Financial Contracts on Electricity in the Nordic Power Market

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Dedicated to
Angela Daum

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München, April 2005 *Wolfgang Kispert*

Contents

Chapter 1

Introduction

Already before the EU Directive 96/92/EG was passed in 1996, some countries in the EU had started their liberalisation process for energy markets. So, in 1991, Norway established a national power market that in 1996 turned into the multinational power exchange Nord Pool, including all Nordic countries today.

In the US, the contemporary largest competitive wholesale electricity market, PJM (Pennsylvania–New Jersey–Maryland electricity market), was approved in 1998 as an independent system operator (ISO); other countries like New Zealand and Australia started with wholesale electricity markets in 1996 and 1998, respectively.

But how can contracts on electricity be priced? How do electricity prices change? What does electricity have in common with other commodities like crude oil or even gold?

Let us give a short overview of electricity pricing literature. As there is a vast amount of economics literature addressing the pricing of electricity and its derivatives, we will concentrate on the following: papers that apply continuous diffusion, jump-diffusion models or GARCH-like processes for electricity prices, or papers that treat the pricing of derivatives like futures, forwards and simple option contracts.[1] For the treatment of the pricing of exotic derivatives like Swing Options, see, for example, the paper of Jaillet, Ronn and Tompaidis (2001).

[1] Of course this review may not be complete, but we included all relevant papers known to us until the final review of this work.

One of the first important papers is that of Eydeland and Geman (1998). In their article, the futures price is determined as a function of expected demand and the futures price w of the combustible used for the generation of electricity. For simplified assumptions, especially a normal distributed demand and a futures price w that is a Brownian motion, the futures price for electricity is then also a Brownian motion.

In Pirrong and Jermakyan (1999) the general idea of Eydeland and Geman (1998) is adopted and processes for demand and fuel are specified. Data from the PJM (Pennsylvania–New Jersey–Maryland electricity market) is used to estimate the parameters in the model via finite difference methods.

In Bessembinder and Lemmon (2002), a specific cost function of the generator is assumed, dependent on the demand for electricity, and profit functions for generators and retailers. The prices in this two-period modelling framework result in market-equilibria.

Skantze and Ilic (2000) also propose an equilibrium-based model, with a futures price that depends not only on the expectation of the spot price but also on its variance.

Barlow (2002) models supply and demand as stochastic processes in order to arrive at a non-linear Ornstein-Uhlenbeck process that is empirically tested on Alberta and California spot prices.

Also in Kanamura and Ohashi (2004), demand and supply are first modelled separately to arrive then at a process for the equilibrium price. Its usefulness for optimal power plant generation and risk management is illustrated.

Lucia and Schwartz (2002) suggest a model with mean reversion and seasonal components. The model is also mentioned in Chap. 5. The authors directly model a stochastic process for the electricity price, without using an equilibrium approach.

This kind of modelling for electricity is already employed by Pilipovic (1998), who incorporates mean reversion in a two-factor model with a stochastic long-term equilibrium price. The author also suggests adding seasonality components.

Bhanot (2000) examines electricity prices for 12 regional markets. He employs and emphasises the importance of modelling seasonality.

Seasonality components can also be found in Elliott, Sick and Stein (2003), Weron, Simonsen and Wilman (2003), Escribano, Peña and Villaplana (2002), Villaplana (2003) or Borovkova (2004), mostly exactly defined like in Lucia and Schwartz (2002). Elliott et al. (2003) do not only model intra-year, but also intra-day seasonal characteristics.

The models of Elliott et al. (2003), Weron et al. (2003), Escribano et al. (2002), Villaplana (2003), Knittel and Roberts (2001), Geman and Roncoroni (2003) and Deng (2000) all contain mean reversion and jumps. The paper of Villaplana (2003) can be seen as an extension with jumps and seasonality of the model of Schwartz and Smith (2000) with a short-term and a long-term factor.

While Elliott et al. (2003) use data from the Alberta Electricity Pool for their empirical investigations, Weron et al. (2003) calibrate their model for spot price data from the Nordic power market Nord Pool. Escribano et al. (2002) examine the electricity markets of Argentina, New Zealand, the Nordic countries and Spain.

In the paper of Knittel and Roberts (2001), prices of the Californian electricity market are examined. Their models include GARCH- and ARMAX-specifications and temperature as an additional regressor.

In the meeting of the European Financial Management Association 2003 Geman and Roncoroni (2003) presented a family of processes with mean reversion and spikes, i.e. upward jumps that are followed by similar-sized drops. They calibrate their models to various US-markets.

Also the model of Borovkova (2004) contains seasonality and spikes. A method is proposed to estimate seasonal risk premia in forward curves.

Deng (2000) uses models with stochastic volatility where mean reversion appear both in the log process and the volatility. Furthermore, he defines a second process that is correlated with the log prices, assumed to represent, for example, prices of fuel. Given plausible parameter values, the author prices a generation plant in a real-option approach.

In Huisman and Mahieu (2001), the authors argue that models with mean reversion and stochastic jump processes might lead to problems in identifying the mean reversion of the process. A model with regime jumps is proposed and applied to various electricity and commodity markets. For the same model, De Jong and Huisman (2002) show how to price European options. They estimate the parameters for their model from Dutch APX spot price data.

Koekebakker and Ollmar (2001) use principal component analysis to explain forward price dynamics of the Nordic Power Market. The futures prices are smoothed by sinusoidal prior continuous forward price functions.

Intra-day electricity prices of New Zealand are examined by Guthrie and Videbeck (2002) with discrete periodic auto-regression (PAR) models.

Barone-Adesi and Gigli (2002) introduce a discrete model with spikes. Binomial and Monte Carlo methods are used for the pricing of American options on electricity. The model is calibrated with spot price data from Nord Pool.

Kellerhals (2001) employs Kalman filters for his estimations. Spot and futures prices from the Californian electricity market are examined. The author calibrates the Heston model with stochastic volatility.[2]

Kåresen and Husby (2000) also work with Kalman filters to calibrate one-factor and multi-factor AR-models to data from Nord Pool.

The same market is examined by Fleten and Lemming (2001). They construct a smooth term structure of futures and forward prices, employing an existing forecast model for this market and observed bid and ask prices.

New approaches for pricing are necessary, keeping in mind the non-storability of electricity and peculiarities of electricity prices, e.g. spikes. We adopt and enhance promising techniques to account for these peculiarities. Our approach is mainly based on that of Lucia and Schwartz (2002); our models belong to the class of models introduced above that contain mean reversion and seasonality components, enhanced by jumps. But while the models of Elliott et al. (2003),

[2] See Heston (1993).

Weron et al. (2003), Escribano et al. (2002) and Geman and Roncoroni (2003) are calibrated with the spot price process of electricity and do not consider derivatives, our interest lies clearly in pricing and calibrating with derivatives. Deng (2000) uses models that lead to stochastic differential equations that can only be solved numerically, whereas our models can be solved analytically up to non-solvable integrals.

One purpose in this work is the employment and further development of already existing models for electricity. A second purpose is to adjust and extend recently proposed models for stock returns for the electricity market. Duffie, Pan and Singleton (2000) introduce stochastic processes with stochastic volatility and jumps. The jumps do not only occur in the price process, but also in the volatility process. The jump sizes in the volatility process are exponentially distributed.

We extend this model class further by allowing the jump sizes in the volatility process to follow a Γ-distribution. This is a generalization of the former model class, because Exponential distributions are a special case of Γ-distributions. The new models are then further extended with seasonality components.

Pricing formulae for derivatives are developed for all these models and extensions. Futures and forward prices are used to estimate implicit parameter values in an empirical part of this work. The estimations for all models are also compared by pricing options by means of Monte Carlo methodology and comparing these theoretical model prices to the observed ones on the market. We use data from Nord Pool, the power exchange for the Nordic countries.

In detail, the monograph is organised as follows:

The first chapter consists of this introduction.

The second chapter states how forward contracts are priced for tradable assets and how some commodity pricing models are adjusted to this pricing approach. The first section ends with a comment on the applicability of this approach for electricity prices. In the second section, necessary technical preliminaries are explained. Then we describe how we price the futures and forwards traded at Nord Pool. The fourth section covers our implicit estimations approach, and in the last section of the second chapter we explain how we compute the prices of options in this market.

The third chapter is about the Nord Pool power exchange. The first section introduces the exchange and the Nordic power market. In the second section, we draw attention to the details of the spot market Elspot. The third section presents the products that are traded on the financial market of Nord Pool.

In the fourth chapter, we list and discuss some descriptive statistics of the spot price and its logarithm, of futures and forward prices, and finally, in the last section, of option prices.

In the fifth chapter, we first present the three model groups in three sections. In the fourth section of this chapter, the pricing equations for futures and forwards are solved, and in the last section we comment on solutions for models with seasonal components.

The sixth chapter shows the empirical results for the model groups. For model groups 1 and 3, summary statistics and regression tables are shown and discussed, out-of sample behaviour is examined and option contracts are priced. The results for model group 2 are listed in the Appendix.

In the last chapter, we give a summary and some conclusions.

Chapter 2

Pricing of Electricity Derivatives

This chapter starts by giving a motivation for pricing electricity derivatives, showing the limits of asset pricing when the asset is not tradable or storable. In the second section, some technical preliminaries are introduced. In the third section, these help us to explain the pricing approach on which we will rely. The fourth and fifth sections show our approach to parameter estimation and option valuation.

2.1 Pricing of Commodities

Consider a tradable asset S with price $S_t, t \in \mathbb{R}_0^+$, and a forward contract on S with price $F(t,T)$ and expiry $T \in \mathbb{R}_0^+, T \geq t$. Assume that the riskless interest rate r is nonstochastic and constant. With a simple replication strategy, we are able to determine the value of this forward contract:

- Period t:
  - Strategy 1: lend money of value S_t for the risk-free rate r and buy asset S.
  - Strategy 2: buy a forward contract with expiry in T for the price $F(t,T)$. The contract must be paid in T.
- Period T:
  - Strategy 1: sell asset S for value S_T and pay back the money that was lent plus the payable interest. The payoff of this strategy is $S_T - \mathrm{e}^{r(T-t)} S_t$.
  - Strategy 2: pay forward contract with the price agreed on in t: $F(t,T)$, and receive S. Immediately sell S. The payoff is $S_T - F(t,T)$.

The *Law of One Price* says that investment assets that have identical cash flows must have the same price.[1] So the value of $F(t,T)$ must be

$$F(t,T) = \mathrm{e}^{r(T-t)} S_t. \tag{2.1}$$

The important point in this argument is the assumption of a tradable asset, included in the notion investment asset.[2] This assumption is definitely correct for securities, but not anymore for commodities like wheat, oil, or electricity. So does the non-validity of this equation lead to a wider range of models, like those with mean reversion or seasonality components, see also Chap. 1. For our Model Group 2, presented in Sect. 5.2, we generalize the risk-neutralized drift for the log-prices, that would be with equation (2.1) $(r - \frac{1}{2}v_t)dt$, to $(\alpha + \beta v_t)dt$, see there for further details.

The discussion above implies, that in the electricity market future and forward contracts cannot be replicated with spot contracts. Therefore, market completeness, regarding the futures and forward markets, can only be shown by using multiple futures and forward contracts, depending on the assumed models. The possibility of using a martingale measure, though, does not depend on market completeness. Despite the lack of unique prices, for a chosen martingale measure arbitrage-free pricing is still assured, see for example Schönbucher (2003, p. 106). See also Sect. 2.2.3.

A simple look at futures prices shows that they are sometimes above (in contango) or below (in backwardation) the spot price.[3] To give an example, we have listed futures prices from the London Metal exchange from 4 May 2001 in Table 2.1.[4] While most contracts tend to be in contango, the 'Primary High Grade

[1]See Hull (1989, pp. 54f), who also gives necessary assumptions: (i) there are no transaction costs, (ii) the same tax rates apply for all, (iii) there is only one risk free rate of interest, for borrowing as for lending, and (iv) arbitrage opportunities are exploited.

[2]For tradable assets the equation causes the process under the Martingale measure to always have a trend of r or $r_t S_t$, respectively, see for example the models of Bakshi, Cao and Chen (1997), or Bates (1991).

[3]Contango is defined as an upward-sloping term structure of futures prices. The opposite is backwardation.

[4]The data can be found at *London Metal Exchange*.

Table 2.1. *Futures Prices from the London Metal Exchange*

Here is an example of commodity prices from the 4 May 2001. Listed are two different kinds of aluminium, copper, lead, nickel, silver, tin and zinc with contract bid-ask prices for the spot market, three-months, 15-months and 27-months futures. The Primary High Grade Aluminium as well as the Nickel contracts are in backwardation (negatively sloped with time to maturity) whereas all other metals are in contango (with a positive slope). Source: *London Metal Exchange*

LME Official prices, US$/tonne (US Cents/fine troy ounce - silver) for 4 May 2001 (Data >1 day old)

	Aluminium Alloy	Primary High Grade Aluminium	Copper Grade A	Standard Lead	Nickel	Silver	Tin	SHG Zinc
Cash buyer	1,255.00	1,585.00	1,697.00	459.50	6,700.00	440.00	5,010.00	948.00
Cash seller & settlement	1,260.00	1,586.00	1,698.00	460.00	6,710.00	450.00	5,015.00	948.50
3-months buyer	1,280.00	1,560.00	1,716.00	476.00	6,635.00	445.00	5,060.00	968.00
3-months seller	1,285.00	1,560.50	1,717.00	477.00	6,645.00	455.00	5,065.00	968.50
15-months buyer	1,355.00	1,548.00	1,758.00	510.00	6,350.00	445.00	5,170.00	1,003.00
15-months seller	1,365.00	1,553.00	1,763.00	515.00	6,400.00	455.00	5,180.00	1,008.00
27-months buyer		1,523.00	1,770.00		6,205.00	445.00		1,022.00
27-months seller		1,528.00	1,775.00		6,255.00	455.00		1,027.00

Aluminium', for example, is in backwardation.[5]

An approach to explain this phenomenon is the introduction of cost-of-carry factors and convenience yields, that in some models are constant, or can also be stochastic.[6] These factors are interpreted as extended storing costs, or the value of having the commodity at hand as an advantage to a forward contract.[7] The 'convenience' for disposing over the commodity can be worth more than interest rates and storing costs, so situations of backwardation can be explained. In situations of contango, interest rates and storing costs higher than the 'convenience' explain the observed futures or forward prices.

[5]For a justification of markets that are in backwardation and an empirical analysis of US oil futures prices, see Litzenberger and Rabinowitz (1995).

[6]See, for example, Brennan and Schwartz (1985) for a constant convenience yield or Gibson and Schwartz (1990), Schwartz (1997, model 3), Miltersen and Schwartz (1998) or Hilliard and Reis (1998) for stochastic convenience yields.

[7]In Hull (1989, p. 661), *Cost of Carry* is defined as "The storage costs plus the cost of financing an asset minus the income earned on the asset".

The usefulness of the adaptation of the above replication strategy and its extensions with cost-and-carry factors and convenience yields strongly depends on the possibility and costs of storing.[8]

Gold and other metals may be quite easy to store, more difficult and more expensive to store are certainly commodities like crude oil, wheat, or cocoa.[9]

Let us consider electricity. Is power storable at all? One could argue that the costs for storing the resources for generating electricity, like oil or gas, or even including the costs for disposing over generating facilities, can be considered equivalent to storing costs.[10] Thus, though a theoretical possibility, storing will not be an alternative for many market participants, in addition to the difficulties in estimating these costs.

We will address the lacking possibilities for storing electricity in two ways:

1. We do not restrict ourselves to models where (2.1) must hold, and
2. Our models try to map the seasonal behaviour of electricity prices that we observe, see Chap. 4, and that is a result of these difficulties in storing.[11]

Many authors agree that models allowing commodity prices to move stochastically around a long-term mean are more appropriate for commodity prices, for example for oil or wheat. This leads to models with mean reversion.[12]
The idea is that production costs are in the long run fixed, but disturbed by factors like, for example, good/bad harvests, high/low consuming rates, or over-/under-production.

We will also address this in Chap. 5 in the context of electricity. One of our model groups contains mean reversion.[13]

[8]Compare also with Bühler, Korn and Schöbel (2000).
[9]See also the comments in Ross (1997, p. 3).
[10]See also Deng (2000).
[11]See also the introduction of this monograph for models with seasonality components.
[12]See for example the models of Schwartz and Smith (2000), Ross (1997), Schwartz (1997) and Lien and Strøm (1999).
[13]See also the introduction of this monograph for models with mean reversion.

2.2 Technical Preliminaries

2.2.1 Kolmogorov's Backward Equation

We begin with Kolmogorov's Backward Equation for time-homogeneous Itô diffusions and then extend it for diffusions with Poisson-distributed jump components.

Theorem 2.1 *Kolmogorov's Backward Equation (KBE)*

Let $(\Omega, \mathcal{F}, P)$ be a probability space with filtration $(\mathcal{F}_t)_{t\geq 0}$. In this probability space, let $\vec{X}_t$ be a time-homogeneous Itô diffusion with values in $\mathbb{R}^n$, i.e. $\vec{X}_t$ is defined as a stochastic differential equation of the form

$$d\vec{X}_t = \vec{\mu}(\vec{X}_t)dt + \vec{\sigma}(\vec{X}_t)d\vec{Z}_t, \tag{2.2}$$

with $\vec{X}_t \in \mathbb{R}^n, \vec{\mu}(\vec{x}) \in \mathbb{R}^n, \vec{\sigma}(\vec{x}) \in \mathbb{R}^{n\times m}$ and a m-dimensional Brownian motion $\vec{Z}_t$.

Furthermore define, for $f \in C^2(\mathbb{R}^n)$ with compact support, the infinitesimal generator A of $\vec{X}_t$ as

$$Af(\vec{x}) = \sum_i \mu_i(\vec{x})\frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}(\vec{\sigma}\vec{\sigma}^\top)_{i,j}(\vec{x})\frac{\partial^2 f}{\partial x_i \partial x_j}. \tag{2.3}$$

(i) Define $u(t,\vec{x}) \in \mathbb{R}$ as

$$u(t,\vec{x}) = E[f(\vec{X}_t)|\vec{X}_0 = \vec{x}]. \tag{2.4}$$

Then

$Au(t,\vec{x})$ exists for all $(t,\vec{x}) \in [0,\infty)\times\mathbb{R}^n$ and

$$Du \;:=\; Au - \frac{\partial u}{\partial t} = 0 \quad \forall (t,\vec{x}) \in [0,\infty)\times\mathbb{R}^n, \tag{2.5}$$

$$u(0,\vec{x}) \;=\; f(\vec{x}) \qquad \forall \vec{x} \in \mathbb{R}^n. \tag{2.6}$$

A is always applied to $\vec{x} \to u(t,\vec{x})$ in this context.

(ii) If $w(t,\vec{x}) \in C^{1,2}(\mathbb{R}\times\mathbb{R}^n)$ is a bounded function also satisfying (2.5) and (2.6), then $w(t,\vec{x})$ is a solution to (2.4), i.e. $w(t,\vec{x}) = u(t,\vec{x}) = E[f(\vec{X}_t)|\vec{X}_0 = \vec{x}]$.

Proof: See Øksendal (2000, p. 132).

The above theorem is very important for models with time-homogeneous Itô diffusions. For models with Poisson-jumps, though, an extension is necessary that adjusts the infinitesimal generator A for these advanced stochastic processes. Compare also with Duffie et al. (2000, pp. 9 ff and Appendix A) and Cheng and Scaillet (2002, p. 11).

Corollary 2.2 *Extension of KBE for jump diffusions*

With the same assumptions as in theorem 2.1 consider a process defined by the differential equation

$$d\vec{X}_t = \vec{\mu}(\vec{X}_t)dt + \vec{\sigma}(\vec{X}_t)d\vec{Z}_t + \vec{J}(\vec{X}_t)dP(\lambda(\vec{X}_t)), \tag{2.7}$$

where the first part corresponds to the Itô diffusion of (2.2), P is a pure jump process with intensity $\lambda(\vec{X}_t) : t \geq 0$ for some $\lambda : \mathbb{R}^n \rightarrow [0, \infty)$, and $\vec{J}(\vec{X}_t)$ is the size of the jump, conditional on a jump occurring. $\vec{J}(\vec{X}_t)$ is assumed to be distributed with density $\vec{\nu} : \mathbb{R}^n \rightarrow \mathbb{R}^m$.[14]

Equivalently to theorem 2.1, define, for $f \in C^2(\mathbb{R}^n)$ with compact support, the infinitesimal generator A of $\vec{X}_t$ as

$$Af(\vec{x}) = \sum_i \mu_i(\vec{x})\frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}(\vec{\sigma}\vec{\sigma}^\top)_{i,j}(\vec{x})\frac{\partial^2 f}{\partial x_i \partial x_j} + \lambda(\vec{x})\int_{\mathbb{R}^n}(f(\vec{x}+\vec{z}) - f(\vec{x}))d\nu(\vec{z}). \tag{2.8}$$

(i) *Define $u(t, \vec{x}) \in \mathbb{R}$ as*

$$u(t, x) = E[f(\vec{X}_t)|\vec{X}_0 = \vec{x}]. \tag{2.9}$$

Suppose that the following integrability conditions hold:

$$E[|u(t,\vec{x})|] \quad < \quad \infty \quad \forall t \in [0, \infty), \tag{2.10}$$

$$E\left[\left(\int_0^t (\vec{\eta}_s\vec{\eta}_s^\top)ds\right)^{1/2}\right] \quad < \quad \infty \quad \forall t \in [0, \infty), \tag{2.11}$$

$$E\left[\int_0^t |\gamma_s| ds\right] \quad < \quad \infty \quad \forall t \in [0, \infty), \tag{2.12}$$

[14] See also Duffie et al. (2000).

where[15]

$$\vec{\eta}_t = \left(\frac{\partial u}{\partial x_1}(t,x), \ldots, \frac{\partial u}{\partial x_n}(t,x) \right) \vec{\sigma} \tag{2.13}$$

and

$$\gamma_t = \lambda(\vec{x}) \int_{\mathbb{R}^n} (u(t, \vec{x} + \vec{z}) - u(t, \vec{x})) d\vec{\nu}(\vec{z}). \tag{2.14}$$

Then

$Au(t, \vec{x})$ *exists for all* $(t, \vec{x}) \in [0, \infty) \times \mathbb{R}^n$ *and*

$$\begin{aligned} Du &:= Au - \frac{\partial u}{\partial t} = 0 \quad \forall (t, \vec{x}) \in [0, \infty) \times \mathbb{R}^n, & (2.15) \\ u(0, \vec{x}) &= f(\vec{x}) \qquad \forall x \in \mathbb{R}^n, & (2.16) \end{aligned}$$

analogous to KBE.

(ii) *If* $w(t, \vec{x}) \in C^{1,2}(\mathbb{R} \times \mathbb{R}^n)$ *is a bounded function also satisfying (2.10), (2.11), (2.12), (2.15) and (2.16), then* $w(t, \vec{x})$ *is a solution to (2.9), i.e.* $w(t, \vec{x}) = u(t, \vec{x}) = E[f(\vec{X}_t) | \vec{X}_0 = \vec{x}]$.

Proof: The proof can be derived from Duffie et al. (2000, Appendix A).

2.2.2 Characteristic Functions

Kolmogorov's Backward Equation gives us a technique to calculate conditional expectations on the stochastic processes we model later on. We will apply this technique for characteristic functions that are defined now.

We will begin with a definition of characteristic functions and then list some properties.

Definition 2.3 *Characteristic Functions*

Let X_t *be a stochastic process adapted to some augmented filtration* $(\mathcal{F}_t)_{t \geq 0}$ *in a probability space* $(\Omega, \mathcal{F}, P)$. *The* conditional characteristic function *of* X_t *is then defined as*

$$\Phi(\omega, \tau, \vec{\vartheta}, X_t) := E[\exp(\mathrm{i}\omega X_{t+\tau}) | \mathcal{F}_t], \tag{2.17}$$

[15] See Cheng and Scaillet (2002, p. 11) for this more generalised formulation.

where $\tau \geq 0$, $\mathrm{i} = \sqrt{-1}$, $\omega \in \mathbb{R}$*; and* $\vec{\vartheta}$ *is the space of parameters. It can be interpreted as the conditional expectation at time t of the exponent of the state variable times* $\mathrm{i}\omega$*,* τ *periods ahead.*

Several properties of characteristic functions can be mentioned:

Properties

- $\Phi(\omega, \tau, \vec{\vartheta}, X_t)$ always exists, because

$$\begin{aligned} |\Phi(\omega, \tau, \vec{\vartheta}, X_t)| &= \left|\int_{-\infty}^{\infty} \exp(\mathrm{i}\omega\tau X_t) dF(X_t + \tau)\right| \\ &\leq \int_{-\infty}^{\infty} |\exp(\mathrm{i}\omega\tau X_t)| dF(X_t + \tau) \\ &= \int_{-\infty}^{\infty} dF(X_t + \tau) \\ &= 1. \end{aligned} \tag{2.18}$$

- If two stochastic processes have the same characteristic function, then they have the same probability distribution; $\Phi(\omega, \tau, \vec{\vartheta}, X_t)$ and the conditional density function $f(X_{t+\tau}; \vec{\vartheta})$ of $X_{t+\tau}$ form a Fourier transform pair:

$$\Phi(\omega, \tau, \vec{\vartheta}, X_t) = \int_{-\infty}^{\infty} \exp(\mathrm{i}\omega X_{t+\tau}) f(X_{t+\tau}; \vec{\vartheta}) dX_{t+\tau}, \tag{2.19}$$

$$f(X_{t+\tau}; \vec{\vartheta}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-\mathrm{i}\omega X_{t+\tau}) \Phi(\omega, \tau, \vec{\vartheta}, X_t) d\omega. \tag{2.20}$$

- Applying an Euler expansion of a complex variable to $\Phi(\omega, \tau, \vec{\vartheta}, X_t)$ leads to a splitting in a real cosine term and an imaginary sine term.
- Furthermore, uncentered moments of X_t are easily obtained by differentiation.

For an application of the two latter aspects, refer, for example, to Chacko and Viceira (2003). They use Euler expansions and calculate uncentered moments of X_t via characteristic functions. For more properties of characteristic functions, as for example $\lim_{c\to\infty} \frac{1}{2c} \int_{-c}^{c} \exp(-\mathrm{i}\omega X_t) \Phi(\omega, \tau, \vec{\vartheta}, X_t) dt = 2\pi \lim_{c\to\infty} \frac{f(X_{t+\tau})}{2c} = 0$, see Stuart and Ord (1987). Even more information is given in Lukacs (1970).

2.2.3 Martingale Measures

We will here give a definition of martingale measures and describe how we will employ them in this work.[16]

Definition 2.4 *In a probability space* $(\Omega, \mathcal{F}, P)$ *with filtration* $(\mathcal{F}_t)_{t\geq 0}$, *a probability measure* Q, *that is equivalent to* P,[17] *is a* Martingale Measure *for an arbitrary asset* X, *if the equality*

$$X_t = E_t^Q[X_T] \quad \forall\ 0 \leq t \leq T < \infty \tag{2.21}$$

holds[18] *and if the Radon–Nikodym derivative* $\mathrm{d}Q/\mathrm{d}P$ *has finite variance.*[19]

The Martingale Measure is often called *Risk-neutral Measure*, because in a risk-neutral world this measure would coincide with the real probabilities. This measure can be very useful, for example for pricing futures, where we know that $F(T,T) \equiv S_T$, i.e. a futures with expiry in T has the value in T of the underlying S.[20] As a result,

$$F(t,T) = E_t^Q[F(T,T)] = E_t^Q[S_T]. \tag{2.22}$$

We can price options the same way.[21] For example European calls with a value at expiry time T of $\max\{S_T - K, 0\}$, where K is the strike price, can be priced as

$$C(t,T,K) = E_t^Q[\max\{S_T - K, 0\}]. \tag{2.23}$$

We will apply these relations in the next section.

The existence of a Martingale measure for all assets in a market is equivalent to the non-existence of arbitrage opportunities, see Musiela and Rutkowski (1997, pp. 246f) and the references therein. Uniqueness, though, depends on the completeness of the market, and completeness can be shown if all contracts can be replicated by other contracts. As was demonstrated in Sect. 2.1 for electricity, the replication of a futures contract is not possible with spot contracts, because storing of electricity is

[16] Compare e.g. with Duffie (1996, p. 108).

[17] The probability measures P and Q are equivalent, when for all $C \in \mathcal{F}$ holds: $P(C) = 0$ if and only if $Q(C) = 0$.

[18] The notation E_t is a short form of $E[\cdot|\mathcal{F}_t]$, i.e. the conditional expectation with respect to the filtration $\mathcal{F}_t, \quad 0 \leq t < T$.

[19] See Duffie (1996, App. C) for Radon–Nikodym derivatives.

[20] For a definition of a futures contract, see Definition 3.1.

[21] For a definition of an options contract, see Definition 3.3.

impossible or at least only at very high costs. Following Lucia and Schwartz (2002, p. 12), though, standard arbitrage arguments nevertheless allow the derivation of a risk-neutral process and also lead to the market prices of risk. As the existence of a martingale measure only depends on the lack of arbitrage possibilities, one can even choose in incomplete markets a martingale measure and use the fact that in the employed pricing system arbitrage possibilities are not possible. See also e.g. Schönbucher (2003, p. 106) and Sect. 2.1.

2.3 Pricing of Nord Pool Futures and Forwards

With this background, our approach is straightforward. In our probability space $(\Omega, \mathcal{F}, P)$ with filtration $(\mathcal{F}_t)_{t\geq 0}$ and state variable X_t, we can define a general version of the conditional characteristic function as

$$\Phi(u, \tau, \vec{\vartheta}, X_t) := E[\exp(uX_{t+\tau})|\mathcal{F}_t], \tag{2.24}$$

with $u \in \mathbb{C}, \tau \in \mathbb{R}_0^+$.[22] $\vec{\vartheta}$ is the space of parameters.

For $u \in \mathrm{i}\mathbb{R}$, there is clearly no convergence problem. For $u \in \mathbb{R}$, one has to assure the existence of $\Phi(u, \tau, \vec{\vartheta}, X_t)$, in our context at least for the value of $u = 1$: for a price process defined as $S_t := \exp(X_t)$, we can easily see that[23]

$$E_t[S_T] = \Phi(1, T-t, \vec{\vartheta}, X_t). \tag{2.25}$$

Let $F(t, T_1, T_n)$ be the price of a futures or forward on day t for delivery days T_1 to T_n, where $n \in \mathbb{N}$.[24] The expected cash settlements, given the information until

[22] For our purposes, it would suffice to define a moment generating function $M(s, \vec{\vartheta}, X_t) := E[\exp(sX_{t+\tau}), s \in \mathbb{R}^+$, or even only to calculate $E_t[\exp(X_{t+\tau})]$. Nevertheless, we choose to embed our work in a more general context, which simplifies comparisons with existing literature and the accomplishments of extensions.

[23] For example, Bakshi and Madan (2000, p. 218) or Zhu (2000, p. 12) mentioned this relation and proposed to set the frequency of the characteristic function to $-\mathrm{i}$, which corresponds to $u = 1$ in our setting. See also Deng (2000, p. 19).

[24] For a specification of the futures and forward contracts of the Nord Pool Financial Market, see definitions 3.1 and 3.2. The prices are quoted as day-prices in the sense of below.

day t, are the following:

$$\begin{aligned}
\text{Day } T_1: &\quad \underbrace{F(t,T_1,T_n) - E_t[S_{T_1}]}_{=:\, Z_1} \\
\text{Day } T_2: &\quad \underbrace{F(t,T_1,T_n) - E_t[S_{T_2}]}_{=:\, Z_2} \\
\vdots &\qquad\qquad \vdots \\
\text{Day } T_n: &\quad \underbrace{F(t,T_1,T_n) - E_t[S_{T_n}]}_{=:\, Z_n}
\end{aligned}$$

Under the risk-neutral or Martingale measure, the present value of these settlements should be zero at day t:

$$\begin{aligned}
& \mathrm{e}^{-r(T_1-t)} Z_1 + \mathrm{e}^{-r(T_2-t)} Z_1 + \cdots + \mathrm{e}^{-r(T_n-t)} Z_n && \stackrel{!}{=} 0 \\
\Leftrightarrow & \\
& \textstyle\sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)} F(t,T_1,T_n) - \sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)} E_t[S_{T_i}] && \stackrel{!}{=} 0. \qquad (2.26)
\end{aligned}$$

A constant interest rate r is assumed here.[25]

Solving for $F(t,T_1,T_n)$ leads to[26]

$$\begin{aligned}
F(t,T_1,T_n) &= \frac{\sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)} E_t[S_{T_i}]}{\sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)}} \\
&= \frac{\sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)} \Phi(1, T_i - t, \vec{\vartheta}, X_t)}{\sum_{i=1}^{n} \mathrm{e}^{-r(T_i-t)}}. \qquad (2.27)
\end{aligned}$$

Therefore, if $\Phi(u, \tau, \vec{\vartheta}, X_t)$ is known and well defined, at least for $u = 1$, then the futures and forward prices are easily calculated with (2.27).[27]

[25] With this assumption and the further characteristics of the Nord Pool Futures and Forwards, see 3.3, we can treat futures and forwards in our context as synonyms that are only differing in expiry days and delivery periods.

[26] For a continuous version of this solution, see Fleten and Lemming (2001).

[27] Note that with (2.27) or (2.22), $F(t,T_i,T_i) = E_t(S_{T_i})$ under the Martingale measure. Now, if (2.1) were valid, $E_t(S_{T_i}) = F(t,T_i,T_i) = e^{r(T_i-t)}S_t$, so the pricing of futures would be completely model-independent (except for the assumption of constant interest rates). Implicit parameter estimations as described in the next section would not be possible with futures and forward prices. For electricity contracts, (2.1) needs not to hold and apparently does not hold. This can e.g. be seen in Figs. 4.6,

2.4 Implicit Estimations

In the estimation process of this work, the parameters are implicitly estimated. The technique follows the way employed by Bakshi et al. (1997), but, instead of options, we use futures and forwards to determine the best fitting parameter values.

For each day t define $F(t, T_1^j, T_n^j)$ as the observed price with delivery period from T_1^j to T_n^j for all futures and forwards $j = 1, \ldots, N$ on that day, and $\hat{F}(t, T_1^j, T_n^j)$ as the theoretical price determined by the respective model. Then the parameter vector $\vec{\vartheta}$ shall be found that minimises the mean of squared errors on day t:

$$MSE(t) \equiv \min_{\vec{\vartheta}} \frac{1}{N} \sum_{j=1}^{N} \left(\hat{F}(t, T_1^j, T_n^j) - F(t, T_1^j, T_n^j) \right)^2 . \tag{2.28}$$

For each day in the sample, a different $MSE(t)$, or equivalently $RMSE(t) = \sqrt{MSE(t)}$, is determined.

Although the variance is an unobservable state variable for some model groups, the daily independent estimations allow estimating it as a regular parameter. This implies that, in conjunction with the other parameters, we are looking for the variance for the precise day that can best explain the observed prices.

Our calculations are all performed in Matlab 6.5. The minimisation function used is ‘fminsearch’, the quadrature function is based on ’quad’ and ‘quadl’. These functions are part of the core package of Matlab.

Because of possible multiple local minima in most models, the estimations often depend on the starting points. So we always started the algorithms with reasonable values for the parameters. Nevertheless, the global minima were probably not always found and the estimations contain many outliers, see Chap. 6.

2.5 Pricing of Nord Pool Options

The options we consider are European-style option contracts on the traded electricity forwards and Asian-style options. The latter options on the arithmetic average of the system price were only listed until 20 April 2001.

Under the Martingale measure, the value of these contracts is for

4.7, and 4.8. Models with mean reversion or seasonality components are possible this way, as well as the models of our Model Group 2, see also Chaps. 1 and 5.

- European calls on forwards:

$$C_e(t, T_e, T_1, T_n, K) = \mathrm{e}^{-r(T_e-t)} E_t[\max\{F(T_e, T_1, T_n) - K, 0\}], \tag{2.29}$$

  where $C_e(t, T_e, T_1, T_n, K)$ is the call price on day t with underlying forward price $F(T_e, T_1, T_n)$ on day T_e, strike K, and option expiry day T_e.

- Asian calls on the system price:

$$C_a(t, T_e, T_1, T_n, K) = \mathrm{e}^{-r(T_e-t)} E_t[\max\{AS(T_1, T_n) - K, 0\}], \tag{2.30}$$

  where

$$AS(T_1, T_n) = (1/n) \sum_{i=T_1}^{T_n} S_i \tag{2.31}$$

  is the average of the spot prices between T_1 and T_n. The remaining parameters are the same as for European options.

Puts are priced analogously, simply swapping the terms of the differences in the above formulae.

Closed-form solutions for option prices with these underlyings are very complex, if they exist at all.[28] For this reason, we employ Monte Carlo methods to price the traded option contracts in our model framework, with the parameters obtained from our implicit estimations from futures and forwards.[29]

To be able to work with discrete processes, we apply Euler approximations.[30] Consider a process of the form

$$dy(t) = a(y(t))dt + b(y(t))dZ_t + JdP_t, \tag{2.32}$$

with a Brownian motion dZ_t and a Poisson process dP_t. J is the jump size distribution and $a(y(t))$ and $b(y(t))$ are the parameter functions. The process is approxi-

[28] The proceeding for European options could be as follows: Starting from the solution for $F(t, T_1, T_n)$, the stochastic differential equation for dF could be derived via Itô's lemma. Then, standard methods like those that were proposed by Scott (1997) or Duffie et al. (2000) could be applied. For the pricing of Asian options, see for example Zhang (1998).

[29] We do not employ observed option prices for parameter estimations, but rather for comparing different parameter estimations and models. See also Sects. 6.1.4, 6.3.4 and Appendix A.4.

[30] See Kubilius and Platen (2002).

mated via

$$y_t = y_{t-1} + a(y_{t-1})\Delta t + b(y_{t-1})\Delta Z_t + J\Delta P_t, \tag{2.33}$$

where $\Delta Z_t \sim N[0, \Delta t]$ and ΔP_t is approximated via a Binomial distribution $Bi[\lambda\Delta t]$. We set $\Delta t = 1/365$, corresponding to daily new available spot prices.

To price the options this way, numerous paths have to be created and the above expectations are then simply averages of simulated returns on option contracts.

Monte Carlo simulations are a time-critical procedure. So, various variance reduction techniques, explained in Hull (1989, pp. 411ff) and Jäckel (2002, pp. 111ff), for example, are used to be able to keep the number of different trial paths at a reasonable level.[31] We employ Antithetic Sampling and Moment Matching techniques.

