end{array}}{A}$$

Here the starred A's function like major premises in the usual E-rules which must now be written as

$$\frac{\begin{array}{c}\vdots\\ * A\wedge B\end{array}}{A}\qquad \frac{\begin{array}{c}\vdots\\ * A\wedge B\end{array}}{B}\qquad \frac{\begin{array}{c}\vdots\\ * A\vee B\end{array}\quad \begin{array}{c}A\\ \vdots\\ C\end{array}\quad \begin{array}{c}B\\ \vdots\\ C\end{array}}{C}\qquad \frac{\begin{array}{c}\vdots\\ * A\to B\end{array}\quad \begin{array}{c}\vdots\\ A\end{array}}{B}\qquad \frac{\begin{array}{c}\vdots\\ * \bot\end{array}}{A}\qquad \frac{\begin{array}{c}\vdots\\ * \forall xA\end{array}}{A[x/t]}\qquad \frac{\begin{array}{c}\vdots\\ * \exists xA\end{array}\quad \begin{array}{c}A[x/y]\\ \vdots\,y\\ B\end{array}}{B}$$

(y not free in ∃xA or B)

For C as the canonical part of the natural deduction calculus having only I-rules as inference rules, a derivation in C+R for a rule of the form (2) is defined as applying R only if the starred premises are derived using an I-rule in the last step, i.e. the starred premises are counted as major premises in a generalized sense. We say that the inversion principle holds for R, or that R is valid, if for all Γ, D: if $\Gamma\vdash_{C+R}D$, then $\Gamma\vdash_{C}D$. It can be shown not only that for all rules of intuitionistic logic I the inversion principle holds (i.e. that they are valid), but also that all rules for which the inversion principle holds are derivable in I; so I is in a certain sense complete.

Assumption Rules We allow not only formulas but also 'assumption rules' of the form $\{A_1,\ldots,A_n\}\Rightarrow_{\underline{x}}A$ to be assumptions on which derivations in natural deduction calculi may depend. (Here the sets $\{A_1,\ldots,A_n\}$ and/or $\underline{x}$ may be empty; in the former case the assumption rule is identified with the assumption A). Assumption rules are applied in a derivation according to the schema

$$\frac{\{A_1,\ldots,A_n\}\Rightarrow_{\underline{x}}A\qquad \begin{array}{c}\vdots\\ A_1[\underline{x}/\underline{t}]\end{array}\ \cdots\ \begin{array}{c}\vdots\\ A_n[\underline{x}/\underline{t}]\end{array}}{A[\underline{x}/\underline{t}]}$$

An assumption rule $\{A_1,\ldots,A_n\}\Rightarrow_{\underline{x}}A$ represents on the object level the metalogical assumption that a derivation of A from $\{A_1,\ldots,A_n\}$ is given whereby eventual further assumptions do not contain any variable of $\underline{x}$ free. The concept of assumption rules allows us to define the derivability of a rule of form (1) as $\{\Gamma_1\Rightarrow_{\underline{x}_1}A_1,\ldots,\Gamma_n\Rightarrow_{\underline{x}_n}A_n\}\vdash A$, analogously to the usual definition of the derivability of a rule of the form $\dfrac{A_1 \ \dots \ A_n}{A}$ as $\{A_1,\dots,A_n\}\vdash A$. If Δ denotes a set of premises of a rule of form (1), Δ' is defined to be the set $\{\Gamma_1 \Rightarrow_{\underline{x_1}} A_1,\dots,\Gamma_n \Rightarrow_{\underline{x_n}} A_n\}$. So a rule $\frac{\Delta}{A}$ is derivable if $\Delta' \vdash A$. (For a systematic treatment of assumption rules also of higher levels see [6]).

<u>An Inversion Principle Based on Elimination Rules</u> The 'harmony' between I- and E-rules has often been emphasized but usually I-rules are chosen to be canonical rules (with the exception of the approach sketched in [4, Appendix A.2] which is somewhat different from the one given here). I shall take the E-rules to be canonical and justify the I-rules by an inversion principle which treats I-rules as inverses of E-rules, dual to the path taken in [7]. This means that we have to formulate counterparts to the concepts defined there. (E.g., counterparts of the major premises of E-rules are now the conclusions of I-rules). Thus we define the canonical part C of the intuitionistic natural deduction calculus I to be the subsystem containing only the E-rules for $\wedge$, $\vee$, $\rightarrow$, $\bot$, $\forall$, $\exists$ (as stated above, but without a star). $\Gamma \vdash_C D$ is defined as usual where Γ may include assumption rules. The general form of an arbitrary rule R is

$$\frac{\begin{matrix}\Gamma_1 \\ \vdots \underline{x_1} \\ A_1\end{matrix} \quad \dots \quad \begin{matrix}\Gamma_n \\ \vdots \underline{x_n} \\ A_n\end{matrix}}{(*)A} \qquad \text{or shortly} \qquad \frac{\Delta}{(*)A}$$

where a non-atomic conclusion A can be starred (premisses must not be starred). A derivation in C+R is a derivation in the calculus resulting from C by addition of R as an inference rule, where, if A is starred, the conclusion of each application of R in the derivation is major premise of an application of an E-rule. We shall say that R fulfils the inversion principle or is valid if for all Γ, D: if $\Gamma \vdash_{C+R} D$, then $\Gamma \vdash_C D$. Since all E-rules of I belong to C, they are trivially valid. The I-rules of I, now to be written as

$$\frac{\begin{matrix}\vdots\\A\end{matrix} \quad \begin{matrix}\vdots\\B\end{matrix}}{* A\wedge B} \quad \frac{\begin{matrix}\vdots\\A\end{matrix}}{* A\vee B} \quad \frac{\begin{matrix}\vdots\\B\end{matrix}}{* A\vee B} \quad \frac{\begin{matrix}A\\\vdots\\B\end{matrix}}{* A\rightarrow B} \quad \frac{\begin{matrix}\vdots y\\A[x/y]\end{matrix}}{* \forall xA} \ \text{(y not free in } \forall xA) \quad \frac{\begin{matrix}\vdots\\A[x/t]\end{matrix}}{* \exists xA} \qquad (\bot \text{ has no I-rule})$$

can be shown to be valid by application of the standard reduction steps. So all inference rules of I are valid. Conversely, we can prove that all valid rules are derivable in I: First we state that all valid rules R without starred conclusions are derivable in C and hence in I. (Take Γ to be Δ'; then $\Delta' \vdash_{C+R} A$ holds trivially and thus $\Delta' \vdash_C A$). Secondly, if a rule R of one of the forms

$$\frac{\Delta}{* A\wedge B} \quad \frac{\Delta}{* A\vee B} \quad \frac{\Delta}{* A\rightarrow B} \quad \frac{\Delta}{* \bot} \quad \frac{\Delta}{* \forall xA} \quad \frac{\Delta}{* \exists xA}$$

is valid, then also

$$\frac{\Delta}{A} \text{ and } \frac{\Delta}{B}, \quad \frac{\Delta \quad \begin{matrix}A\\\vdots\\C\end{matrix} \quad \begin{matrix}B\\\vdots\\C\end{matrix}}{C} \text{ for all C}, \quad \frac{\Delta \quad \begin{matrix}\vdots\\A\end{matrix}}{B}, \quad \frac{\Delta}{C} \text{ for all C}, \quad \frac{\Delta}{A[x/t]} \text{ for all t}, \quad \frac{\Delta \quad \begin{matrix}A[x/y]\\\vdots y\\B\end{matrix}}{B}$$

(y not free in $\exists xA$ or B) for all B, respectively, are valid and hence derivable in I. (E.g. in the third case, by replacing each application of

$$\frac{\Delta \quad \begin{matrix}\vdots\\A\end{matrix}}{B} \quad \text{by} \quad \frac{\dfrac{\Delta}{A\rightarrow B} \quad \begin{matrix}\vdots\\A\end{matrix}}{B}$$

, we obtain a derivation in C+R, from which R can be eliminated). By application of I-rules in I the derivability of R in I then follows. (E.g. in the second case, we have a derivation of $A\vee B$ from $\Delta' \cup \{A \Rightarrow A\vee B,\ B \Rightarrow A\vee B\}$ in I; replacing all applications of the assumption rules $A \Rightarrow A\vee B$ and $B \Rightarrow A\vee B$ by applications of the $\vee$-I-rules we obtain a derivation of $A\vee B$ from Δ'). If we denote by $\Gamma \Vdash D$ that D is derivable from Γ only by use of valid rules, we have established:

<u>Theorem</u> $\Gamma \Vdash D$ iff $\Gamma \vdash_{I} D$.