[31]For each model, the number of trial paths was augmented independently until a certain level of accuracy was obtained.

Chapter 3

Nord Pool

Before we continue and introduce our models that we use to price futures, forwards and options, we will now, in this chapter, take a look at the Nord Pool Power Exchange, its physical spot market and its financial market where various power contract types are traded.[1]

3.1 Nord Pool ASA and the Nordic Power Market

Nord Pool ASA operates the oldest non-mandatory power-exchange and was the first to trade financial derivatives on electricity. The present Nordic power market began in 1991 with a national Norwegian power market; Sweden joined in 1996, Finland in 1998, western Denmark in 1999, and eastern Denmark in 2000.

Nord Pool offers the following services:

- A spot market, called *Elspot*, on which contracts for physical delivery are traded. It is the main market for trading products that are physically delivered. Units are always the 24 hours of the next day, traded separately. *Elbas* is another spot market, operating in Finland and Sweden. In Elbas, hourly contracts, until up to one hour before delivery, are traded.
- Financial markets, called *Eltermin* and *Eloption*, where futures, forwards, contracts for difference and options are traded.

[1]For more details about Nord Pool and its products see *Nord Pool ASA*, *Nord Pool Products*, *Nord Pool Information* and *Nord Pool Product Reports* (2002), where most of the following was found. Section 3.2 is also based on *Standard Terms for Trading And Clearing in Nord Pool Spot AS' Physical Markets*, whereas *Nord Pool Rulebook for The Financial Electricity Market* (2002) and *Nord Pool Options* (2002) were also used for Sect. 3.3. We drew heavily from these sources. Other sources are given in the sections and subsections themselves.

- Clearing services. Nord Pool established the *Nordic Electricity Clearing House ASA (NECH)* for the clearing of financial contracts. Clearing by NECH is obligatory for financial products traded at Nord Pool's financial market, but the clearing services are also offered for bilaterally traded standardised products.

3.1.1 Key Figures Nord Pool

The total turnover of Nord Pool in 2002 was 461 billion Norwegian Kronor (NOK) with a volume of 3232 TWh.[2] Most of the turnover consisted of clearing services at the bilateral or over-the-counter (OTC) market with NOK 254 billion or 1747.6 TWh.

The volume traded at the financial market was 1019 TWh with a value of NOK 180 billion, on the physical market only 124 TWh were traded, with a value of NOK 27 billion. The volumes traded on the physical and the financial markets increased by 11.9% in 2002, while the volumes that were cleared increased by 19.5%. The cleared volumes on the bilateral market with 2089 TWh were nearly twice the volumes of 1143 TWh directly traded on the exchange. The traded and cleared volumes increased to a total of 3232 TWh compared to 2769 TWh in 2001; this is a growth of 16.7%.

The average spot or system price, the price on the spot market if capacity restrictions are ignored, was 8.1% higher in 2002 with a peak of NOK 686/MWh while the highest price in 2001 reached NOK 633/MWh.

The highest monthly volumes traded on the spot market with 13.72 TWh were recorded for September 2002; the highest monthly volumes traded on the financial market already took place in April 2002 with 141.48 TWh.[3]

For detailed information about Nord Pool's key figures for 2002, see *Nord Pool Press Release 02/2003* and Table 3.1.

[2] See *Nord Pool Press Release 02/2003*. The figures there are also converted to euro with an exchange rate of 7.8 NOK/€. We cite all prices only in NOK for consistency purposes, because our empirical investigations in Chap. 6 are performed for the prices in NOK or rather the logarithm of these prices.

[3] For more details about the spot price and futures / forward / option prices between October 1999 and September 2002 see Chap. 4.

Nord Pool Key figures 2002

Activity by product area	2002	2001	Percent change 2002-2001	2000	1999	1998	1997	1996
Volume traded, physical market (TWh)	124	112	11,2 %	97	76	57	44	41
Value of volume traded, physical market (NOK billion)	27	21	25,4 %	11	9	7	6	10
Volume, financial market (TWh)	1 019	910	12,0 %	359	216	89	53	43
Value, financial market (NOK billion)	180	157	14,5 %	43	28	13	9	11
Total volume traded (TWh)	1 143	1 022	11,9 %	456	292	146	97	83
Total value traded (NOK billion)	207	178	15,8 %	54	37	20	15	21
Volume, clearing of bilateral-market trade (TWh)	2 089	1 748	19,5 %	1 180	684	373	147	
Value, clearing of bilateral market trade (NOK billion)	254	234	8,5 %	123	88	56	25	
Total volume, traded + bilateral clearing (TWh)	3 232	2 769	16,7 %	1 635	975	519	244	
Total value, traded + bilateral clearing (NOK billion)	461	412	11,8 %	177	124	75	40	
Number of participants								
as of 31 December	302	295	2,4 %	281	264	250	199	148
Avarage system price								
Avarge system price pr year NOK/MWh	201	186	8,1 %	103	112	116	136	254
Highest systemprice NOK/MWh	686	633	8,3 %	388	226	266	262	
Lowest systemprice NOK/MWh	81	119	-32,3 %	32	50	21	58	

Highest volumes	2002	2001		2000
Volume, Elspot 1-hour contracts (MWh/h) Hour 18, 2.January	21 926	21 045	4,2 %	16 887
Daily volume, Elspot (TWh) 31. December	0,48	0,46	4,0 %	0,35
Weekly vol, Elspot (TWh) Week 51	3,15	2,87	9,8 %	2,34
Monthly vol, Elspot (TWh) December	13,72	11,90	15,3 %	9,70
Daily volume, Financial market (TWh) 22. April	21,10	13,22	59,6 %	6,89
Weekly vol, Financial market (TWh) Week 41	45,36	34,18	32,7 %	16,28
Monthly vol, Financial market (TWh) April	141,48	114,07	24,0 %	52,33
Daily vol, bilateral clearing (TWh) 23 October	26,81	26,70	0,4 %	19,92
Weekly vol,, bilateral clearing (TWh) Week 41	91,97	77,00	19,4 %	60,37
Monthly vol, bilateral clearing (TWh) October	330,42	241,50	36,8 %	177,81

Figure 3.1. *Key Figures Nord Pool*

Enclosure to *Nord Pool Press Release 02/2003*

3.1.2 Generation of Electricity in the Nordic Countries

The power in the different countries is produced and consumed as follows:[4]

- Denmark, with a population of 5.34 million people and a consumption of about 34.61 TWh a year, uses 90% fossil fuel-based generation and 10% wind power.
- Norway produces nearly 100% of its energy by hydropower. The country has 4.49 million inhabitants and consumes about 113.09 TWh per year.

[4]See also *Energy Indicators per Country*.

- Sweden and Finland both use a mix of hydropower, nuclear power, and fossil fuel-powered or thermal generation. With a population of 8.86 million people, Sweden is the most populated of the Nordic countries, and its population consumes about 138.91 TWh per year. Finland, with a population of 5.18 million people, consumes about 79.12 TWh a year.

All Nordic countries are still generating most of the electricity for their populations themselves, but the price for the power traded at Nord Pool reflects the price for the cheapest capacities available in the whole region at each specific hour.

Hydropower generation can be very volatile: the volatility is about 20 TWh a year, more than half of Denmark's yearly consumption. And though the Nordic power market leads to a smoothing of local power prices, there still remains a high volatility.[5] Changing water reservoir levels used for hydropower generation are also the main reason in the Nordic market for seasonal effects in the price curve. Low water reservoir levels in the north imply that nearly all traded electricity is expensively produced in the southern part of the Nordic countries, resulting in high system prices.

Trade with other European countries, i.e. with Germany, Poland and Russia, already plays an important role, see, for example, the map of the trade between the countries on 9 May 2003 in Fig. 3.2. It shows the exchange of electricity for this one day in MWh. The figure 14452 on the arrow between Finland and Sweden, for example, means that on 9 May 2003 Finland exported 14452 MWh to Sweden.

3.2 Nord Pool Spot Market

The physical power market is organised via Elspot and Elbas. Elspot is a day-ahead market; the products traded are contracts for physical delivery on the next day.

In 1993, the Nord Pool Elspot market was established in Norway, but soon it turned into an exchange for the whole Nordic region.

The trading is organised via an auction system. Bids for purchases and sales of power contracts of a duration of one hour cover all hours of the following day. The 24 calculated spot prices are based on the balance of all bids and offers.

[5] For a graph of the system price, which is an average price ignoring capacity restrictions, see Fig. 4.1.

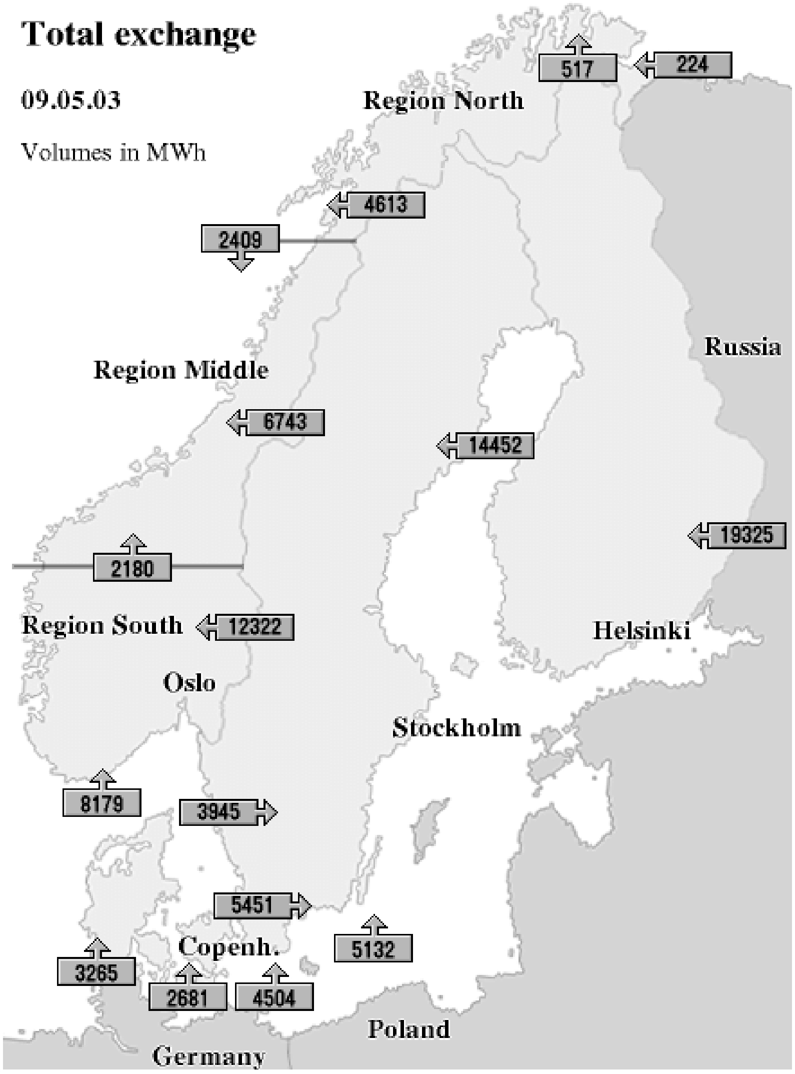

Figure 3.2. *Exchange of Electricity*
Example of the exchange of electricity between countries belonging to the Nordic region and their neighbouring countries. Shown is also the exchange between the different areas in the Nordic region. As an example, the 9 May 2003 was chosen arbitrarily. The figure 14452 on the arrow between Finland and Sweden, for example, means that on 9 May 2003 Finland exported 14452 MWh to Sweden. Source:*Nord Pool ASA – Market Information*

Elspot is also the primary marketplace in the Nordic countries to handle predictable grid congestions. Congestions occur when the theoretical transmission capacity on a part of the grid is reached or surpassed. Capacity constraints in the Norwegian power grid and between the countries are controlled by a price mechanism that can lead to different prices in different areas.

For Sweden and Finland, Elbas is the organised balance adjustment market. The participants can adjust their power balances for each hour after the trade on Elspot is finished. As an hour-ahead market, members can trade contracts on Elbas up until one hour before the delivery hour. New contracts are opened when the Elspot prices for the next day have been set at 2 pm. At the same time, the trading for the Elbas contracts ends for this day.

3.2.1 Elspot Bidding

On Elspot, the traded products are physical-delivery electricity contracts for the following day. Contract duration is one hour or one block, depending on the type of bid. Blocks are defined as several consecutive hours with one price.

The participants have to submit their bids for the next 24 hours on a bidding form like shown in Table 3.1. The minimum size of the contracts is 0.1 MWh/h; contracts are obligations for physical delivery or take-off for one hour or one block. Bids consist of price/volume pairs. If a participant wants to purchase electricity, a positive price has to be submitted; if he wants to sell power, the price bid must be negative. This way, the same forms for purchase as well as for sale bids can be used, for example to purchase power at low price levels and to sell power at high levels. At intermediate price levels, participants normally do not want to buy or sell, so 0 MWh/h are bid for these prices.

For each of the 24 hours, separate bids can be set. The participants can freely set their own price steps, and between the price steps, the volumes are linearly interpolated. See for example the bidding form in Table 3.1: if the area price between 3 pm and 4 pm (Hour 15) would be determined at NOK 160, then the volume the participant would sell is $60\text{MWh} + \frac{160-150}{170-150}(80-60)\text{MWh} = 70$ MWh. There can be 62 price steps between the price limits determined by Nord Pool.

A second example, one for the easiest bidding form, is in Table 3.2: The participant wants to purchase 100 MWh/h for all 24 hours the next day, independent from the price. If the 24 resulting spot prices were different, the participant would pay a different price for each hour.

The way market participants bid depends on various factors:

- Production plans, either power generation or production of goods or services that are very electricity-intensive.
- Demands of consumers that retailing companies are expecting.

Table 3.1. *Part of a bidding form*

In bidding forms, for each of the 24 hours of a day, a separate volume can be bid for price ranges freely chosen. For prices between the chosen price steps, the volumes are linearly interpolated. A positive volume means that the participant wants to purchase the volume for the specified price, a negative volume means analogously an offer of electricity for this price. The prices are in NOK, volumes in MWh/h.

Hour/Price	0	119	120	149	150	170	400	10000
1	0					0	-100	-100
⋮								
15	100	100	0	0	-60	-80	-100	-100
16	100	100	0	0	-80	-90	-150	-150
17	90	80	0	0	-50	-70	-150	-150
⋮								
24	0					0	-100	-100

Table 3.2. *Simple bidding form*

In this bidding form the participant wants to purchase 100 MWh/h from hour 1 to hour 24, for all prices between NOK 0 and NOK 10000.

Hour/Price	0	10000
1-24	100	100

- Open positions on the OTC or bilateral market with physical delivery, or other commitments that are influencing the need or availability of power.

The bidding results themselves then influence the further plans of the participants, for example, the volumes of power that have to be generated the next 24 hours.

3.2.2 Calculation of the System Price

The hourly system price is calculated as the intersection between a sale and a demand curve for each hour. The curves are determined by aggregating all sell bids to one curve and all demand bids to the other curve. Looking back at the individual bidding forms, the volumes purchased or sold are also determined by the above intersection.

Examples for bid-and-ask curves and the resulting price can be viewed in Figs. 3.3, 3.4 and 3.5. The first two refer to the same day, 11 June 2002, but different hours. Figure 3.5 shows the curves for 12 am to 1 pm on 6 September 2002.

If, after determination of an hourly system price, no constraints are violated, i.e. the contractual flow between the bidding areas does not exceed the capacity allowed

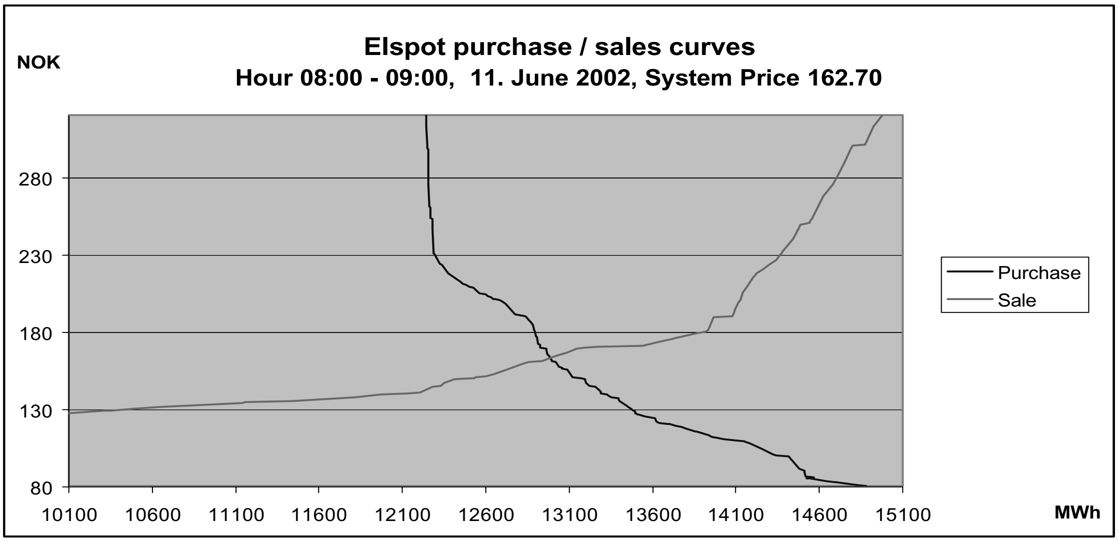

Figure 3.3. *Example 1 of a bid-ask-curve on 11 June 2002*

The system price is determined each hour as the intersection between the purchase and the sales curve. For 11 June 2002, between 8 am and 9 am, the intersection is at a price of NOK 162.70/MWh at about 13000 MWh. Source: *Nord Pool ASA*

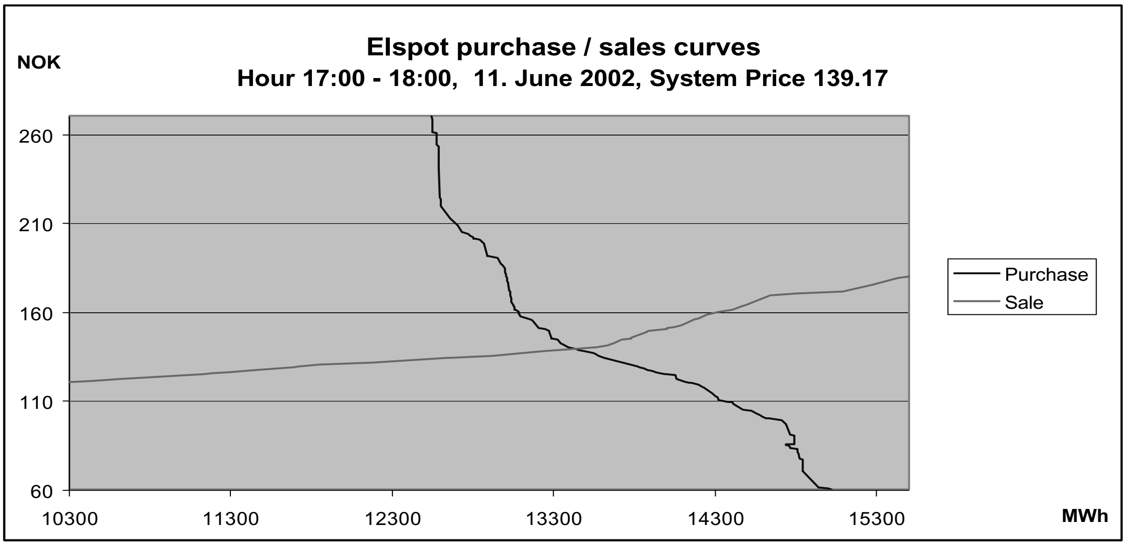

Figure 3.4. *Example 2 of a bid-ask-curve on 11 June 2002*

The system price is determined each hour as the intersection between the purchase and the sales curve. For 11 June 2002, between 5 pm and 6 pm, the intersection is at a price of NOK 139.17/MWh at about 13400 MWh. Source: *Nord Pool ASA*

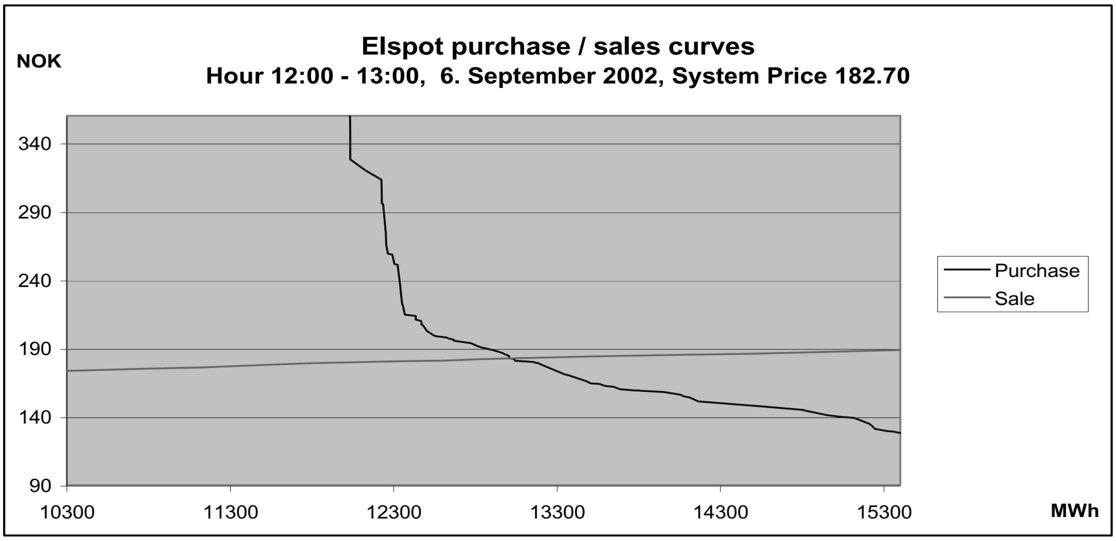

Figure 3.5. *Example of a bid-ask-curve on 6 September 2002*

The system price is determined each hour as the intersection between the purchase and the sales curve. For 6 September 2002, between 12 am and 1 pm, the intersection is at a price of NOK 182.70/MWh at about 13200 MWh. Source: *Nord Pool ASA*

during this time, all area prices will equal the system price. In the case of possible bottlenecks, further calculations resulting in different area prices are made.[6]

The average of the 24 hourly system prices is also called system price or spot price. It serves, either directly — or indirectly for options — as the reference price for all derivatives on the financial market. Therefore, it is of special interest for us and will be further examined in Sect. 4.1.

3.3 Nord Pool Financial Market

The financial market, divided in Eltermin and Eloption, was established in 1993 by Nord Pool as a forward market with physical delivery.

Since autumn 1995, all products are due to cash settlement. The trading time horizon is up to four years. All contract positions can be held until maturity or be closed any time via a countertrade. The continuously traded products of Nord Pool compete with the contracts traded on the bilateral or OTC market.

[6]Different bidding areas that can convert into different price areas are: Norway, with various bidding areas, Finland, Sweden and Denmark with the bidding areas Denmark East and Denmark West. See also Fig. 3.2.

NECH is the obligatory clearing institution for all products traded on Nord Pool and also clears a substantial proportion of financial contracts in the bilateral markets, see also Sect. 3.1.1.

3.3.1 Products – Overview

The products traded at Nord Pool are futures, forwards, contracts for difference and options. The delivery time starts after expiration of the contracts, and the contracts are settled each delivery day, futures are already settled mark-to-market from the day on which the contract is purchased. Forward contracts accumulate the price differences until the delivery period. Then the difference between agreed price and market price is settled in equal shares each day. Experiences on the OTC market and in the exchange have shown, that, for large periods, forwards without daily settlements are preferred, while for short-time contracts mark-to-market settlement seems to be more attractive. Therefore, the forward contracts now listed at Nord Pool cover larger delivery periods, seasons and years, while the futures contracts cover delivery periods of days, weeks and blocks (four or five weeks).[7]

The prices listed for the different contracts are quoted as price / contract day, i.e. a week contract with a quoted price of NOK 120, for example, has a contract value of 7 days x NOK 120 = NOK 840.

The above contracts are based on the system price. But in different areas, there might exist different area prices. Thus, to enable better risk management for the participants, Nord Pool offers a forward contract on the difference between the system price and area prices. This instrument is called *Contract for Difference (CfD)*.[8] CfDs enable the hedging of the price difference between the areas defined by Nord Pool and the system price.[9]

The reference price for the options traded at Nord Pool is not the system price, but are the prices of forward contracts, or the average of the system price for Asian-style options, respectively. There are options on the seasonal forwards and on the

[7]The trade of day futures contracts is not very liquid in the time series considered in the empirical part of this study, so the day futures are ignored.

[8]If in the following the term forward is used, CfDs shall be excluded.

[9]As CfDs are not regarded in our empirical studies, they will be ignored in the following.

yearly forwards as well as – until 20 April 2002 – Asian-style options on the averages of the system price that cover the delivery times of futures.[10]

Every day, Nord Pool calculates closing prices for all listed products. If, for at least ten consecutive trading days, there are no open positions in a contract series, Nord Pool can de-list this contract.

3.3.2 Futures

Definition 3.1 *Futures*

A futures contract is an agreement between two parties to trade a product in the future for a price fixed in this agreement. The price is not to be paid in advance, but each day the changes of the market price of the product result in payments, called mark-to-market settlements.

Nord Pool established financially settled futures in 1995. The first contracts were listed on 25 September 1995.

The futures listed by Nord Pool are all base load contracts, for one day, one week, or four to five weeks, called blocks. Also season contracts were listed until 30 December 1999.

Four weeks before delivery time, block contracts are split into week contracts, and 10 days before delivery time, the week contracts are again split into day contracts. When the delivery time of the contracts of a season starts, new block contracts for the next season are listed.

Before delivery time, the contracts are daily mark-to-market settled; the difference of the closing price of a contract from one day to another is credited or debited on the account of the contract holder.

The splitting of the contracts causes only day futures to remain on the due dates. When they are due, they are settled with the last closing price against the reference price, i.e. the system price of Elspot, on the delivery day.

With a futures contract, a market participant can fix the price for electricity on a specific day. For example, assume a participant wants to buy an amount of 1 MWh/h for 24 hours on date T.[11] Assume the futures price for a daily contract for

[10] An actual product calendar can be viewed in *Nord Pool Product Calendar*.

[11] The example can be generalised for a day futures, because all other futures split until expiry into day futures.

that day is $F(t, T)$.[12] The participant can now fix the price for his power demand on day t to $F(t, T)$ by entering into this contract: The settlement sums add up to the difference $F(t, T) - S(T)$ until day T, if interest rates are ignored. So his costs for purchasing the electricity are

- the price he pays on Elspot: $S(T)$,
- plus the gain/loss of his activity on the financial market: $F(t, T) - S(T)$.

This results in an effective price of his power contract of $S(T) + F(t, T) - S(T) = F(t, T)$.

3.3.3 Forwards

Definition 3.2 *Forwards*

Forwards at Nord Pool are very similar to futures. The daily mark-to-market differences are summed up as daily losses or profits, but they are not realised through the whole trading period. Once a contract is due, it behaves similar to a futures contract. But, the total differences between the purchasing price of the forward and the reference prices are credited/debited on the participant's account on the delivery days.[13]

Nord Pool introduced cash settled forward contracts on 27 October 1997. They are contracts with delivery times of seasons (Winter 1, Summer, Winter 2) and years. In contrary to the splitting of futures, forward contracts are never split.

Assume again that a participant wants to buy an amount of 1 MWh / h for 24 hours on date T_1 until T_n. Assume further the forward price for a daily contract for that day is $F(t, T_1, T_n)$. The participant can again fix the price on day t to $F(t, T_1, T_n)$ by entering into this contract: There is no settlement until date T_1. From then on, the sum due is $F(t, T_1, T_n) - S(T)$ each day until T_n. So the costs of the participant are

- the price he pays on Elspot each day: $S(T_1), \ldots, S(T_n)$,
- plus the gain/loss of his activity on the financial market each day: $F(t, T_1, T_n) - S(T_1), \ldots, F(t, T_1, T_n) - S(T_n)$.

[12] To be consistent in our notations, we should write here $F(t, T, T)$ instead of $F(t, T)$.

[13] With these definitions of Nord Pool Futures and Forwards and the assumption of constant interest rates, we can treat futures and forwards as synonyms that differ only in expiry days and delivery periods, see also page 17.

On day i, the price for the power he purchases is $S(T_i) + F(t, T_1, T_n) - S(T_i) \equiv F(t, T_1, T_n)$.

If an open position in a forward contract is closed by a counter-trade, the difference of the price at the time the contract was opened to the price at the time, when it was closed, has to be paid/is credited each day of the delivery time.

3.3.4 Options

Definition 3.3 *Options*

An option is the right to buy or sell an underlying contract at a previously specified price, called strike or exercise price, on a specified date, called expiration or exercise time.

The trading of options at Nord Pool began on 29 October 1999. Option contracts were introduced as further instruments for price hedging and risk management.

All options are European-style. The underlyings are the different forward contracts. New option series are listed on the first trading day after the exercise day of the previous contract series.

Until 20 April 2001, Asian-style option contracts were also traded, but beyond block 5 from 2001 with delivery time from 23 April 2001 to 20 May 2001, no new contracts were issued. The underlying of this kind of option is always the average of a reference price during a specified period; in Nord Pool, this is the arithmetic average on the system price during the specified delivery time.

Options are listed for the two nearest season contracts and the two nearest year contracts as the underlying forward contracts. Asian-style options were listed with delivery periods equivalent to the last already split futures block contract and the two following futures block contracts. New series are always offered on the first trading day after the exercise day of the previous contract series. Exercise day is the third Thursday of the month before the first delivery month of the underlying forward. For Asian options, exercise day was the first trading day after the last day of the period that determined the average.

Options are automatically exercised unless the holder of the option decides otherwise. In this case, he is able to exercise it manually.

The strike or exercise price is set by Nord Pool. When a series is first listed, Nord Pool sets five different strike prices for each call and put option. One of the strike prices is equal to the closing price of the previous day of the underlying, rounded off to always get the same intervals. Two strike prices are set above this price and two below. The spread between them depends on the absolute value of the underlying:

Forward price		Option spread
	price < NOK 100:	NOK 2,
NOK 100 <	price < NOK 200:	NOK 5,
NOK 200 <	price:	NOK 10.

So the strike prices are for example: NOK 96, NOK 98, NOK 100, NOK 105, NOK 110, when the forward price is NOK 100; or NOK 195, NOK 200, NOK 210, NOK 220, NOK 230, when the forward price is NOK 210.

When the underlying is traded below or above the second lowest or highest strike price, respectively, a new contract below/above the lowest/highest exercise price is listed the next day.[14]

[14]We do not give an example for the risk management possibilities with options here. For electricity options they are exactly the same as for securities. For example refer to Hull (1989).

Chapter 4

Descriptive Statistics

For the empirical work we use data supplied by Nord Pool ASA from 1 October 1999 to 30 September 2002. We choose this time interval, because options were introduced in the Nord Pool Financial Market on 29 October 1999. Furthermore, a three year time interval seems long enough to capture the characteristics of the spot price and the futures and forward curves.

We will first take a look at the spot price and the logarithm of the spot price.[1] In the second section, we will examine futures and forward prices, and in the last section, we address the option contracts of Nord Pool.

The prices are all in Norwegian Kronor (NOK) per MWh.

4.1 Spot Price and Log Spot Price

We will begin with the spot price or Elspot system price and the log spot price, see Figs. 4.1 and 4.2.[2] The high volatility of the two processes is quite obvious. We can detect evidence of seasonal behaviour of the spot price in the figures, but it is more apparent if we examine Table 4.1, where monthly average spot and log spot prices are listed. With some exceptions, the prices in winter are normally higher than the prices in summer, for example in February 2001 with NOK 222.16/MWh, compared to August 2001 with NOK 172.09/MWh.

[1] In the following, we will always use the term 'log spot price' for the logarithm of the spot price.

[2] For a description of the Elspot system price, see Sect. 3.2.2. Also compare the key figures in Fig. 3.1. We examine all spot prices and not only the prices on the days when the financial market was open. This explains having a sample of 1096 days here, while in our later daily examinations in Chap. 6 we only use 750 different samples or trading days.

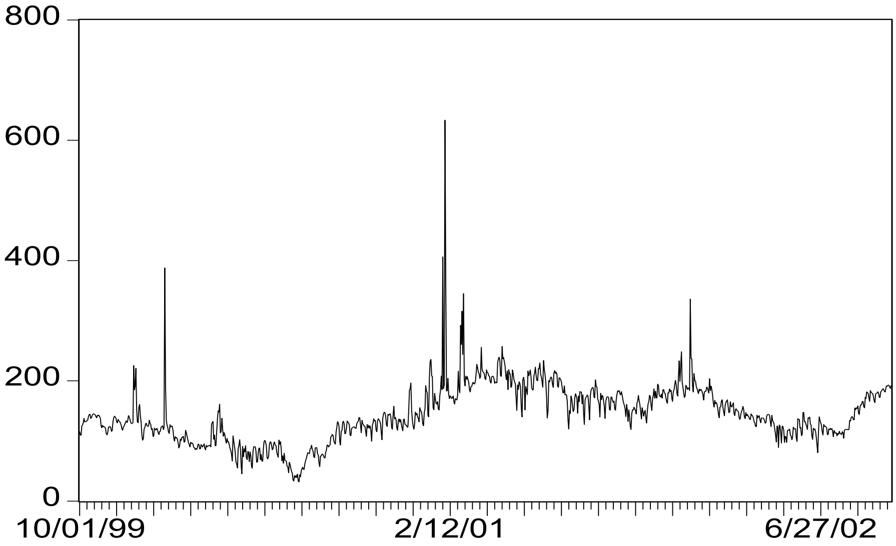

Figure 4.1. *Spot Price*

Spot price from 1 October 1999 to 30 September 2002. The prices are in NOK/MWh.

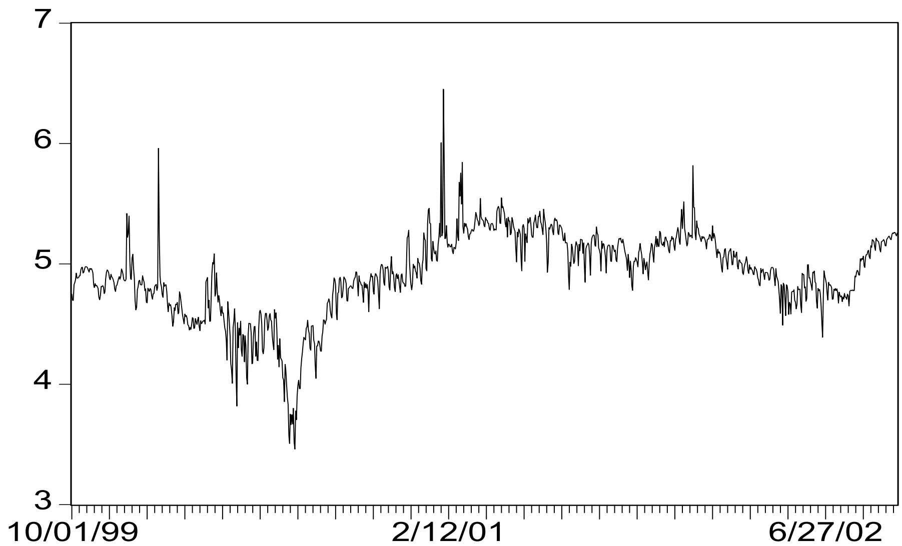

Figure 4.2. *Log Spot Price*

Logarithm of the spot price from 1 October 1999 to 30 September 2002.

We can also see on Table 4.1 that the yearly average prices have grown every year since 2000, from NOK 103.35/MWh in 2000 to NOK 186.51/MWh in 2001 to finally NOK 201.03/MWh in 2002.[3]

In the histograms,[4] we can see a standard deviation of 47.99 for the prices and

[3]Also compare with Fig. 3.1. The price of NOK 112.11/MWh for 1999 is the average price for the whole year, while only the monthly average prices for October to December are listed, since the other months are not in the time period regarded in this study. Analogously is the average price of NOK 201.03/MWh in 2002 the average of the whole year 2002. At the end of 2002 the prices reached all-time maxima with a price in December of NOK 544.34/MWh. This explains the high average price for 2002.

[4]See Figs. 4.3 and 4.4.

Table 4.1. *Monthly average prices*

In this table, the average system prices of Nord Pool are printed. The prices are in NOK/MWh. The average price of the year 1999 is the average price of the whole year of 1999, though only the monthly prices of October 1999 until December 1999 are given. The same applies for the average price of 2002.

	January	February	March	April	May	June
1999						
2000	131.65	104.46	95.59	104.32	77.99	86.21
2001	168.63	222.16	211.29	214.78	192.72	200.82
2002	194.74	158.34	143.67	132.75	114.98	121.90

	July	August	September	October	November	December	Year
1999				134.72	125.84	140.62	112.11
2000	51.95	79.31	113.95	123.58	134.12	137.74	103.35
2001	180.25	172.09	167.20	152.90	169.84	188.97	186.51
2002	116.09	150.72	181.67				201.03

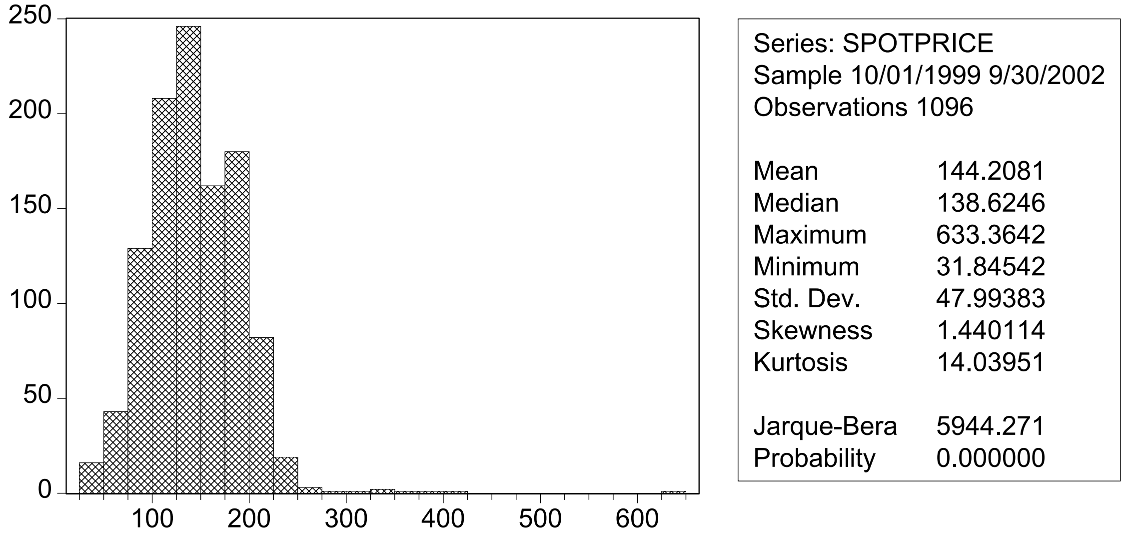

Figure 4.3. *Histogram Spot Price*

Histogram of the spot price for the sample from 1 October 1999 to 30 September 2002, including some basic statistics.

0.35 for the log prices, translating into yearly volatilities of 916.85 and 6.69. The values for minimum and maximum are also striking. The range of the prices is between NOK 31.85/MWh and NOK 633.36/MWh.

The mean that is higher than the median in the spot price series suggests high outliers that have already been seen in Figs. 4.1 and 4.2. Mean and median in the log spot prices are nearly the same with 4.91 and 4.93.

For random numbers that are i.i.d. normally distributed skewness is 0.00 and kurtosis is 3.00; in our sample, these values are quite different for the spot prices: (1.44, 14.03) in contrast to (-0.72, 4.66) for the log spot prices. Examining the

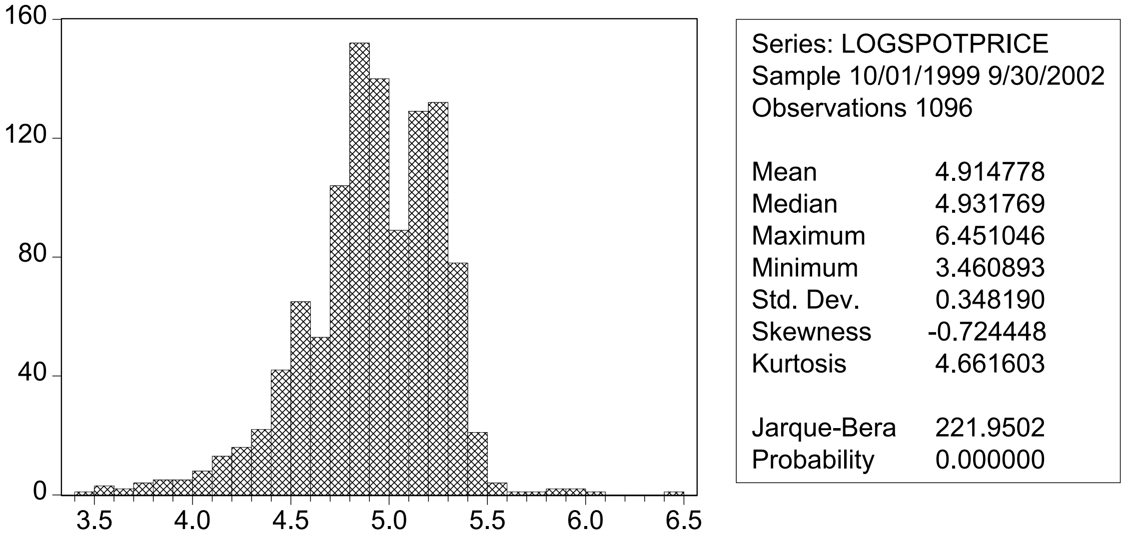

Figure 4.4. *Histogram Log Spot Price*

Histogram of the log spot price for the sample from 1 October 1999 to 30 September 2002, including some basic statistics.

Jarque–Bera statistics, the hypothesis of a normal distribution can clearly be rejected for both series, the statistic of the prices is even a multiple of the statistic of the log prices.

This result is partly supported by the QQ-Plots, see Figs. 4.1 and 4.2. In Fig. 4.2, the non-normality is not apparent.

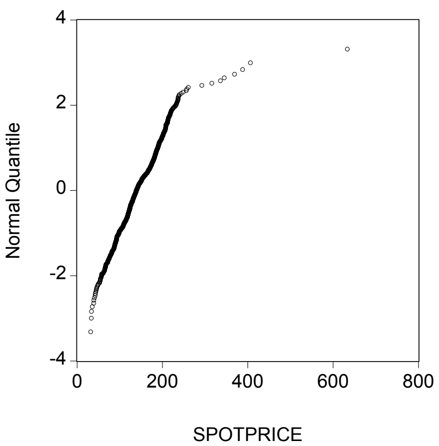

Figure 4.1. *QQ-Plot Spot Price*

The empirical quantiles of the spot price are plotted against the quantiles of a Normal distribution.

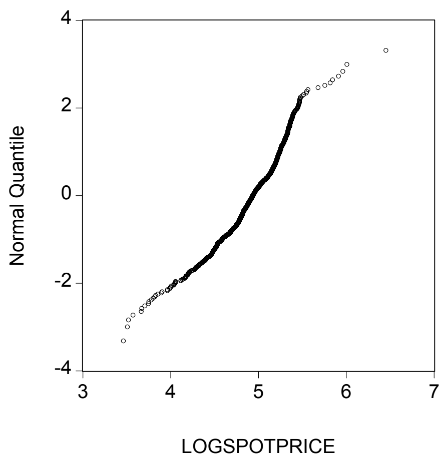

Figure 4.2. *QQ-Plot Log Spot Price*

The empirical quantiles of the log spot price are plotted against the quantiles of a Normal distribution.

If we once again refer to the graphs in Figs. 4.1 and 4.2, the prices seem to be stationary, and so the results of the Augmented Dickey Fuller (ADF) tests in Tables 4.2 and 4.3 are not surprising. The hypothesis of a unit root for the spot price is rejected at the 1% level, for the log spot price at the 5% level. We

Table 4.2. *Augmented Dickey Fuller Test for a Unit Root in the Spot Price*

The report shows the results that Eviews gives for the spot prices. The hypothesis of a unit root can be rejected at the 1% level.

Augmented Dickey-Fuller Unit Root Test on SPOTPRICE

ADF Test Statistic	-4.215283	1% Critical Value*	-3.4391
		5% Critical Value	-2.8646
		10% Critical Value	-2.5684

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SPOTPRICE)
Method: Least Squares
Date: 07/04/03 Time: 17:16
Sample(adjusted): 10/06/1999 9/30/2002
Included observations: 1091 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPOTPRICE(-1)	-0.068056	0.016145	-4.215283	0.0000
D(SPOTPRICE(-1))	-0.371823	0.032182	-11.55379	0.0000
D(SPOTPRICE(-2))	-0.395578	0.033765	-11.71552	0.0000
D(SPOTPRICE(-3))	-0.087004	0.032943	-2.641031	0.0084
D(SPOTPRICE(-4))	-0.096122	0.030223	-3.180404	0.0015
C	9.941809	2.433328	4.085685	0.0000

R-squared	0.231049	Mean dependent var	0.059432
Adjusted R-squared	0.227506	S.D. dependent var	26.72542
S.E. of regression	23.48942	Akaike info criterion	9.156462
Sum squared resid	598651.9	Schwarz criterion	9.183931
Log likelihood	-4988.850	F-statistic	65.20266
Durbin-Watson stat	2.023169	Prob(F-statistic)	0.000000

also performed Phillips–Perron tests. In these, the hypothesis for a unit root was rejected at the 1% significance level for the spot price as well as for the log spot price.

4.2 Futures and Forward Prices

Now we will focus on the futures and forward prices.[5] From the range of futures with delivery periods from days to seasons (1/3 of a year) and forwards with delivery periods from seasons to years, we ignore daily futures because of low trading activity on them. We also ignore Contracts for Difference (CfDs), because we are not interested in area prices and the difference between area prices and the system price. Therefore, during the period we regard, there remain between 19 and 29

[5]For a theoretical description of the contracts and an explanation of the traded products, see Sects. 3.3.2 and 3.3.3.

Table 4.3. *Augmented Dickey Fuller Test for a Unit Root in the Log Spot Price*

The report shows the results that Eviews gives for the log spot prices. The hypothesis of a unit root can be rejected at the 5% level.

Augmented Dickey-Fuller Unit Root Test on LOGSPOTPRICE

ADF Test Statistic	-3.075142	1% Critical Value*	-3.4391
		5% Critical Value	-2.8646
		10% Critical Value	-2.5684

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOGSPOTPRICE)
Method: Least Squares
Date: 07/04/03 Time: 17:17
Sample(adjusted): 10/06/1999 9/30/2002
Included observations: 1091 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOGSPOTPRICE(-1)	-0.032885	0.010694	-3.075142	0.0022
D(LOGSPOTPRICE(-1)	-0.238527	0.030811	-7.741605	0.0000
D(LOGSPOTPRICE(-2)	-0.287628	0.031237	-9.208076	0.0000
D(LOGSPOTPRICE(-3)	-0.130532	0.030929	-4.220431	0.0000
D(LOGSPOTPRICE(-4)	-0.136515	0.030070	-4.539965	0.0000
C	0.162377	0.052673	3.082745	0.0021

R-squared	0.126475	Mean dependent var	0.000378
Adjusted R-squared	0.122449	S.D. dependent var	0.125918
S.E. of regression	0.117957	Akaike info criterion	-1.431510
Sum squared resid	15.09652	Schwarz criterion	-1.404040
Log likelihood	786.8886	F-statistic	31.41870
Durbin-Watson stat	2.068276	Prob(F-statistic)	0.000000

futures and forwards that were listed and traded on the 750 trading days between 1 October 1999 and 30 September 2002.

To get an idea of the futures and forward prices, three graphs were plotted, see Figs. 4.6, 4.7 and 4.8.[6] In the first graph, the term structure of the futures is plotted. In the second graph, the term structure of the season forwards, and in the third graph, we see the term structure of the year forwards. In these graphs, one axis represents the observation days of all prices, another axis the delivery periods of the contracts, and the third axis is the price axis in NOK/MWh. I.e., the contracts are plotted on a particular observation day on the x-axis, with the same price for the whole delivery period on the y-axis. For example, refer to Fig. 4.8: If you fix a point on the x-axis called observation date, you can always detect on the y-axis three different year contracts lasting one year. A year contract has one price for the whole delivery period that is one year, so we see one price on the z-axis called Price for one contract on that day. The other graphs can be similarly understood. For example, we can find a lot of different contracts for each observation day on Fig. 4.6. The spot price is additionally plotted in all graphs. It can be interpreted

[6]We do not plot them in one figure because of intersections in the delivery time.

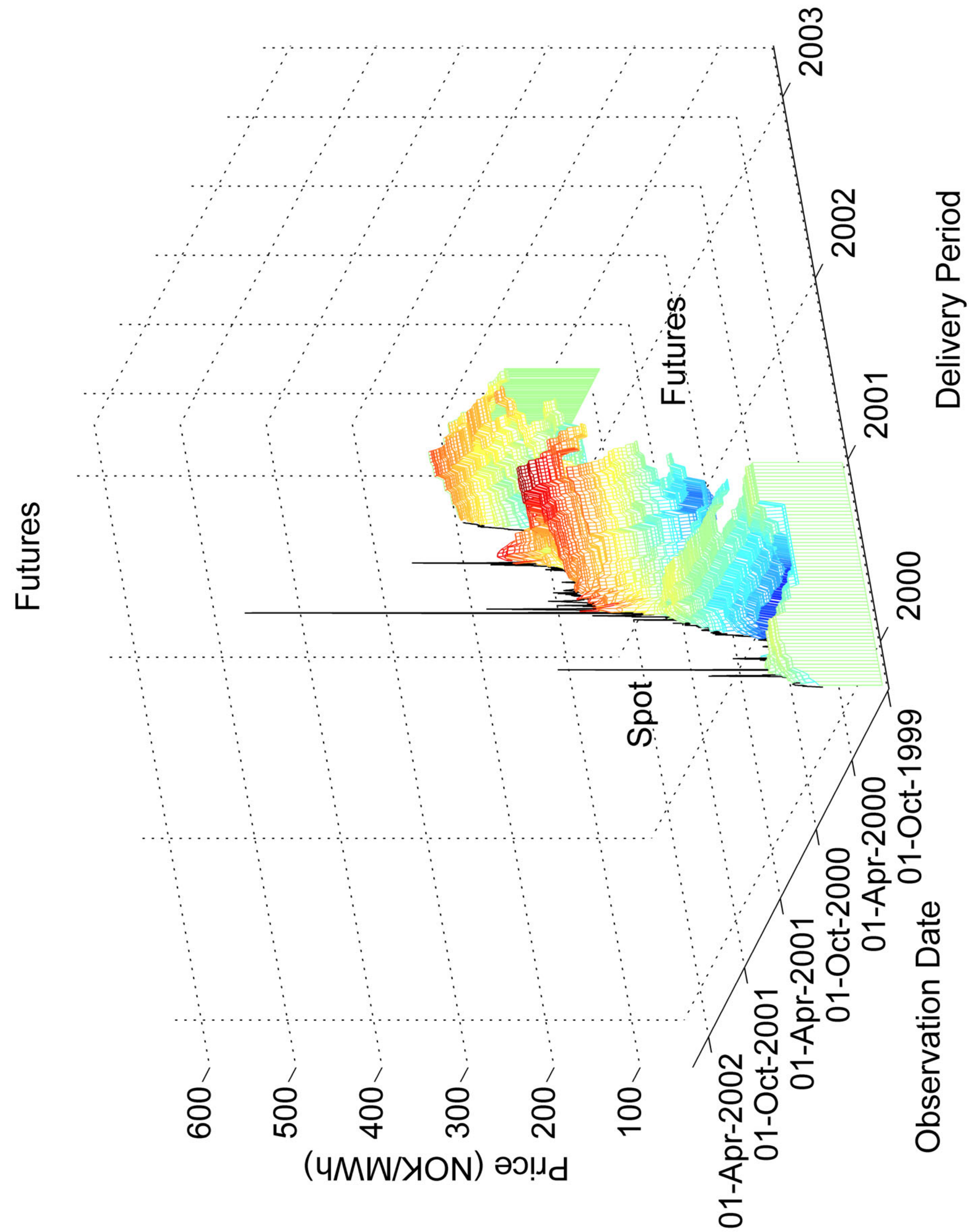

Figure 4.6. *Term Structure Futures*

All futures prices are plotted here. One axis represents the observation days, the second the delivery periods, and on the third axis, the prices are plotted. To draw comparisons, the spot price is added.

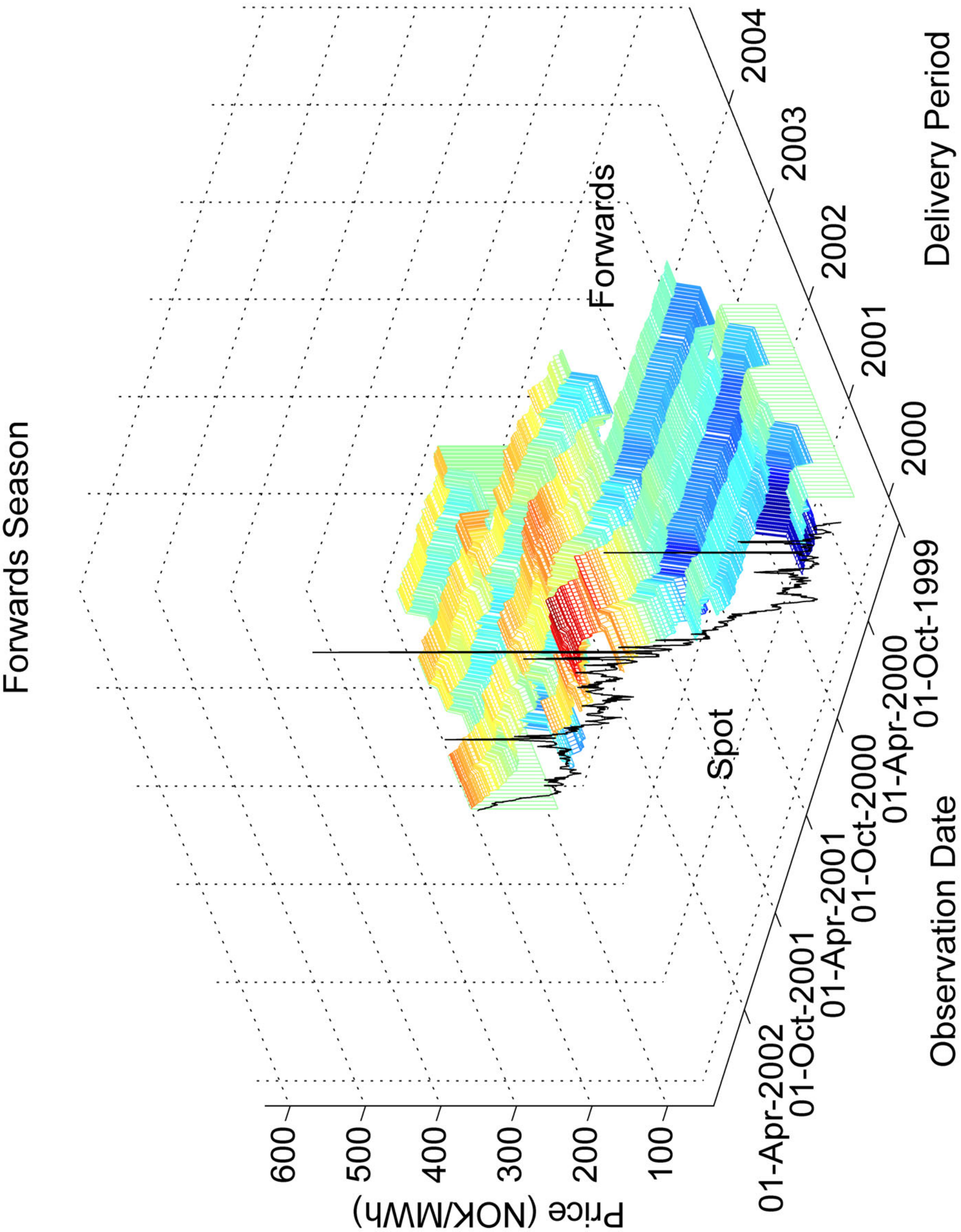

Figure 4.7. *Term Structure Season Forwards*

All Season Forward prices are plotted here. One axis represents the observation days, the second the delivery periods, and on the third axis, the prices are plotted. To draw comparisons, the spot price is added.

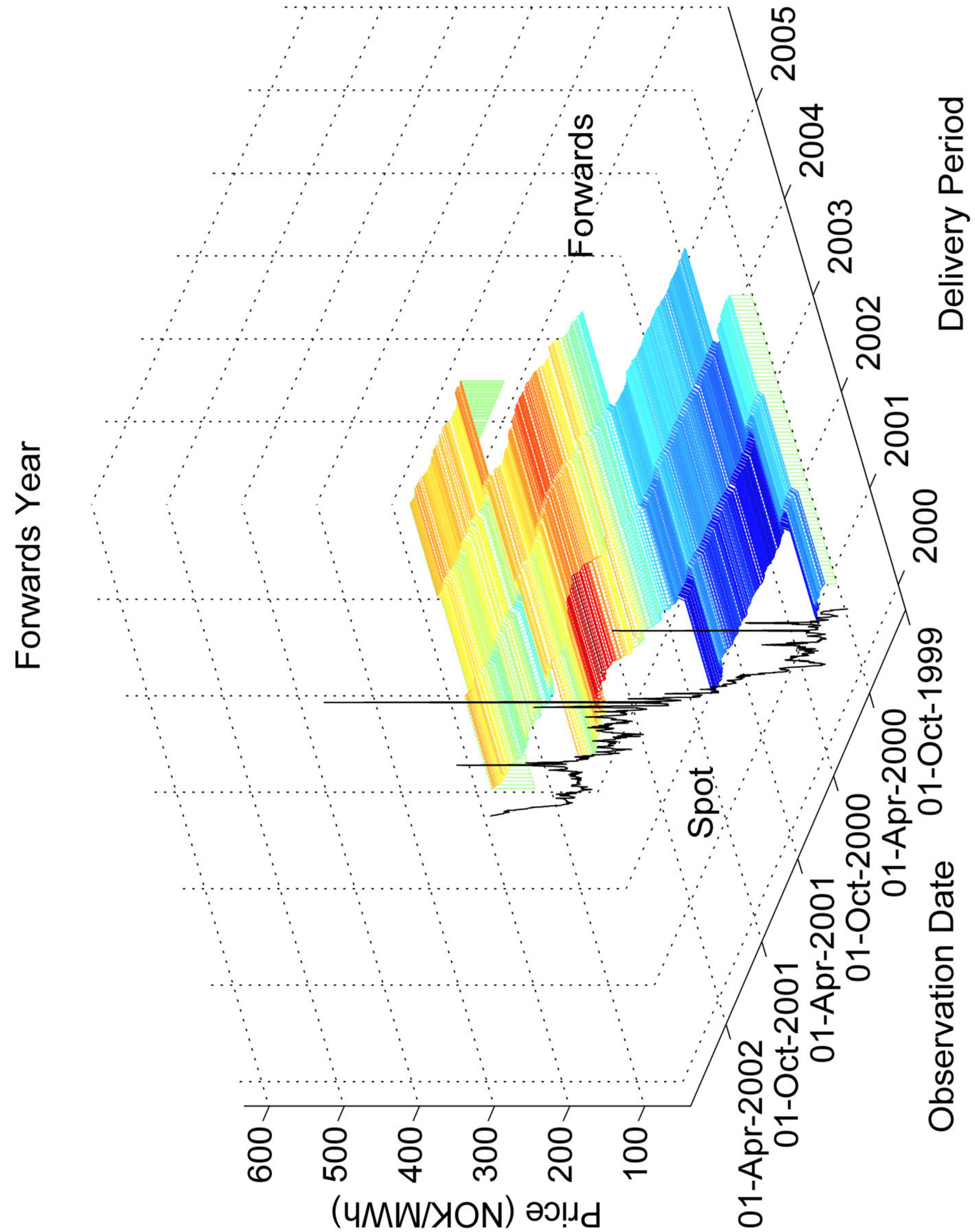

Figure 4.8. *Term Structure Year Forwards*

All Year Forward prices are plotted here. One axis represents the observation days, the second the delivery periods, and on the third axis, the prices are plotted. To draw comparisons, the spot price is added.

as futures for the next day. A further plot shows the whole term structure for the randomly chosen day 17 February 2000.[7]

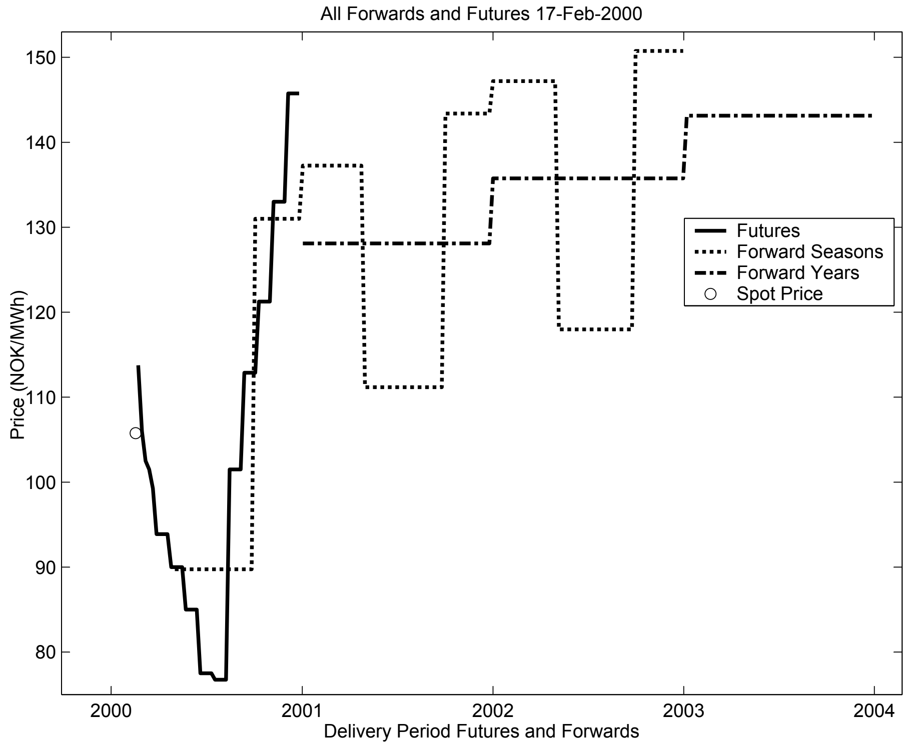

Figure 4.9. *Term Structure Futures and Forwards on 17 February 2000*
The figure shows that the different contracts intersect in their delivery times. We can observe the seasonal behaviour of the futures and season forwards, and the positive trend in the year forwards. Note that the spot price of NOK 105.77/MWh is below the futures price of NOK 113.75/MWh of the contract that starts four days later, on 21 February 2000.

As a comparison with the spot price shows, sudden price moves that are not permanent are not perceived in the futures and forward prices. The insensitivity to 'noisy' movements in the spot becomes more obvious the longer the delivery periods last.

Though a seasonal pattern is not clearly distinctive in the time series of the spot price or the log spot price, partially because the time interval was quite short, such a pattern can easily be distinguished in the term structures for the futures and seasonal forwards curves. The yearly forwards cannot exhibit seasonal patterns, they show clearly the phenomenon of an expected positive price trend or a risk premium that is increasing with time to maturity, see Figs. 4.8 and 4.9.

[7] See Fig. 4.9.

This is affirmed if we study the summary statistics in Table 4.4. The mean

Table 4.4. *Futures and Forwards*

The table shows the number of observations, mean, median and standard deviation for all futures and forwards, split in the different contract types. Season futures were only listed until 30 December 1999, so the number of observations is very low.

	Futures Week	Futures Block	Futures Season	All Futures
# obs.	4118	6836	72	11026
mean	148.81	151.74	146.32	150.61
median	149.38	148.82	148.88	149.00
stddev	40.24	36.81	8.06	38.05

	Forwards Season	Forwards Year	All Forwards	All Futures and Forwards
# obs.	5270	2232	7502	18528
mean	156.07	156.41	156.17	152.86
median	152.95	158.75	153.75	151.45
stddev	25.61	17.31	23.45	33.04

value of the contracts is higher for delivery periods that are further in the future and longer. This characteristic can be illustrated by comparing the means of the futures with NOK 150.61/MWh with the means of the season forwards with NOK 152.95/MWh and the means of the year forwards with NOK 156.41/MWh, though the absolute differences are not high.

Obviously, contracts with shorter delivery time, which are furthermore closer to the observation day, have higher standard deviations. For example, the week futures have a standard deviation of 40.24, whereas the year forwards have a standard deviation of 17.31. On one hand this can be explained by the observation that the short-term contracts with shorter delivery periods adopt, as already mentioned before, more of the volatility of the spot prices than the long-term contracts with longer delivery periods (see again Figs. 4.6, 4.7 and 4.8). Contracts with longer delivery periods are always less volatile than contracts with shorter delivery periods. On the other hand, this higher volatility is due to the seasonality effects in the short-term contracts that are not observed in year futures, as also previously discussed. Season futures, that were only traded until 30 December 1999, have for the same reason a very low standard deviation of 8.06, because our sample of this contract only covers two months, i.e. seasonal behaviour is not included in these prices.

If we compare the spot prices with the contract prices of futures and forwards, mean and median for the spot price are NOK 144.21/MWh and NOK 138.62/MWh,[8] and for the futures and forwards NOK 152.86/MWh and NOK 151.45/MWh.

The difference of spot and derivative prices is not very high, compared, for example, with the range of different spot prices from NOK 31.85/MWh to NOK 633.36/MWh in our period of three years, see again Fig. 4.3. This suggests that either

(i) there might be a small risk premium in the derivative prices, or
(ii) the prices are expected to rise slightly, or
(iii) both effects have an impact on the futures and forward prices, possibly in opposite directions.

4.3 Option Prices

Now let us investigate the option prices. Unfortunately, there is no real liquid option market until now at Nord Pool.[9] Most contracts are traded very rarely, see Table 4.5, where only prices were counted when trades took place.[10] It is obvious,

Table 4.5. *Option Contracts: Number of Trades*
The numbers refer to trades of option contracts on the exchange. The trading activity is very low. Asian-style options were only traded from 29 October 1999 to 20 April 2001.

	European	Asian	All
Calls	72	34	106
Puts	71	3	74
All	143	37	180

that, with these few trades, daily implicit estimations in the sense of Bakshi et al. (1997) are not possible. In addition, it is a question, if such a small sample can be of any empirical use at all. In any case, all examinations have to be conducted with caution.[11]

[8] See Fig. 4.3.
[9] For a description of the options traded at Nord Pool see Sect. 3.3.4.
[10] See *Nord Pool Rulebook for The Financial Electricity Market* for a description how closing prices are determined in the case of no trade.
[11] In Sects. 6.1.4, 6.3.4 and Appendix A.4, the data is used to assess the goodness of our estimations from futures and forwards, but only as one of various procedures.

Asian-style options were only traded between 29 October 1999 and 20 April 2001, that is one and a half years, but trade on calls took place only 34 times, and trade on Asian puts took place only three times during this period. European puts are more liquid in Nord Pool than their Asian counterparts. Between the end of October 1999 and the end of September 2002, 72 European calls and 71 European puts were traded. Since the time period was twice the period that Asian-style options were traded, we can see that European options are about as liquid as were the Asian calls. The total of closing prices of options is 180. The average we get in 750 trading days is a trading activity of about one trade every four days.

The option prices are also plotted. Figures 4.10 and 4.11 show all prices for

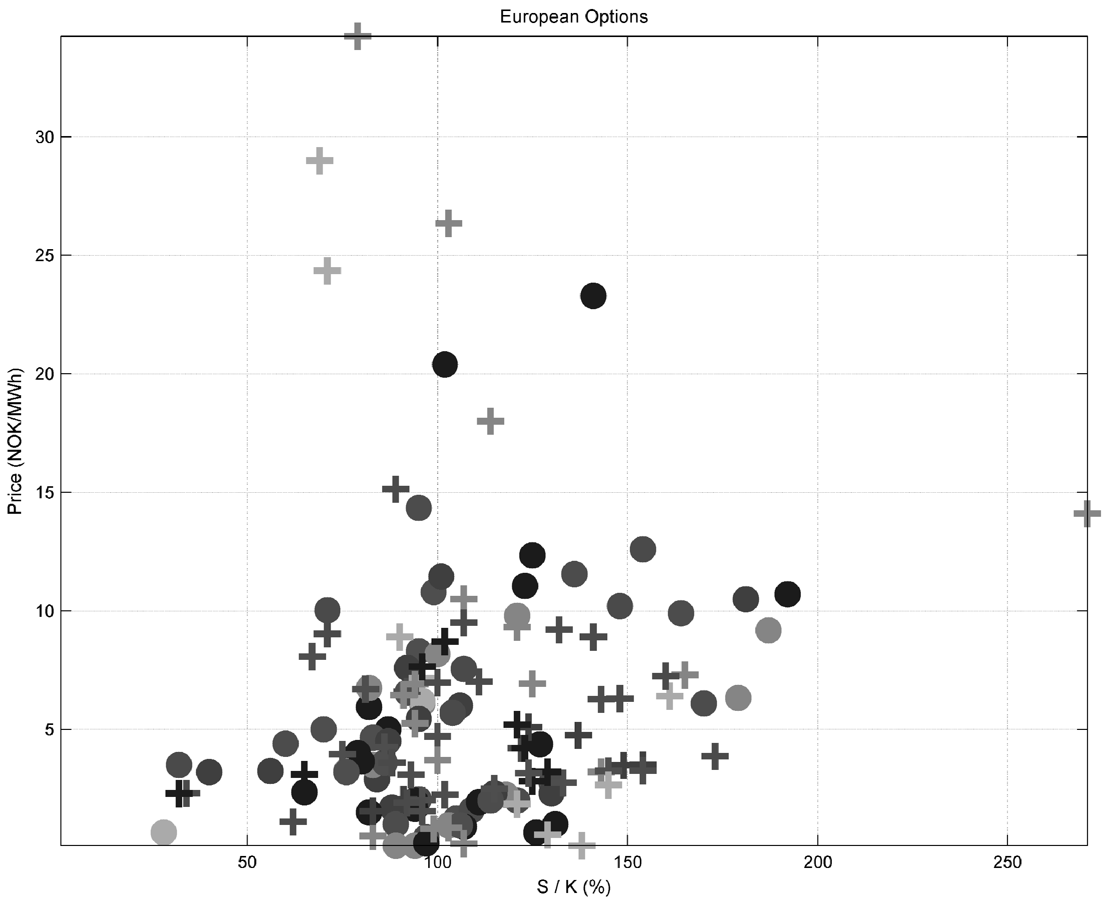

Figure 4.10. *Term Structure European Options*

One axis shows the quotient spot price / exercise price, the second axis the contract price. The different grey shadings correspond to different expiry dates, i.e. the darker the data point, the earlier expiry. Calls are represented as circles, puts as pluses.

calls and puts. One axis shows the quotient S/K, i.e. the actual spot price divided

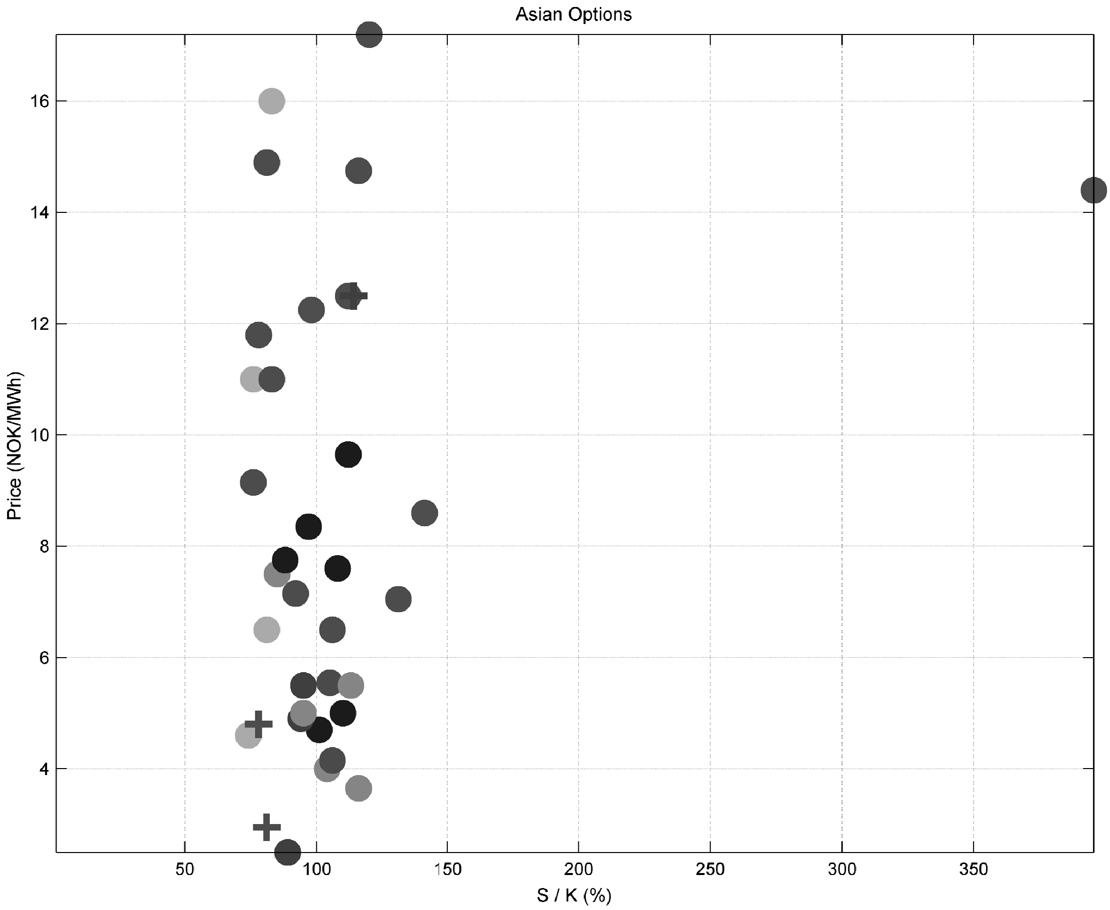

Figure 4.11. *Term Structure Asian Options*
One axis shows the quotient spot price / exercise price, the second axis the contract price. The different grey shadings correspond to different expiry dates, i.e. the darker the data point, the earlier expiry. Calls are represented as circles, puts as pluses. Asian-style puts were only traded three times, calls 34 times, see also Table 4.5.

by the strike price, and the second axis is the axis of option prices.[12] Calls are plotted as circles, and put options are marked as pluses. Options with the same dates of expiry have the same grey shading, i.e. the darker ones have earlier expiry days than the lighter ones. The graphs reveal that, while most European options are traded at the money or with a coefficient of S/K of over 100%, most Asian options are traded at the money, i.e. with a coefficient of S/K of about 100%.

[12]For calls, S/K is also called moneyness, for puts, moneyness is defined as K/S. It may be a matter of taste and is of no relevance if the quotient S/K for both, calls and puts, or moneyness is used; we decided to use S/K.

Chapter 5

Estimated Models

In this chapter, the models are first introduced and then $E_t[S_T]$ is calculated with Corollary 2.2. All models directly describe the assumed process of the log prices X_t, instead of first proposing models for the price process S_t and then transforming to $X_t = \log(S_t)$ or equivalents.

5.1 Model Group 1 – Ornstein–Uhlenbeck Processes

The basic model for this group is an Ornstein–Uhlenbeck process (OU) already used by Vasicek (1977) to model the term structure of interest rates. It is also used for log prices in Schwartz (1997) in order to price and hedge copper, oil, and gold. Furthermore, it is employed as the stochastic process that describes the movement of log prices of oil in Bühler et al. (2000). Note also, that these models, noted under their risk-neutral probability measures, do not evolve with the drift term $r_t - \frac{1}{2}v_t$, as is necessary for tradable assets. See the discussion of this topic in Sect. 2.1. For further comments on this see also the next subsection.

Model OU

The stochastic differential equation is

$$dX_t = \kappa(\theta - X_t)dt + \sigma dZ_t, \tag{5.1}$$

with $X_t = \log(S_t)$. The vector of parameters that has to be estimated is $\vec{\vartheta} = (\kappa, \theta, \sigma)$, and Z_t is a standard Brownian motion. The state variable X_t moves around its long-term mean θ, and it has normal disturbances with standard deviations of σ.