<u>Remarks</u> 1. This theorem does not include that each rule <u>derivable</u> in I is valid. For example the rule R

$$\frac{\begin{matrix}\vdots & \vdots & \vdots\\ A & B & C\end{matrix}}{*\,(A \wedge B) \wedge C}$$

which can be derived in I by twofold application of $\wedge$-I is not valid in our sense. Its application is not eliminable e.g. from the derivation

$$\frac{\dfrac{A \quad B \quad C}{(A \wedge B) \wedge C}}{A \wedge B}$$

in C+R. Thus if $\Delta' \Vdash D$, $\frac{\Delta}{D}$ need not be valid. Combination of valid rules does not always yield a valid rule. So the proposed inversion principle is weaker than the definitions of validity Prawitz proposed in [4,5], which are transitive in the sense that combination of valid rules always yield valid rules. A completeness proof for intuitionistic logic with respect to Prawitz' concept of validity (or a related concept) would be more informative than the one given here, but is still a desideratum.

2. We would obtain an analogous result for classical logic if we took C to include

$$\frac{\begin{matrix}A \to \bot\\ \vdots\\ \bot\end{matrix}}{A} \qquad \text{instead of} \qquad \frac{\bot}{A}\ .$$

If we wanted to give reasons for preferring intuitionistic logic to classical logic in our framework, we would have to argue for a certain choice of canonical E-rules (e.g. that major premises of E-rules must not depend on assumptions). The completeness result ifself does not provide reasons for such a preference.

<u>References</u> [1] H. Hermes, Zum Inversionsprinzip der operativen Logik, in: A. Heyting (ed.), Constructivity in Mathematics, Amsterdam 1959, 62-68. [2] P. Lorenzen, Einführung in die operative Logik und Mathematik, Berlin 1955, 2nd ed. 1969. [3] D. Prawitz, Natural Deduction. A Proof-Theoretical Study, Stockholm 1965. [4] D. Prawitz, Ideas and Results in Proof Theory, in: J. E. Fenstad (ed.), Proceedings of the Second Scandinavian Logic Symposium, Amsterdam 1971, 235-307. [5] D. Prawitz, Towards a Foundation of a General Proof Theory, in: P. Suppes et al. (eds.), Logic, Methodology and Philosophy of Science IV, Amsterdam 1973, 225-250. [6] P. Schroeder-Heister, Untersuchungen zur regellogischen Deutung von Aussagenverknüpfungen, Dissertation, Bonn 1981. [7] P. Schroeder-Heister, The Completeness of Intuitionistic Logic with Respect to a Validity Concept Based on an Inversion Principle, J. Philos. Log., in press.

An asymmetry between introduction and elimination inferences

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The symmetry between introduction (I) and elimination (E) inferences in natural deduction or between right-introduction ($\vdash *$) and left-introduction ($* \vdash$) inferences in sequent calculi is normally considered a central feature of Gentzen systems. Both philosophical and mathematical investigations have tried to point out a uniform relationship or duality between I or $\vdash *$ and E or $* \vdash$ inferences, always in connection with normalization and cut elimination. This is not being questioned here. However, a certain characteristic asymmetry will be pointed out that has to do with the notion of discharging assumptions. Let $X[A]$ express that the formula A occurs at a certain place in a list X of formulae, and let $X[Y]$ denote the result of replacing this occurrence of A in X by the list Y. Then, for example, the schema of implication introduction in sequent-style natural deduction should be formulated as

$$\frac{X, A \vdash B}{X \vdash A \to B}$$

and *not* as

$$\frac{X[A] \vdash B}{X \vdash A \to B} \ ,$$

whereas the schema of disjunction elimination should be formulated as

$$\frac{X \vdash A \vee B \quad Y[A] \vdash C \quad Y[B] \vdash C}{Y[X] \vdash C}$$

and *not* as

$$\frac{X \vdash A \vee B \quad Y, A \vdash C \quad Y, B \vdash C}{Y, X \vdash C} \ .$$

Similarly, in the multiple-conclusion sequent calculus the schema of implication introduction on the right should be formulated as

$$\frac{X, A \vdash B, Y}{X \vdash A \to B, Y}$$

and *not* with A or B bracketed, whereas the schema of disjunction introduction on the left should be formulated as

$$\frac{Y[A] \vdash C \quad Y[B] \vdash C}{Y[A \vee B] \vdash C}$$

rather than

$$\frac{Y, A \vdash C \quad Y, B \vdash C}{Y, A \vee B \vdash C} \ ,$$

and analogously for other connectives.
These claims are based on the following principles:

1. Rules for logical constants should be uniform and independent of the structural principles assumed.
2. Normalization (for natural deduction systems) and cut elimination (for sequent calculi) should hold.
3. In the multiple-conclusion case symmetry should not be forced by providing a mechanism that permits to move formulae between the two sides of a sequent.

Proceedings of the Workshop on Linear Logic and Logic Programming, Washington, DC, 14 November1992 (following the 1992 Joint International Conference and Symposium on Logic Programming). Ed. by Dale Miller (University of Pennsylvania Department of Computer and Information Science Technical Report No. MS-CIS-92-80), https://repository.upenn.edu/cis_reports/301

Rules of definitional reflection in logic programming

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Given a set $\mathbf{D}$ of clauses of the form

$$F \Rightarrow A,$$

where F is a formula of some logic and A is an atom, it is natural to extend the sequent calculus for that logic by a rule like

$$\frac{\Gamma \vdash F}{\Gamma \vdash A} \ (\vdash \mathbf{D}),$$

yielding a logic over $\mathbf{D}$. This idea has been used in proof-theoretic interpretations and extensions of definite Horn clause programming, notably λ-Prolog, by giving a computational reading to ($\vdash$ $\mathbf{D}$), which corresponds to resolution if the clauses in $\mathbf{D}$ are of a particular form.

In systems like GCLA, a principle dual to ($\vdash$ $\mathbf{D}$) is considered in addition, yielding a fully symmetric sequent calculus. It is called "definitional reflection" since it is based on reading the database $\mathbf{D}$ as a definition. There are two main options for formulating definitional reflection. The rule on which GCLA is based is the following:

$$\frac{\{\Gamma, F\sigma \vdash G : F \Rightarrow B \in \mathbf{D} \ \mathit{and}\ A = B\sigma\}}{\Gamma, A \vdash G} \ (\mathbf{D} \vdash).$$

An alternative rule which has been considered by Eriksson and which seems also to be the one Girard is favoring, has the following form:

$$\frac{\{\Gamma\sigma, F\sigma \vdash G\sigma : F \Rightarrow B \in \mathbf{D} \ \mathit{and}\ \sigma = mgu(A, B)\}}{\Gamma, A \vdash G} \ (\mathbf{D} \vdash)^*.$$

As they stand, $(\mathbf{D} \vdash)^*$ is stronger than $(\mathbf{D} \vdash)$ (in the non-propositional case) - a standard example being the derivations of the axioms of ordinary first-order equality theory. Computationally, however, they rest on different intuitions. The first rule considers free variables as *existentially* quantified from outside, for which an appropriate substitution has to be computed. The second rule considers them as *universally* quantified from outside rather than something for which an substitution has still to be found. By means of unification it takes into account all possible substitution instances of the atom A, which can be inferred according to the given definition $\mathbf{D}$, thus corresponding to some kind of ω-rule.

Therefore, the extension of logic programming systems by computational variants of $(\mathbf{D}\vdash)$ and $(\mathbf{D}\vdash)^*$ leads to conceptually different approaches. A combination of $(\mathbf{D}\vdash)$ and $(\mathbf{D}\vdash)^*$ with both existential and universal variables, as proposed by Eriksson, would be a most desirable feature of a logic programming system with definitional reflection. There are certain algorithmic problems involved in such a combination that have still to be solved.

In any case, whether one considers $(\mathbf{D}\vdash)$ or $(\mathbf{D}\vdash)^*$ or a combination of both, cut-elimination fails for the full system but holds if the definition $\mathbf{D}$ does not contain implications in clause bodies or if the underlying logic is contraction-free (e.g., linear). We argue that the failure of cut-elimination is a matter of the definition $\mathbf{D}$ considered rather than a defect of the underlying logic.

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Cut Elimination for Logics with Definitional Reflection and Restricted Initial Sequents

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The failure of cut elimination in general has sometimes be considered a deficiency of systems with definitional reflection. If the system is contraction-free or if the definition considered does not contain implication, then the system admits cut elimination (see [5]). Based on considerations unrelated to cut elimination, Kreuger [4] has proposed to restrict initial sequents

$$a \vdash a$$

in the logic of definitional reflection to the case where a is an atomic formula which is not properly defined by the given definition $\mathcal{D}$ in the sense that $a \Leftarrow a$ is the only clause for a in $\mathcal{D}$. It will be shown that in such systems, which contain contraction, cut is eliminable.

Slightly differently, without using clauses like $a \Leftarrow a$, we can describe the situation as follows: Let $\mathcal{D}$ be a definition and $\mathcal{U}$ a distinguished set of atoms which are not defined by $\mathcal{D}$, i.e., which are not head of any clause in $\mathcal{D}$. Elements of $\mathcal{U}$ are called 'uratoms'. Then we consider the system of definitional reflection described in [6] with both thinning and contraction, but replace the rule (I) with

$$(I)_{\mathcal{U}} \ \frac{}{a \vdash a} \quad a \in \mathcal{U}$$

and the rule $(\mathcal{D}\vdash)$ with

$$(\mathcal{D}\vdash)_{\mathcal{U}} \ \frac{\{\Gamma, C \vdash A : C \in \mathcal{D}(a)\}}{\Gamma, a \vdash A} \quad a \notin \mathcal{U}$$

(of course with the usual proviso that guarantees closure under substitution). Furthermore, due to the presence of contraction, we just consider a single conjunction $\wedge$. We call this system $\mathbf{DR}_{\mathcal{U}}(\mathcal{D})$. We show that cut is admissible in this system.

*Draft (Februar 1994) — Comments welcome)

First we transform $\mathbf{DR}_{\mathcal{U}}(\mathcal{D})$ into a system $\mathbf{DR}^*_{\mathcal{U}}(\mathcal{D})$, in which contraction and thinning are no longer explicit rules but built into the other rules. For simplicity, we here only consider the propositional part:

$$(I)\ \frac{}{\Gamma, a \vdash a}\ a \in \mathcal{U}$$

$$(\top)\ \frac{}{\Gamma \vdash \top} \qquad (\bot)\ \frac{}{\Gamma, \bot \vdash A}$$

$$(\vdash\wedge)\ \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \qquad (\wedge\vdash)\ \frac{\Gamma, A, B \vdash C}{\Gamma, A \wedge B \vdash C}$$

$$(\vdash\vee)\ \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \qquad (\vee\vdash)\ \frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C}$$

$$(\vdash\to)\ \frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \qquad (\to\vdash)\ \frac{\Gamma, A \to B \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \to B \vdash C}$$

$$(\vdash\mathcal{D})\ \frac{\Gamma \vdash C}{\Gamma \vdash a}\ C \in \mathcal{D}(a) \qquad (\mathcal{D}\vdash)_{\mathcal{U}}\ \frac{\{\Gamma, C \vdash A : C \in \mathcal{D}(a)\}}{\Gamma, a \vdash A}\ a \notin \mathcal{U}$$

It is obvious that thinning is admissible in $\mathbf{DR}^*_{\mathcal{U}}(\mathcal{D})$. For contraction we argue as follows: We can show that $(\mathcal{D}\vdash)_{\mathcal{U}}$ is invertible in the sense that, if $\Gamma, a \vdash A$ is derivable with n applications of $(\mathcal{D}\vdash)_{\mathcal{U}}$, then $\Gamma, C \vdash A$ is derivable for any $C \in \mathcal{D}(a)$ with $< n$ applications of $(\mathcal{D}\vdash)_{\mathcal{U}}$. Here it is crucial that $\Gamma, a \vdash A$ cannot result from $(I)_{\mathcal{U}}$, since a is no uratom. The admissibility of contraction then follows by induction on the number of applications of $(\mathcal{D}\vdash)_{\mathcal{U}}$ and the length of derivations.

Remark If we had to formulate a system with implicit contraction, but with the rule (I) unrestricted, we would have to take

$$\frac{\{\Gamma, a, C \vdash A : C \in \mathcal{D}(a)\}}{\Gamma, a \vdash A}$$

with the a repeated above the inference line as a primitive rule. Otherwise, for example, from the definition $p \Leftarrow p \to \bot$ the sequent $p \vdash \bot$ would not be derivable, which is derivable with explicit contraction.

Now in the system $\mathbf{DR}^*_{\mathcal{U}}(\mathcal{D})$ we can easily eliminate cuts: we use induction on the triple $\langle d, c, l \rangle$, where the $\mathcal{D}$-rank d is the maximum number of applications of $\mathcal{D}$-rules in all branches leading to the conclusion of the cut, the cut-degree c is the complexity of the cut formula, and the cut-length l is the number of rule applications above the conclusion of the cut. In the main reductions with $\to$ the $\mathcal{D}$-rank is not increased since in its definition we have taken the maximum and not the sum of applications of $\mathcal{D}$-rules. In the main reduction of $\mathcal{D}$ the $\mathcal{D}$-rank is decreased since we have counted both $(\mathcal{D}\vdash)_{\mathcal{U}}$- and $(\vdash\mathcal{D})$-inferences.

Remarks

1. In the system with unrestricted (I) and implicit contraction within $(\mathcal{D}\vdash)$ we cannot perform main reductions of $\mathcal{D}$, since the $\mathcal{D}$-rank is not necessarily decreased. If we have no contraction at all, then it is possible to work with sums in the computation of the $\mathcal{D}$-rank and just count $(\mathcal{D}\vdash)$-applications (as in [5]).[1]

2. If we use the ω-version of definitional reflection with $(\mathcal{D}\vdash)_\omega$ instead of $(\mathcal{D}\vdash)$, no additional problems arise in principle. The rule $(\mathcal{D}\vdash)_\omega$ is invertible, if we do not require contraction.[2]

3. Jäger and Stärk [3], who work with a multiple succedent calculus with negation as primitive, have proved a result similar to the one given here. The differences between their sequent system and ours are not very important as far as cut elimination is concerned. They translate proofs in the original system into a system with ramified $\mathcal{D}$-rules, for which the cut elimination proof is completely standard, and then retranslate cut-free proofs. Such a translation and retranslation is possible if identity (I) is lacking. This method — which also works if contraction is missing, but full identity is present — can easily be carried over to the situation considered here. Jäger and Stärk arrive at their system without identity from a different point of view, considering the three-valued semantics of logic programs with negation as failure. Kreuger motivated the restrictions on (I) by considerations concerning the operational interpretation of definitional reflection as implemented in GCLA.

4. We do not think that the issue of cut elimination is of any relevance as to whether to restrict (I) (or analogously, whether to reject contraction). The rules of the system have to be justified independently. Unlike Girard [1] we have always taken the view that eliminability of cuts is a feature of the particular definition $\mathcal{D}$ under consideration, and not something that has to be made sure from the beginning. According to Hallnäs [2] a partial inductive definition $\mathcal{D}$ is called total, if the consequence relation generated by $\mathcal{D}$ is transitive (i.e., if we can eliminate cuts). Whether a partial inductive definition is properly partial or whether it is total is something that may (or may not) be proved after stating the definition. This is quite analogous to the definition of a partial recursive function which later on may (or may not) turn out to be total. It seems impossible to single out the definitions which are total by a simple syntactic criterion.

[1] So in that case we either use the maximum und count both $(\vdash\mathcal{D})$- and $(\mathcal{D}\vdash)$-applications, or we use sums and count $(\mathcal{D}\vdash)$. In $\mathbf{DR}^*_\mathcal{U}(\mathcal{D})$we have to use the maximum.

[2] This is proved in [6, Lemma 4]. Actually, there the invertibility of $(\mathcal{D}\vdash)_\omega$ was mistakenly claimed also for the system with contraction. The validity of the theorems of that paper is not affected by this fault.

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Abstracts of the 11th International Congress of Logic, Methodology and Philosophy of Science, Cracow, Poland, August 1999, p. 449

Section 16: History of Logic, Methodology, and Philosophy of Science

GENTZEN-STYLE FEATURES IN FREGE

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As has been recently observed ([4],[5]), there is a striking resemblance between certain features of Frege's formal systems and Gentzen-style formulations of logical calculi, although they are historically unrelated. This is especially significant as Frege-style and Gentzen-style systems are normally considered to be fundamentally different, the former being prototypes of Hilbert-type calculi.

Kutschera [4] showed that the first-order fragment of the system proposed by Frege in his *Grundgesetze der Arithmetik* [2] can be understood as a sequent-style natural deduction system. For the implicational part this result may be put as follows. Consider the Gentzen-style system with the following rules of inference, where in our linear notation the slash '/' denotes an inference line:

$/ \ A, B \Rightarrow A$ with $/ \ A \Rightarrow A$ as a limiting case

$\Gamma \Rightarrow A \ / \ \Gamma' \Rightarrow A$ (Γ' permutation of Γ)

$\Gamma[A \ldots A] \Rightarrow B \ / \ \Gamma[A] \Rightarrow B$ (Contraction of two or more occurrences of A)

$\Gamma, A \Rightarrow B \ / \ \Gamma \Rightarrow A \to B$ ($\to$ introduction)

$\Gamma \Rightarrow A \quad \Delta \Rightarrow A \to B \ / \ \Delta, \Gamma \Rightarrow B$ ($\to$ elimination)

Let the *Frege counterpart* of a sequent $A_1, \ldots, A_n \Rightarrow A$ be the implicational formula $A_1 \to (\ldots \to (A_n \to A)\ldots)$ (with A being the Frege counterpart of $\Rightarrow A$) — of course to be written two-dimensionally in Frege's original notation. Then any derivation in the Gentzen-style calculus yields a derivation in Frege's system. We just have to replace every sequent with its Frege counterpart und to delete all applications of ($\to$ introduction) (whose premiss and conclusion is translated into the same implicational formula).