κ measures the strength of the mean reversion, it determines how fast X_t is "pulled back" in the direction of θ.

For the valuation of electricity futures and forwards we also need the instantaneous risk rate r. Conforming with Nord Pool's Clearing House NECH we chose this rate to be a constant 6% for all models.[1] Bakshi et al. (1997) include stochastic interest rates in their models, but have to recognise that they only improve their models for long-term options. While they regard stock options, we assume that in our models of electricity derivatives, shifts in the interest rates have even less impact on the prices than for derivatives on tradable assets.

Model OUJ

Our second model is based on the Ornstein–Uhlenbeck process. We get the model

$$dX_t = \kappa(\theta - X_t)dt + \sigma dZ_t + JdP_t, \tag{5.2}$$

where P_t follows a Poisson distribution with intensity λ, J is the jump size that is normally distributed with parameters μ_J and σ_J, and the rest is similar to model OU. The Brownian motion, the Poisson distribution and the jump size distribution are all pairwise uncorrelated. The 'J' in OUJ is used as abbreviation for 'Jump'. $\vec{\vartheta} = (\kappa, \theta, \sigma, \lambda, \mu_J, \sigma_J)$.

This model was chosen by Das (2002) for interest rates. He estimates the parameters from a time series of the Fed Funds rate via Maximum Likelihood and uses the conditional characteristic function to calculate the moments of his process.

Models OU and OUJ were employed by Lien and Strøm (1999) for pricing commodities. They priced wheat prices, modelling the prices like the X_t above, whereas we consider the state variable X_t to be the log prices.

Model OUS

To improve our results, we expand the first two models to be able to map seasonal features in our data. This model is the Ornstein–Uhlenbeck process OU, but X_t is

[1]See *Nord Pool Rulebook for The Financial Electricity Market* (2002, p. 55).

defined as $X_t = \log(S_t - f(t))$, i.e. $S_t = f(t) + \exp(X_t)$, where $f(t)$ is a deterministic function defined as

$$f(t) = s_0 \cos\left(\frac{2\pi}{365}(t + 365 s_1)\right), \tag{5.3}$$

s_0 and s_1 being further parameters.[2] With $f(t)$, we want to map the seasonal behaviour of electricity prices. The parameter s_0 controls the deterministic amount that the prices fluctuate over the course of the year (as a factor in front of the cosine term, that changes continuously from -1 to 1 and back); and s_1 determines the point of the cosine where the deterministic function $f(0)$ starts. The 'S' for 'Season' in OUS emphasises this new characteristic. The vector of parameters is $\vec{\vartheta} = (\kappa, \theta, \sigma, s_0, s_1)$.

The first who applied sinusoidal curves to the seasonal regularities of electricity prices were Lucia and Schwartz (2002), see also Chap. 1. One of their models is

$$S_t = f(t) + X_t, \tag{5.4}$$

with X_t defined as

$$dX_t = -\kappa X_t dt + \sigma dZ_t. \tag{5.5}$$

Another model specifies the log spot price as $\log S_t = f(t) + X_t$.
One of their specifications for the deterministic function $f(t)$ is

$$f(t) = \alpha + \beta D_t + \gamma \cos\left(\frac{2\pi}{365}(t + \delta)\right), \tag{5.6}$$

where D_t is defined as

$$D_t = \begin{cases} 1; & \text{if date t is a holiday or weekend} \\ 0; & \text{otherwise} \end{cases} \tag{5.7}$$

[2] s_1 is defined as lying between 0 and 1, mainly because of numerical reasons.

The models are very similar.[3] To be able to continue using the Ornstein–Uhlenbeck process as in model OU, we chose to omit Lucia and Schwartz's (2002) α in (5.6). Furthermore, we do not use the dummy variable D_t, because intraweekly patterns of the electricity prices are irrelevant for us. There are three reasons for this:

1. When implicitly estimating from futures and forward prices, we are only estimating on trading, i.e. working days.
2. The delivery times of the contracts are either multiple weeks or cover large periods such as seasons or years. Then it is irrelevant or negligible if the contracts start or end on weekdays or weekends.
3. Introducing D_t only for holidays does not result in significant estimates.

Model OUJS

Model OUJ is extended like OU. Model OUJS is the same as model OU with the jump component of OUJ and the seasonal component of OUS. $\vec{\vartheta} = (\kappa, \theta, \sigma, \lambda, \mu_J, \sigma_J, s_0, s_1)$.

5.2 Model Group 2 – Introducing Stochastic Volatility

With models of this group, stochastic volatility is introduced.[4] We rely here on a model that is similar to the model that is examined by Heston (1993). Some extensions of this model with stochastic volatility are employed by Duffie et al. (2000) in a similar form to ours. The authors suggest their models with exponential distributed jumps in the variance process for S&P-futures. Their aim is to improve the models used by Bakshi et al. (1997) that include stochastic volatility, jumps in

[3]Advanced approaches to map seasonality effects are presented for example in Anderson (1971, Chap. 4.3). Our aim in this work was to keep the number of parameters for seasonality low, to be able to emphasize our study more on stochastic components. Furthermore, the approach taken here seemed quite promising and was applied in similar form, apart from Lucia and Schwartz (2002), e.g. by Escribano et al. (2002), see also Chap. 1.
Stochastic seasonality modelled with ARIMA-processes cannot be easily included in this framework, because they violate in general the Markov property. For more details on ARIMA models, see Box and Jenkins (1976).

[4]Following other authors, we often speak of stochastic volatility though the stochastic process defined is the stochastic process for the variance. Because the volatility is the square root of the variance, transformations are straightforward, see e.g. in Heston (1993), and we will not address this topic further. For models with explicit use of volatility see Schöbel and Zhu (1999).

the spot price and stochastic interest rates. Our extensions of Heston (1993) can be seen as generalisations of the models of Duffie et al. (2000).

Model H

This model with stochastic volatilities is abbreviated H because Heston (1993) was the first to find a closed-form solution for the conditional characteristic function for this kind of model. It is defined as follows:

$$\begin{aligned} dX_t &= (\alpha + \beta v_t)dt + \sqrt{v_t}dZ_t^1, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dZ_t^2. \end{aligned} \tag{5.8}$$

Z_t^1 and Z_t^2 are standard Brownian motions with $dZ_t^1 dZ_t^2 = \rho dt$. The parameters that have to be estimated are $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho)$. Note that $\sqrt{v_t}$ is estimated like a parameter.[5] This means that we are looking for the daily volatility that can explain the futures and forwards prices best, given the spot price S_t.

Consider the special case $\alpha = r$ and $\beta = -\frac{1}{2}$ that are implied by tradable assets, see (2.1), and that lead to the models used for example in Bakshi et al. (1997). Like was shown in Sect. 2.1, (2.1) does not need to hold for electricity, and, like can easily be seen e.g. in Figs. 4.6, 4.7, and 4.8, apparently does not hold for electricity. The variation of futures and forwards obviously cannot be explained by the variation of interest rates. Therefore, we allow in this model group for free parameters α and β, that are estimated in our minimization algorithm.[6]

Model HXJ

We extend — like in our first model group — our basic model and add jumps. In Model HXJ, used in a similar form already by Bakshi et al. (1997), a jump with a normal distributed jump size is added to the log spot price. XJ in the model name

[5] See also Bakshi et al. (1997) and Sect. 2.4.

[6] The first model group, presented in the last section, does not allow for the drift term $(r - \frac{1}{2})dt$, neither, but, apart from the seasonality components, we proposed a drift term with mean reversion $\kappa(\theta - X_t)$, see also Sect. 5.1. Mean reversion is in accordance with other models for commodities and electricity, see e.g. Bühler et al. (2000), Lucia and Schwartz (2002) and Chap. 1. See also the last footnote in Sect. 2.3.

is used to express jumps in X_t.

The process is

$$\begin{aligned} dX_t &= (\alpha + \beta v_t)dt + \sqrt{v_t} dZ_t^1 + J dP_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma \sqrt{v_t} dZ_t^2, \end{aligned} \tag{5.9}$$

where everything from model H applies and the jump component is defined as in model OUJ, i.e. P_t follows a Poisson distribution with intensity λ and $J \sim N(\mu_J, \sigma_J^2)$. The correlation of the Brownian motions is defined like before as $dZ_t^1 dZ_t^2 = \rho dt$. The Brownian motions are uncorrelated with the Poisson distribution and the jump size distribution. The latter two are also uncorrelated. $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J)$.

Model HVJ

In this model, see also Duffie et al. (2000), jumps are not modelled for the log spot price, but for the variance process:

$$\begin{aligned} dX_t &= (\alpha + \beta v_t)dt + \sqrt{v_t} dZ_t^1, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma \sqrt{v_t} dZ_t^2 + J dP_t, \\ dZ_t^1 dZ_t^2 &= \rho dt. \end{aligned} \tag{5.10}$$

In the model name, VJ is used to express jumps in v_t. However, the jump size is not normally distributed but rather exponentially with mean η. The correlations are similar to the last model. $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \eta)$.

The domain of the exponential distribution is $[0, \infty)$, with mode at zero, and mean and variance η and η^2, respectively. η must of course be positive. An example for a pdf of an Exponential distribution can be viewed in Fig. 5.1.

This kind of jump is very convenient for our purposes, because the jump sizes are always positive, so the variance cannot become negative. Furthermore, the Exponential distribution is analytically very tractable, and only one parameter which determines mean as well as variance has to be estimated. And, it can easily be generalised, as we can see in the next model.

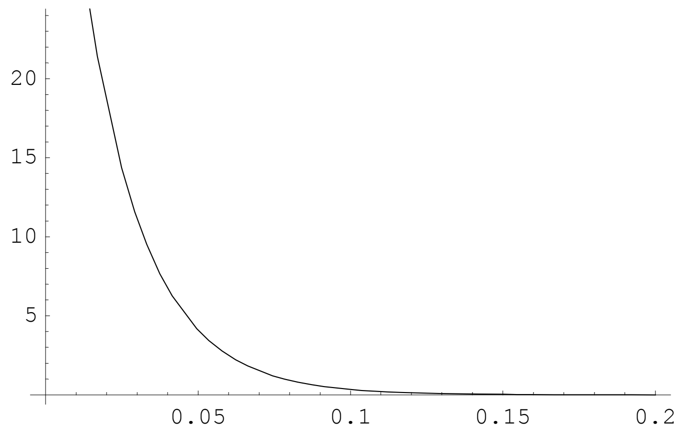

Figure 5.1. *Example of an Exponential distribution*
pdf of an Exponential distribution, $\eta = 0.02$.

Model HVJG

This model is the same as model HVJ, but we define here a jump size in the variance process that is distributed $\sim \Gamma(\frac{1}{\eta}, \gamma)$. G for Gamma in the model name illustrates this. $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \eta, \gamma)$.

The domain of the Gamma distribution is like that of the exponential distribution $[0, \infty)$, with $\eta, \gamma > 0$. Mean and variance are $\eta\gamma$ and $\eta^2\gamma$, but the mode for $\gamma > 1$ is not simply zero like in the exponential distribution but rather $\eta(\gamma - 1)$.[7] For examples of Gamma distributions, see the pdfs in Figs. 5.2 and 5.3.

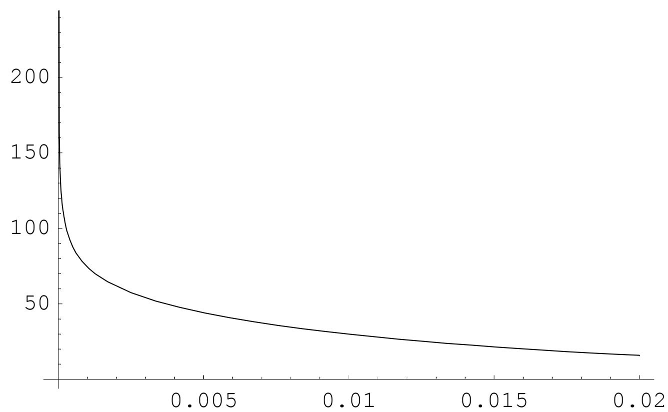

Figure 5.2. *Example 1 of a Gamma distribution*
pdf of a Gamma distribution, $(\eta, \gamma) = (0.02, 0.80)$.

[7] For the characteristics of the Gamma distribution see, for example, Resa Corporation and Licensors (2000–2003).

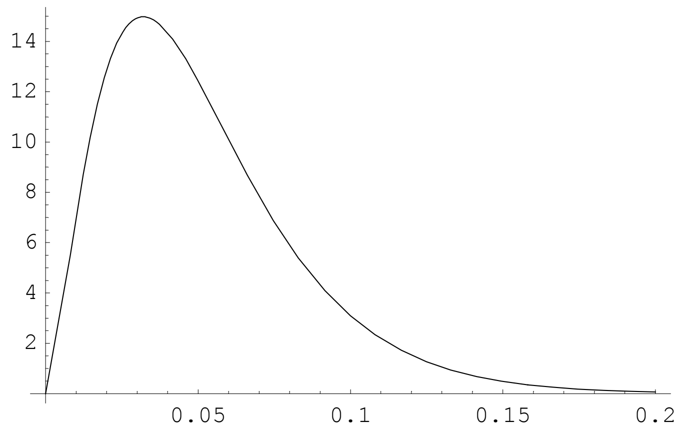

Figure 5.3. *Example 2 of a Gamma distribution*
pdf of a Gamma distribution, $(\eta, \gamma) = (0.02, 2.6)$.

The following connection between the exponential and the Gamma distribution can be pointed out:
If

$$Y_i \quad i.i.d. \sim \text{Exp}(\frac{1}{\eta}), \quad i = 1 \dots r, \tag{5.11}$$

then

$$\sum_{i=1}^{r} Y_i \sim \Gamma(\frac{1}{\eta}, r). \tag{5.12}$$

This connection is of course only valid for $r \in \mathbb{N}$, but γ in $\Gamma(\frac{1}{\eta}, \gamma)$ is defined for all $\gamma \in \mathbb{R}^+$. Thus, the Gamma distribution is not only more flexible than the Exponential, but a Gamma distributed jump J can also be seen as the sum of simultaneously occurring Exponential distributed jumps, or one jump with Gamma distribution. If the model were overspecified with this extension, we would expect estimates for γ that are close to 1.

Model HXVJ

In this model, that is again similar to one model in Duffie et al. (2000), we assume that jumps in the log spot price and the variance process occur simultaneously.

$$\begin{aligned} dX_t &= (\alpha + \beta v_t)dt + \sqrt{v_t}dZ_t^1 + J_X dP_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dZ_t^2 + J_v dP_t \\ dZ_t^1 dZ_t^2 &= \rho dt. \end{aligned} \tag{5.13}$$

The jump size in the variance process is exponentially distributed with mean η. Given a jump in v_t of z_v, the jump size in X_t is normally distributed with mean $\mu_J + \rho_J z_v$ and variance σ_J^2. The Poisson-distributed jump intensity and jump sizes are uncorrelated with the Brownian motions. Jump intensity and sizes are also uncorrelated. $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J, \eta, \rho_J)$.

Note that the instantaneous correlation coefficient of jump sizes in X_t and in v_t is not ρ_J, but can be calculated as[8]

$$\frac{\rho_J \eta}{\sqrt{\sigma_J^2 + \rho_J^2 \eta^2}}. \tag{5.14}$$

Model HXVJG

This model is the same as model HXVJ, but we define here again the jumps in v_t to be distributed $\sim \Gamma(\frac{1}{\eta}, \gamma)$. The vector of parameters for this model is $\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J, \eta, \gamma, \rho_J)$.

The correlation coefficient of the jump sizes of simultaneous jumps in X_t and in v_t is

$$\frac{\rho_J \eta \sqrt{\gamma}}{\sqrt{\sigma_J^2 + \rho_J^2 \eta^2 \gamma}}. \tag{5.15}$$

5.3 Model Group 3 – Stochastic Volatility and Season

In model group 3 we extend model group 2 with an additional deterministic function to capture the seasonality of electricity prices. $S_t = f(t) + \exp(X_t)$, and

$$f(t) = s_0 \cos\left(\frac{2\pi}{365}(t + 365 s_1)\right), \tag{5.16}$$

like first mentioned in model OUS, see p. 50. To characterize the models, they are described with the parameter vector $\vec{\vartheta}$:

Model HS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, s_0, s_1)$.

Model HXJS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J, s_0, s_1)$.

[8] See Duffie et al. (2000, p. 25).

Model HVJS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \eta, s_0, s_1)$.

Model HVJGS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \eta, \gamma, s_0, s_1)$.

Model HXVJS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J, \eta, \rho_J, s_0, s_1)$.

Model HXVJGS

$\vec{\vartheta} = (\alpha, \beta, \sqrt{v_t}, \kappa, \theta, \sigma, \rho, \lambda, \mu_J, \sigma_J, \eta, \gamma, \rho_J, s_0, s_1)$.

5.4 Solutions for $E_t[S_T]$

In the next two subsections, $E_t[S_T]$ is solved for all models. We need it for the calculation of the futures and forward prices, see (2.27).

5.4.1 Solutions for Models OU and OUJ

We restrict ourselves to the solution for model OUJ; for model OU, the solution is then obtained by setting $\lambda = \mu_J = \sigma_J = 0$.

The solution can also be derived from Das (2002) who employs a similar model to (OUJ) for interest rates. Though he does not need the characteristic function for his time series estimations, $\Phi(u, \tau, \vec{\vartheta}, X_t)$ is used to derive the moments of the distribution of X_t, see also Sect. 2.2.2.

In order to find a closed-form solution of the general conditional characteristic function $\Phi(u, \tau, \vec{\vartheta}, X_t)$, as defined in (2.24), we apply Corollary 2.2 for model OUJ to obtain

$$\begin{aligned} D\Phi &= \kappa(\theta - X_t)\frac{\partial \Phi}{\partial X_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 \Phi}{\partial X_t^2} \\ &\quad + \lambda \int_{\mathbb{R}} (\Phi(u, \tau, \vec{\vartheta}, X_t + J) - \Phi(u, \tau, \vec{\vartheta}, X_t)) d\nu(J) \\ &\quad - \frac{\partial \Phi}{\partial \tau} \stackrel{!}{=} 0, \end{aligned} \tag{5.17}$$

and boundary condition $\Phi(u, 0, \vec{\vartheta}, X_t) = \exp(uX_t)$.

Assuming that $\Phi(u,\tau,\vec{\vartheta},X_t)$ has the form $\Phi(u,\tau,\vec{\vartheta},X_t) = \exp(A(u,\tau,\vec{\vartheta})X_t + B(u,\tau,\vec{\vartheta}))$, and because the size of a jump J at time T is independent from $\{X_s, 0 \leq s < T\}$, the following ordinary differential equations (ODE) have to be solved:

$$\begin{aligned} \kappa\theta A(u,\tau,\vec{\vartheta}) + \frac{1}{2}\sigma^2 A^2(u,\tau,\vec{\vartheta}) + & \\ \lambda \int_{\mathbb{R}} (\exp(A(u,\tau,\vec{\vartheta})J) - 1) d\nu(J) - \frac{\partial B(u,\tau,\vec{\vartheta})}{\partial \tau} &\stackrel{!}{=} 0, \quad (5.18) \\ -\kappa A(u,\tau,\vec{\vartheta}) - \frac{\partial A(u,\tau,\vec{\vartheta})}{\partial \tau} &\stackrel{!}{=} 0, \quad (5.19) \end{aligned}$$

with boundary conditions $A(u,0,\vec{\vartheta}) \stackrel{!}{=} u$ and $B(u,0,\vec{\vartheta}) \stackrel{!}{=} 0$.
Being

$$E[\exp(cJ)] = \exp(\mu_J c + \frac{1}{2}\sigma_J^2 c^2) \quad (5.20)$$

for all $c \in \mathbb{C}$ and $J \sim N(\mu_J, \sigma_J^2)$, and the solution of $A(u,\tau,\vec{\vartheta})$ being

$$A(u,\tau,\vec{\vartheta}) = u\mathrm{e}^{-\kappa\tau}, \quad (5.21)$$

$\int_{\mathbb{R}} (\exp(A(u,\tau,\vec{\vartheta})J) - 1) d\nu(J)$ can be simplified to

$$\int_{\mathbb{R}} (\exp(u\mathrm{e}^{-\kappa\tau}J) - 1) d\nu(J) = \exp(\mu_J u\mathrm{e}^{-\kappa\tau} + \frac{1}{2}\sigma_J^2 u^2 \mathrm{e}^{-2\kappa\tau}) - 1, \quad (5.22)$$

and the solution for $B(u,\tau,\vec{\vartheta})$ is:

$$\begin{aligned} B(u,\tau,\vec{\vartheta}) &= u\theta(1 - \mathrm{e}^{-\kappa\tau}) + \frac{\sigma^2 u^2}{4\kappa}(1 - \mathrm{e}^{-2\kappa\tau}) \\ &\quad -\lambda\tau + \lambda \int_0^\tau \exp(\mu_J u\mathrm{e}^{-\kappa t} + \frac{1}{2}\sigma_J^2 u^2 \mathrm{e}^{-2\kappa t}) dt. \quad (5.23) \end{aligned}$$

The last integral must be solved numerically for $u = 1$. Then, as already shown in Chap. 2, $E_t[S_T] = \Phi(1, T-t, \vec{\vartheta}, X_t)$.

The following remarks are in order:

- For all models of this model group, $\Phi(u,\tau,\vec{\vartheta},X_t)$ is bounded and well-defined for all finite parameters and $\kappa > 0$, so $E_t[S_T]$ is always finite and $\in \mathbb{R}$.

- Note that already for the basic model, $E_t[S_T] = \exp(\mathrm{e}^{-\kappa\tau}X_t + (1-\mathrm{e}^{-\kappa\tau})\theta + \frac{\sigma^2}{4\kappa}(1-\mathrm{e}^{-2\kappa\tau}))$, and therefore $E_t[S_T]$ is not independent from σ.
- $\log(E_t[S_T])$ will not necessarily lie between X_t and θ: $0 < \mathrm{e}^{-\kappa\tau} < 1$ for $\kappa, \tau > 0$, and $\alpha X_t + (1-\alpha)\theta$ is for $0 \le \alpha \le 1$ the line between X_t and θ. So $\log(E_t[S_T])$ might be bigger than X_t and θ because and only because of the term $\frac{\sigma^2}{4\kappa}(1-\mathrm{e}^{-2\kappa\tau}) > 0$.

5.4.2 Solutions for Models HXJ, HVJ, HVJG, HXVJ and HXVJG

Without specifying any type of jumps and with $\vec{Y}_t = (X_t, v_t)'$, Corollary 2.2 for $\Phi(u, \tau, \vec{\vartheta}, \vec{Y}_t)$ leads to

$$\begin{aligned} D\Phi &= (\alpha+\beta v_t)\frac{\partial\Phi}{\partial X_t} + \kappa(\theta - v_t)\frac{\partial\Phi}{\partial v_t} + \frac{1}{2}v_t\frac{\partial^2\Phi}{\partial X_t^2} \\ &+\frac{1}{2}\sigma^2 v_t\frac{\partial^2\Phi}{\partial v_t^2} + \rho\sigma v_t\frac{\partial^2\Phi}{\partial X_t\partial v_t} \\ &+\lambda\int_{\mathbb{R}^2}(\Phi(u,\tau,\vec{\vartheta},\vec{Y}_t+\vec{J}) - \Phi(u,\tau,\vec{\vartheta},\vec{Y}_t))d\nu(\vec{J}) - \frac{\partial\Phi}{\partial\tau} \stackrel{!}{=} 0. \end{aligned} \tag{5.24}$$

Assuming that the solution for $\Phi(u, \tau, \vec{\vartheta}, \vec{Y}_t)$ is

$$\Phi(u,\tau,\vec{\vartheta},\vec{Y}_t) = \exp(uX_t + A(u,\tau,\vec{\vartheta})v_t + B(u,\tau,\vec{\vartheta})), \tag{5.25}$$

we can write (5.24) as

$$\begin{aligned} D\Phi &= (\alpha+\beta v_t)u\Phi + \kappa(\theta - v_t)A(u,\tau,\vec{\vartheta})\Phi + \frac{1}{2}v_t u^2\Phi \\ &+\frac{1}{2}\sigma^2 v_t A^2(u,\tau,\vec{\vartheta})\Phi + \rho\sigma v_t u A(u,\tau,\vec{\vartheta})\Phi \\ &+\lambda\int_{\mathbb{R}^2}(\Phi(u,\tau,\vec{\vartheta},\vec{Y}_t+\vec{J}) - \Phi(u,\tau,\vec{\vartheta},\vec{Y}_t))d\nu(\vec{J}) \\ &-\frac{\partial A(u,\tau,\vec{\vartheta})}{\partial\tau}v_t\Phi - \frac{\partial B(u,\tau,\vec{\vartheta})}{\partial\tau}\Phi \stackrel{!}{=} 0. \end{aligned} \tag{5.26}$$

The size of a jump $\vec{J}$ at time T is independent from $\{Y_s, 0 \le s < T\}$, therefore we can simplify $\int_{\mathbb{R}^2}(\Phi(u,\tau,\vec{\vartheta},\vec{Y}_t+\vec{J}) - \Phi(u,\tau,\vec{\vartheta},\vec{Y}_t))d\nu(\vec{J})$ to $\Phi(u,\tau,\vec{\vartheta},\vec{Y}_t)\int_{\mathbb{R}^2}(\exp(uJ_x+$

$A(u,\tau,\vec{\vartheta})J_v) - 1)d\nu(J_x, J_v)$, where (J_x, J_v) are the jump sizes in the log spot price and the variance process, respectively.

The following ODEs remain:

$$\begin{aligned}
\frac{1}{2}\sigma^2 A^2(u,\tau,\vec{\vartheta}) + (\rho\sigma u - \kappa)A(u,\tau,\vec{\vartheta}) & \\
+\frac{1}{2}u^2 + \beta u - \frac{\partial A(u,\tau,\vec{\vartheta})}{\partial \tau} &\stackrel{!}{=} 0, \qquad (5.27)\\
\kappa\theta A(u,\tau,\vec{\vartheta}) + \alpha u & \\
+\lambda \int_{\mathbb{R}^2} (\exp(uJ_x + A(u,\tau,\vec{\vartheta})J_v) - 1)d\nu(J_x, J_v) - \lambda & \\
-\frac{\partial B(u,\tau,\vec{\vartheta})}{\partial \tau} &\stackrel{!}{=} 0, \qquad (5.28)
\end{aligned}$$

with boundary conditions $A(u,0,\vec{\vartheta}) \stackrel{!}{=} 0$ and $B(u,0,\vec{\vartheta}) \stackrel{!}{=} 0$.

Ignoring $\int_{\mathbb{R}^2}(\exp(uJ_x + A(u,\tau,\vec{\vartheta})J_v) - 1)d\nu(J_x, J_v)$ at first, we can apply the solution of Heston (1993) to the rest, leading to

$$\begin{aligned}
A(u,\tau,\vec{\vartheta}) &= \frac{-h+d}{\sigma^2}\left(\frac{1-\mathrm{e}^{d\tau}}{1-g\mathrm{e}^{d\tau}}\right),\\
B(u,\tau,\vec{\vartheta}) &= \alpha u\tau + \frac{\kappa\theta}{\sigma^2}\left((-h+d)\tau - 2\log\left(\frac{1-g\mathrm{e}^{d\tau}}{1-g}\right)\right)\\
&\quad -\lambda\tau + \lambda\int_0^\tau \left(\int_{\mathbb{R}^2} (\exp(uJ_x + A(u,\tau,\vec{\vartheta})J_v))d\nu(J_x, J_v)\right) ds,\\
g &= \frac{-h+d}{-h-d},\\
d &= \sqrt{h^2 - \sigma^2(2\beta + u^2)},\\
h &= \rho\sigma u - \kappa. \qquad (5.29)
\end{aligned}$$

Consider the case $\beta = -\frac{1}{2}$ and $\lambda = 0$. $\varPhi(1,\tau,\vec{\vartheta},X_t)$ reduces then to

$$\varPhi(1,\tau,\vec{\vartheta},X_t) = \exp(X_t + \alpha\tau). \qquad (5.30)$$

Indeed, this would imply that the futures prices only depend on α, valid for tradable assets with $\alpha \equiv r$, see (2.1). So, in our empirical studies, we test for $\beta = -\frac{1}{2}$, but unfortunately, caused by high standard errors, the results are not very clear. Nevertheless, the estimations for β are not close to $-\frac{1}{2}$, see Sect. 6.3.

The rest of the solution depends on the type of jump. For each model with jump, we first calculate $\int_{\mathbb{R}^2}(\exp(uJ_x + A(u,\tau,\vec{\vartheta})J_v) - 1)d\nu(J_x,J_v)$ and then $\int_0^\tau \left(\int_{\mathbb{R}^2}(\exp(uJ_x + A(u,t,\vec{\vartheta})J_v) - 1)d\nu(J_x,J_v)\right) dt$.
The solution of $E_t[S_T]$ is $\Phi(1, T-t, \vec{\vartheta}, X_t)$.

For models HXJ, HVJ and HXVJ compare also to Duffie et al. (2000).

Model HXJ

For this model $J_x \sim N(\mu_J, \sigma_J^2)$ and $J_v \equiv 0$.

$$\int_{\mathbb{R}}(\exp(uJ_x))d\nu(J_x) = \exp(\mu_J u + \frac{1}{2}\sigma_J^2 u^2), \tag{5.31}$$

and

$$\int_0^\tau \left(\int_{\mathbb{R}}(\exp(J_x))d\nu(J_x)\right) dt = (\exp(\mu_J + \frac{1}{2}\sigma_J^2))\tau. \tag{5.32}$$

Model HVJ

Here, $J_x \equiv 0$, and $J_v \sim \mathrm{Exp}(\frac{1}{\eta})$, equivalently to Duffie et al. (2000).

$$\int_{\mathbb{R}^+}(\exp(A(u,\tau,\vec{\vartheta})J_v))d\nu(J_v) = \frac{1}{1-\eta A(u,\tau,\vec{\vartheta})}, \tag{5.33}$$

and

$$\int_0^\tau \left(\int_{\mathbb{R}^+}(\exp(A(u,t,\vec{\vartheta})J_v))d\nu(J_v)\right) dt = \frac{\sigma^2\left(d\left(-\eta k + g\sigma^2\right)\tau - (1-g)k\eta\log(m)\right)}{d\left(\eta k - \sigma^2\right)\left(\eta k - g\sigma^2\right)}, \tag{5.34}$$

where

$$\begin{aligned} m &= \frac{(1-g)\sigma^2}{-\eta k + \sigma^2 + \mathrm{e}^{d\tau}\left(\eta k - g\sigma^2\right)}, \\ k &= d - h. \end{aligned} \tag{5.35}$$

Model HVJG

Similar to model HVJ, $J_x \equiv 0$, but $J_v \sim \Gamma(\frac{1}{\eta}, \gamma)$.

$$\int_{\mathbb{R}^+} (\exp(A(u,\tau,\vec{\vartheta})J_v))d\nu(J_v) = (1-\eta A(u,\tau,\vec{\vartheta}))^{-\gamma}, \tag{5.36}$$

$\int_0^\tau \left((1-\eta A(u,t,\vec{\vartheta}))^{-\gamma}\right) dt$ must be solved numerically.

Model HXVJ

In this model, the jumps occur simultaneously, but the jump sizes are $J_v \sim \text{Exp}(\frac{1}{\eta})$, and $J_{x|J_v=z_v} \sim N(\mu_J + \rho_J z_v, \sigma_J^2)$, i.e. the distribution of the jump size in x, given a realisation of z_v of the jump in the variance process, is $N(\mu_J + \rho_J z_v, \sigma_J^2)$. This is again analogous to a model in Duffie et al. (2000).

$$\begin{aligned}
\int_{\mathbb{R}\times\mathbb{R}^+} \exp(uJ_x &+ A(u,\tau,\vec{\vartheta})J_v)d\nu(J_x,J_v) \\
&= \int_{\mathbb{R}^+}\left(\int_{\mathbb{R}} \exp\left(uJ_x + A(u,\tau,\vec{\vartheta})z_v\right) d\nu_{J_x|J_v=z_v}\right) d\nu_{J_v} \\
&= \int_{\mathbb{R}^+} \exp\left((\mu_J+\rho_J z_v)u + \frac{1}{2}\sigma_J^2 u^2\right) \exp\left(A(u,\tau,\vec{\vartheta})z_v\right) d\nu_{J_v} \\
&= \int_{\mathbb{R}^+} \exp\left(\mu_J u + \frac{1}{2}\sigma_J^2 u^2\right) \exp\left(u\rho_J z_v + A(u,\tau,\vec{\vartheta})z_v\right) d\nu_{J_v} \\
&= \exp\left(\mu_J u + \frac{1}{2}\sigma_J^2 u^2\right) \int_{\mathbb{R}^+} \exp\left((u\rho_J + A(u,\tau,\vec{\vartheta}))z_v\right) d\nu_{J_v} \\
&= \frac{\exp\left(\mu_J u + \frac{1}{2}\sigma_J^2 u^2\right)}{1-\eta(u\rho_J + A(u,\tau,\vec{\vartheta}))} \\
&= \frac{\exp\left(\mu_J u + \frac{1}{2}\sigma_J^2 u^2\right)}{1-\eta\rho_J u - \eta A(u,\tau,\vec{\vartheta})}.
\end{aligned} \tag{5.37}$$

$\nu(J_x, J_v)$ is the common density of J_x and J_v, $\nu_{J_x|J_v=z_v}$ is the conditional density of J_x, and ν_{J_v} is the density of J_v.

The integral of this conditional expectation for $u = 1$ is

$$\int_0^\tau \left(\int_{\mathbb{R}\times\mathbb{R}^+} (\exp(uJ_x + A(u,t,\vec{\vartheta})J_v))d\nu(J_x,J_v)\right)_{|u=1} dt = \exp(\mu_J + \frac{1}{2}\sigma_J^2)\beta(1-\eta\rho_J), \tag{5.38}$$

where

$$
\begin{aligned}
\beta(c) &= \frac{\sigma^2 \left(d\left(-\eta k + gc\sigma^2\right)\tau - (1-g)k\eta \log(m(c))\right)}{d(\eta k - c\sigma^2)(\eta k - gc\sigma^2)}, \\
m(c) &= \frac{(1-g)c\sigma^2}{-\eta k + c\sigma^2 + \mathrm{e}^{d\tau}(\eta k - gc\sigma^2)}, \\
k &= d - h. \qquad (5.39)
\end{aligned}
$$

As it can easily be seen, $\int_0^\tau E_{t-}[\exp(A(1,t,\vec{\vartheta})J_v)]dt$ in (5.34) is the same as $\beta(1)$.

Model HXVJG

The jump size J_v in the variance process of this model is a Gamma instead of an Exponential, i.e. $J_v \sim \Gamma(\frac{1}{\eta},\gamma)$. J_x is as before, i.e. $J_{x|J_v=z_v} \sim N(\mu_J + \rho_J z_v, \sigma_J^2)$ for a given variance jump realisation z_v.

$$
\begin{aligned}
\int_{\mathbb{R}\times\mathbb{R}^+} (\exp(uJ_x &+ A(u,\tau,\vec{\vartheta})J_v))d\nu(J_x,J_v) \\
&= \int_{\mathbb{R}^+} \left(\int_{\mathbb{R}} ((\exp(uJ_x + A(u,\tau,\vec{\vartheta})z_v))d\nu_{J_x|J_v=z_v} \right) d\nu_{J_v} \\
&= (\exp(\mu_J u + \frac{1}{2}\sigma_J^2 u^2))(1 - \eta A(u,\tau,\vec{\vartheta}))^{-\gamma}. \qquad (5.40)
\end{aligned}
$$

$\int_0^\tau \left((\exp(\mu_J u + \frac{1}{2}\sigma_J^2 u^2))(1 - \eta A(u,t,\vec{\vartheta}))^{-\gamma} \right) dt$ must be solved numerically.

Positivity of v_t and real-valued Solutions

If $2\kappa\theta \geq \sigma^2$, the variance v_t is assured to stay positive.[9] This should not only be valid for model H and for the models with jumps in X_t, but also for the models HVJ, HVJG, HXVJ and HXVJG, because we modelled only variance jumps with positive sizes.

However, problems can arise for all models, if $h^2 - \sigma^2(2\beta+1) < 0$, or $\frac{1-g\mathrm{e}^{d\tau}}{1-g} < 0$ for some or all τ.

[9] See Feller (1951).

Example 5.1 *Assume that* $\alpha = 10, \beta = 10, \kappa = 1, \theta = 10, \sigma = 1, \rho = 0$. *Then, for* $u = 1$, $h = \rho\sigma - \kappa = -1$, $d = \sqrt{h^2 - \sigma^2(2\beta + 1)} = \sqrt{-20} = 4.4721\mathrm{i}$,

$$\begin{aligned} g &= \frac{-h + d}{-h - d} \\ &= \frac{1 + 4.4721\mathrm{i}}{1 - 4.4721\mathrm{i}} \\ &= -0.9048 + 0.4259\mathrm{i}. \end{aligned} \tag{5.41}$$

For $\tau = 1$,

$$\begin{aligned} A(1, 1, \vec{\vartheta}) &= \frac{1 + 4.4721\mathrm{i}}{1}\left(\frac{1.2379 + 0.9713\mathrm{i}}{0.3710 - 0.7774\mathrm{i}}\right) \\ &= -8.3706, \end{aligned} \tag{5.42}$$

and, for Model H,

$$\begin{aligned} B(1, 1, \vec{\vartheta}) &= 10 + \frac{10}{1}(1 + 4.4721\mathrm{i} - 2\log(0.9896 - 0.5122\mathrm{i})) \\ &= 36.3583 + 62.8319\mathrm{i} \end{aligned} \tag{5.43}$$

$\Phi(1, \tau, \vec{\vartheta}, X_t)$ *is then*

$$\begin{aligned} \Phi(1, \tau, \vec{\vartheta}, X_t) &= \exp(X_t + A(1, 1, \vec{\vartheta})v_t + B(1, 1, \vec{\vartheta})) \\ &= \exp(X_t - 8.3706v_t + 36.3583 + 62.8318\mathrm{i}). \end{aligned} \tag{5.44}$$

For the models with jumps, we have more restrictions. But, as our optimisation algorithm only considers parameters where real and finite solutions are assured for $E_t[S_T]$, these conditions are automatically fulfilled.