Conversely, by writing implicational formulas $A_1 \to (\ldots \to (A_n \to A)\ldots)$ as sequents $A_1, \ldots, A_i \Rightarrow A_{i+1} \to (\ldots \to (A_n \to A)\ldots)$ (with $\Rightarrow A_1 \to (\ldots \to (A_n \to A)\ldots)$ being a limiting case), a derivation in the above Gentzen-style system is obtained from any derivation in Frege's system. To cope with the ambiguity of splitting up an implicational formula at a particular place A_i, it may be necessary to insert applications of ($\to$ introduction), as well as applications of ($\to$ elimination) with the left (minor) premiss being of the form $A \Rightarrow A$.

This paper discusses whether this resemblance is just a technical coincidence with no deeper bearing on the notion of a logical system, or whether it shows that Frege anticipated certain ideas later developed by Gentzen [3].

In spite of the fact that in Frege there is no syntactical distinction between implications and sequents, and that Frege repeatedly and explicitly rejects the idea of a conceptual difference between assumptions in proofs and hypotheses of implications, it can be argued that to some extent he is aware of the Gentzen-style features mentioned. Actually, his metalinguistic distinction between the 'Oberglied' (= *succedens*) and the 'Unterglieder' (= *antecedentia*) of an implicational formula crucially enters his formulation of the inference rules in [2]. Thus Frege's formalism may appropriately be called a 'metalinguistically specified sequent system'. To give an analogy, we may refer to certain formalisms considered by Schütte [6] which are not sequent calculi in the syntactic sense, but are specified as sequent systems by means of a metalinguistic classification of formula parts as 'positive' or 'negative' .

This view is further supported by the fact that Frege's distinction between 'Oberglied' and 'Unterglieder' is drawn only at the uppermost level of formula construction, but never at the level of embedded implications. All this adds to the strong *prima facie* plausibility our interpretation gains from Frege's explicit choice of structural rules ('Vertauschung', 'Zusammenziehung') as primitive rules of inference in the *Grundgesetze*, which renders this system fundamentally different from the *Begriffsschrift* [1] system.

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Section 5: Logic & Scientific Method

POPPER'S INFERENTIAL DEFINITIONS OF LOGICAL CONSTANTS

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This paper attempts to shed some new light on Popper's little-known articles of 1947-49 on the foundations of (deductive) logic.[1] These articles suffer from the fact that they were written without awareness of the state of the art in mathematical logic and, in particular, of Gentzen type inference systems. They nevertheless contain ideas which are particularly interesting from a more modern perspective, and which deserve to be better known.

Popper's framework is based on an inference relation which essentially has the structural features of Gentzen's sequent arrow (identity, weakening and cut). Logical operations are defined metalinguistically by the inferential role they play, independently of whether they are syntactically represented by means of a connective. For example, an (arbitrarily formed) sentence *A* is called a *disjunction of B and C*, if for any *D*: *D* can be inferred from *A* if and only if *D* can be inferred from *B* as well as from *C*.

These definitions are not to be understood as a new sort of semantics. A semantics would start with a formal language, define a central semantical notion for its sentences such as truth, and *justify* an inference relation on the basis of such a definition. Rather, given an already established inference relation, an inferential definition *singles out* certain operations by calling them conjunctions, disjunctions, negations etc. of sentences.

We shall argue that this idea is highly original, in spite of the flaws in Popper's presentation. It is closely related to modern attempts to specify logical constants or logical systems in terms of consequence or implication relations[2] [3], and in particular to Koslow's structuralist theory of logic[4]. We shall also compare Popper's characterization of the underlying inference relation with ideas developed by Hertz and Gentzen in the 1920s and 1930s.[5]

Although inferential definitions in Popper's sense can be a powerful *descriptive* tool, in particular when different logical systems are investigated, they seem to us not suited to provide a *foundation* for logic (if there is such a thing at all). We shall discuss in detail the interrelationship between inferential definitions, semantical considerations and questions concerning the logicality of operations.

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Section A.2 Philosophical Logic

DEFINITIONAL REFLECTION AND CIRCULAR REASONING

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The theory of definitional reflection provides a novel framework for studying logical features of circular, and especially paradoxical reasoning. Definitional reflection originated from reading clauses for atoms as definitions, thereby extending ideas concerning elimination rules in natural deduction [2, 3]. In the simplest (propositional) case, given a definition for an atom a of the form $\mathbb{D}: \begin{cases} a \Leftarrow \Delta_1 \\ \quad\vdots \\ a \Leftarrow \Delta_n \end{cases}$, the rule of definitional reflection $(\mathbb{D}\vdash)\dfrac{\{\Gamma, \Delta_i \vdash C\}_i}{\Gamma, a \vdash C}$ is associated with a as a left introduction rule. If individual variables are present, and for computational purposes, the rule becomes more complicated [3, 5]. $(\mathbb{D}\vdash)$ is considered as introducing an atomic assumption a according to its definitional meaning given by $\mathbb{D}$. This is the *specific* way of introducing a as an assumption, which is distinguished from the *unspecific* way by means of an initial sequent $(a \vdash a)$. As in logic programming, $\mathbb{D}$ may contain arbitrary atoms, even a itself, without any well-foundedness requirement. Unlike definite Horn clause programming, the definientia Δ_i of a are not restricted to lists of atoms but may include, e.g., implications. This enables us to study, besides circular reasoning based on clauses like $a \Leftarrow a$, also paradoxical reasoning using clauses like $a \Leftarrow \neg a$ (i.e., $a \Leftarrow (a \to \bot)$). Considering the definition $\mathbb{D} := \{a \Leftarrow \neg a\}$, which may be regarded as an "abridged" form of a logical or set-theoretical paradox, we can distinguish three possible strategies, each of which blocks the derivation of absurdity $(\vdash \bot)$.

(1) We expect a derivation of absurdity to be direct (i.e., normal or without cuts). There is no such derivation, as all derivations of absurdity we can produce from $\mathbb{D}$ are indirect. This was discovered by Hallnäs [2] and is related to Ekman's paradox [1].

(2) We allow for assumptions to be introduced in an unspecific way only if no specific way of introducing them is available, i.e., if there is no appropriate definitional clause in $\mathbb{D}$. This corresponds to the requirement often made in the sequent calculus that initial sequents must be atomic. (Note that "atomic" in the logical sense corresponds here, where we are only dealing with atoms, to the fact that no definitional clause is available.) This idea is due to Kreuger [4, 6].

(3) We prohibit the identification of assumptions of the same shape but of a different kind (i.e., of assumptions introduced in an unspecific way vs. assumptions introduced by definitional reflection). This can be done by globally forbidding contraction, corresponding to the dealing with paradoxes in the tradition of BCK logic (Fitch, Curry, Ackermann, Grishin), or, preferably, by a more sophisticated procedure which keeps track of the origin of assumptions.

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Sektion 1: Logik

On the notion of *assumption* in logical systems

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Logical calculi, in particular natural deduction systems, exhibit a certain asymmetry between assumptions and assertions. There is a variety of rules for asserting a formula depending on which form this formula has (introduction rules) or from which this formula is inferred (elimination rules), but there is just a single trivial rule for making assumptions, namely by asserting a formula A as depending on itself.

This asymmetry can be removed by carrying ideas from the sequent calculus over to natural deduction. The left introduction rules of the sequent calculus might then be read as rules which introduce assumptions in a specific way depending on their form. For example, the rule of $\wedge$-introduction on the left side of the sequent sign can be interpreted in natural deduction as a rule for introducing $A \wedge B$ as an assumption (assuming that derivations with either A or B as assumptions are available). The result is a natural-deduction-style sequent calculus, in which the role of assumptions is symmetric to that of assertions. In this calculus, major premisses of elimination rules only occur in top position (i.e., as assumptions).

Our next step is to investigate what happens when different sorts of assumptions, those introduced in an *unspecific* way (by just stating A as an assumption) and those introduced by a *specific* assumption introduction rule (which depends on the form of A) are kept apart, as they rely on a different sense of "assumption". There are various strategies at hand to achieve this goal. One is to prohibit the contraction of several occurrences of the same formula into a single one, if these occurrences result from different (specific vs. unspecific) ways of making an assumption.

Finally, these strategies are applied to circular reasoning as it takes place in connection with antinomies. It turns out that the different treatment of specific and unspecific assumptions blocks the derivation of contradictions from circular constructions (within minimal logic). This sheds new light on logical aspects of handling contradictions which add to the proof-theoretic peculiarities which arise in the derivation of an outright contradiction $A \wedge \neg A$ (or $\bot$) from the proposition $A \leftrightarrow \neg A$ which just expresses circularity.

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Sektion 1: Logik

On the notion of *assumption* in logical systems

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1. The asymmetry between assumptions and assertions

When we *assume* a formula A in a logical derivation, we mean that A as well as subsequent formulae inferred using A *depend* on A. In certain types of logical calculi, especially natural deduction systems, assumptions can also be *discharged*, i.e., the dependence on certain assumptions can be removed. This happens, for example, when an implication $A \to B$ is inferred given a derivation of B which depends on A. The introduction of an assumption A is normally *unspecific* in the sense that there are no restrictions as to the form of A or the context in which A occurs. In principle, just any formula A can serve as an assumption.

This is different with *assertions* made in a derivation. There is, of course, an unspecific way of asserting A, viz., when A is asserted as depending on itself as an assumption. But there are normally also many *specific* ways of asserting a formula, depending on its form or its context. Any introduction inference in natural deduction gives an example for that: We can assert $A \wedge B$ given derivations of both A and B, we can assert $\exists x A(x)$ given a derivation of A(t) for some t, and so on. Even the elimination inferences constitute a specific way of making assertions, where "specific" now applies to the premisses and therefore to the context in which the assertion is made: We can assert A(t) given a derivation of $\forall x A(x)$, we can assert C given derivations of $A \vee B$, of C depending on A, and of C depending on B, and so on. There is a variety of specific inference rules for making assertions, but just a single unspecific rule for making assumptions.

2. Removing the asymmetry: Natural-deduction-style sequent calculus

I claim that this asymmetry should be removed. There is no reason why assertions should be better off in logic than assumptions. In any case it is interesting to see what conceptual insights we gain from considering a more symmetric system. Fortunately, such a system is at hand in the form of the sequent calculus. By "sequent calculus", I mean the symmetric sequent calculus with introductions both on the right and on the left side of the sequent sign ("⇒" in our notation), not sequent-style natural deduction with introductions and eliminations only on its right side. If we transform this calculus into natural deduction format, we can read the left introduction rules as *specific* assumption introduction rules. For example, the ($\wedge \Rightarrow$)- rule

$$\frac{\Gamma, A \Rightarrow C}{\Gamma, A \wedge B \Rightarrow C}$$

allows one *to introduce* $A \wedge B$ *as an assumption* from which C can be inferred (in a context Γ), given that C can be inferred from the assumption A (in the context Γ), and the (∃ ⇒)-rule

$$\frac{\Gamma, A(y) \Rightarrow C}{\Gamma, \exists x A(x) \Rightarrow C}$$

allows one *to introduce* $\exists x A(x)$ *as an assumption*, from which C can be inferred (in a context Γ), given that C can be inferred from the assumption A(y) (in the context Γ, modulo certain eigenvariable conditions), etc.

In the resulting natural deduction system, which might be called a *natural-deduction-style sequent calculus*, major premisses of elimination rules are only allowed to occur in top position, i.e., as assumptions introduced in a *specific* way. Besides that we still have the *unspecific* way of introducing assumptions (and assertions) by means of just assuming (and at the same time asserting) a formula A, which corresponds to initial sequents $A \Rightarrow A$ in the sequent calculus. Obviously, now the situation is completely symmetric with respect to assumptions and assertions: both of them can be introduced either in a *specific* way (by applying an inference rule governing the main operator of the assumption) or in an *unspecific* or *trivial* way (by just stating them). Correspondingly, we shall speak of *specific* and *unspecific* (or *trivial*) *assumptions*.

There have been some proof-theoretic investigations of such systems (e.g., by von Plato 2001), and there have also been strong extensions of similar systems beyond pure logic in theories of definitional reflection (e.g., by Hallnäs 1990 and Schroeder-Heister 1993), but their philosophical significance has not been fully appreciated so far.

3. Keeping apart specific and unspecific assumptions

I do not only want to propagate the view that assumptions deserve equal rights as compared to assertions. I should also like to draw certain philosophical consequences from the distinction between specific and unspecific assumptions, when they are treated in a different way. Specific assumptions are introduced according to their meanings whereas unspecific assumptions are just stated without special regard. Therefore one might argue that they have to be kept apart. For this to achieve I see three possible strategies:

(1) We require that any assumption which can in principle be introduced in a specific way, i.e., for which a specific assumption introduction rule is available, *must not* be introduced in an unspecific way, i.e. as a trivial assumption. In standard logical systems this just means that only atomic formulae can function as trivial assumptions, which in the sequent calculus corresponds to the restriction often imposed that in initial sequents $A \Rightarrow A$ the formula A has to be atomic. In general, this is a kind of well-foundedness condition on assumptions. If there is a specific assumption introduction rule for A, then A can only be assumed via that rule, which presupposes that certain other propositions occurring in the premisses of that rule have already been assumed, and so on. Trivial assumptions represent, so to speak, the base case of this chain. (This approach corresponds to a principle proposed for an extension of logic programming by P. Kreuger, see Schroeder-Heister 1994.)

(2) We disallow contracting different occurrences of the same formula A to a single A, if the two occurrences originate from different sorts of assumptions (i.e. one from a specific assumption and the other one from a trivial one). Here, in natural deduction format, contraction means discharging more than one occurrence of the same formula at the same time. However, it is technically difficult to make precise what

"originate" should mean. A clearcut case is only given if one occurrence of A is specific whereas the other one is not. The case of a logically complex A, with subexpressions originating from different sorts of assumptions, needs special consideration.

(3) We prohibit contraction at all, i.e. we use contraction-free logic. Although this is a very crude way of keeping different sorts of assumptions distinct, which is definitely not fully satisfying, our reasoning concerning the notion of assumption gives at least some partial philosophical justification for contraction-free systems, which for different purposes have been considered in various areas.

4. Application to antinomies

As an application I consider circular reasoning as it arises in connection with antinomies. Normally, the main step in antinomies is to derive, for a certain formula A, (i) ¬A from A, and (ii) A from ¬A (for example by taking A to be R∈R for the Russell set R in naïve set theory). Then, in pure logic, we proceed as follows to derive a contradiction: (i) yields ¬A, and with (ii) we also obtain A. However, if we apply our programme of keeping specific and unspecific assumptions apart, the following happens, depending on which strategy we choose.

Ad (1): We cannot derive (i), as there are rules for specifically assuming A (in the case of Russell's antinomy: rules for introducing ∈), which cannot be applied because their premisses cannot be assumed.

Ad (2): Given (i), we cannot derive ¬A, as in the derivation of ¬A from (i), we have to use A as an unspecific assumption to be contracted with the specific assumption A in the derivation of (i).

Ad (3): Given (i), we cannot derive ¬A, as contraction is blocked anyway. This is an *a fortiori* consequence of the previous case.

Whereas strategy (1) presents a fresh look at antinomies based on the well-foundedness of assumption rules, strategies (2) and (3) challenge the logical step from the circular formula A↔¬A to the outright contradiction A∧¬A or to absurdity ⊥ (in intuitionistic or minimal logic, of course). It should be remarked that, even without any restriction concerning assumptions and contraction, the natural deduction derivation from A↔¬A to ⊥ (i.e., the derivation of ¬(A↔¬A) in propositional logic) has peculiar features and is by no means trivial (see Ekman 1998).

This is no solution to the antinomy problem (if there is a problem at all), but it illuminates certain logical, and especially proof-theoretic, aspects of circular reasoning which have not been studied very deeply so far. I conjecture that the phenomena mentioned are not restricted to particular antinomies such as Russell's but that something similar happens with most, if not all, mathematical and semantical antinomies.

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[above p. 13]

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Generalized Rules, Direct Negation, and Definitional Reflection

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By "generalized rules" in logic (more specifically: in natural deduction) I mean uniform elimination rules for logical constants, given certain introduction rules. E.g., if

$$\frac{\Delta_1(p_1,\ldots,p_n)}{c(p_1,\ldots,p_n)} \quad \cdots \quad \frac{\Delta_m(p_1,\ldots,p_n)}{c(p_1,\ldots,p_n)}$$

are the introduction rules for an n-ary propositional connective c, the elimination rule may be presented uniformly as

$$\frac{c(p_1,\ldots,p_n) \qquad \begin{matrix}\Delta_1(p_1,\ldots,p_n)\\ C\end{matrix} \quad \cdots \quad \begin{matrix}\Delta_m(p_1,\ldots,p_n)\\ C\end{matrix}}{C}.$$

Obviously, this schema is modelled after the elimination rules for disjunction. Making this idea more precise requires specifying the exact form of the premisses $\Delta_i(p_1,\ldots,p_n)$ of the introduction rules. In [7] I proposed including some sort of structural implication which may be contained in the Δ's, leading to a theory of rules of higher levels.