5.5 Solutions for the Models with Season

If, for a model without seasonal component, $E_t[S_T]$ is known, the solution for the respective model with a seasonal component is straightforward: $E_t[S_T^{\text{with Season}}] = E_t[S_T^{\text{without Season}}] + f(T)$. X_t has to be calculated as $\log(S_t - f(t))$, as already mentioned in the specification of model OUS.

The problem of real and finite solutions of the models with season is the same as of the respective models without the deterministic function $f(t)$, and it is also assured by the optimisation algorithm.

Chapter 6

Empirical Results

In our empirical investigations we first examine model group 1, the four models without stochastic volatility (models OU, OUJ, OUS and OUJS). The results of model group 2, the models with stochastic volatility without seasonal modelling (models H, HXJ, HVJ, HVJG, HXVJ and HXVJG) are only briefly discussed and the results are plotted in Appendix A. In the third section, we present the results of model group 3, the models with stochastic volatility and seasonal modelling (models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS).[1] The chapter ends with a short discussion of all models.

6.1 Model Group 1 (Models OU, OUJ, OUS and OUJS)

In this section, we will first compare the RMSEs[2] and the summary statistics for each parameter in each model. Then we will take a look at the regression statistics, and finally conduct further examinations.

6.1.1 Summary Statistics

For an overview of our results, summary statistics of the four models OU, OUJ, OUS and OUJS are shown in Table 6.1. For each model, for the RMSE and each parameter occurring therein, mean, median and standard deviation are given.

If we examine the RMSEs in Table 6.1, we see that introducing seasonality greatly improves the models. The RMSEs of models OUS and OUJS are less

[1] The models are presented in detail in Chap. 5.

[2] The RMSE is the root of the mean of all squared errors for a day. The squared errors for each day are the function that was minimised in our algorithm, see (2.28). So the RMSE shows how well the models fit the data.

Table 6.1. *Summary Statistics models OU, OUJ, OUS and OUJS*

For each trading day between 01 October 1999 and 30 September 2002, all parameters for the relevant models were independently estimated. Futures and forward contracts lead to implicit estimates minimising the RMSE (root mean squared error) for all contracts. In this summary statistics table, mean, median and standard deviation of each parameter as well as of the RMSE are reported. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

	κ	θ	σ	λ	μ_J	σ_J	s_0	s_1	RMSE
Model OU									
Mean	106,76	4,42	3,85						14,11
Median	6,10	4,62	2,32						13,74
Std. Dev.	397,15	1,07	7,61						3,05
Model OUJ									
Mean	42,32	4,15	3,15	13,56	0,18	0,18			14,03
Median	5,65	4,39	2,29	4,36	0,12	0,12			13,85
Std. Dev.	149,58	1,07	4,51	56,29	0,36	0,19			2,98
Model OUS									
Mean	135,54	4,80	4,29				26,57	0,64	5,31
Median	2,43	4,85	1,21				26,22	0,64	4,98
Std. Dev.	432,83	2,67	9,71				4,33	0,03	2,02
Model OUJS									
Mean	145,74	4,33	2,52	27,36	0,07	0,26	25,47	0,64	5,71
Median	3,10	4,68	1,27	4,16	0,12	0,15	25,57	0,64	5,45
Std. Dev.	1501,16	1,02	5,47	113,33	0,35	0,47	5,20	0,03	2,07

than a half of the counterpart models OU and OUJ, while adding normal-distributed jumps do not improve models OU and OUS. Since models OU and OUS are embedded in models OUJ and OUJS for the limits $\lambda \to 0$ or $\mu_J \to 0$ and $\sigma_J \to 0$, the RMSEs of the latter should be at least as good as those of OU and OUS. The poor values can be explained by the need for calculations of numerical integrals that are used in model OUJ as well as in model OUJS, see Sect. 5.4.1. The numerical problems are also evident if we look at the histograms in Fig. 6.1, at least for OUJS: while this model often has lower RMSEs, there are also a lot of outliers at very high values.

The differences between the models with and without season are also obvious in the histograms: the models without seasonal modelling are not able to explain observed prices up to a level that models OUS and OUJS easily can, i.e. in the models with season, the values of the RMSEs are more focused around 4.5 to 5.0, while in the models without seasonal modelling, most estimates seem to be around 14.

If we compare means and medians of the RMSEs in Table 6.1, they do not differ much for either model, and also the standard deviations are in an acceptable range.

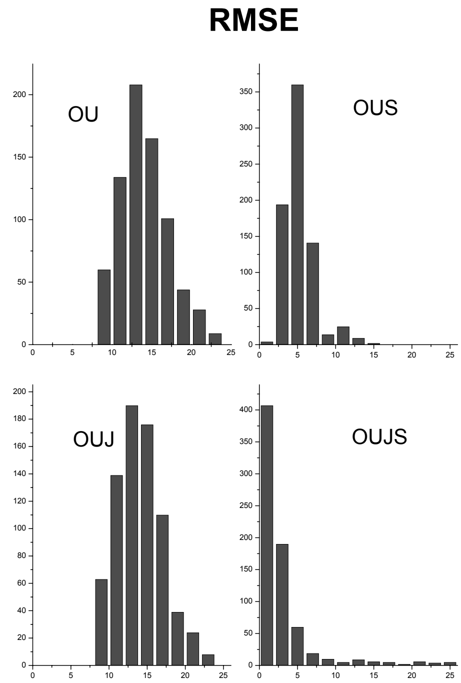

Figure 6.1. *Histograms RMSEs*
The RMSEs for all estimations are plotted in a histogram for each model. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

The standard deviations diminish if we change from model OU over OUJ to OUS. The slightly worse standard deviation for OUJS of 2.07 in comparison to 2.02 of model OUJS is again due to the numerical problems stated before. We already see here that modelling seasonal behaviour is very important. Jumps also could improve the models if numerical problems could be overcome.

If we examine the parameters in Table 6.1, we see that mean and median greatly differ for κ, λ, and, to a lesser extent, σ, μ_J and σ_J. The estimations do not seem to be robust. Comparing the standard deviations supports these findings.

One of the parameters that seems to be relative stable is θ, whose means and medians lie between 4.15 and 4.85 for all models. θ is the parameter of the long-term mean of the time series, i.e. the log spot price. The mean and median of the log spot price are 4.91 and 4.93, refer to Fig. 4.4. Thus, θ lies within a range that seems reasonable. The difference can emerge from the different time horizons of historical and implicit data, but also from risk premia. For histograms of the estimations of θ, see Fig. 6.2. The most robust values of θ seems to have model OUS.

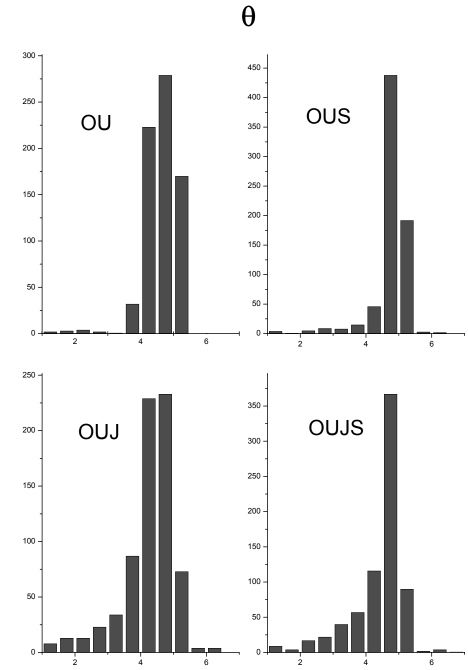

Figure 6.2. *Histograms θ*

The parameter values of θ for all estimations are plotted in a histogram for each model. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

The parameter κ shows the velocity of the data to return to the long-term mean θ. The means of κ seem quite high, but the medians that are between 2.43 for model OUS and 6.10 for model OU are in a more encouraging range. Histograms of the estimates can be viewed in Fig. 6.3. The histograms show that κ in models OUS and OUJS is more biased against zero than in OU and OUJ.

The values for σ, the standard deviation of the process, are most of the times below the standard deviation of the log spot price of 6.69 (compare to the annualized standard deviation on p. 38). But they are quite similar for all models, higher for OU and OUJ than for OUS and OUJS, at least in the medians. See Fig. 6.4 for histograms of the estimates. The histograms show that in OU and OUJ the estimates are more dispersed than in the models OUS and OUJS. But some estimates are near 6 and 7 for all models.

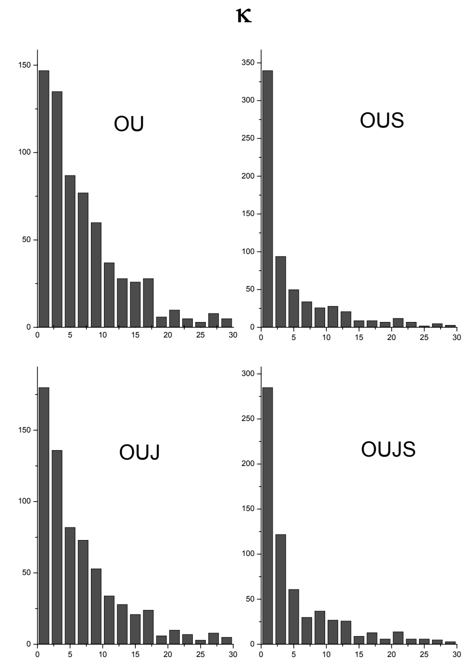

Figure 6.3. *Histograms κ*

The parameter values of κ for all estimations are plotted in a histogram for each model. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

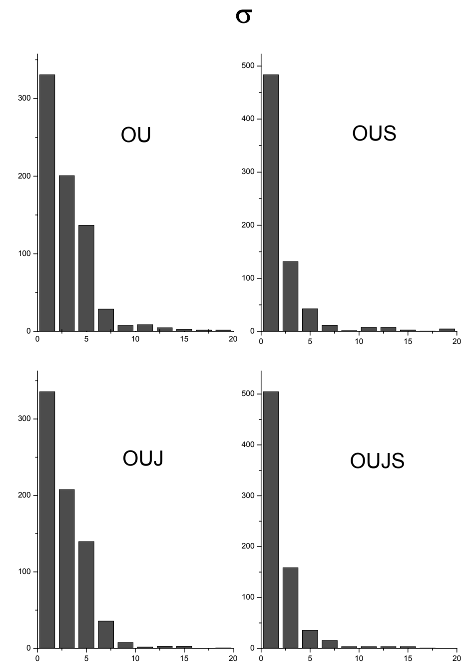

Figure 6.4. *Histograms σ*

The parameter values of σ for all estimations are plotted in a histogram for each model. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

λ can be interpreted as the expected number of jumps per year, and more than four jumps ($\approx$ the medians of models OUJ and OUJS), but less than 27 ($\approx$ the mean of model OUJS) seem realistic. The histograms of λ in Fig. 6.5 show that

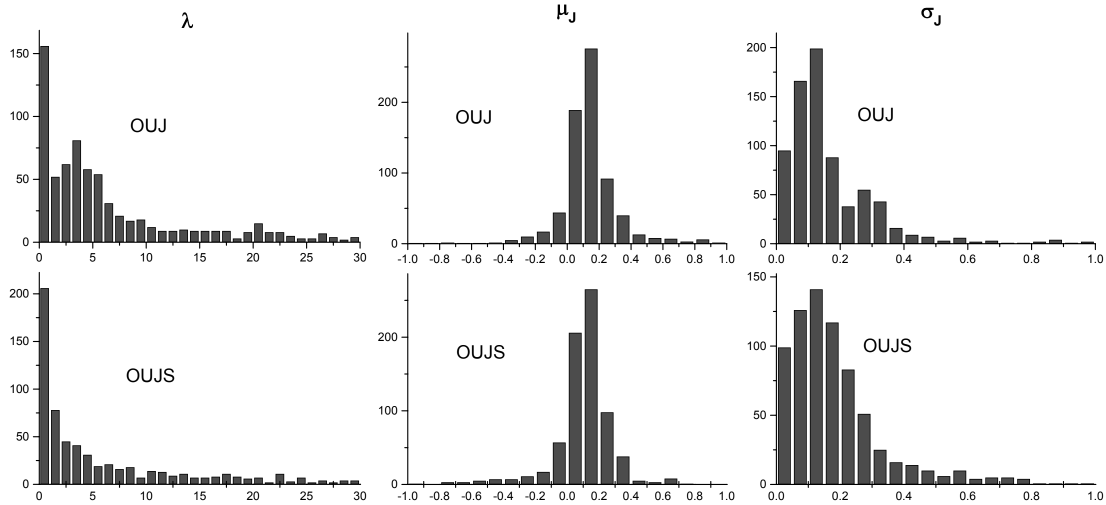

Figure 6.5. *Histograms λ_J, μ_J and σ_J*

The parameter values of λ_J, μ_J and σ_J for all estimations are plotted in a histogram for each parameter and for each model containing a jump. The models OUJ and OUJS refer to an Ornstein–Uhlenbeck process with jump and to an Ornstein Uhlenbeck process with season and jump.

most estimates are very close to zero, with a second mode around 3.0 in model OUJ.

The mean jump size μ_J of model OUJ of 0.18 in the mean is much higher than the mean of μ_J in model OUJS with 0.07, but the medians in both models are the same with 0.12. Also the standard deviations of μ_J are nearly the same. For OUJS, the mean of the σ_J, that is the standard deviation of the jump size, of 0.26 is above that of OUJ with 0.18,[3] but the medians again do not differ very much. The estimations of σ_J in model OUJS with a standard deviation of 0.47 are more volatile than in model OUJ with a standard deviation for σ_J of 0.19.

The histograms for μ_J and σ_J in Fig. 6.5 support the findings that the estimations for OUJ and OUJS are quite similar.

The seasonality parameters s_0 and s_1 are quite similar in the two models. In model OUS, s_0 is 26.57 and 26.22, and this is both in mean and median slightly higher than in model OUJS with 25.47 and 25.57. If we plot the spot price, and the

[3]The rounded means and medians for μ_J and σ_J are really the same, this is no typing error!

spot price at $t_0 = 10/01/99$ plus the deterministic functions, calculated from the mean of s_0 and s_1 of the two models, like in Fig. 6.6, the deterministic functions of models OUS and OUJS cannot be distinguished on the graph.

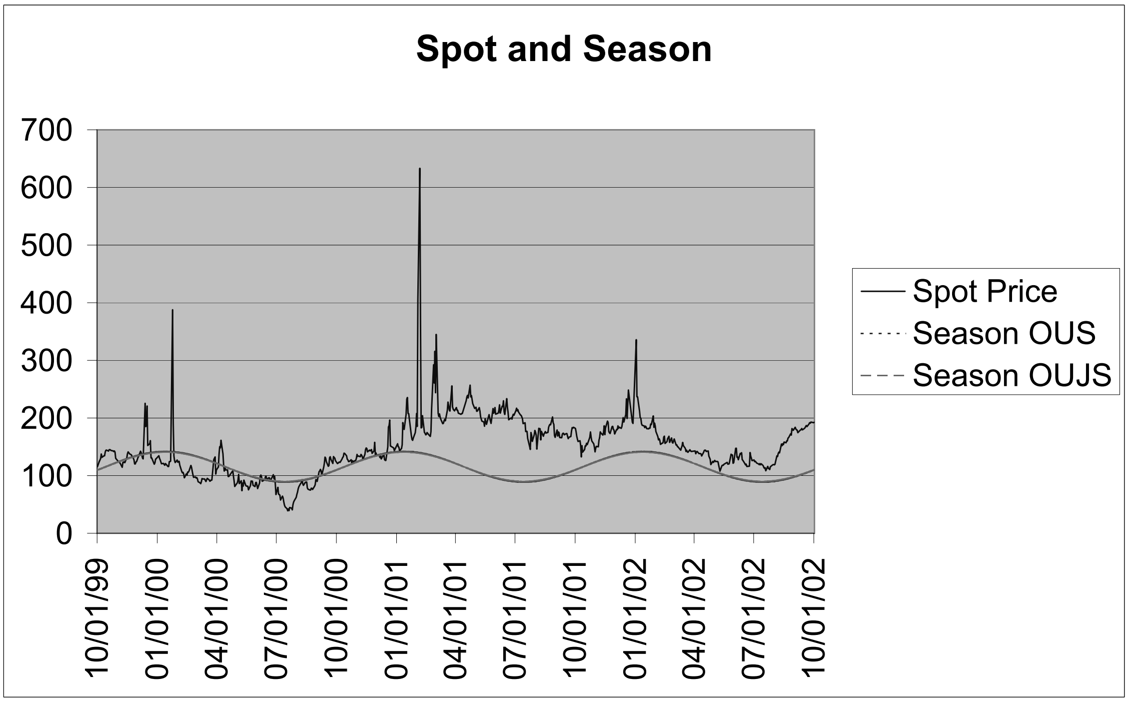

Figure 6.6. *Spot Price and Season Estimates*
The spot price is plotted together with the deterministic function $f(t)$ $(+S_0 - f(0))$. $f(t)$ maps the seasonality of the spot price and is defined as $f(t) = s_0 \cos((2\pi/365)(t + 365s_1)$. S_0 is the spot price on 1 October 1999. For the calculation of the deterministic function, the means of s_0 and s_1 are used. The resulting curves for the two models OUS and OUJS are not distinguishable on the graph. OUS and OUJS refer to an Ornstein–Uhlenbeck process with season and to an Ornstein Uhlenbeck process with season and jump.

For model OUS the value of about 26.6 of s_0 in Table 6.1 means, that +/- NOK 26.6 of the variation of the spot price is deterministic, i.e. predictable. The same applies for model OUJS with +/- NOK 25.5. The parameter value of s_1 indicates on which point in the cosine function we started and so is dependent on t_0, the 1 October 1999. Its value can be interpreted, that the models estimate, that the 10 January is the coldest day in the year, or at least the one with the highest electricity consumption.[4] This seems to be a realistic estimation. The histograms in Fig. 6.7 show that the estimates for both parameters and models are robust.

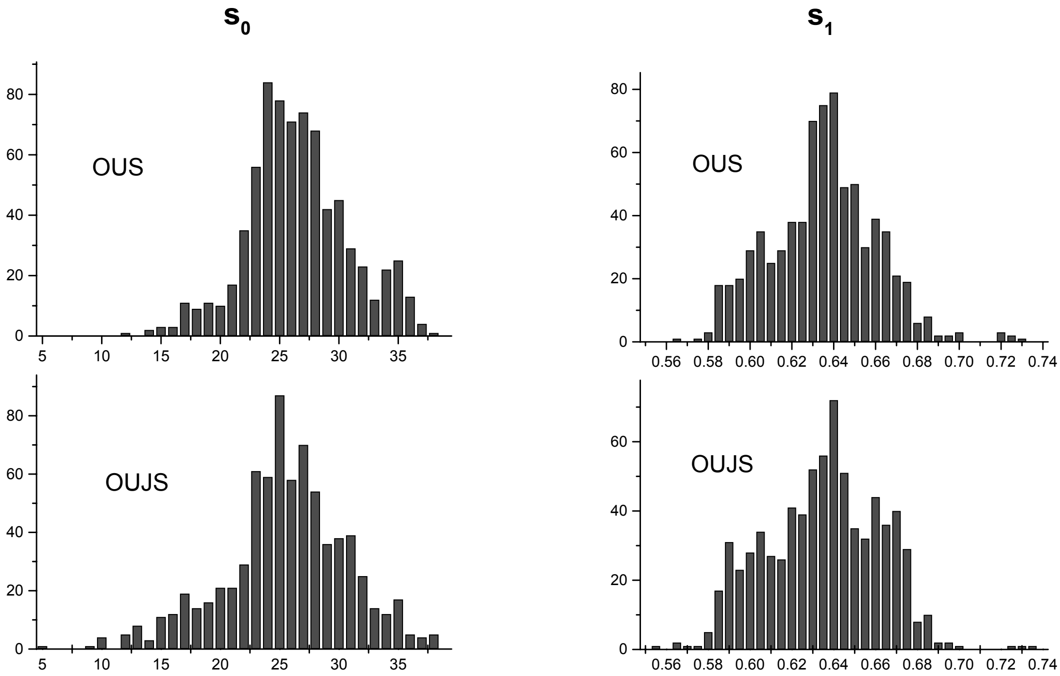

Figure 6.7. *Histograms s_0 and s_1*

The parameter values of s_0 and s_1 for all estimations are plotted in a histogram for each parameter and for each model with seasonal components. The models OUS and OUJS refer to an Ornstein–Uhlenbeck process with season and to an Ornstein Uhlenbeck process with season and jump.

[4]The calculation is as follows: $\cos((2\pi/365)((t_0 + 29) + 365s_1) = 1$, or (with $s_1 = 0.64$) $t_0 = 102$, computed with the Matlab date (mod 365) 29 for 1 October 1999. 1 October +102 days corresponds to 10 January.

6.1.2 Regression Statistics

Our implicit estimations of parameters can be interpreted as nonlinear regressions in the following way:

In a linear regression, the equation is $\vec{Y} = \vec{A}\vec{\beta} + \vec{\varepsilon}$, where $\vec{Y} \in \mathbb{R}^N$ is the dependent variable, $\vec{A} \in \mathbb{R}^{M \times N}$ is the matrix of independent variables, $\vec{\beta} \in \mathbb{R}^M$ the coefficient vector and $\vec{\varepsilon}$ the vector of residuals. The equation is solved by finding the minimal $\vec{\varepsilon}$ as $min_{\vec{\beta}}(\vec{Y} - \vec{A}\vec{\beta})^2$. In a nonlinear regression, $\vec{A}\vec{\beta}$ is replaced by $f(\vec{\beta})$, with $f : \mathbb{R}^M \to \mathbb{R}^N$.

If we now set $\vec{Y}$ to the vector of observed futures prices on day t, the parameter vector $\vec{\vartheta} = \vec{\beta}$ and f to the vector of theoretic futures prices, i.e. our equation is now, with the notations of Sect. 2.4,[5]

$$\begin{pmatrix} \hat{F}(t, T_1^1, T_n^1) \\ \hat{F}(t, T_1^2, T_n^2) \\ \cdots \\ \hat{F}(t, T_1^N, T_n^N) \end{pmatrix} = \begin{pmatrix} F(t, T_1^1, T_n^1; X_t, \vec{\vartheta})) \\ F(t, T_1^2, T_n^2; X_t, \vec{\vartheta})) \\ \cdots \\ F(t, T_1^N, T_n^N; X_t, \vec{\vartheta})) \end{pmatrix} + \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \\ \cdots \\ \varepsilon^N \end{pmatrix}.$$

It is solved as

$$\min_{\vec{\vartheta}} \sum_{j=1}^{N} \left(\hat{F}(t, T_1^j, T_n^j) - F(t, T_1^j, T_n^j; X_t, \vec{\vartheta}) \right)^2.$$

The last equation is equivalent to $N\times$ MSE, the value we already minimized in our implicit estimations, see (2.28). So we can understand the implicit estimations as nonlinear regressions. The error terms $\varepsilon^j, j = 1, \ldots, N$ are calculated as $\varepsilon^j = \hat{F}(t, T_1^j, T_n^j) - F(t, T_1^j, T_n^j; X_t, \vec{\vartheta_{\text{opt}}})$, where $F(t, T_1^j, T_n^j; X_t, \vec{\vartheta_{\text{opt}}})$ are the theoretic futures values with optimal parameter values.

We can run now the analysis done for linear regressions for our nonlinear regressions. As documented for example in *Eviews 3.1 Help* (1999, Sect. *L*east Squares (Weighted, Two-Stage, Nonlinear)), the so-determined statistical results and tests are asymptotically valid.[6]

[5] In contrast to (2.28), we denote $F(t, T_1^2, T_n^2)$ as $F(t, T_1^j, T_n^j; X_t, \vec{\vartheta})$ to stress the dependency of the model prices of the independent variable X_t and the parameter vector $\vec{\vartheta}$.

[6] See also there for a detailed description on linear and non-linear least squares (LS or NLS, respective).

In the following of this subsection we will first regard the statistics of the parameters and the F-statistic for each model. Then we will compare the additional statistics like the R^2 for all models.

Let us take a look at Tables 6.2, 6.3, 6.4 and 6.5.[7] For our analysis, we chose the median values of the regression statistics, because the mean values seemed very biased for some parameters, caused by many high outliers. Nevertheless, the conclusions drawn in this section would be nearly the same for the means.[8] The p-values refer to two-sided t-tests, i.e. $= 0$ vs. $\neq 0$ for parameters that are allowed to become negative, and one-sided t-tests, i.e. ≤ 0 vs. > 0, for parameters that are by definition positive.[9] In the columns 'sign. 5%' and 'sign. 1%' are the percentages of the estimations for the specific parameters, for that the two-sided and one-sided t-statistics are significant at the 5% and the 1% level, respectively. 'Mean dependent var' and 'S.D. dependent var'[10] correspond to the price of the futures and forward contracts. They are therefore the same in all models, 150.83 and 19.65, respectively.

In Table 6.2 for model OU, we see that the parameter θ is significantly different from zero, in the median as well as in a high percentage of all cases. σ is in the median still significant at the 10% level, and in 48.67% of all days significant at the 5% level. κ seems difficult to estimate. In the median its p-value is 0.1215, and only in 39.73% of all days, i.e. in 298 out of 750, it could be estimated as significant at the 5% level. The F-statistic is significant in the median at the 1% level, which means, that the hypothesis of a total misspecification of the model is rejected.

In model OUJ, see Table 6.3, the t-statistics for all parameters are very low, the standard errors for λ with 730.96, for example, very high. Nevertheless, in the

[7] The calculated statistics correspond to the statistics shown in Eviews as result of LS or NLS estimations. See the *Eviews 3.1 Help* (1999) for details. We omitted the sum of squared residuals and the standard error of regression because of their similarity to the RMSE. Furthermore, the Durbin–Watson statistic cannot be interpreted here in a pure time series context. The supposed regression is not on time series, but on futures and forward series that are ordered respective their delivery periods; so the Durbin–Watson statistic refers to the correlation between futures residuals of subsequent delivery times.

[8] Remember that we performed individual parameter estimations for each trading day considered (from 1 October 1999 until 30 September 2002, i.e. 750 trading days). So we also performed the regression analysis for each of these days separately, resulting in 750 R^2, for example.

[9] In Eviews 3.1, the given p-values are always the p-values for two-sided t-tests.

[10] Mean of the dependent variables and Standard Deviation of the dependent variables.

Table 6.2. *Regression Statistics Model OU*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. The model OU refers to an Ornstein–Uhlenbeck process.

Medians Model OU

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
κ	6,10	3,73	1,20	0,1215	39,73%	22,13%
θ	4,62	0,51	9,63	0,0000	84,00%	82,93%
σ	2,32	2,46	1,46	0,0791	48,67%	40,93%

F-Statistic	Prob.	sign. 5%	sign. 1%
7,14	0,0041	72,00%	56,93%

R-squared	0,40	Mean dependent var	150,83
Adjusted R-squared	0,34	S.D. dependent var	19,65
Akaike info criterion	8,32	Schwarz criterion	8,47
Log likelihood	-101,75	Durbin-Watson stat	1,22

median we get for θ, for example, similar values as in OU. The problem for the estimations seems to be the extension for jumps that make numerical quadrature procedures necessary to determine the value of integrals. The F-statistic is not so clear as in model OU, misspecification is not rejected at the 5% level.

As already suspected when comparing the RMSEs, the introduction of season in the models proves to be very important. In model OUS, the deterministic components have low standard errors and very high t-statistics, see Table 6.4. In 99.87% or 96.93% of all estimations, they were significant at the 1% level. Also the F-statistic is the highest so far. θ can again be significantly estimated, even at the 1% level in the median, but κ and σ do not have t-statistics as good as in model OU.

Table 6.3. *Regression Statistics Model OUJ*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. The model OUJ refers to an Ornstein–Uhlenbeck process with jump.

Medians Model OUJ

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
κ	5,65	6,47	0,70	0,245	16,67%	7,33%
θ	4,39	161,28	0,03	0,4897	1,20%	0,27%
σ	2,29	82,18	0,03	0,4873	0,80%	0,67%
λ	4,36	730,96	0,01	0,4976	0,00%	0,00%
μ_J	0,12	241,71	0,0005	0,9996	0,00%	0,00%
σ_J	0,12	343,97	0,0004	0,4999	0,00%	0,00%

F-Statistic	Prob.	sign. 5%	sign. 1%
2,58	0,0621	47,47%	33,73%

R-squared	0,41	Mean dependent var	150,83
Adjusted R-squared	0,25	S.D. dependent var	19,65
Akaike info criterion	8,57	Schwarz criterion	8,86
Log likelihood	-101,58	Durbin-Watson stat	1,23

In model OUJS, we have the same problems with the significance of the parameters as in model OUJ, see Table 6.5. While s_0 and s_1, the parameters for seasonality, are clearly significant at the 1% level in the median, all other parameters are not. Nevertheless, the values of the medians themselves are encouraging.

Comparing the R^2 of the four models, the conclusions drawn before, regarding the RMSEs, can be repeated: The models improve most by changing from the models OU and OUJ to their counterparts OUS and OUJS with season. Little or

Table 6.4. *Regression Statistics Model OUS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. The model OUS refers to an Ornstein–Uhlenbeck process with season.

Medians Model OUS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
κ	2,43	7,84	0,35	0,3637	26,80%	20,93%
θ	4,85	1,88	2,59	0,0090	61,60%	50,27%
σ	1,21	5,56	0,22	0,4160	26,13%	22,80%
s_0	26,22	2,46	10,94	0,0000	99,87%	99,87%
s_1	0,64	0,01	43,15	0,0000	96,93%	96,93%

F-Statistic	Prob.	sign. 5%	sign. 1%
60,44	0,0000	99,87%	99,87%

R-squared	0,93	Mean dependent var	150,83
Adjusted R-squared	0,91	S.D. dependent var	19,65
Akaike info criterion	6,45	Schwarz criterion	6,70
Log likelihood	-74,99	Durbin-Watson stat	1,46

no improvement is achieved by adding jumps. The Adjusted R^2 changes from 0.34 in model OU to 0.91 in model OUS, while in OUJ, it diminishes to 0.25. Finally, the model OUJS has an Adjusted R^2 of 0.88.

The log likelihoods can only be compared for nested models, i.e. models OUJ and OUS cannot be compared in this context. Again model OUS, compared with OU and OUJS, proves to be the best, while of the triple OU, OUJ and OUJS, model OUJS has the highest log likelihood. Also the Durbin–Watson statistic seems to be closest to 2.00 for OUS, though 1.46 is still very low. The Akaike Information Criterion (AIC) and the Schwarz Criterion (SC) draw the

Table 6.5. *Regression Statistics Model OUJS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. The model OUJS refers to an Ornstein–Uhlenbeck process with jump and season.

Medians Model OUJS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
κ	3,10	8,44	0,39	0,3516	18,80%	13,87%
θ	4,68	74,93	0,06	0,4762	2,80%	1,47%
σ	1,27	54,53	0,02	0,4902	0,13%	0,13%
λ	4,16	334,86	0,01	0,4955	0,67%	0,27%
μ_J	0,12	66,63	0,001	0,9988	0,13%	0,13%
σ_J	0,15	104,64	0,17	0,4995	0,13%	0,13%
s_0	25,57	3,46	7,67	0,0000	98,40%	95,47%
s_1	0,64	0,02	31,70	0,0000	99,47%	99,47%

F-Statistic	Prob.	sign. 5%	sign. 1%
25,18	0,0000	94,59%	0,97%

R-squared	0,91	Mean dependent var	150,83
Adjusted R-squared	0,88	S.D. dependent var	19,65
Akaike info criterion	6,88	Schwarz criterion	7,27
Log likelihood	-77,13	Durbin-Watson stat	1,40

same conclusions: The SC, that penalises additional coefficients, has, as well as the AIC, lower values for the models without jumps, compared to those with jumps.

If we make a short conclusion of our analysis so far, we can say that the basic model OU can be extended to clearly better map the characteristics of the data.

The most important improvements are the deterministic or seasonal enhancements. According to most statistics, adding jumps does not really improve the models, perhaps the model without seasonal modelling, OU, but not model OUS. The parameters are more difficult to estimate, the more complex the models are. This can be seen by the increasing number of outliers in the estimates and sometimes higher standard deviations. The problem could be the necessity to solve numerical integrals for OUJ and OUJS in the optimisation process, see also Sect. 5.

6.1.3 Out-of-Sample Behaviour

Further analysis of the models can be done. The following can be of practical relevance:

- How much do I misprice the contracts of today, if I use the implied parameters of the day before (denoted in the following as $\vec{\vartheta}$(t-1))?
- How much do I misprice the contracts of today, if I use the median of the implied parameters of all days before, starting at 1 October 1999 (denoted in the following as median($\vec{\vartheta}$(1:t-1)))?
- How much do I misprice the contracts of today, if I use the median of all implied parameters, estimated during the whole studied period (denoted in the following as median($\vec{\vartheta}$(1:end)))?

Although the last pricing approach is no real out-of-sample consideration, it might still be of interest to compare it with the other results.

For this examination, which is similar to Bakshi et al. (1997), we list mean, median and standard deviation of the RMSEs. The RMSEs of Table 6.1 are again listed for a better comparison. The results can be viewed in Table 6.6.

The following observations are in order: RMSE($\vec{\vartheta}$(t-1)) is always only about a half of the values of RMSE(median($\vec{\vartheta}$(1:t-1))) and RMSE(median($\vec{\vartheta}$(1:end))). Good models should capture the behaviour of the series over time and using more information should lead to better results. Thus, important structure is missing in all of these models, also in the models with seasonal modelling. With the exception of model OU, RMSE(median($\vec{\vartheta}$(1:end))) gives the worst results. The parameter estimations are changing too much to give reasonable values for the whole period.

Table 6.6. *RMSEs – Out-of-Sample*

In this summary statistics table, mean, median and standard deviation of the RMSEs with the parameters determined by minimising the RMSEs are reported, the RMSEs calculated with the parameters that were optimal in the estimations of the day before, called here RMSE($\vec{\vartheta}$(t-1)), the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until day t-1 (RMSE(median($\vec{\vartheta}$(1:t-1)))), and the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until the last day (30 September 2002), called RMSE(median($\vec{\vartheta}$(1:end))). The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

		RMSE	RMSE(ϑ(t-1))	RMSE(median(ϑ(1:t-1)))	RMSE(median(ϑ(1:end)))
OU	**Mean**	14.11	14.79	38.58	32.07
	Median	13.74	14.30	35.12	28.96
	Std. Dev.	3.05	3.55	16.46	14.72
OUJ	**Mean**	14.03	14.70	37.17	40.33
	Median	13.85	14.30	32.32	38.38
	Std. Dev.	2.98	3.49	17.73	16.97
OUS	**Mean**	5.31	6.94	15.78	16.10
	Median	4.98	6.02	14.05	13.77
	Std. Dev.	2.02	7.35	12.08	10.28
OUJS	**Mean**	5.71	7.07	16.59	17.59
	Median	5.45	6.34	14.19	15.25
	Std. Dev.	2.07	6.29	10.34	10.43

If we regard only time periods of a day, i.e. the values of RMSE($\vec{\vartheta}$(t-1)), the models perform better. The values are all not far away from the RMSEs which lead to the optimal parameters, for all models. Earlier observations are confirmed: seasonal modelling is very important, jumps may be reasonable to add. The results of 14.70 and 14.30 from model OUJ in RMSE($\vec{\vartheta}$(t-1)) in comparison to OU with 14.79 and 14.30 are better in the mean but the same in the median. The values of RMSE($\vec{\vartheta}$(t-1)) for model OUJS are higher than those of model OUS. Regarding the standard deviations of RMSE($\vec{\vartheta}$(t-1)), we can say that those of models OU and OUJ with 3.55 and 3.49 are lower than those of OUS and OUJS with 7.35 and 6.29; so the size of the pricing errors for the first two models does not change the amount that it does for the third and fourth model.

The conclusion is the same as before: seasonal modelling is important; jumps cannot be estimated or are not really relevant for Ornstein–Uhlenbeck-type models.

6.1.4 Option Pricing

We now employ models OU, OUJ, OUS and OUJS to price the European and Asian options of Nord Pool and compare them to the observed prices. For this, we use only option prices that were the results of real trades, see also the comments in Sect. 4.3. The theoretical option prices are calculated by Monte Carlo simulations, see also Sect. 2.5. The results can be seen in Table 6.7. The values that are

Table 6.7. *RMSEs of option pricing*

In this summary statistics table, mean, median and standard deviation of the RMSEs of the comparison of calculated and observed option prices are reported. In RMSE($\vec{\vartheta}$(t)), the parameters of day t, implicitly estimated by futures and forward prices, are used; in RMSE(median($\vec{\vartheta}$(1:t)), the medians of all parameters until day t, implicitly estimated by futures and forward prices, are used; and finally, in RMSE(median($\vec{\vartheta}$(1:end)), the medians of all parameters, implicitly estimated by futures and forward prices, are used. The models OU, OUJ, OUS and OUJS refer to an Ornstein–Uhlenbeck process, an Ornstein–Uhlenbeck process with jump, with season and with season and jump.

		RMSE(ϑ(t))	RMSE(median((ϑ(1:t)))	RMSE(median(ϑ(1:end)))
OU	**Mean**	25.03	20.32	22.50
	Median	24.08	17.48	20.83
	Std. Dev.	18.07	13.14	14.79
OUJ	**Mean**	27.56	20.52	20.23
	Median	15.00	18.13	17.88
	Std. Dev.	104.89	22.59	28.11
OUS	**Mean**	26.99	20.50	23.79
	Median	16.04	15.57	22.70
	Std. Dev.	26.93	16.65	15.61
OUJS	**Mean**	30.97	16.77	16.61
	Median	12.32	14.04	13.87
	Std. Dev.	114.96	16.78	19.15

compared are again the RMSEs, i.e. for option prices

$$RMSE(t) = \sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t}\left(\hat{O}(t,j) - O(t,j)\right)^2}, \qquad (6.1)$$

where $\hat{O}(t,j)$ are the theoretical option prices on day t, and $O(t,j)$ the observed ones, $j = 1,\ldots,N_t$. N_t is the total number of all traded options on day t. $t = 1,\ldots 750$, where $1 \mathrel{\hat{=}} 1$ October 1999 and $750 \mathrel{\hat{=}} 30$ September 2002.