The principle of *definitional reflection* ([3, 4, 8]) generalizes this approach. Here arbitrary clauses (with variables as in logic programming, and possibly also with embedded implication and quantification) are treated like introduction rules which can be inverted by means of this principle. Due to the presence of variables and function symbols, inversion is more complicated, the logical elimination rules just being a limiting case. At the same time it is more powerful, leading to a significant extension of logic programming, and allowing to deal with non-wellfounded phenomena such as semantical and mathematical paradoxes.

In this talk I consider the situation which arises when *direct negation* in the sense of Nelson's logic of constructible falsity [6] is added (the term "direct logic" is due to v. Kutschera [5]). Besides positive introduction rules we also have negative introduction rules for the rejection of logically compound formulas. In the more general case we would consider clauses with negated heads as in extended logic programs ([1, 2]). The logical case is relatively easy to deal with, as it is clear from the very beginning how the rejection rules for logically compound formulas should look like ([5, 9]). One would just have to add elimination rules for negated formulas. For example, in the case of implication, the rejection rule corresponding to implication introduction has the form

$$\frac{p \quad {\sim}q}{{\sim}(p \supset q)},$$

so that we would just add

$$\frac{\sim(p\supset q) \qquad \begin{matrix}[p \quad \sim q] \\ C\end{matrix}}{C}$$

as an elimination inference.

However, with generalized clauses, we would like to consider *arbitrary* positive and negative clauses in our database which are not related with each other in such a specific way. As in the theory of extended logic programming, this may even lead to inconsistent databases. In extended logic programming, no inversion principle like definitional reflection has been considered so far. If we want to add definitional reflection to a system containing both positive and negative clauses we have to address questions such as the following:

1. Due to the rejection operator we can dualize clauses, generating positive from negative clauses and vice versa. Should we distinguish between *primary* and *secondary* definitional clauses, the secondary ones being generated by dualization from the primary ones?

2. Are secondary clauses to be treated on par with primary ones, when it comes to definitional reflection?

3. How is dualization and inversion (definitional reflection) to be defined, if function constants are present, i.e. if not necessarily finitely many clauses are generated?

4. Which role do the "paradoxes of implication", in particular the absurdity principle, play in the context of dualizing definitional rules?

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Abstract submitted for GAP6, Sixth International Congress of the Society for Analytical Philosophy (Berlin, 11 - 14 September 2006) [Participation cancelled for personal reasons]

Identical Abstract submitted for GAP7, Seventh International Congress of the Society for Analytical Philosophy (Bremen, 14 - 17 September 2009)

Sektion 1: Logik und Wissenschaftstheorie

***Assertion* and *Denial* in Proof-Theoretic Semantics**

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Proof-theoretic semantics is an attempt to define logical consequence and, more generally, analytic reasoning in terms of proof rather than truth. By its very nature – in emphasizing *proof* rather than *refutation* – it is assertion-driven. It defines what counts as a valid proof of an *assertion*, and even when it deals with *assumptions*, it considers them to be placeholders for valid proofs. Alternative versions of proof-theoretic semantics give the notion of an *assumption* a stronger stance, considering assumption inferences to be on the same level as assertion inferences. However, even then there remains an asymmetry between proofs and refutations or between assertions and denials. This is reflected by the fact that in such frameworks negation is defined indirectly by reduction to absurdity rather than by a notion in its own right.

Corresponding to ideas developed in extended logic programming, we propose a clausal logic of assertions and denials, in which clauses have the form

$$(\sim)A \Leftarrow (\sim)B_1,\ldots,(\sim)B_n$$

Here '~' is a rejection operator which indicates the denial of a proposition and which may only occur in outermost position, i.e. cannot be iterated. The parentheses indicate that the rejection operator may be either present or missing.

Dealing with generalized reasoning systems of this kind leads to novel *symmetry or harmony principles* which go beyond the well-known harmony principles for natural deduction or sequent systems. This is due to the fact that by means of dualization, given ('primary') *assertion* rules lead to associated ('secondary') *denial* rules and *vice versa*. We may now ask how secondary rules relate to primary ones laid down by definition, whether the primary rules comprise the secondary ones, etc. We investigate corresponding harmony principles and relate them to questions of nonmonotonicity and general questions of the foundations of proof-theoretic semantics. We also indicate how the idea of incorporating formal proofs and formal refutations in a *uniform system* can illuminate general questions of rationality, in particular concerning the role of foundational reasoning in constructivist epistemologies in comparison with Popper's refutation-based approach.

Extended Summary submitted for the GAP6 conference (Berlin, 11 - 14 September 2006)
Identical to Abstract submitted for the congress of UNILOG'07 Xi'an (20-22 August 2007)
Apart from additional references identical to Extended Summary for GAP7 (Bremen, 14 - 17 September 2009)

Sektion 1: Logik und Wissenschaftstheorie

***Assertion* and *Denial* in Proof-Theoretic Semantics**

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1. The asymmetry between proofs and refutations

Proof-theoretic semantics is an attempt to define logical consequence and, more generally, analytic reasoning in terms of proof rather than truth (Schroeder-Heister 2006). By its very nature – in emphasizing *proof* rather than *refutation* – it is assertion-driven. It defines what counts as a valid proof of an *assertion*, and even when it deals with *assumptions*, it considers them to be placeholders for valid proofs. Alternative versions of proof-theoretic semantics give the notion of an *assumption* a stronger stance, considering assumption inferences to be on the same level as assertion inferences (Schroeder-Heister 2004). However, even then there remains an asymmetry between proofs and refutations or between assertions and denials. This is reflected by the fact that in such frameworks negation is defined indirectly by reduction to absurdity rather than by a notion in its own right.

2. Constructive duality

We argue that this asymmetry should be removed. Actually, duality arguments show that there is no proper advantage of assertion over denial. In classical truth-condition semantics such duality arguments are well known: There truth with respect to the standard connectives under a valuation *v* is the same as falsity with respect to the dual connectives under the complementary valuation *v'* (which interchanges truth and falsity), and *vice versa*. This fact can be used as an argument that it is not possible to fix both the meaning of truth and falsity and the meaning of the logical connectives at the same time by means of truth conditions. What is not so well known is the fact that even for proof-theoretic semantics, which is a constructive approach leading to intuitionistic logic, some related indeterminacy of meaning can be demonstrated. If one changes the basic concepts of proof-theoretic semantics such as “canonical proof”, “proof depending on open (not yet proved) assumptions” etc. into refutation concepts such as “canonical refutation”, “refutation leading to open (not yet refuted) conclusions”, the meaning of the standard connectives is turned in that of connectives dual to them. In this sense duality is not lost when passing from truth-condition semantics to proof-theoretic semantics. This means that in proof-theoretic semantics, as in truth-condition semantics, there is no fundamental semantic principle available which favours assertion. In any case it is interesting to see which conceptual insight we gain from considering a more symmetric system.

3. Clausal logic of assertions and denials

Therefore it is only natural to consider the possibility of incorporating proofs and refutations, or assertions and denials in a single framework. If one bases such a framework on systems of clauses ('programs', 'definitions'), one should consider assertion and denial clauses depending on assertions and denials. One approach is to consider a special negation '~' as a denial operator which can only occur in outermost position, and allow for both unnegated and negated atoms in the heads and bodies of clauses. This means that clauses have the form

$$(\sim)A \Leftarrow (\sim)B_1,\ldots,(\sim)B_n$$

where the parentheses indicate that the rejection operator may be either present or missing. Certain aspects of dealing with such clauses can be handled according to the model of extended logic programming, where heads and bodies of clauses may contain negations (see Damásio & Pereira 1998).

4. Balanced sets of clauses

Dealing with such generalized reasoning systems leads to novel *symmetry or harmony principles* which go beyond the harmony between assertions and assumptions in sequent systems or between introduction and elimination rules in natural deduction. By means of dualization, assertion rules lead to associated denial rules, whereas rejection rules lead to associated assertion rules. This means that we can distinguish between assertion and denial principles just laid down by definition (*primary* assertion and denial), and those obtained from these principles by dualization (*secondary* assertion and denial). Now we may ask whether the primary principles are such that they cover the secondary principles, i.e. contain their own dual. Reasoning systems based on sets of clauses with this property are called *balanced*. Balanced sets of clauses exhibit a maximum degree of explicitness in the sense that reasoning with the primary clauses suffice to obtain everything that can be extracted from these clauses. When investigating balanced reasoning systems, we discuss the following questions:

(1) Are balanced systems monotone with respect to balanced extensions?
(2) Does the property of being balanced imply that the system is total, which technically means that cut elimination holds?
(3) Does the converse hold, or are there total systems which are not balanced?

Question (1) receives a negative answer, at least when implication is admitted as a connective in the bodies of clauses; hence we remain in the realm of nonmonotonic reasoning. The relationship between totality and being balanced ((2) and (3)) turns out to be more intricate. A positive answer would show that being balanced is a strong indicator for a reasoning system to be 'well-behaved'. In particular, non-wellfounded phenomena such as paradoxes would be excluded. We also relate the systems proposed to reasoning systems with strong negation (see Schroeder-Heister 2005a).

The problems discussed are naturally relevant to the relationship between foundational reasoning in constructivist epistemologies and Popper's refutation-based approach (Schroeder-Heister 2005b). On the basis of the logical and semantic arguments given, there is no reason to prioritise one of the two approaches. On the contrary, the results support the idea of a uniform framework of proofs and refutations, at least when viewed from a semantic perspective.

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Schroeder-Heister, P. On the Notion of *Assumption* in Logical Systems. In: R. Bluhm & C. Nimtz (eds.), Selected Papers Contributed to the Sections of GAP5, Fifth International Congress of the Society for Analytical Philosophy, Bielefeld, 22-26 September 2003, Paderborn: mentis 2004, 27-48. Online-publication of the conference volume: http://www.gap5.de/proceedings

Schroeder-Heister, P. Generalized Rules, Direct Negation, and Definitional Reflection. Abstract submitted for the 1st World Congress on Universal Logic (UNILOG 2005a), (Montreux, Switzerland). pdf [above p. 29]

Schroeder-Heister, P. Begründungsrationalität und Logik. In: M. Carrier, G. Wolters (eds.), Homo Sapiens und Homo Faber: Epistemische und technische Rationalität in Antike und Gegenwart. Festschrift für Jürgen Mittelstraß, Berlin: de Gruyter 2005, 285-296. pdf [https://doi.org/10.15496/publikation-72306]

Schroeder-Heister, P. Validity concepts in proof-theoretic semantics. In: R. Kahle, P. Schroeder-Heister (eds.), Proof-Theoretic Semantics. Special Issue, Synthese 146 (2006), 525-571. pdf [https://doi.org/10.1007/s11229-004-6296-1]

Abstract for the 1st World Congress on the Square of Opposition,
Montreux, Switzerland, 1-3 June 2007.
Full paper under the title "Definitional reasoning in proof-theoretic semantics and the square of opposition" in: The Square of Opposition: A General Framework for Cognition. Ed. by Jean-Yves Béziau and Gillman Payette. Bern: Peter Lang 2012, pp. 323-349. https://doi.org/10.15496/publikation-72333

Direct negation in proof-theoretic semantics and the square of opposition

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The standard approach to negation in proof-theoretic semantics is via its intuitionistic interpretation using *falsum* as a logical constant. The inference rule *ex falso quodlibet* is then obtained from the fact that no canonical way of proving *falsum* is available, so that it is vacuously true that every canonical proof of *falsum* can be transformed into a proof of any proposition whatsoever. While this point is itself related to the interpretation of the *square of opposition* (see Wagner de Campos Sanz' contribution to this conference), I would like to relate the *square* to the treatment of direct or explicit negation in proof-theoretic semantics. By *direct negation* I mean negation given through explicit denial rules governing the refutation of propositions, in contradistinction to the indirect treatment via a *falsum* constant.

Suppose a rule-based definition is given, consisting of clauses with positive heads ('assertion clauses') and clauses with negative heads ('denial clauses'). They are called clauses for *primary assertion and denial*. Then by a procedure very close to *inversion* or *definitional reflection*, corresponding inferences for *secondary assertion and denial* can be generated, the secondary denial of A saying that all canonical conditions for the primary assertion of A can be refuted, whereas the secondary assertion of A says that all of the canonical conditions for the primary denial of A are refutable. The system as a whole is called *balanced*, when secondary assertion and denial can be inferred from primary assertion and denial, respectively.

In my very tentative talk, I would like reach a result of the following kind: Primary assertion and denial are contraries, secondary assertion and denial are subcontraries, secondary assertion and denial are subalterns to the corresponding primary judgements, and (primary assertion)/(secondary denial) and (primary denial)/(secondary assertion) are contradictories.

Proof-theoretic semantics — Peter Schroeder-Heister

Advanced Course, ESSLLI, Bordeaux, 27–31 July 2009

Contents

0. Preliminaries
   - Gentzen-style natural deduction
   - Philosophical preliminaries

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   - Natural deduction with atomic rules

II. Proof-theoretic validity
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Summary

Proof-Theoretic Semantics (PTS) is an alternative to model-theoretic (or truth-condition) semantics. It is based on the idea that the central notion in terms of which meanings are assigned to expressions is “proof” rather than “truth”. In this sense PTS is inferential rather than denotational in spirit. Although the claim that meaning is use has been quite prominent in philosophy for more than half a century, the model-theoretic approach has always dominated formal semantics. This is, as I see it, due to the fact that for denotational semantics very sophisticated formal theories are available, going back to Tarski’s definition of truth, whereas “meaning is use” has often been just a slogan without much formal underpinning. However, within general proof theory several formal approaches to PTS have been developed which promise to provide a “real” alternative to the model-theoretic approach. They are all based on ideas of Gentzen-style proof-theory, which are then turned into logico-philosophical principles.

After recalling certain basics from the theory of natural deduction, this course presents in its first part the idea of generalized introduction and elimination rules for logical or non-logical (atomic) constants, discusses adequacy criteria for such rules and investigates, as a case study, the example of negative circularity as it occurs with the paradoxes.

In its second part it develops and discusses the Dummett–Prawitz approach to PTS and their definition of proof-theoretic validity. It discusses various options of how to define the validity of proofs and relates them to corresponding notions of logical consequence. It puts particular emphasis on the “universal” aspects of these ideas, dealing with general proof structures and arbitrary proof reduction systems as models with respect to which validity is defined.

The third part is devoted to definitional and clausal approaches to PTS as developed by the instructor himself jointly with Lars Hallnäs (Gothenburg) using the principle of “definitional reflection”. This approach puts the validity of rules and inference steps (rather than that of whole proofs) first. As compared to the Dummett–Prawitz approach, it is local rather than global and does not require that global properties of proofs such as normalization or cut elimination hold in every possible case. This approach is is not restricted to logical constants but uses clausal definitions as the basis of reasoning, which means that it goes far beyond logic in the narrower sense. Interesting applications are theories of equality, circular reasoning, universal theories of denial and negation, and extensions of logic programming.

The fourth part deals with the treatment of denial and negation in the general framework developed. After making precise in which sense duality principles, which are well-known from classical logic, also hold in the constructive realm, it pleads for a “direct” treatment of negation in terms of rules for the denial of sentences, where the denial operator only occurs in outermost position (and thus cannot be iterated). This leads to a framework of clausal definitions for assertion and denial, formally related to extended logic programming. Principles of definitional reflection and definitional closure with respect to such definitions are discussed. Overall, this approach is intended as an alternative to the “indirect” approach to negation prevailing in the intuitionistic tradition.

The approach favoured is “bidirectional” in that assertions and assumptions are treated on par. Technically, this implies a shift from natural deduction to the sequent calculus as the basic model of reasoning, or at least to some bidirectional variant of natural deduction. Therefore, in the final fifth part, the idea of definitional reflection is used to deal with the symmetry features of the sequent calculus, in which the duality between assertions and assumptions is much more explicit than in natural deduction. Various approaches are discussed and related to existing theories. Particular emphasis is given to substructural issues.

A tutorial comprising (essentially) parts I-III of the present course was given at the School on Universal Logic, Xi’an (China) in August 2007. Parts IV and V are new.

Materials

General

- Proof-Theoretic versus Model-Theoretic Consequence. In: M. Peliš (ed.), The Logica Yearbook 2007, Prague: Filosofia 2008, 187–200. https://doi.org/10.15496/publikation-70827

Part I: E rules as functions of I rules

- A natural extension of natural deduction. Journal of Symbolic Logic 49 (1984), 1284–1300. https://doi.org/10.2307/2274279
- Structural Frameworks with Higher-Level Rules. Philosophical Investigations on the Foundations of Formal Reasoning. (Habilitationsschrift.) Konstanz 1987. https://doi.org/10.15496/publikation-69827

Part II: Proof-theoretic validity

- Validity Concepts in Proof-Theoretic Semantics. In: R. Kahle, P. Schroeder-Heister (eds.), Proof-Theoretic Semantics. Synthese 148 (2006), 525–571. https://doi.org/10.1007/s11229-004-6296-1.