The different listed RMSEs are similar to those in the last subsection, but we are now using the parameters of or until the same day an option is traded. This way, we can assess the consistency of the models that should be able to price options as good as futures and forwards. In RMSE($\vec{\vartheta}$(t)), we use the parameters estimated implicitly by futures and forward prices on the same day a particular option is traded. In RMSE(median($\vec{\vartheta}$(1:t))), we employ the medians of all parameters that were the solutions to our implicit estimations by futures and forwards, until day t where a particular option price is observed. In RMSE(median($\vec{\vartheta}$(1:end)), we rely on the medians of all parameters estimated implicitly by futures and forwards, even though we assume that we have even more information this way than a trader on that market.

First note that the results for the three different RMSEs do not differ very much as was the case in the last subsection. RMSE($\vec{\vartheta}$(t)) does not always have lower values than the other two statistics, but mean and standard deviation are even always the highest, often to an enormous extent. Though the additional information does not improve the values of RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))), they also do not worsen them. But the values are always substantially higher than their counterparts in Sect. 6.1.3.

Surprisingly, the best values are observed for the medians of OUJS. With values of 12.32, 14.04 and 13.87 for RMSE($\vec{\vartheta}$(t)), RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))) they are clearly below the medians of OUS with 16.04, 15.57 and 22.70. Also the means, apart from RMSE($\vec{\vartheta}$(t)) with the worst value for OUJS, can be classified this way.

For both models with jump, OUJ and OUJS, it can be said that their RMSEs have the highest standard deviations, most clearly for RMSE($\vec{\vartheta}$(t)) with 104.89 for OUJ and 114.96 for OUJS.

It must be clearly noted that the values of OUJ and OUS do not differ very much. So is the mean of RMSE($\vec{\vartheta}$(t)) of OUJ with 27.56 higher than the mean of OUS with 26.99, but in the median the order is the opposite: 15.00 for OUJ and 16.04 for OUS. For RMSE(median($\vec{\vartheta}$(1:t))) mean and median of OUJ are above those of OUS, though only by 0.02 for the mean of RMSE(median($\vec{\vartheta}$(1:t))). In contrast, comparing the values of mean and median of RMSE(median($\vec{\vartheta}$(1:end))), model OUJ performs better than model OUS.

Obviously the least favorable model in this pricing approach is OU. With the exceptions of the median of 25.03 in RMSE($\vec{\vartheta}$(t)) and of 17.48 in RMSE(median($\vec{\vartheta}$(1:t))), it always performs the worst, with the lowest standard deviations.

The results in this subsection differ from the previous ones. Whereas before model OUS, followed by OUJS, performed best, here we get the best values for OUJS, if we ignore the mean of RMSE($\vec{\vartheta}$(t)). Model OUJ and OUS follow. The inadequacy of the timely behaviour of the models does not seem to be important for option pricing, RMSE($\vec{\vartheta}$(t)) is not lower than the other statistics in general. We want to emphasise again, that only few option prices were available, so the results must be carefully interpreted.

Concluding the whole section, we can repeat that the most important feature in the models is the seasonal modelling. Jumps cannot clearly improve the models, maybe because of numerical reasons. The best model so far seems to be OUS.

6.2 Model Group 2 (Models H, HXJ, HVJ, HVJG, HXVJ and HXVJG)

The results for this model group are not discussed in detail. As was foreseeable when comparing models OU and OUJ with OUS and OUJS in the first model group, adding components for seasonality is essential.

So are the RMSEs for all models of this group below those of model OUS and OUJS of model group 1, as well as below those of model group 3. They are, in fact, near OU and OUJ of model group 1. R^2 is always very low, and for models HXJ, HXVJ and HXVJG, the Adjusted R^2 is even negative. The F-statistic is never significant at the 5% level in the median, so the hypotheses of misspecifications of all models are not rejected.

Nevertheless, all statistics, histograms and other tables are listed for model group 2 as for model groups 1 and 3, and also the out-of-sample behaviour and the option pricing fit have been conducted. All tables and histograms can be viewed in Appendix A.

6.3 Model Group 3 (Models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS)

We will again, for this model group, compare the RMSEs and the summary statistics for each parameter in each model. Then we will continue with the regression statistics and end with out-of-sample calculations and option pricing.

6.3.1 Summary Statistics

The approach here is like in Sect. 6.1. We will begin with the summary statistics and discuss the RMSEs first and then each parameter, showing all histograms also.

Examine Table 6.8 and its second part 6.9. Upon first glance of the RMSEs, we already recognise that the models in this model group perform at least twice as well as the models in the previous group without seasonal modelling. While the values in models H, HXJ, HVJ, HVJG, HXVJ and HXVJG are all around 14–16 (see Table A.2 in Appendix A), in models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS, they are all around 5–8. This confirms the observations we already made in Sect. 6.1, that modelling seasonality is very important.

Disappointing is that the RMSEs of this model group are not lower than those of model group 1 with season. All the sophisticated enhancements of the Heston-type model can only help to bring the RMSE to the same level as a simple OU-process with season, model OUS.

Opposed to the OU-models with or without season, the H-models definitely do improve if we add jumps. The RMSE of HS improves from 7.82 in the mean and 5.92 in the median to 6.04 and 4.96 for model HXJS, i.e. the model with jump in the returns. HVJS, the model with an exponential-distributed jump size in the volatility, has similar RMSEs like HXJS, with a mean of 6.02 and a median of 5.08. Model HVJGS seems even more interesting, with values of 5.33 and 4.62,

Table 6.8. *Summary Statistics models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS*

For each trading day between 01 October 1999 and 30 September 2002, all parameters for the relevant models were independently estimated. Futures and forward contracts lead to implicit estimates minimising the RMSE (root mean squared error) for all contracts. In this summary statistics table, mean, median and standard deviation of each parameter are reported, as well as of the RMSE. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

	α	β	√v	κ	θ	σ	ρ
Model HS							
Mean	-2,73	0,01	2,59	98,46	5,29	0,39	0,27
Median	-1,22	0,01	2,30	79,08	2,44	0,25	0,67
Std. Dev.	5,06	0,01	1,72	83,28	9,92	0,68	0,80
Model HXJS							
Mean	-3,15	0,001	2,36	78,19	6,31	0,41	0,42
Median	-1,80	0,001	2,07	49,62	2,95	0,27	0,61
Std. Dev.	4,78	0,01	1,81	73,34	15,21	0,64	0,62
Model HVJS							
Mean	-2,78	0,002	2,63	103,76	5,18	0,50	0,02
Median	-1,37	0,0001	2,33	94,31	2,69	0,36	0,08
Std. Dev.	4,71	0,02	1,96	81,77	7,69	0,77	0,77
Model HVJGS							
Mean	-3,32	0,002	2,29	66,24	6,04	0,34	0,28
Median	-1,68	0,0004	2,07	37,01	2,99	0,24	0,46
Std. Dev.	5,92	0,02	1,69	70,71	10,17	0,45	0,68
Model HXVJS							
Mean	-3,22	0,001	2,52	77,99	8,29	0,33	0,36
Median	-1,79	0,0004	2,12	46,98	2,99	0,26	0,54
Std. Dev.	5,24	0,01	1,96	74,83	19,66	0,31	0,62
Model HXVJGS							
Mean	-2,80	0,003	2,50	85,02	5,87	0,25	0,27
Median	-1,78	0,002	2,11	59,53	3,10	0,20	0,45
Std. Dev.	5,87	0,01	1,87	76,84	13,62	0,26	0,63

respectively. With the lowest RMSE in mean and median, though not by a large amount, HVJGS performs the best. Model HXVJS, with simultaneous jumps in log spot price and volatility, comes near with mean and median of 5.37 and 4.93. With 2.51, it even has a lower standard deviation than HVJGS and also the other models, i.e. the pricing errors are less fluctuating than in all other models. Model HXVJS is embedded in model HXVJGS and so the RMSE for the latter should be lower than that of HXVJS. However, we might have ended in HXVJGS with similar

Table 6.9. *Summary Statistics models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS – Continued*

	λ	μ_J	σ_J	η	γ	ρ_J	s_0	s_1	RMSE
Model HS									
Mean							22,74	0,64	7,82
Median							23,34	0,64	5,92
Std. Dev.							7,65	0,04	8,64
Model HXJS									
Mean	4,83	0,08	0,20				25,90	0,64	6,04
Median	1,60	0,09	0,15				25,62	0,64	4,96
Std. Dev.	11,30	0,24	0,19				6,99	0,03	5,62
Model HVJS									
Mean	20,31			0,10			24,90	0,64	6,02
Median	2,23			0,02			25,14	0,64	5,08
Std. Dev.	125,42			0,35			6,58	0,03	4,32
Model HVJGS									
Mean	13,96			0,08	8,94		25,62	0,64	5,33
Median	3,44			0,02	2,60		25,43	0,64	4,62
Std. Dev.	66,36			0,36	24,81		5,61	0,02	4,39
Model HXVJS									
Mean	8,13	0,05	0,18	0,08		0,69	25,50	0,64	5,37
Median	1,55	0,07	0,14	0,02		0,73	25,42	0,64	4,93
Std. Dev.	19,94	0,29	0,16	0,62		1,04	5,15	0,02	2,51
Model HXVJGS									
Mean	3,37	0,01	0,21	0,05	6,97	0,89	25,74	0,64	5,98
Median	0,97	0,03	0,18	0,02	2,53	0,86	25,17	0,64	5,10
Std. Dev.	9,39	0,18	0,18	0,11	17,09	1,19	7,41	0,03	4,02

numerical problems like in OUJS, because here again numerical integrals had to be solved during the optimisation process, compare to Sect. 5.4.2.[11]

A look at the histograms of RMSEs in Fig. 6.8 confirms the larger dispersion of 8.64 of HS compared to the other models whose standard deviations all lie around 4, with exception of HXVJS with a standard deviation of 2.51, as already noted. This smaller deviation of model HXVJS, however, can hardly be recognised in the histograms.

We will take a short look at each parameter. The values for α, the parameter for the constant trend of x_t, are very similar for all models, with means around -3 and medians around -1.5. The histograms in Fig. 6.9 are, even more than for the RMSEs, very similar and show that α was clearly estimated below zero for all models, with a mode around -0.5 – 0. Negative estimations for α are contrary to observations in the stock market, see e.g. the empirical investigations of Bakshi et al. (1997). However, as we already learned in Sect. 2.1, electricity markets are different. Since the estimations are implicit or under the Martingale-measure, there might be

[11] For model HVJGS, numerical integrals have to be solved, too. But in this case, these calculations do not seem to have disturbed the results so severely.

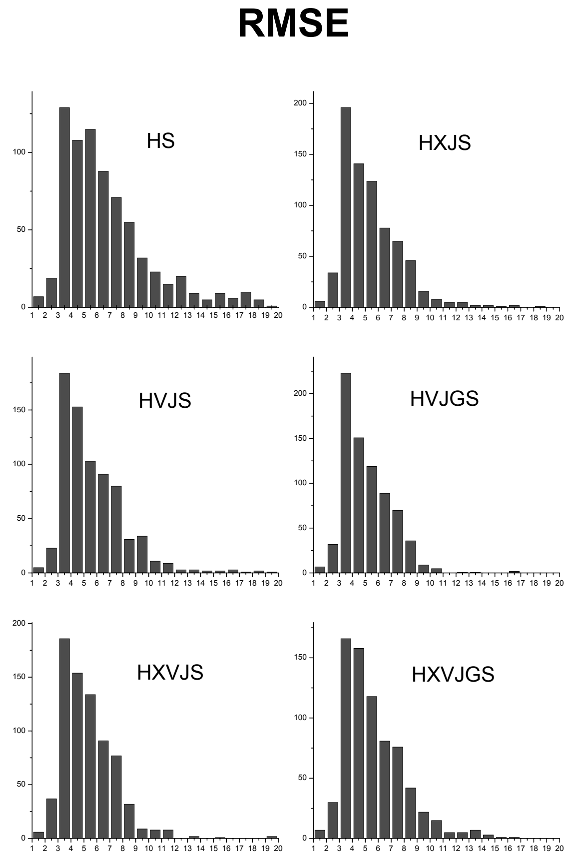

Figure 6.8. *Histograms RMSEs*

The RMSEs for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

various explanations for this phenomenon. The most obvious would be that the market participants indeed expect the prices to fall, caused by a still continuing liberalisation of the market or growing numbers of participants in the Nord Pool Spot Market. An explanation could also be a negative jump risk premium, though an economic interpretation of such a negative premium is not easy to find.

The parameter β is also part of the trend of x_t, but it is the part that is dependent on v_t, the variance at the same time. In all markets with futures-arbitrage

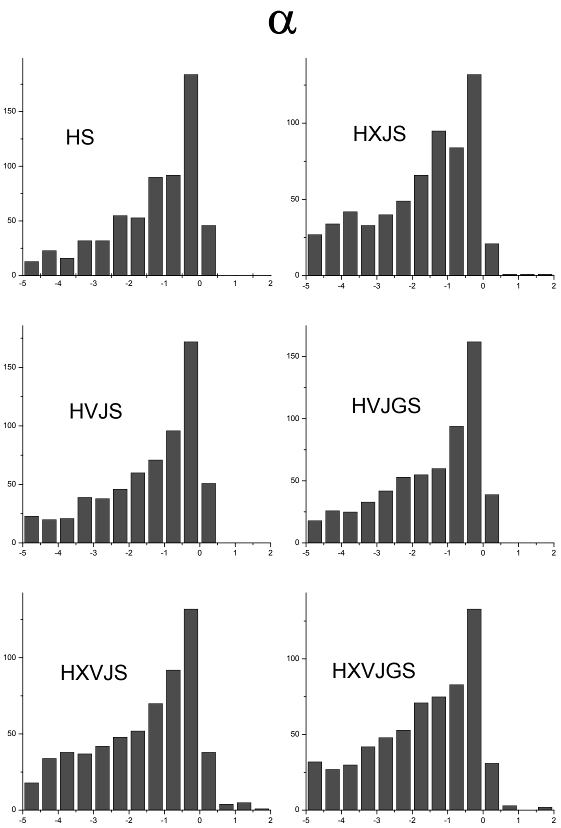

Figure 6.9. *Histograms α*

The parameter values of α for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

opportunities, β should be equal to -0.5, see (2.1) and Sect. 5.30. But, for all models, it is very close to zero, in the means as well as in the medians, and the standard deviations are not particularly high. The histograms 6.10 show that β is around zero with a positive bias for all models, with the strongest bias for model HS.

The variance v or v_t is a very important parameter in our estimations.[12] v_t

[12]In the following we speak of the variance as well as of the volatility. Though we modelled the variance in our processes, the parameter estimated and plotted in the tables is the volatility $\sqrt{v}$, the square root of the variance. Its value is more convenient for economic interpretations.

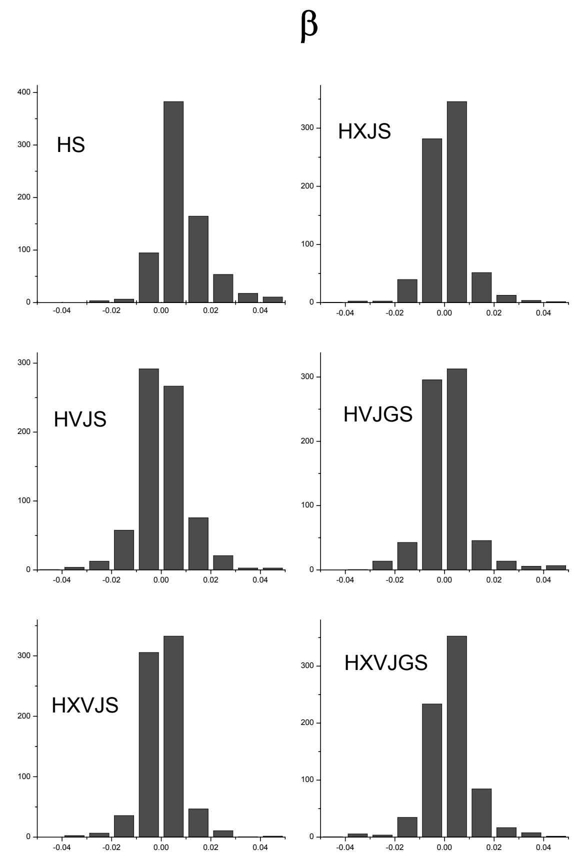

Figure 6.10. *Histograms β*

The parameter values of β for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

is a hidden state variable that can only be estimated as a parameter because we estimate implicitly on daily data. It is directly related to the parameters κ (the velocity of v_t to return to θ), θ (the long term mean of v_t), σ (the standard deviation of v_t), and even ρ (the correlation coefficient between the log prices and v_t). For the models with jumps in the variance process, also λ, η, γ, and ρ_J (as indicator for the correlation between simultaneous jumps in the log prices and the variance) are linked to the volatility, i.e. the square root of the variance.

For all models, mean and median of $\sqrt{v}$ are relatively stable between 2.07 (the median of HXJS and HVJGS) and 2.59 (the mean of HS) with all standard deviations below 2.00, see Table 6.8. In the histograms in Fig. 6.11, two modes for all

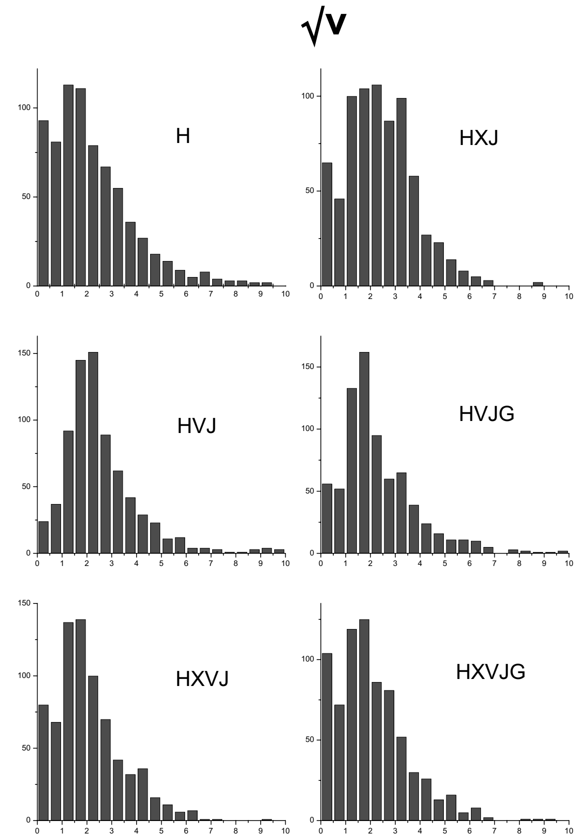

Figure 6.11. *Histograms* $\sqrt{v}$

The parameter values of $\sqrt{v}$ for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

models can be distinguished: the first close to zero, and the second around two.

Before we turn to κ, let us begin with θ, the value that should be the long-term mean of the variance process. Mean and median are between 2.44 (the median of model HS) and 8.29 (the mean of model HXVJS) for all models. The root of these two values is 1.56 and 2.88, and the means and medians of $\sqrt{v}$ all lie between these two values, which can be seen as a good indicator for well specified models. The standard deviations of θ, however, are quite high for all models, the minimum being at 7.69 for model HVJS and the maximum at 19.66 for HXVJS, the model where mean, with 8.29, and median, with 2.99, already differ the most. The histograms for θ in Fig. 6.12 confirm the observation of quite dispersed estimates.

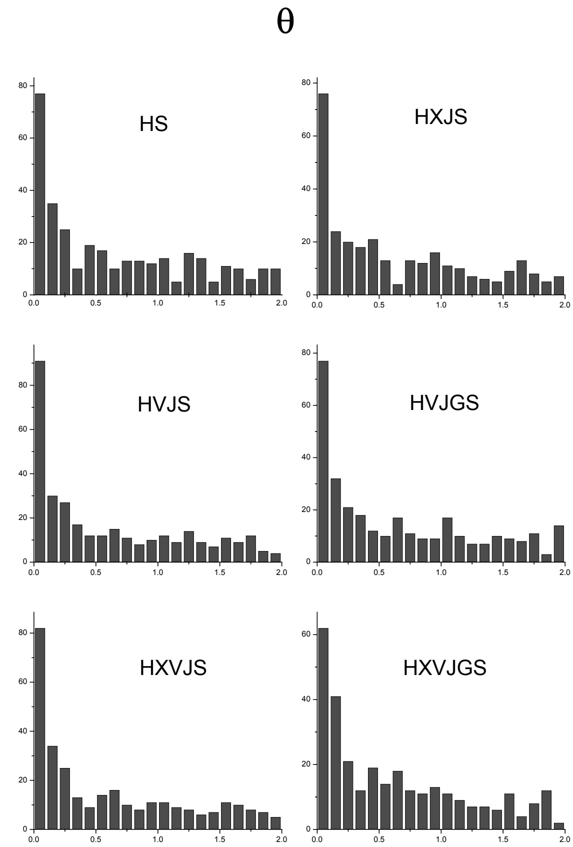

Figure 6.12. *Histograms θ*

The parameter values of θ for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

Regarding κ now, we recognise a mean and median of 66.24 and 37.01, with a standard deviation of 70.71, for model HVJGS, the model that proved to have the lowest RMSEs. All other models estimate even higher values for κ, with a maximum of 103.76 and 94.31 in mean and median for model HVJS. The standard deviation for κ is, with 83.28, the highest for model HS. In the histograms in Fig. 6.13, we again notice that all models behave similarly, and we can recognise at

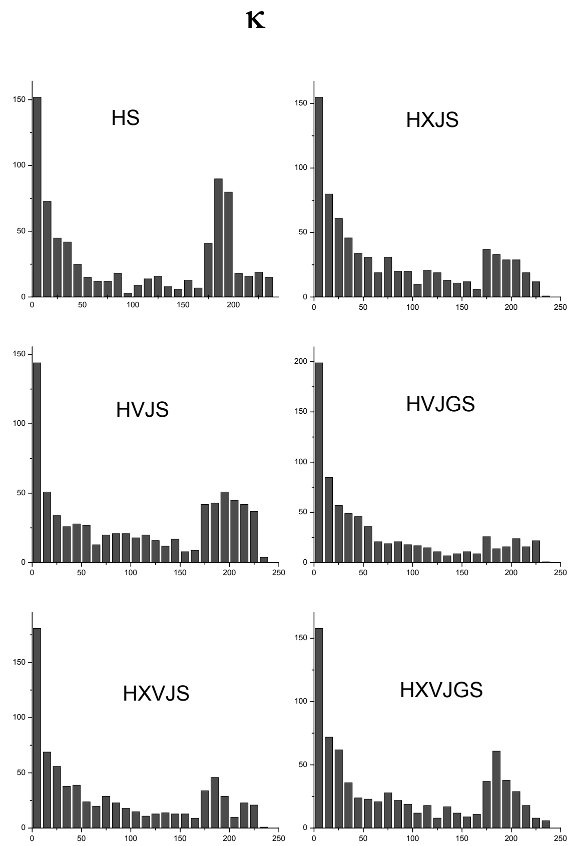

Figure 6.13. *Histograms κ*

The parameter values of κ for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

least two modes: one close to zero, the other around 200. The value of κ expresses the velocity of the variance process to return to the long-term mean θ, and with a high κ, the variance should almost immediately return to a value near θ. Since the standard deviation of the estimations of $\sqrt{v}$ is relatively low in comparison to our other estimates, even high values of κ seem reasonable.

But the dispersion of v does not only depend on how fast the variance returns to its mean θ, but, perhaps even more, on the standard deviation σ. In Table 6.8 we see values of the means and medians of σ, from 0.20 for the median of model HXVJGS to 0.50 for the mean of model HVJS. The standard deviations of σ range from 0.26 in model HXVJGS to 0.77 in model HVJS, i.e. the models with the lowest/highest values in mean/median are also the models with the lowest/highest values of the standard deviation of σ. So – as we would expect – the low dispersion of v is accompanied by high values for κ and low values for σ. The histograms for σ in Fig. 6.14 show a second mode near zero for all models. A value of zero would mean, in the case of no jumps in the variance process, that there is no stochastic volatility, or, if the models contain jumps in the volatility process, that this process is only driven by this jump process without a diffusion component. Some estimation results seem to suggest this.

Let us now proceed with the last parameter, apart from the seasonal parameters, that is common to all models of this model group, ρ. Particularly interesting would be values of -1, 0 or 1, and, indeed, for model HVJS, mean and median are, with 0.02 and 0.08, quite close to zero, meaning, that log prices and variance are nearly uncorrelated. However, this first impression is misleading. Consider the histograms of ρ in Fig. 6.15. We see that most estimates of ρ in model HVJS are near -1 or 1, only averaging a value near zero. So either perfect negative or perfect positive correlation is assumed in many estimation results. All other models have means and medians between 0.27 (the mean of model HS and HXVJGS) and 0.67 (the median of model HS). So they are all positive. This means that high returns correspond to a high variance and vice versa. Although this is contrary to stock markets, we expect this behaviour. When electricity prices are very high, their fluctuation is also relatively high. A look again at Fig. 6.15 shows that with the exception of HVJS, correlations of -1 are relatively rare for all models.

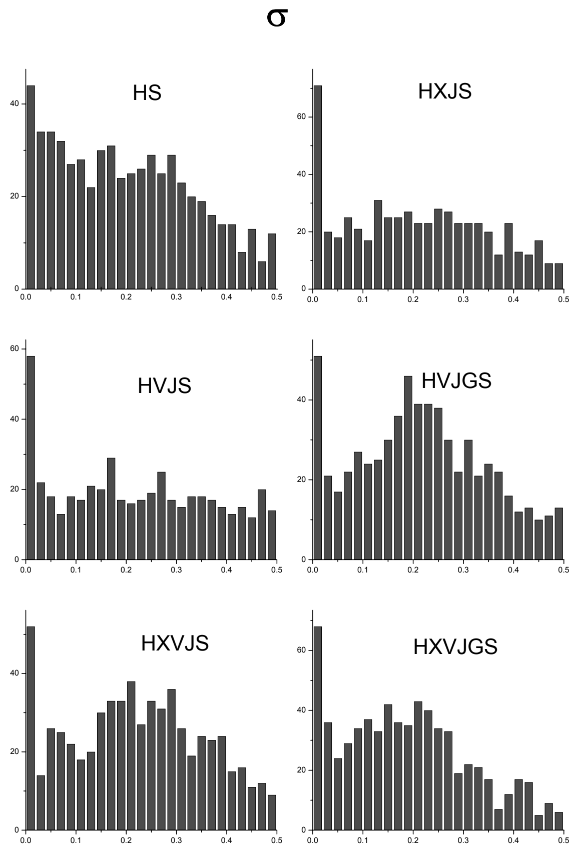

Figure 6.14. *Histograms σ*

The parameter values of σ for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

The next parameter that we consider is λ. It always represents the frequency of jumps per year, but we have to clearly distinguish the type of jump in each model: in model HXJS, only jumps in the log prices are modelled, in HVJS, only exponential distributed jumps in the variance process. HVJGS is the same as model HVJS, but the jump sizes in the variance process follow a Γ-distribution. In models HXVJS and HXVJGS, finally, λ is the frequency of simultaneous jumps in the log spot prices and the variance process. In the first model, the jump sizes in the

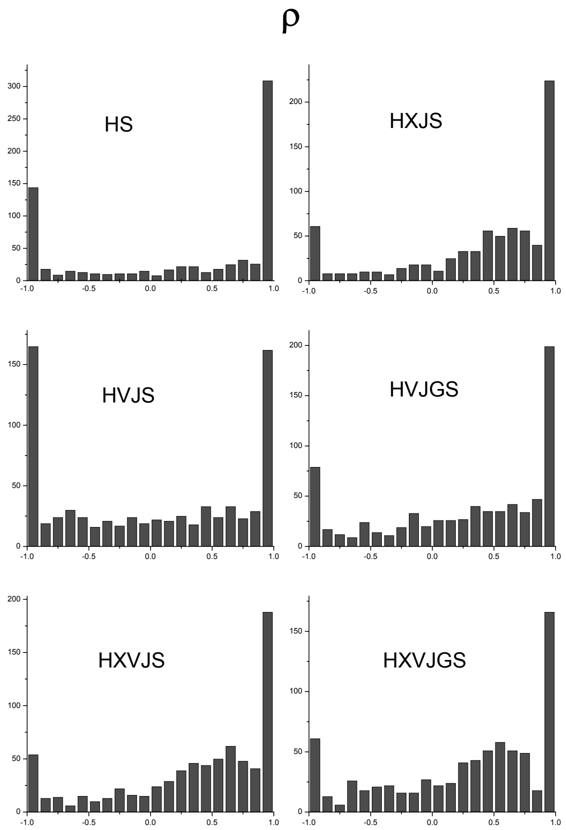

Figure 6.15. *Histograms ρ*

The parameter values of ρ for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

variance process are again exponentially distributed, in HXVJGS Γ-distributed. So the estimations are also very different: in models with jump in the log price, λ is relatively small: the lowest values are estimated for HXVJGS with a mean 3.37 and a median 0.97. Also the standard deviation of the estimations is the smallest for this model with 9.39. The second lowest values for λ are in model HXJS, the model where jumps only occur in the log price. Here, mean, median and standard deviation are 4.83, 1.60, and 11.30. The highest values occur in HVJS and HVJGS, the models with jumps only in the variance process. The means and medians range there from 2.23 in the median of model HVJS to 20.31 in the mean of the same model. Also the standard deviation for HVJS is by far the highest with 125.41. The estimations for HVJGS seem more stable. The enormous differences in the models are, however, not so clear in the histograms in Table 6.16. However, we can detect

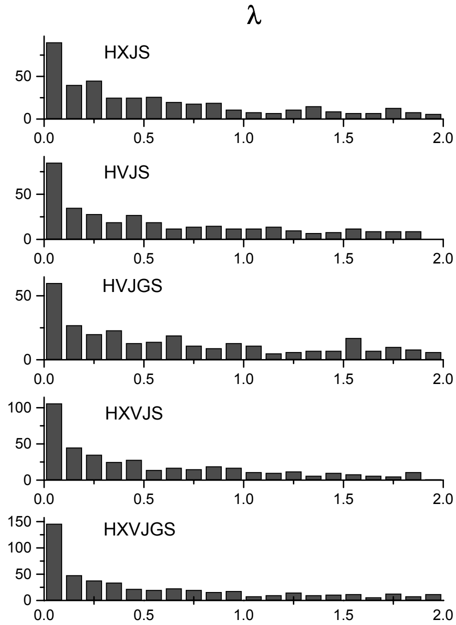

Figure 6.16. *Histograms λ*

The parameter values of λ for all estimations are plotted in a histogram for each model. The models are:

- HXJS: a Heston-like model with season, with a normal-distributed jump in the first state variable,
- HVJS: the same Heston-like model with season, with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

that for model HXVJGS, most estimations are very close to zero, in comparison to model HVJGS, for example.

μ_J and σ_J are only defined in the models with jumps in the log price, i.e. models HXJS, HXVJS and HXVJGS. They are the parameters for the mean and standard deviations of the normal distributed jump sizes in x. Having in mind high price spikes of the electricity prices, i.e. sudden movements of the price to a multiple of its previous level, with a nearly immediate return to this level, the estimated values are disappointing. The jumps are between 0.01 (the mean of model HXVJGS) and 0.09 (the median of model HXJS) in the mean and median for all three relevant models. The standard deviation for μ_J is also the lowest in model HXVJGS with 0.18, and the highest in model HXJS with 0.24. The histograms for μ_J in Fig. 6.17 seem to show normal distributions, with a mean of the μ_J of all models slightly

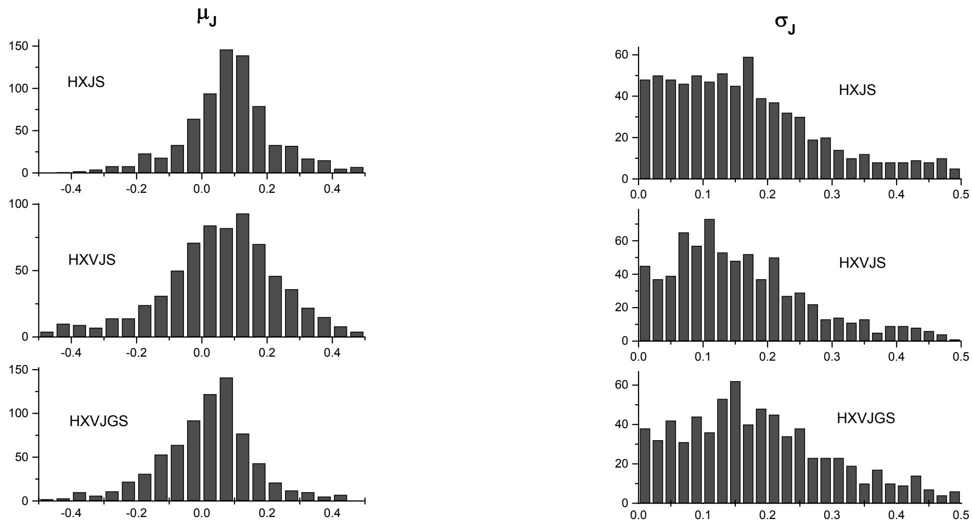

Figure 6.17. *Histograms μ_J and σ_J*
The parameter values of μ_J and σ_J for all estimations are plotted in a histogram for each parameter and for each model. The models are:

- HXJS: a Heston-like model with season, with a normal-distributed jump in the first state variable,
- HXVJS: the same Heston-like model with season, with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

positive.

For all relevant models, the estimations of σ_J seem to be very stable, around 0.2, and even their standard deviations are all very similar from 0.16 (for model HXVJS) to 0.19 (for model HXJS). The histograms for σ_J in Fig. 6.17 show, that for all models most estimations are between 0 and 0.2; for model HXJS, the σ_J even seem to be uniformly distributed between 0 and 0.2.

η is the mean of the exponential- or Γ-distributed jump sizes of the variance process in models HVJS, HVJGS, HXVJS and HXVJGS. It is the lowest for model HXVJGS with a mean of 0.05, and the highest for the model without jump in the log price and exponentially distributed jump size in the variance process, model HVJS. There, the value for the mean of η is 0.10. The medians are always 0.02. The histograms in Fig. 6.18 show that for all models, a large amount of estimations

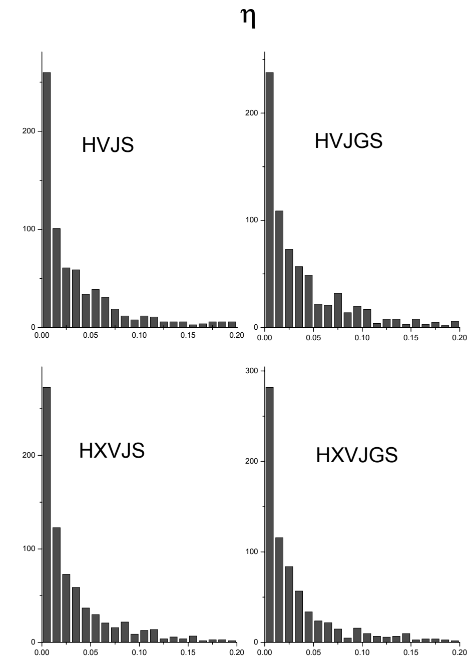

Figure 6.18. *Histograms η*

The parameter values of η for all estimations are plotted in a histogram for each model. The models are:

- HVJS: a Heston-like model with season, with an exponential-distributed jump in the second state variable,
- HVJGS: the same Heston-like model with season, with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

is close to zero.

An exponential distribution is a Γ-distribution with the parameter $\gamma \equiv 1$.[13] However, if we examine Table 6.9, the means for models HVJGS and HXVJGS are 8.94 and 6.97, respectively. The medians are also relatively far away from 1.00 with 2.60 and 2.53. But the estimations of γ are not very robust: in model HVJGS the standard deviation of all estimations is 24.81, for model HXVJGS 17.09. The histograms in Fig. 6.19 show high outliers for both models, up to the value of 30.

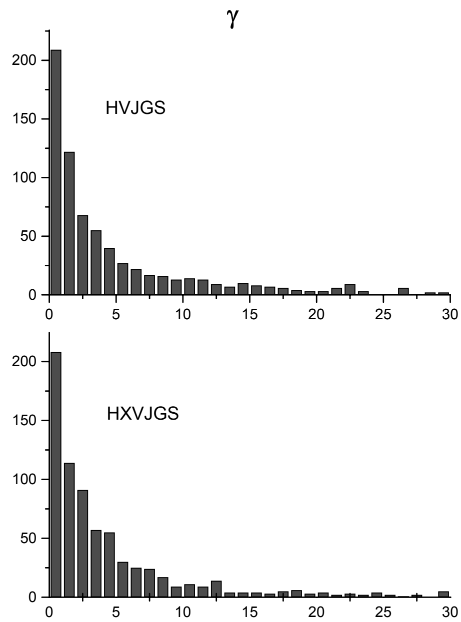

Figure 6.19. *Histograms γ*

The parameter values of γ for all estimations are plotted in a histogram for each model. The models are:

- HVJGS: a Heston-like model with season, with a Γ-distributed jump in the second state variable,
- HXVJGS: the same Heston-like model with season, with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

In models with simultaneous jumps in log price and variance, the value that is particularly interesting is ρ_J, an indicator for the correlation between the simultaneous jumps in log price and variance. The correlations of these jumps can be calculated with (5.14) and (5.15). So, with a mean value of ρ_J of 0.69 in model HXVJ and 0.89 in model HXVJG, we get, using also the means of the other parameters that are needed, correlation coefficients of 0.29 and 0.49, respectively. If we

[13] For an explanation of the Gamma-distribution and the parameter γ, see Sect. 5.2 p. 55 or Resa Corporation and Licensors (2000–2003).

employ the medians for all variables, values of 0.10 and 0.15, respectively, result. This is a very low correlation. If simultaneous jumps in the log prices and the variance process occur, then they seem to have a low correlation. See Fig. 6.20 for the

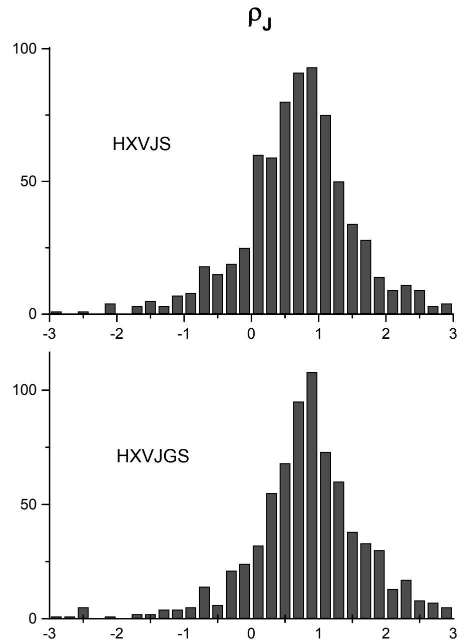

Figure 6.20. *Histograms ρ_J*

The parameter values of ρ_J for all estimations are plotted in a histogram for each model. The models are:

- HXVJS: a Heston-like model with season with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: the same model, but with a Γ-distributed jump in the second state variable.

histograms of ρ_J, where we see that most values are positive and around a mode near 1, translating to low correlation values.[14]

Now to the parameters that are responsible for capturing the seasonal behaviour of the electricity prices, s_0 and s_1. These two parameters have already proven their great importance in the OU-type models and that they are consistent to estimate. As we have already remarked earlier, the inclusion of these seasonal components could also reduce the RMSEs for the Heston-type models by a large amount.