Part III: Local inversion and reflection

- Rules of definitional reflection. In: Proceedings of the 8th Annual IEEE Symposium on Logic in Computer Science (Montreal 1993), Los Alamitos 1993, 222–232. https://doi.org/10.1109/LICS.1993.287585.
- Generalized Definitional Reflection and the Inversion Principle. Logica Universalis 1 (2007), 355–376. https://doi.org/10.1007/s11787-007-0018-7

Part IV: Assertion and denial

- Definitional Reasoning in Proof-Theoretic Semantics and the Square of Opposition. Proceedings of the International Congress on the Square of Opposition (Montreux, June 1–3, 2007). URL of full paper: https://doi.org/10.15496/publikation-72333

Part V: Generalized definitional reflection and the sequent calculus

- Sequent Calculi and Bidirectional Natural Deduction: On the Proper Basis of Proof-Theoretic Semantics. In: M. Peliš (ed.), The Logica Yearbook 2008, London: College Publications 2009. https://doi.org/10.15496/publikation-70817
- Wagner de Campos Sanz & Thomas Piecha, Inversion by Definitional Reflection and the Admissibility of Logical Rules. Review of Symbolic Logic 2 (2009), 550-569 . https://doi.org/10.1017/S1755020309990165

GAP8, 8th International Congress of the Society for Analytical Philosophy, Konstanz, 17-20 September 2012,

Colloquium: "Consequence and Consequences – What's truth got to do with it?" (18 September 2012)

Organisation: Heinrich Wansing, Manfred Kupffer

Participants: Jc Beall, Graham Priest, Peter Schroeder-Heister
Introduction by Heinrich Wansing

Full colloquium program: Congress program booklet, pp. 40-41
http://www.gap8.de/mentis-gap8-druck-endversion.pdf

What is the proper logic of consequence?

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According to the standard view, consequence means transmission of truth: A consequence statement is valid if its conclusion is true given its premisses are true. In this sense truth is the fundamental concept on which consequence is based. From the point of view of proof-theoretic semantics I argue for reversing this order: Consequence should be taken as a 'primordial' hypothetical concept which does not rely on a categorical concept such as truth. The proper logic of consequence then becomes the logic of consequences, i.e., the logic of consequence statements. I will discuss the implications of this view for standard laws of consequence as well as for notions of inference and (semantic) completeness.

Gabriele Heister, Walter H. Ehrenstein and Peter Schroeder-Heister:
Spatial S-R compatibility with two-finger choice reactions

Abstract for the 9th European Vision Conference, Bad Nauheim, Germany,
17-20 September 1986

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Ninth European Vision Conference, Bad Nauheim, West Germany, 17-20 September 1986. Abstracts. *Perception* 15 (1986), Number 1, pp. A1-A44.

Spatial S-R compatibility with two-finger choice reactions

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Spatial stimulus-response (S-R) compatibility usually refers to the fact that choice reactions are shorter when the spatial position (left, right) is the same for the visual stimulus and the responding hand. We studied whether spatial S-R compatibility also obtains when choice reactions are made with two fingers of the same hand. Eight right-handed subjects reacted as quickly as possible to a 100 ms flash of light that was presented 5 deg to the left or right of a fixation point. Using the index and middle fingers of their left or right hand, subjects pressed either the spatially-same (compatible) key or the spatially-different (incompatible) key. In condition A the subjects' palms faced down; in condition B the palms faced up so that the spatial order of the fingers was reversed. Strong S-R compatibility was found in both conditions: responses were always faster when finger and light were on the same side. Compatible reaction times were shorter than incompatible by 52 ms in condition A and by 61 ms in condition B. The results suggest a coding hypothesis of spatial S-R compatibility (Wallace, 1971 *Journal of Experimental Psychology* **88** 354; Umiltà and Nicoletti, 1985 *Attention and Performance XI* Lawrence Erlbaum, Hillsdale, NJ).

Gabriele Heister, Peter Schroeder-Heister and Walter H. Ehrenstein:
Spatial stimulus-response (S-R) compatibility under head tilt: evidence for a factorial model

Abstract for the 10th European Conference on Visual Perception, Varna, Bulgaria,
21-24 September 1987

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10th European Conference on Visual Perception, Varna, Bulgaria, 21–24 September 1987. Abstracts. Perception 16 (1987), Number 2, pp. A1-A52 (= pp. 225-276).
https://doi.org/10.1068/p160225

Spatial stimulus-response (S-R) compatibility under head tilt: evidence for a factorial model

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Choice reaction times are shorter when the spatial positions (left, right) of the stimulus (S) and the response (R) are the same than when they are different. We investigated how this S-R compatibility is affected by head tilt, that is, when egocentric and environmental frames of reference are dissociated. Subjects responded with their right or left index finger to a light presented 10 deg to the right or left of a fixation point. In alternating blocks of trials responses were made on the same side as the stimulus or on the opposite side. In experiment 1 hands were held normally; in experiment 2 they were crossed. Three conditions were tested: (i) upright head position; (ii) head tilted 90° to the right; and (iii) head tilted 90° to the left. A spatial compatibility effect was obtained for all conditions in experiment 1 and for the head-upright condition of experiment 2. In the head-tilted conditions of experiment 2 the spatial compatibility effect significantly decreased but did not reverse. The data indicate that, under head tilt, stimuli are coded within the environmental frame of reference. For crossed hands, head tilt may weaken the factor of spatial coding and strengthen that of anatomical hand mapping. These findings, together with recent results for orthogonal S-R arrangements, favour a factorial model that modifies the coding hypothesis of spatial S - R compatibility.

Walter H. Ehrenstein, Peter Schroeder-Heister and Gabriele Heister:
Spatial visuo-motor compatibility with an orthogonal stimulus-response arrangement

Abstract for the 11th European Conference on Visual Perception, Bristol, England, 31 August - 3 September 1988

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11th European Conference on Visual Perception, Bristol, England, 31 August - 3 September 1988. Abstracts. Perception 17 (1988), Number 3, pp. A1-A84 (= pp. 339-422).

Spatial visuo-motor compatibility with an orthogonal stimulus-response arrangement

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Eight subjects responded as quickly as possible with either the index or the middle finger of one hand to a bicolour light emitting diode according to whether it was red or green. The stimuli appeared randomly 5 deg to the left or right of fixation; their position was irrelevant for the subject's task but essential for data analysis. The response keys were oriented (i) right and left, parallel to the stimuli; or orthogonal to the stimuli in either (ii) the horizontal or (iii) the vertical midsaggital plane. Spatial compatibility effects of similar magnitude were found in each condition. In (i), responses were 54 ms faster when the stimulus and the responding finger were on the same side. Rotating the hands had virtually no effect: the finger that had previously responded more rapidly continued to have an advantage. This supports a spatioanatomical mapping hypothesis (eg the index finger of the right hand is mapped as spatially left irrespective of hand orientation). However, in a fourth condition, with response keys as in (ii) but inverted (subjects responding with the palm upward), the spatioanatomical stimuius/finger relationship was reversed, as if inverting the hand had caused its 'sidedness' to reverse, suggesting a modified mapping hypothesis.

Walter H. Ehrenstein, Gabriele Heister and Peter Schroeder-Heister:
Spatial visuomotor compatibility as a function of retinal eccentricity

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17 - 22 September 1989

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12th European Conference on Visual Perception, Zichron Yaakov, Israel,
17 - 22 September 1989. Abstracts. Perception 18 (1989), Number 4, pp. A1-A75
(= pp. 483-557).

Spatial visuomotor compatibility as a function of retinal eccentricity

W H Ehrenstein, G Heister ¶, P Schroeder-Heister # (Institut für Arbeitsphysiologie, Universität Dortmund, Ardeystrasse 67, D-4600 Dortmund 1, FRG; ¶ Abteilung Neurophysiologie, Universitätsspital Zürich, CH-8091 Zürich, Switzerland; # Fachgruppe Philosophie, Universität Konstanz, D-7750 Konstanz, FRG)

Reaction times of choice responses depend on the spatial relationship between stimulus (S) and response (R), eg left-left (compatible) S-R pairing tends to be faster than left-right (incompatible) ones. It has been proposed that the stage at which these spatial S - R compatibility effects are generated is that of response selection rather than that of stimulus encoding. To test this proposal we varied the location of a stimulus light. Twelve subjects responded with their left or right index finger to a small light, presented left or right of a fixation point, at one of six eccentricities: 0.5, 2.5, 5, 10, 20, or 40 deg. There was a clear dependence of response time on strength of S-R compatibility. The compatibility effect (incompatible reaction time minus compatible reaction time) was 29.5 ms at 0.5 deg, increased linearly to 43 ms at 10 deg, and decreased again at higher eccentricities to 33 ms at 40 deg, resulting in an inverted U-shaped (quadratic) function for eccentricities between 2.5 and 40 deg. The results provide evidence that the perceptual stage of stimulus encoding may determine spatial S-R compatibility to a large extent, and may thus interact with that of response selection.