Let us start with s_0. Remember that in our first model group the values were all around 25.6 to 26.5. Apart from model HS, we do have quite similar values here,

[14] A value of 3 for ρ_J, only very rarely estimated, would lead to a correlation coefficient of 0.8, if the other parameters are given as the means of model HXVJS.

only a bit lower. In model HS, mean and median are 22.74 and 23.34, values that are still not far away from the values of the other models. We perceive the highest values in model HXJS, with a mean of 25.90 and a median of 25.62. The standard deviations are all under 8.00; model HS has the highest standard deviation with a value of 7.65. This shows that extending model HS does not overspecify the model, but even leads to a better recognition of the seasonal parameters.

This is confirmed by looking at the standard deviation of s_1, which is also the highest for HS. The values of mean and median are, rounded to two digits, 0.64 for all models, as it has already been the case for s_1 in models OUS and OUJS of model group 1.

So, the histograms in Figs. 6.21 and 6.22 are not really surprising and they are similar for all models. A graph with spot price and the spot price at $t_0 = 10/01/99$ plus the deterministic functions, like done in Fig. 6.6 of Sect. 6.1, is omitted here because of the similarity of the parameters.

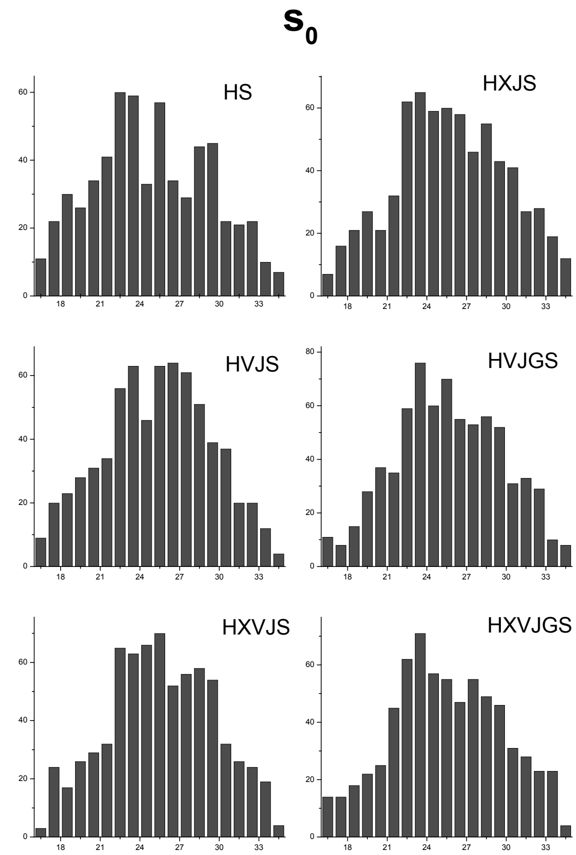

Figure 6.21. *Histograms* s_1

The parameter values of s_1 for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

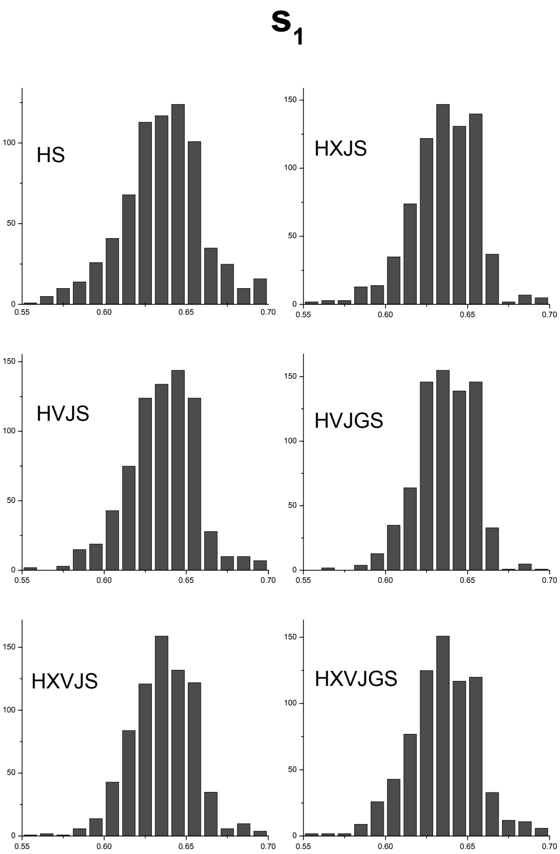

Figure 6.22. *Histograms s_1*

The parameter values of s_1 for all estimations are plotted in a histogram for each model. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

6.3.2 Regression Statistics

We will proceed with the regression statistics, examining each model in detail. Unlike in Sect. 6.1, we will comment on all statistics at once when we discuss a particular model, referring always to the previous models.

In Table 6.10 we see that the t-statistics are only significantly different from zero for κ and the parameters s_0 and s_1 that account for the seasonal behaviour. The same phenomenon has already been observed for s_0 and s_1 in models OUS and OUJS in Sect. 6.1. Unfortunately, all other parameters are at a very low significance level, with the worst values for α, θ, β and $\sqrt{v}$. The high standard error of θ is surprising, being even higher than the standard errors of α and $\sqrt{v}$. The reason for these unsatisfying values might be the difficulties of the optimisation algorithm to find clear extrema with such a huge amount of parameters in these highly non-linear problems. For the following models with even more parameters, we will expect that the t-statistics and standard deviations still get worse. So the parameter values themselves should be taken with care. In contrast, the rest of the statistics, for example the F-test (showing the adequacy of the models) and R^2 etc. prove the relevance of the models and justify their discussion, as we will see further on.

Thus, the F-statistic for model HS in the median is significant at the 1% level, as it was for 87.73% of all days, i.e. 658 out of 750. R^2 and the Adjusted R^2 are with 0.91 and 0.85 quite high, and also the Akaike Information Criterion (AIC) and the Schwarz Criterion (SC) are with 7.16 and 7.61 near model OUS, that seemed to perform best until now. The Durbin–Watson statistic is, with 1.35, still far away from 2.00, as it was for all models before.

In model HXJS, jumps in the deseasonalised log prices were added. The results in Table 6.11 are similar to those of model HS in Table 6.10. κ, s_0 and s_1 are significant at the 1% level in 73.07%, 88.00% and 99.07% of all days, whereas all other parameters have very low t-statistics. Again, the standard deviation for θ

Table 6.10. *Regression Statistics Model HS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HS refers to a Heston-like model with season.

Medians Model HS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,22	247,04	0,003	0,9977	0,00%	0,00%
β	0,01	0,54	0,01	0,9919	2,53%	1,73%
√**v**	2,30	199,14	0,01	0,4954	3,07%	2,93%
κ	79,08	0,70	85,33	0,0000	77,47%	74,93%
θ	2,44	457,45	0,003	0,4987	2,40%	2,40%
σ	0,25	0,50	0,46	0,2871	37,73%	33,47%
ρ	0,67	0,73	1,04	0,3132	41,20%	36,80%
s_0	23,34	3,51	7,03	0,0000	86,93%	81,73%
s_1	0,64	0,02	28,07	0,0000	96,27%	95,07%

F-Statistic	Prob.	sign. 5%	sign. 1%
17,98	0,0000	90,13%	87,73%

R-squared	0,91	Mean dependent var	150,83
Adjusted R-squared	0,85	S.D. dependent var	19,65
Akaike info criterion	7,16	Schwarz criterion	7,61
Log likelihood	-78,78	Durbin-Watson stat	1,35

Table 6.11. *Regression Statistics Model HXJS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXJS refers to a Heston-like model with season, with a normal-distributed jump in the first state variable.

Medians Model HXJS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,80	517,75	0,002	0,9986	0,13%	0,13%
β	0,001	0,54	-0,005	0,9962	1,07%	0,80%
√**v**	2,07	225,55	0,01	0,4971	1,47%	1,33%
κ	49,62	0,93	43,88	0,0000	76,13%	73,07%
θ	2,95	533,45	0,002	0,4989	1,33%	1,33%
σ	0,27	0,51	0,56	0,2680	37,20%	31,47%
ρ	0,61	0,94	0,62	0,5479	36,27%	33,33%
λ	1,60	590,90	0,001	0,4996	1,20%	1,20%
μ_J	0,09	404,90	0,0002	0,9998	0,40%	0,27%
σ_J	0,15	591,87	0,0002	0,4999	1,20%	1,20%
s_0	25,62	3,70	7,52	0,0000	94,53%	88,00%
s_1	0,64	0,02	28,90	0,0000	99,33%	99,07%

F-Statistic	Prob.	sign. 5%	sign. 1%
17,00	0,0000	92,00%	84,13%

R-squared	0,94	Mean dependent var	150,83
Adjusted R-squared	0,88	S.D. dependent var	19,65
Akaike info criterion	7,02	Schwarz criterion	7,61
Log likelihood	-74,25	Durbin-Watson stat	1,50

is very high with 533.45, like the standard deviations for α, $\sqrt{v}$ and the jump parameters λ, μ_J and σ_J.

This model seems more appropriate for the data than model HS. The F-Statistic is also significant at the 1% level in the median and in 84.13% of all days. R^2 and the Adjusted R^2 are with 0.94 and 0.88 higher in model HXJS than in model HS. The values of AIC, log likelihood and Durbin–Watson statistic favour model HXJS, too. With 7.61, the SC is the same for both models. The Durbin–Watson statistic and R^2 are also above the values of OUS, but the other statistics are still better for OUS.[15]

The following models are of particular interest, because models with jumps in the variance process were, as far as we know, only empirically tested by Duffie et al. (2000), who first supposed those types of jumps, themselves, while Γ-distributed jump sizes in the variance process are introduced by us, as far as we know.

Let us start with Exponential jump sizes and model HVJS. In Table 6.12, we again see similar t-statistics for our parameters as before, and also the high standard errors can be perceived for the same parameters. η cannot be more significantly estimated than μ_J and σ_J in model HXJS.

As for the models of this previous group, the F-statistic is significant at the median at the 1% level. R^2 is with 0.93 below the R^2 of HXJS with 0.94, but the Adjusted R^2 is equal with 0.88 in both models, caused by the fact that HVJS uses only eleven parameters, instead of twelve, the number of parameters used in model HXJS. The values of the AIC and of the SC, that penalises additional coefficients, are both lower for HVJS than for HXJS, with 6.98 and 7.51, so the model with jumps in the variance process is favoured. The log likelihood in HVJS is lower than in model HS, supporting the extension of HVJS. Also the Durbin–Watson statistic is better for model HVJS, with 1.43, than for HS, with 1.35, but lower than for HXJS with a value of 1.50.

[15] Log likelihood of models HXJS and OUS cannot be compared because HXJS is not embedded in OUS.

Table 6.12. *Regression Statistics Model HVJS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HVJS refers to a Heston-like model with season, with an exponential-distributed jump in the second state variable.

Medians Model HVJS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,37	250,52	0,003	0,9980	0,00%	0,00%
β	0,0001	0,40	0,009	0,9932	2,13%	1,73%
√v	2,33	215,68	0,008	0,4966	0,80%	0,67%
κ	94,31	0,54	146,38	0,0000	80,00%	78,13%
θ	2,69	482,33	0,002	0,4991	0,67%	0,53%
σ	0,36	0,34	0,71	0,2372	38,40%	34,53%
ρ	0,08	0,56	0,95	0,3573	42,13%	37,87%
λ	2,23	871,82	0,001	0,4995	1,73%	1,07%
η	0,02	458,02	0,00003	0,5000	0,67%	0,67%
s_0	25,14	3,37	7,99	0,0000	94,27%	88,93%
s_1	0,64	0,02	30,01	0,0000	98,93%	98,67%

F-Statistic	Prob.	sign. 5%	sign. 1%
17,80	0,0000	93,60%	87,20%

R-squared	0,93	Mean dependent var	150,83
Adjusted R-squared	0,88	S.D. dependent var	19,65
Akaike info criterion	6,98	Schwarz criterion	7,51
Log likelihood	-75,53	Durbin-Watson stat	1,43

Now consider one of the two models with γ-sized jumps. The coefficients of model HVJGS in Table 6.13 have similar statistics to the previous models: The seasonal parameters s_0 and s_1 are the most significant ones, in 91.87% and 99.87%, respectively, of all days, i.e. in 689 and 749 out of 750 days, they were significant at the 1% level. The only parameter that is also significant in the median at the 1% and even the 5% and 10%-level is κ. And though the new parameter in this model, γ, is also not significantly estimated and has a high standard error of 231.03, at least it diminishes the standard errors of λ and η, in comparison to model HVJS.

The F-statistic is significant at the 1% level in the median, as in the previous models of this model group. R^2 is, with 0.94, higher than in model HVJS and equal to model HXJS, but the Adjusted R^2 is the same for all three models, caused by the penalisation of the Adjusted R^2 for the extra parameter in HXJS and HVJGS. The AIC, with 6.87, and the SC, with 7.46, slightly favour model HVJGS. The log likelihood that can only be compared with model HS and HVJS, is with -73.91 lower for model HVJGS than for model HS with -78.78 and for model HVJS with -75.53. The Durbin–Watson statistic is, with 1.51, closer to 2.00 than in all other models. For some statistics, the values are even better in HVJGS than for model OUS in Table 6.4 (in R^2 and the Durbin–Watson statistic).

Now see Table 6.14 for the model with simultaneous jumps in the log price and variance process. The comments on the t-statistics and standard deviations are the same as for the other models of this model group, and also ρ_J has a very high standard error with 665.96.

Also, like in the other models, the F-statistic is significant at the 1% level in the median. Though model HXJS is embedded in model HXVJS, i.e. HXVJS is HXJS in the limit when $\eta = 0$, the R^2 of model HXVJS is, with 0.93, lower than the R^2 of HXJS with 0.94. This phenomenon was already observed in Sect. 6.1 with models OU and OUJ, and OUS and OUJS, respectively. All other statistics may be better for the less complex models because of penalisation terms for the number of parameters. Thus all statistics of HXVJS are topped by those of HXJS and usually by those of HVJS. Regarding these statistics, simultaneous jumps in

Table 6.13. *Regression Statistics Model HVJGS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HVJGS refers to a Heston-like model with season, with a Γ-distributed jump in the second state variable.

Medians Model HVJGS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,68	257,45	0,004	0,9972	0,13%	0,13%
β	0,0004	0,59	0,01	0,9959	2,67%	2,27%
√v	2,07	197,41	0,01	0,4969	1,20%	1,20%
κ	37,01	0,94	39,55	0,0000	73,87%	71,33%
θ	2,99	495,13	0,003	0,4988	1,20%	1,20%
σ	0,24	0,46	0,32	0,3527	33,60%	28,40%
ρ	0,46	0,94	0,54	0,5985	34,67%	30,67%
λ	3,44	653,14	0,002	0,4990	1,20%	1,20%
η	0,02	62,52	0,0003	0,4999	1,73%	1,73%
γ	2,60	231,03	0,01	0,4962	5,47%	4,80%
s_0	25,43	3,63	7,38	0,0000	96,80%	91,87%
s_1	0,64	0,02	29,73	0,0000	99,87%	99,87%

F-Statistic	Prob.	sign. 5%	sign. 1%
17,55	0,0000	96,27%	89,47%

R-squared	0,94	Mean dependent var	150,83
Adjusted R-squared	0,88	S.D. dependent var	19,65
Akaike info criterion	6,87	Schwarz criterion	7,46
Log likelihood	-73,91	Durbin-Watson stat	1,51

Table 6.14. *Regression Statistics Model HXVJS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXVJS refers to a Heston-like model with season, with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously.

Medians Model HXVJS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,79	691,38	0,001	0,9990	0,13%	0,00%
β	0,0004	0,52	0,005	0,9964	2,93%	2,27%
$\sqrt{v}$	2,12	341,10	0,01	0,4973	1,73%	1,60%
κ	46,98	0,87	69,70	0,0000	77,60%	74,53%
θ	2,99	681,81	0,002	0,4991	1,87%	1,73%
σ	0,26	0,46	0,46	0,3156	33,87%	27,60%
ρ	0,54	0,80	0,75	0,4659	30,13%	25,60%
λ	1,55	785,60	0,001	0,4995	1,60%	1,47%
μ_J	0,07	568,35	0,0002	0,9999	1,33%	1,33%
σ_J	0,14	730,06	0,0001	0,5000	1,33%	1,33%
η	0,02	573,82	0,00002	0,5000	1,33%	1,33%
ρ_J	0,73	665,96	0,001	0,9994	1,33%	1,33%
s_0	25,42	4,34	6,14	0,0001	92,40%	82,93%
s_1	0,64	0,03	23,65	0,0000	98,67%	98,40%

F-Statistic	Prob.	sign. 5%	sign. 1%
11,71	0,0002	89,60%	78,93%

R-squared	0,93	Mean dependent var	150,83
Adjusted R-squared	0,85	S.D. dependent var	19,65
Akaike info criterion	7,17	Schwarz criterion	7,85
Log likelihood	-74,38	Durbin-Watson stat	1,46

the log price and the variance process do not seem to improve the models, at least if the jump size in the variance process is exponentially distributed. However, remember that the values for the RMSE for this model are the lowest with model HVJGS.

What if the jump size in the variance process is Γ-distributed? Let us consider Table 6.15. The t-statistics and standard deviations are comparable to those of the other models of the model group 3.

And though the F-statistic is lower than in all other models with seasonal components, it is still significant at the 1% level, rejecting the hypothesis of misspecification. But while the model HVJS is improved by including Γ-sized jumps instead of Exponential ones (which results in better statistics for model HVJGS), this is not the case when comparing HXVJS to HXVJGS. Indeed, all statistics, for example the AIC with 7.32 in HXVJGS in comparison to 7.17 in HXVJS, are better for model HXVJS. The same conclusions can be drawn by comparing models HXVJGS and HVJGS.

To conclude so far, the results of the regression statistics show, as we have already suspected in the previous subsection, that model HVJGS seems the most promising, whereas simultaneous jumps in the log price and the variance process, though having a low RMSE, are disappointing in the regression statistics tables, the models are even penalised for their additional parameters.[16]

Before we give a final statement and compare model group 1 and model group 3, we continue with our investigations for model group 3.

[16] We also performed further t-tests for ρ for all models with the hypotheses $\rho = -1$ vs. $\rho > -1$ and $\rho = 1$ vs. $\rho < 1$. Furthermore, we also computed these tests for the correlation of the jump components, i.e. $\frac{\rho_J \eta}{\sqrt{\sigma_J^2 + \rho_J^2 \eta^2}}$ for HXVJS and $\frac{\rho_J \eta \sqrt{\gamma}}{\sqrt{\sigma_J^2 + \rho_J^2 \eta^2 \gamma}}$ for HXVJGS, using the standard error of ρ_J as approximation for the standard error of the correlation. And last, we tested the hypothesis whether $\beta = -0.5$ vs. $\beta \neq -0.5$, for all models. Unfortunately, clearer rejections were not more often possible than for the above t-tests, again caused by the high standard errors, so we omitted these results here.

Table 6.15. *Regression Statistics Model HXVJGS*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXVJGS refers to a Heston-like model with season, with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps occurring simultaneously.

Medians Model HXVJGS

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,78	1075,68	0,001	0,9993	0,27%	0,13%
β	0,002	0,36	0,01	0,9916	2,27%	2,00%
$\sqrt{v}$	2,11	451,50	0,004	0,4985	2,00%	1,73%
κ	59,53	0,51	109,68	0,0000	81,20%	79,33%
θ	3,10	895,90	0,001	0,4995	1,87%	1,87%
σ	0,20	0,31	0,44	0,3200	32,93%	26,13%
ρ	0,45	0,52	0,88	0,3973	36,67%	31,73%
λ	0,97	1098,20	0,0003	0,4999	2,00%	1,87%
μ_J	0,03	928,76	0,0001	1,0000	0,00%	0,00%
σ_J	0,18	1185,31	0,0001	0,5000	1,33%	1,33%
η	0,02	367,40	0,00003	0,5000	1,73%	1,73%
γ	2,53	664,76	0,003	0,4988	3,20%	2,80%
ρ_J	0,86	634,25	0,001	0,9991	0,67%	0,40%
s_0	25,17	5,23	5,00	0,0007	84,53%	72,53%
s_1	0,64	0,03	20,48	0,0000	98,53%	97,47%

F-Statistic	Prob.	sign. 5%	sign. 1%
8,68	0,0012	79,33%	67,60%

R-squared	0,93	Mean dependent var	150,83
Adjusted R-squared	0,82	S.D. dependent var	19,65
Akaike info criterion	7,32	Schwarz criterion	8,06
Log likelihood	-75,44	Durbin-Watson stat	1,45

6.3.3 Out-of-Sample Behaviour

Like for model group 1, see Sect. 6.1.3, we show the RMSEs not only with the optimal parameters for day t, but we computed it also with the parameters that were optimal on day t-1 (RMSE($\vec{\vartheta}$(t-1))), with the medians of optimal parameters from day 1 to t-1 (RMSE(median($\vec{\vartheta}$(1:t-1)))), and with the medians of all parameters (RMSE($\vec{\vartheta}$(1:end))). The results are shown in Table 6.16.

Table 6.16. *RMSEs – Out-of-Sample*

In this summary statistics table, mean, median and standard deviation of the RMSEs are reported, with the parameters determined by minimising the RMSEs, and the RMSEs calculated with the parameters that were optimal in the estimations of the day before, called here RMSE($\vec{\vartheta}$(t-1)), the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until day t-1 (RMSE(median($\vec{\vartheta}$(1:t-1)))), and the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until the last day (30 September 2002), called RMSE(median($\vec{\vartheta}$(1:end))). The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

		RMSE	RMSE(ϑ(t-1))	RMSE(median((ϑ(1:t-1)))	RMSE(median(ϑ(1:end)))
HS	**Mean**	7.82	12.15	22.16	23.04
	Median	5.92	8.46	16.83	17.14
	Std. Dev.	8.64	18.15	27.76	26.79
HXJS	**Mean**	6.04	11.19	28.40	31.69
	Median	4.96	7.46	24.26	29.91
	Std. Dev.	5.62	18.99	22.88	20.32
HVJS	**Mean**	6.02	11.18	23.13	23.57
	Median	5.08	7.72	18.75	19.21
	Std. Dev.	4.32	18.99	24.68	23.93
HVJGS	**Mean**	5.33	10.76	28.87	34.21
	Median	4.62	7.15	26.28	32.34
	Std. Dev.	4.39	17.77	22.98	19.66
HXVJS	**Mean**	5.37	10.65	29.85	30.91
	Median	4.93	7.32	26.14	28.95
	Std. Dev.	2.51	16.96	25.85	20.60
HXVJGS	**Mean**	5.98	11.22	33.97	31.04
	Median	5.10	7.51	32.31	29.20
	Std. Dev.	4.02	17.84	20.22	20.56

Comparing first $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$, $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:t-1})))$ and $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:end})))$, we can draw similar conclusions as in model group 1. For all models, $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$ is the lowest with values of a half to a third of the other two. In addition, with exclusion of the means and medians of model HXVJGS with 33.97 and 32.31 for $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:t-1})))$ and 31.04 and 29.20 for $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:end})))$, $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:t-1})))$ is always slightly lower than $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:end})))$. The models are more appropriate for the data for short time intervals than for longer periods.

Now we will look at each model. The difference in $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$ between the models of model group 3 is not very high, the medians are in a range from 7.15 for model HVJGS to 8.46 for model HS, the means are between 10.65 for model HXVJS and 12.15 for model HS. The standard deviations are around 18 and 19 for all models. Though still very low, the values of $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$ are not as close to the RMSEs that resulted in the optimisation process, as the values of $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$ in Sect. 6.1.3. The out-of-sample fitting is worse.

The differences between the models of model group 3 are more obvious if we look at $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:t-1})))$ and $\text{RMSE}(\text{median}(\vec{\vartheta}(1\text{:end})))$. Although HS is the worst model for $\text{RMSE}(\vec{\vartheta}(\text{t-1}))$, it now turns out to be the clear favourite. With means of 22.16 and 23.04 and medians of 16.83 and 17.14, the values are clearly lower than, for example, for model HVJGS with means of 28.87 and 34.21 and medians of 26.28 and 32.43, or model HXVJS with means of 29.85 and 30.91 and medians of 26.14 and 28.95. Model HVJS seems to be the second best, with means of 23.13 and 23.57 and medians of 18.75 and 19.21. Model HXJS has values of the same size as HVJGS, HXVJS and HXVJGS.

If we compare the results to model group 1 up to now, the suspicion arises that the models in model group 3 might be overspecified. For model group 1, the models OUS and OUJS that had the best fit, i.e. the lowest RMSEs, are also the preferred ones in the out-of-sample analysis; this is clearly not the case for models HVJGS and HXVJS. The values here are also absolutely higher for the latter two models than for models OUS and OUJS of model group 1. Furthermore, the models

HVJGS and HXVJS only show a good performance for RMSE($\vec{\vartheta}$(t-1)); for the other calculations they are as inappropriate as all other extensions of HS.

6.3.4 Option Pricing

The procedures followed for models HS, HXJS, HVJS, HVJGS, HXVJS and HXVJGS are the same as for models OU, OUJ, OUS and OUJS of model group 1. The results are listed in Table 6.17.

Table 6.17. *RMSEs of option pricing*

In this summary statistics table, mean, median and standard deviation of the RMSEs of the comparison of calculated and observed option prices are reported. In RMSE($\vec{\vartheta}$(t)), the parameters of day t, implicitly estimated by futures and forward prices, are used; in RMSE(median($\vec{\vartheta}$(1:t)), the medians of all parameters until day t, implicitly estimated by futures and forward prices, are used; and finally, in RMSE(median($\vec{\vartheta}$(1:end)), the medians of all parameters, implicitly estimated by futures and forward prices, are used. The models are:

- HS: a Heston-like model with season,
- HXJS: the same model with a normal-distributed jump in the first state variable,
- HVJS: with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJS: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJGS: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

		RMSE(ϑ(t))	RMSE(median((ϑ(1:t)))	RMSE(median(ϑ(1:end)))
HS	**Mean**	44.24	29.86	27.93
	Median	17.48	13.39	11.72
	Std. Dev.	101.14	43.07	44.01
HXJS	**Mean**	42.86	35.11	31.86
	Median	11.06	12.76	14.62
	Std. Dev.	90.88	43.38	43.97
HVJS	**Mean**	36.75	22.34	26.59
	Median	12.64	7.23	8.96
	Std. Dev.	51.24	43.27	46.54
HVJGS	**Mean**	30.09	32.14	30.20
	Median	8.80	12.88	10.62
	Std. Dev.	37.98	43.93	44.20
HXVJS	**Mean**	28.38	34.26	31.09
	Median	8.36	12.87	13.20
	Std. Dev.	37.69	43.14	43.25
HXVJGS	**Mean**	37.76	35.34	31.21
	Median	10.01	12.89	13.39
	Std. Dev.	79.68	43.33	43.09

Although it seems difficult to detect at first, the results here support the results of Sects. 6.3.1, 6.3.2, 6.3.3, as well as showing similarities to Sect. 6.1.4.

The difference compared to the last subsection is, which agrees with Sect. 6.1.4, that the values for RMSE($\vec{\vartheta}$(t)) are not always the lowest. The statistic with the lowest values is dependent on the model: sometimes it is RMSE(median($\vec{\vartheta}$(1:t))), RMSE($\vec{\vartheta}$(t)), or sometimes even RMSE(median($\vec{\vartheta}$(1:end))). The absolute values are often lower than those of Table 6.7,and the standard deviations, ignoring some exceptions, are higher. Also, the means are always two times or more the values of the medians, indicating high outliers.

The model with the lowest means and medians for RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))) is HVJS, with values of 22.34 and 7.23 for RMSE(median($\vec{\vartheta}$(1:t))) and 26.59 and 8.96 for RMSE(median($\vec{\vartheta}$(1:end))), respectively. The mean and median of RMSE($\vec{\vartheta}$(t)) are 36.75 and 12.64 for this model. RMSE($\vec{\vartheta}$(t)) has the lowest values for model HXVJS, with a mean of 28.38 and a median of 8.36. It is the only model for which RMSE($\vec{\vartheta}$(t)) has a mean below 30.00. RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))) also have means below 30.00, apart from model HVJS, for model HS, with 29.86 for RMSE(median($\vec{\vartheta}$(1:t))) and 27.93 for RMSE(median($\vec{\vartheta}$(1:end))). A further remarkable model, that seems to have acceptable values for all of the statistics, is HVJGS, with medians of 8.80, 12.88 and 10.62 for RMSE($\vec{\vartheta}$(t)), RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))).

Thus, the results favour models HVJS, HVJGS and HXVJS, but we also get relatively low means for RMSE(median($\vec{\vartheta}$(1:t))) and RMSE(median($\vec{\vartheta}$(1:end))) for model HS, the clear favourite in the last subsection. The validity of the models for short time intervals only does not play a role here, as it has already been noticed for model group 1 in Sect. 6.1.4.

In general, the models of this model group are often not clearly distinctive for the quality of their results. Jumps in the variance process seem to improve model HS, and sometimes also $\Gamma-$distributed size jumps are preferred over exponential-distributed ones, but only if they are not combined with jumps in the deseasonalised log prices x. Models HXJS and HXVJGS, the model with jumps in x and the model with simultaneous jumps in x and the variance process v, with Γ-distributed jump sizes in v, show the poorest performance.

6.4 Conclusions of Empirical Results

In this chapter, we compared various model groups. The characteristic of the first model group was its mean reversion, i.e. the assumption that the spot price would always move around a deterministic value θ. The basic model was extended with jumps and with deterministic components to account for the seasonal behaviour of electricity prices. The last extension proved to be very important while jumps could not really improve the model, also due to numerical problems.

The basic model of our second model group was a model with stochastic volatility like proposed by Heston (1993). This base model was extended with jumps in the log spot price x as well as with jumps in the variance process v. The jump sizes in v were either exponentially or Γ-distributed. The model was further extended with simultaneous jumps in x and v, and the jumps in v again were exponentially or Γ-distributed. The seasonal characteristics of the prices were ignored. The fitting of all of these models did not really exceed the fitting results for model group 1 without seasonal modelling. Furthermore, not even one F-statistic was significant at the 5% level, so we decided not to discuss the model group, but to list all figures and tables in Appendix A.

The third model group exactly corresponds to model group 2, but with the inclusion of seasonal modelling as was already proven to be very useful in model group 1. This improves all results of model group 2 to the same extent that the models in model group 1 without seasonal components were improved by adding them. All F-statistics were then significant at the 1% level, at least at the median. Nevertheless, the results could not clearly surpass the optimal values of model group 1, and in out-of-sample considerations, the models of model group 3 even seemed to be overfitted. The absolute values in the subsection, where options were priced, were in general better for the last model group, though the results of option pricing have to be carefully interpreted, because the available sample of observed option prices was very low.

The most important feature in the models were the components for the seasonal behaviour of the spot prices. Mean reversion was only modelled in model group 1, and it already led to results which were difficult for the complex models of model group 3 to cope with. Moreover, signs of overfitting for this model group were noticed in out-of-sample considerations.

For future research, we would therefore propose models that contain advanced deterministic components for the mapping of, for example, seasonal behaviour. Maybe other regressors like water reservoir levels could be included. Models that include mean reversion are preferable. If no analytic solutions for models with mean reversion in the first state variable and stochastic volatility can be found, numerical solutions for this combination should be examined. The possibilities still remaining are challenging.

Chapter 7

Summary and Conclusions

The purpose of this work was twofold. The electricity market is looking for pricing models that can explain the characteristics of its price movements with sometimes tremendous up-moves followed soon after by down-moves to the previous levels, movements that are also called spikes. Furthermore, we observe that for some seasons in the year, the prices are always higher than for other seasons, in the Nordic countries, for example, electricity is more expensive in winter than in summer, caused by a higher consumption of heating. This was one challenge for us.

Moreover, we wanted to apply the models, that were introduced by Duffie et al. (2000) for stock markets, for electricity. We adapted these models for electricity prices and added components and generalisations where it seemed reasonable and possible.

This work proceeded as follows:

First, we examined the validity of restrictions on pricing models that apply to tradable assets for electricity prices. With less restrictions, the set of possible models becomes larger. Futures and forward prices can be used to implicitly estimate parameters that cannot be estimated from futures and forward prices on tradable assets.

After having explained some preliminaries, we could present our approach for pricing and estimation.

Next, we regarded the Nordic power exchange Nord Pool. The spot as well as the financial market were analysed and the available products and contracts were explained. The spot prices and the term structure of futures/forward and option prices were examined and visualised.

Then we presented the models which we used. We employed a model that is well known in pricing commodities, an Ornstein–Uhlenbeck (OU) process, and extended it in various ways: first, we added Poisson-jumps with Gaussian-distributed jump sizes to the log prices, and then we enriched the price process with a deterministic function responsible for catching deterministic characteristics of electricity prices, especially seasonal characteristics.

But this was not the only model group used. We also employed a model with stochastic volatility, similar to the one proposed by Heston (1993). Recent extensions introduced by Duffie et al. (2000) for the stock market were adopted and extended. These are jumps that occur in the volatility process, and jumps that simultaneously occur in the log prices and in the volatility process. Whereas the jump sizes in the volatility process, or better variance process, in Duffie et al. (2000) are distributed with an Exponential, we also take Γ-distributions into consideration, that are a generalisation of Exponential distributions. Furthermore, as the extensions for seasonality in the OU-models were very successful, these were also applied to the Heston-type models.

The models were all presented and the necessary calculations for estimation and pricing were performed. Often, though, closed-form solutions are not available and integrals had to be solved numerically.

In the next chapter, we presented the empirical results and compared the different models and model groups using various statistics and out-of-sample considerations. Also, options were priced by Monte Carlo methods and compared with the traded option contracts in Nord Pool.

The results show the importance of the inclusion of features that can map the seasonal behaviour of electricity prices. Mean reversion also seems important and similar results for Heston-like processes can only be obtained by adding more structure, as for example jumps in the volatility process. In the cases where no closed-form solutions were available and integrals had to be solved numerically, the results contained high outliers. Nevertheless, if these extensions could be combined with mean reversion in the log prices, further advances could be accomplished.

For further research on electricity prices, we recommend extending OU models with more elaborate seasonal components or further regressors. Models with stochastic volatility and jumps, like proposed by Duffie et al. (2000) and enhanced by us, can be combined with mean reversion. More generalisations and extensions are possible.

It would also be of interest to employ the model extensions of Γ-distributed jump sizes that we introduced, for the stock market or other financial or commodity markets. Not being specifically tailored to electricity prices, these extensions can also improve the models of Duffie et al. (2000) for markets with tradable assets.

Appendix A

Appendix to Empirical Results, Model Group 2

A.1 Summary Statistics

The summary statistics are in Tables A.1 and A.2, the histograms in Figs. A.1, A.2, A.3, A.4, A.5, A.6, A.7, A.8, A.9, A.10, A.11, A.12 and A.13.

Table A.1. *Summary Statistics models H, HXJ, HVJ, HVJG, HXVJ and HXVJG*

For each trading day between 01 October 1999 and 30 September 2002, all parameters for the relevant models were independently estimated. Futures and forward contracts lead to implicit estimates minimising the RMSE (root mean squared error) for all contracts. In this summary statistics table, mean, median and standard deviation of each parameter are reported, as well as of the RMSE. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

	α	β	√v	κ	θ	σ	ρ
Model H							
Mean	-4,24	0,01	2,56	52,83	8,65	0,60	0,59
Median	-1,01	0,01	1,90	13,50	1,86	0,44	0,95
Std. Dev.	11,57	0,02	2,76	70,01	25,81	0,71	0,65
Model HXJ							
Mean	-2,73	0,01	2,42	84,96	5,85	0,47	0,23
Median	-0,78	0,01	2,27	39,42	1,06	0,24	0,57
Std. Dev.	9,04	0,02	1,55	84,67	25,37	0,82	0,79
Model HVJ							
Mean	-4,97	0,01	2,66	52,24	10,14	0,63	0,45
Median	-0,96	0,01	2,20	11,81	1,66	0,29	0,71
Std. Dev.	12,19	0,02	1,82	70,47	37,30	1,34	0,69
Model HVJG							
Mean	-3,86	0,01	2,30	47,14	6,46	0,40	0,60
Median	-0,65	0,003	1,89	10,56	1,09	0,30	0,80
Std. Dev.	8,01	0,01	1,56	68,83	13,71	0,37	0,55
Model HXVJ							
Mean	-3,37	0,002	2,10	37,15	6,28	0,39	0,47
Median	-1,30	0,001	1,80	9,61	1,57	0,28	0,64
Std. Dev.	5,56	0,01	1,44	59,32	14,14	0,50	0,60
Model HXVJG							
Mean	-3,45	0,005	2,16	45,59	8,09	0,25	0,20
Median	-1,54	0,003	1,81	11,08	1,68	0,16	0,28
Std. Dev.	6,42	0,02	1,99	66,78	34,37	0,53	0,59

Table A.2. *Summary Statistics models H, HXJ, HVJ, HVJG, HXVJ and HXVJG – Continued*

	λ	μ_J	σ_J	η	γ	ρ_J	RMSE
Model H							
Mean							15,31
Median							14,48
Std. Dev.							6,20
Model HXJ							
Mean	3,51	0,12	0,29				16,42
Median	0,62	0,13	0,18				15,02
Std. Dev.	15,12	0,30	0,43				7,65
Model HVJ							
Mean	12,15			0,28			14,65
Median	2,98			0,02			14,06
Std. Dev.	40,98			1,43			4,05
Model HVJG							
Mean	7,54			0,08	18,04		14,20
Median	2,64			0,02	2,20		13,82
Std. Dev.	28,15			0,29	294,07		2,86
Model HXVJ							
Mean	3,66	0,14	0,29	0,12		0,98	14,20
Median	1,00	0,13	0,20	0,03		0,86	13,86
Std. Dev.	15,77	0,22	0,39	0,96		1,37	2,79
Model HXVJG							
Mean	6,24	0,07	0,24	0,06	12,69	0,67	14,45
Median	3,34	0,09	0,20	0,01	1,73	0,80	13,97
Std. Dev.	18,71	0,29	0,20	0,67	103,28	1,49	3,60

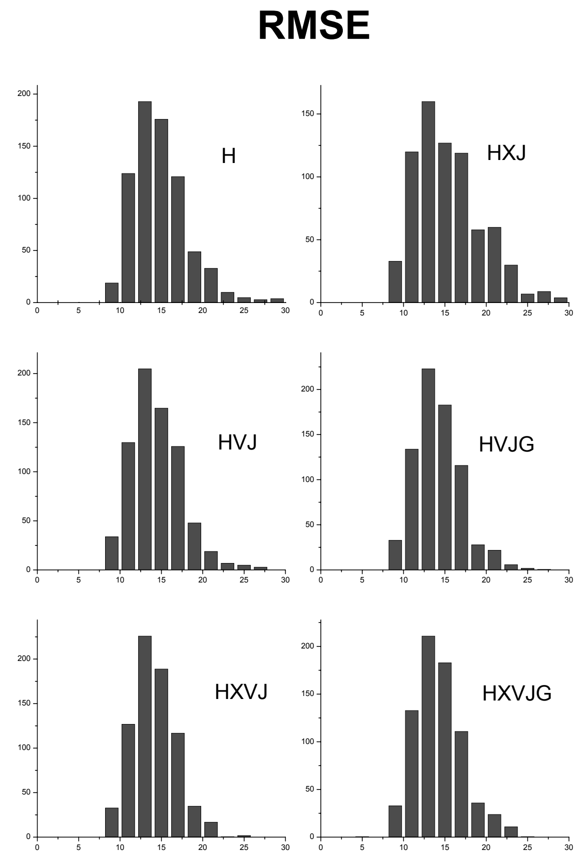

Figure A.1. *Histograms RMSEs*

The RMSEs for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

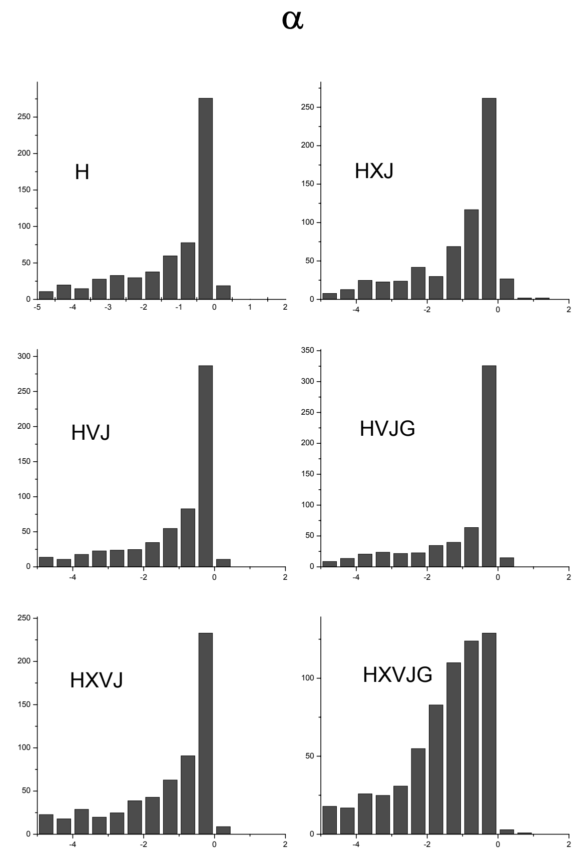

Figure A.2. *Histograms α*

The parameter values of α for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

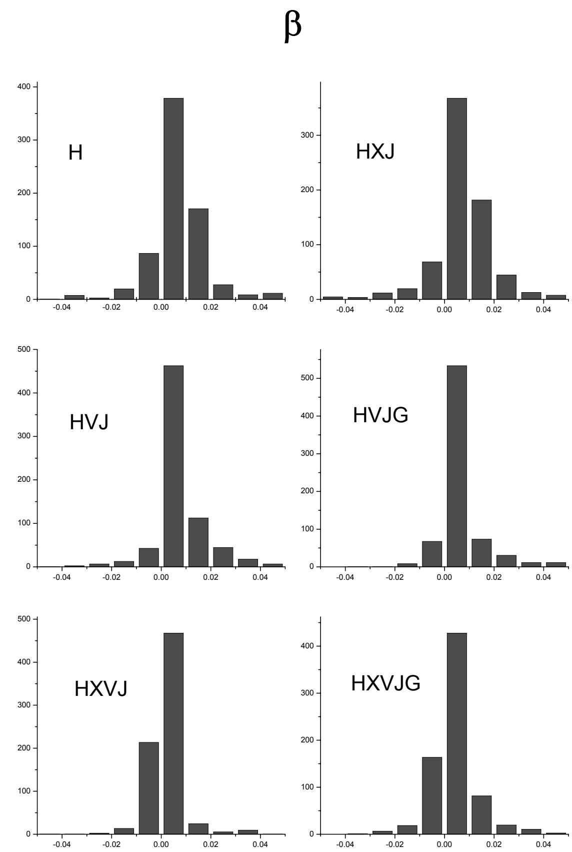

Figure A.3. *Histograms β*

The parameter values of β for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

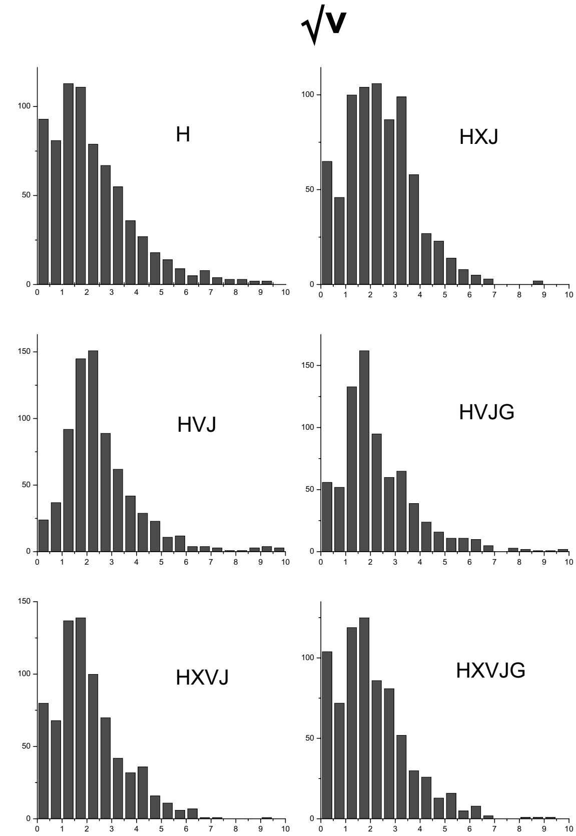

Figure A.4. *Histograms* $\sqrt{v}$

The parameter values of $\sqrt{v}$ for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

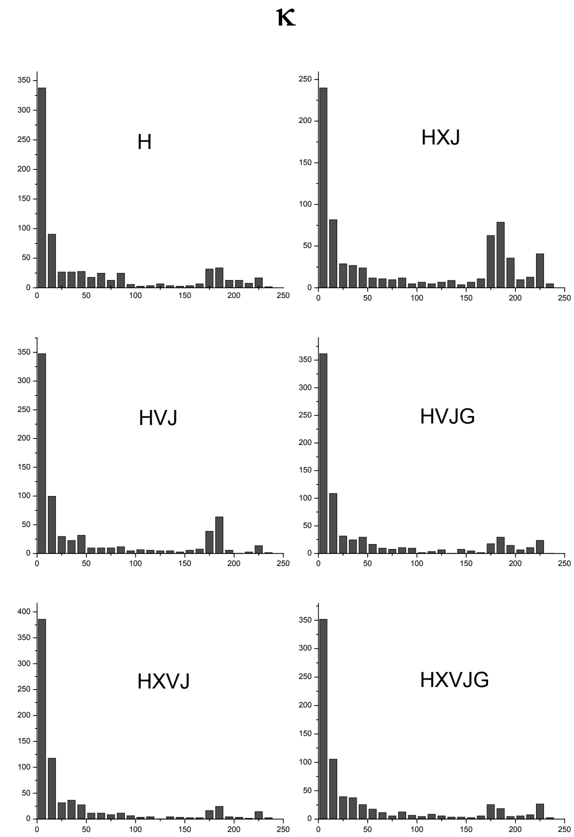

Figure A.5. *Histograms κ*

The parameter values of κ for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

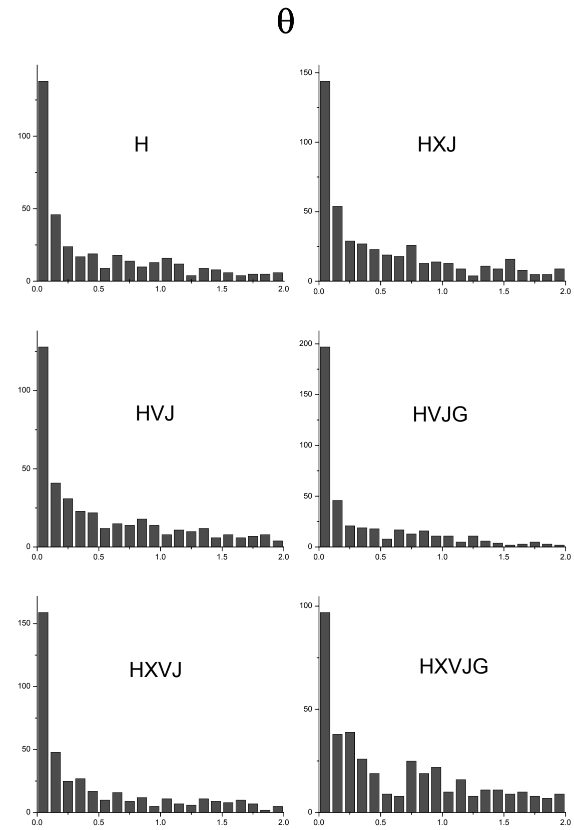

Figure A.6. *Histograms θ*

The parameter values of θ for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

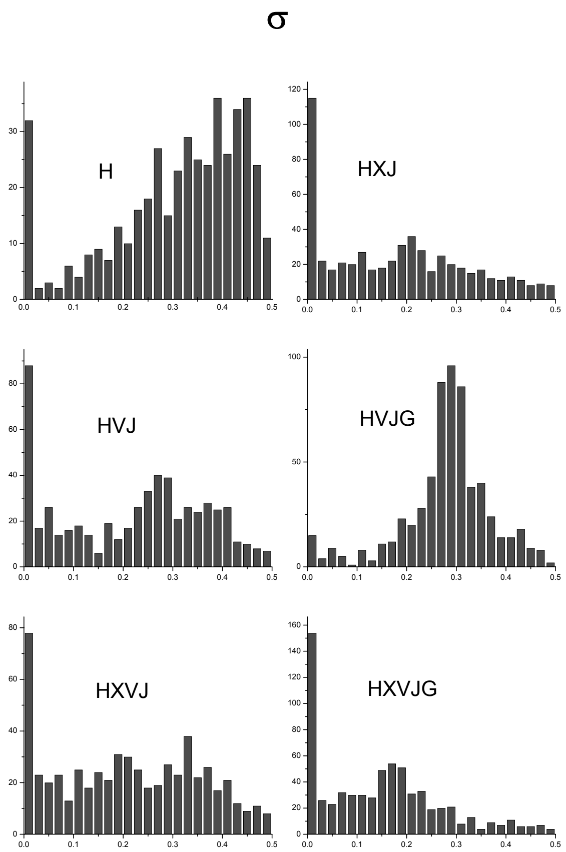

Figure A.7. *Histograms σ*

The parameter values of σ for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

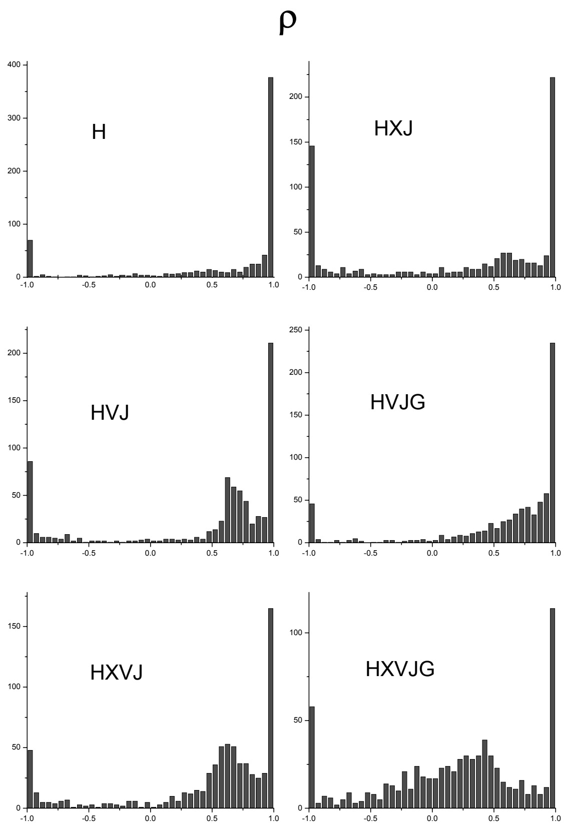

Figure A.8. *Histograms ρ*

The parameter values of ρ for all estimations are plotted in a histogram for each model. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

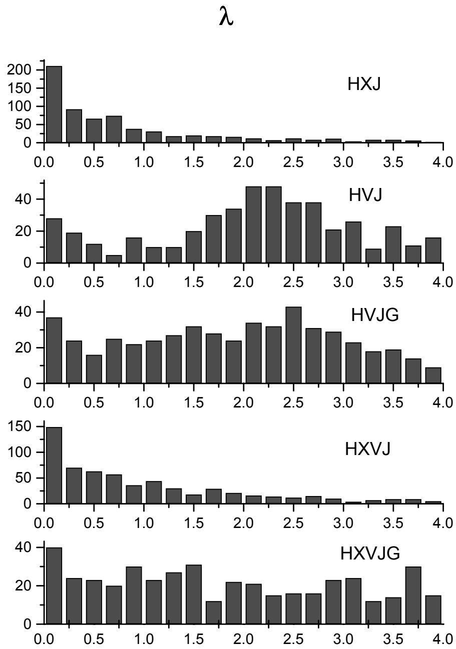

Figure A.9. *Histograms λ*

The parameter values of λ for all estimations are plotted in a histogram for each model. The models are:

- HXJ: a Heston-like model with a normal-distributed jump in the first state variable,
- HVJ: the same Heston-like model with an exponential-distributed jump in the second state variable,
- HVJGS: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

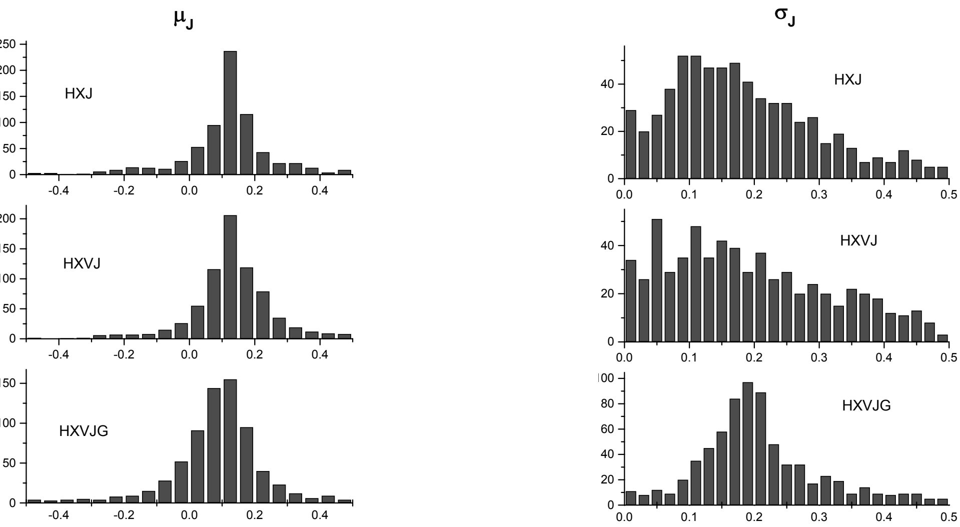

Figure A.10. *Histograms μ_J and σ_J*

The parameter values of μ_J and σ_J for all estimations are plotted in a histogram for each parameter and for each model. The models are:

- HXJ: a Heston-like model with a normal-distributed jump in the first state variable,
- HXVJ: the same Heston-like model with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

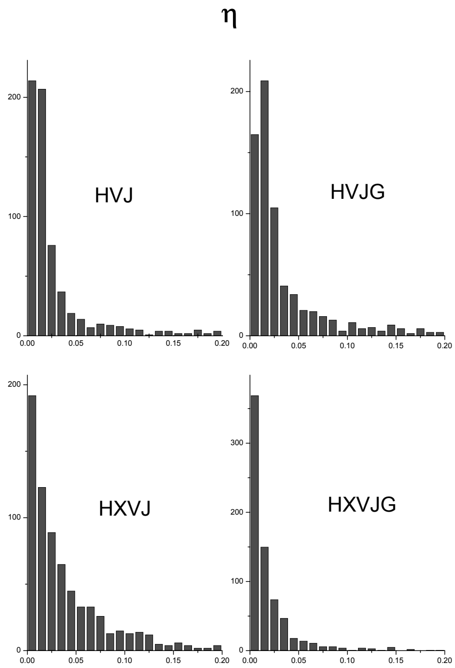

Figure A.11. *Histograms η*

The parameter values of η for all estimations are plotted in a histogram for each model. The models are:

- HVJ: a Heston-like model with an exponential-distributed jump in the second state variable,
- HVJG: the same Heston-like model with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

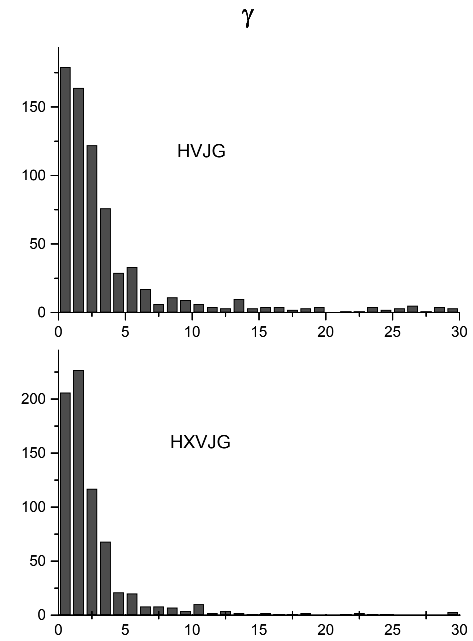

Figure A.12. *Histograms* γ

The parameter values of γ for all estimations are plotted in a histogram for each model. The models are:

- HVJG: a Heston-like model with a Γ-distributed jump in the second state variable,
- HXVJG: the same Heston-like model with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

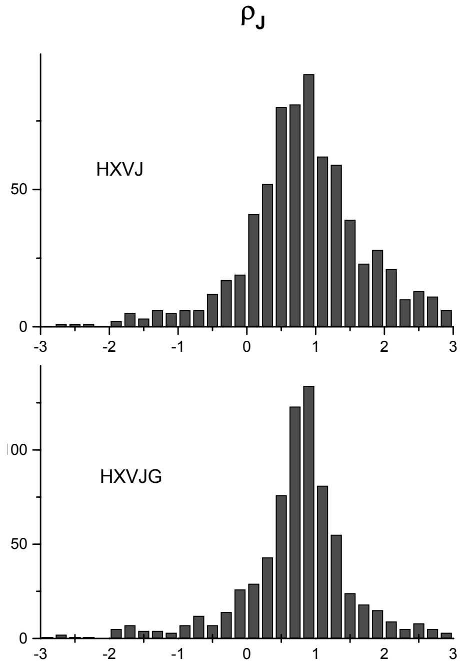

Figure A.13. *Histograms ρ_J*

The parameter values of ρ_J for all estimations are plotted in a histogram for each model. The models are:

- HXVJ: a Heston-like model with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: the same model, but with a Γ-distributed jump in the second state variable.

A.2 Regression Statistics

The regression statistics are in Tables A.3, A.4, A.5, A.6, A.7 and A.8.

Table A.3. *Regression Statistics Model H*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. H refers to a Heston-like model.

Medians Model H

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,01	392,52	0,002	0,9982	0,00%	0,00%
β	0,01	11,57	0,001	0,9995	0,27%	0,13%
√v	1,90	305,51	0,01	0,4976	0,08%	0,00%
κ	13,50	10,94	1,26	0,0822	48,27%	44,13%
θ	1,86	706,56	0,002	0,4991	1,87%	1,87%
σ	0,44	10,67	0,04	0,4811	16,67%	15,20%
ρ	0,95	13,67	0,06	0,9549	15,87%	13,60%

F-Statistic	Prob.	sign. 5%	sign. 1%
1,68	0,1934	32,80%	19,87%

R-squared	0,36	Mean dependent var	150,83
Adjusted R-squared	0,15	S.D. dependent var	19,65
Akaike info criterion	8,74	Schwarz criterion	9,08
Log likelihood	-103,07	Durbin-Watson stat	1,17

Table A.4. *Regression Statistics Model HXJ*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXJ refers to a Heston-like model with a normal-distributed jump in the first state variable.

Medians Model HXJ

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-0,78	1381,76	0,0003	0,9997	0,27%	0,27%
β	0,01	3,02	0,0023	0,9982	1,07%	0,80%
$\sqrt{v}$	2,27	668,50	0,003	0,4989	0,08%	0,00%
κ	39,42	4,10	6,79	0,0000	61,07%	58,80%
θ	1,06	1369,34	0,0003	0,4999	0,67%	0,67%
σ	0,24	2,78	0,05	0,4810	15,20%	12,40%
ρ	0,57	3,95	0,16	0,8790	22,27%	18,00%
λ	0,62	1564,42	0,0002	0,4999	0,40%	0,40%
μ_J	0,13	1310,87	0,0001	0,9999	0,00%	0,00%
σ_J	0,18	1595,54	0,0001	0,5000	0,53%	0,53%

F-Statistic	Prob.	sign. 5%	sign. 1%
0,66	0,7342	16,67%	9,60%

R-squared	0,29	Mean dependent var	150,83
Adjusted R-squared	-0,15	S.D. dependent var	19,65
Akaike info criterion	9,06	Schwarz criterion	9,55
Log likelihood	-103,82	Durbin-Watson stat	1,05

Table A.5. *Regression Statistics Model HVJ*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HVJ refers to a Heston-like model with an exponential-distributed jump in the second state variable.

Medians Model HVJ

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-0,96	520,60	0,001	0,9992	0,53%	0,40%
β	0,01	8,13	0,001	0,9993	1,60%	1,20%
√v	2,20	312,52	0,01	0,4976	0,08%	0,00%
κ	11,81	7,70	1,90	0,0343	51,60%	47,20%
θ	1,66	996,98	0,001	0,4997	1,47%	1,20%
σ	0,29	7,49	0,03	0,4867	11,20%	9,73%
ρ	0,71	10,23	0,06	0,9500	18,00%	15,20%
λ	2,98	1134,62	0,002	0,4992	1,20%	1,07%
η	0,02	779,88	0,00002	0,5000	0,67%	0,67%

F-Statistic	Prob.	sign. 5%	sign. 1%
1,26	0,3331	23,47%	14,40%

R-squared	0,39	Mean dependent var	150,83
Adjusted R-squared	0,08	S.D. dependent var	19,65
Akaike info criterion	8,84	Schwarz criterion	9,28
Log likelihood	-102,02	Durbin-Watson stat	1,26

Table A.6. *Regression Statistics Model HVJG*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HVJG refers to a Heston-like model with a Γ-distributed jump in the second state variable.

Medians Model HVJG

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-0,65	557,67	0,001	0,9994	0,27%	0,27%
β	0,003	14,40	0,0003	0,9998	0,93%	0,67%
√v	1,89	421,89	0,004	0,4985	0,08%	0,00%
κ	10,56	11,01	1,04	0,1482	44,53%	41,07%
θ	1,09	1031,00	0,001	0,4997	0,67%	0,67%
σ	0,30	11,63	0,03	0,4891	14,53%	12,13%
ρ	0,80	16,12	0,05	0,9611	15,47%	12,67%
λ	2,64	1210,80	0,002	0,4992	0,93%	0,93%
η	0,02	202,98	0,0001	0,5000	0,67%	0,67%
γ	2,20	510,82	0,004	0,4983	2,80%	2,40%

F-Statistic	Prob.	sign. 5%	sign. 1%
1,21	0,3594	20,80%	12,40%

R-squared	0,42	Mean dependent var	150,83
Adjusted R-squared	0,07	S.D. dependent var	19,65
Akaike info criterion	8,90	Schwarz criterion	9,39
Log likelihood	-101,80	Durbin-Watson stat	1,33

Table A.7. *Regression Statistics Model HXVJ*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXVJ refers to a Heston-like model with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously.

Medians Model HXVJ

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,30	1307,78	0,001	0,9994	0,40%	0,40%
β	0,001	7,60	0,0003	0,9998	0,53%	0,40%
√v	1,80	510,16	0,003	0,4987	0,08%	0,00%
κ	9,61	7,69	1,18	0,0882	47,73%	43,87%
θ	1,57	1368,48	0,001	0,4997	2,13%	2,00%
σ	0,28	6,37	0,02	0,4903	17,20%	15,07%
ρ	0,64	9,32	0,07	0,9418	16,00%	12,53%
λ	1,00	1536,50	0,0005	0,4998	1,60%	1,60%
μ_J	0,13	1315,05	0,0001	0,9999	0,00%	0,00%
σ_J	0,20	1513,14	0,0001	0,5000	1,60%	1,60%
η	0,03	1126,91	0,00002	0,5000	2,13%	2,00%
ρ_J	0,86	1390,21	0,001	0,9996	0,40%	0,40%

F-Statistic	Prob.	sign. 5%	sign. 1%
0,83	0,6187	12,40%	4,53%

R-squared	0,42	Mean dependent var	150,83
Adjusted R-squared	-0,09	S.D. dependent var	19,65
Akaike info criterion	9,08	Schwarz criterion	9,67
Log likelihood	-102,01	Durbin-Watson stat	1,29

Table A.8. *Regression Statistics Model HXVJG*

The medians of all coefficient estimates and statistics are presented. The statistics are the standard errors for the parameter estimates, the t-Statistics, and their p-value. The proportions of the significant estimates are given, at the 5% and the 1% level. Moreover, the F-Statistic for the model is plotted, also with p-value and the proportion of significant estimates at the 5% and the 1% level. Relevant statistics are further R^2, the Adjusted R^2, Akaike and Schwarz information criterions, the log likelihood and the Durbin–Watson statistic. The mean and standard deviation of the dependent variables $F(t, T_1, T_2)$ are given for completeness. They are of course independent from the chosen model. HXVJG refers to a Heston-like model with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps occurring simultaneously.

Medians Model HXVJG

Name	Coefficient	Std. Error	t-Statistic	Prob.	sign. 5%	sign. 1%
α	-1,54	2018,52	0,0005	0,9996	1,07%	1,07%
β	0,003	3,35	0,001	0,9990	1,60%	0,93%
√v	1,81	712,04	0,002	0,4992	0,08%	0,00%
κ	11,08	4,03	3,24	0,0030	58,00%	52,27%
θ	1,68	1693,41	0,0004	0,4998	2,00%	1,73%
σ	0,16	3,22	0,03	0,4894	12,93%	10,67%
ρ	0,28	4,37	0,09	0,9274	16,00%	13,20%
λ	3,34	1991,49	0,001	0,4995	1,47%	1,47%
μ_J	0,09	1127,66	0,0001	0,5000	0,53%	0,53%
σ_J	0,20	1975,38	0,0001	0,5000	0,80%	0,67%
η	0,01	344,48	0,00003	0,5000	0,80%	0,80%
γ	1,73	582,93	0,002	0,4991	2,53%	2,00%
ρ_J	0,80	595,67	0,001	0,9990	1,60%	1,20%

F-Statistic	Prob.	sign. 5%	sign. 1%
0,68	0,7479	8,67%	3,47%

R-squared	0,42	Mean dependent var	150,83
Adjusted R-squared	-0,20	S.D. dependent var	19,65
Akaike info criterion	9,16	Schwarz criterion	9,79
Log likelihood	-102,02	Durbin-Watson stat	1,27

A.3 Out-of-Sample Behaviour

For the RMSEs with lagged estimates and medians of estimates see Table A.9.

Table A.9. *RMSEs – Out-of-Sample*

In this summary statistics table, mean, median and standard deviation of the RMSEs with the parameters determined by minimising the RMSEs are reported, and the RMSEs calculated with the parameters that were optimal in the estimations of the day before, called here RMSE($\vec{\vartheta}$(t-1)), the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until day t-1 (RMSE(median($\vec{\vartheta}$(1:t-1)))), and the RMSEs calculated with the medians of optimal parameters from day 1 (1 October 1999) until the last day (30 September 2002), called RMSE(median($\vec{\vartheta}$(1:end))). The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

		RMSE	RMSE(ϑ(t-1))	RMSE(median((ϑ(1:t-1)))	RMSE(median(ϑ(1:end)))
H	**Mean**	15.31	18.77	36.30	32.27
	Median	14.48	16.20	30.10	26.73
	Std. Dev.	6.20	15.35	32.71	27.55
HXJ	**Mean**	16.42	19.53	36.80	36.77
	Median	15.02	16.71	29.57	31.19
	Std. Dev.	7.65	15.78	26.04	24.47
HVJ	**Mean**	14.65	18.39	40.04	34.38
	Median	14.06	15.89	35.66	30.36
	Std. Dev.	4.05	14.74	31.48	28.13
HVJG	**Mean**	14.20	17.91	39.98	33.48
	Median	13.82	15.62	37.42	28.77
	Std. Dev.	2.86	14.08	32.43	28.27
HXVJ	**Mean**	14.20	17.99	48.85	47.45
	Median	13.86	15.65	47.22	44.25
	Std. Dev.	2.79	14.10	26.72	20.87
HXVJG	**Mean**	14.45	18.08	43.23	42.38
	Median	13.97	15.65	38.05	38.29
	Std. Dev.	3.60	14.01	25.78	22.36

A.4 Option Pricing

The results of the option pricing are in Table A.10.

Table A.10. *RMSEs of option pricing*

In this summary statistics table, mean, median and standard deviation of the RMSEs of the comparison of calculated and observed option prices are reported. In RMSE($\vec{\vartheta}$(t)) the parameters estimated implicitly by futures and forward prices are used, in RMSE(median($\vec{\vartheta}$(1:t)) the medians of all implicitly with futures and forward prices estimated parameters until day t are used, and finally, in RMSE(median($\vec{\vartheta}$(1:end)), the medians of all implicitly with futures and forward prices estimated parameters are used. The models are:

- H: a Heston-like model,
- HXJ: the same model with a normal-distributed jump in the first state variable,
- HVJ: with an exponential-distributed jump in the second state variable,
- HVJG: with a Γ-distributed jump in the second state variable,
- HXVJ: with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously,
- HXVJG: with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously.

		RMSE(ϑ(t))	**RMSE(median((ϑ(1:t)))**	**RMSE(median(ϑ(1:end)))**
H	**Mean**	44.43	26.36	25.74
	Median	14.95	10.35	9.16
	Std. Dev.	180.24	43.49	46.03
HXJ	**Mean**	47.26	21.55	23.56
	Median	8.58	7.26	9.09
	Std. Dev.	259.31	43.75	44.19
HVJ	**Mean**	70.87	25.27	26.72
	Median	21.11	8.29	9.57
	Std. Dev.	208.80	43.61	45.46
HVJG	**Mean**	37.40	29.08	29.57
	Median	10.89	11.56	10.57
	Std. Dev.	57.31	43.91	46.33
HXVJ	**Mean**	31.68	28.83	26.98
	Median	13.06	13.46	12.80
	Std. Dev.	37.71	42.69	42.79
HXVJG	**Mean**	32.83	35.10	30.48
	Median	9.62	12.84	14.43
	Std. Dev.	41.65	43.41	43.71

Notation and Symbols

In the following, the most frequent notation and symbols are listed and explained.

ADF	Augmented Dickey Fuller test
AIC	Akaike Information Criterion
A	the generator of an Itô or jump diffusion X_t
$AS(T_1, T_n)$	arithmetic average of S from T_1 to T_n
billion	$1,000,000,000$
CfD	Contract for Difference
$C(\cdot)$, $P(\cdot)$, $O(\cdot)$	Call-, Put-, Option price
$C_e(\cdot)$, $P_e(\cdot)$, $C_a(\cdot)$, $P_a(\cdot)$	European-, Asian-style call or put
$C^m(\mathbb{R}^n)$	m-times differentiable functions from $\mathbb{R}^n$ to $\mathbb{R}$, with continuous m-th derivative
$C^{1,2}(\mathbb{R} \times \mathbb{R}^n)$	the functions $f(t,x) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ which are C^1 with respect to t, and C^2 with respect to x.
$\mathbb{C}$	complex numbers
dt	infinitesimal time increment
dP	infinitesimal Poisson-distributed increments
dZ_t	infinitesimal Brownian increment

D	$Du = Au - \frac{\partial u}{\partial t}$
$\mathrm{Exp}(1/\eta)$	Exponential distribution with mean η and variance η^2
$E[\cdot]$, $E[\cdot\|\cdot]$, $E_t[\cdot]$	expectation operator, conditional expectation
$f(t)$	deterministic function for seasonal behaviour of electricity prices, in our models $f(t) = s_0 \cos\left(\frac{2\pi}{365}(t + 365 s_1)\right)$
$F(t,T)$, $F(t,T_1,T_n)$	futures or forward price
$\hat{F}(\cdot)$, $\hat{O}(\cdot)$	theoretical prices of futures/forwards or options
$\mathcal{F}_t, t \geq 0$	filtration, information set up to time t
$\Gamma(1/\eta, \gamma)$	Gamma or Γ-distribution with mean $\eta\gamma$ and variance $\eta^2\gamma$
H	a Heston-like model
HXJ	a Heston-like model with a normal-distributed jump in the first state variable
HVJ	a Heston-like model with an exponential-distributed jump in the second state variable
HVJG	a Heston-like model with a Γ-distributed jump in the second state variable
HXVJ	a Heston-like model with a normal-distributed jump in the first state variable and an exponential-distributed jump in the second state variable, both jumps occurring simultaneously

HXVJG	a Heston-like model with a normal-distributed jump in the first state variable and a Γ-distributed jump in the second state variable, both jumps also occurring simultaneously
HS, HXJS, HVJS, HVJGS, HXVJS, HXVJGS	the same models as H, HXJ, HVJ, HVJG, HXVJ and HXVJG, but with additional seasonal modelling
i.i.d.	independently and identically distributed
i	imaginary unit, $\sqrt{-1}$
$\vec{J}$, J_X, J_v	(randomly distributed) jump size, in X, in v
K	strike or exercise price
KBE	Kolmogorov's Backward Equation
κ, θ, σ, λ, μ_J, σ_J, α, β, $\sqrt{v}$, ρ, η, γ, ρ_J, s_0, s_1	parameters in models
log price, log spot price	natural logarithm of spot price
$\log(\cdot)$	natural logarithm
MSE	Mean Squared Error
MWh, MWh/h	Mega-watt Hours, Mega-watt Hours per Hour
NECH	Nordic Electricity Clearing House
NOK	Norwegian Kronor, monetary unit of Norway
$N(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2.
$\mathbb{N}$	the natural numbers

$\vec{\nu}$, $\nu(J_x, J_v)$, ν_{J_X}, ν_{J_v}	density of jump size or jump sizes
OU	Ornstein–Uhlenbeck process model
OUJ	Ornstein–Uhlenbeck process model with jump
OUS	Ornstein–Uhlenbeck process model with season
OUJS	Ornstein–Uhlenbeck process model with jump and season
OTC	over-the-counter
$(\Omega, \mathcal{F}, P)$	probability space
pdf	probability density function
prob.	probability
PJM	Pennsylvania–New Jersey–Maryland Electricity Market
$Po(\lambda)$	Poisson distribution with mean λ
$\Phi(\cdot)$	characteristic function
RMSE	Root Mean Squared Error
r	riskless interest rate
$\mathbb{R}$	the real numbers
$\mathbb{R}_0^+$	the real positive numbers, including 0
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	the $n \times m$ real-valued matrices
sign. 5%, sign. 1%	significant at the 5% level, at the 1% level
Std. Error	Standard Error
SC	Schwarz Criterion
S.D.	standard deviation

S, S_t, $S(t)$	asset, electricity price or stochastic process of asset or electricity price at time t
t-stat	t-statistic
t	variable for time
T, $T_1, \ldots, T_n$	variable for expiry/delivery day or days
TWh	Tera-watt Hours
$\vec{\vartheta}$	vector of parameters
τ	time to maturity, $T - t$
var	variable
vs.	versus, against
v, v_t	parameter for variance or variance process
$\sqrt{v}$, $\sqrt{v_t}$	parameter for volatility
X_t, x_t	stochastic process, logarithm of S_t or logarithm of $S_t - f(t)$
$\vec{Y}_t$	$= (X_t, v_t)'$
Z_t	standard Brownian motion
$\forall$	for all
$\equiv$	equivalent to
$\approx$	approximately
$\sim$, $Y \sim N(\mu, \sigma^2)$	distributed with, Y is distributed with $N(\mu, \sigma^2)$
$_{\mid}$, $A_{\mid B}$	restricted to, A restricted to B

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References

Anderson, T. W.: 1971, *The Statistical Analysis of Time Series*, Wiley.

Bakshi, G., Cao, C. and Chen, Z.: 1997, Empirical performance of alternative option pricing models, *Journal of Finance* **52(5)**, 2003–2049.

Bakshi, G. and Madan, D.: 2000, Spanning and derivative-security valuation, *Journal of Financial Economics* **55**, 205–238.

Barlow, M. T.: 2002, A diffusion model for electricity prices, *Mathematical Finance* **12(4)**, 287–298.

Barone-Adesi, G. and Gigli, A.: 2002, Electricity derivatives. Working Paper, USI, Lugano, CH.

Bates, D.: 1988, Pricing options under jump-diffusion processes. Working Paper Rodney L. White Center for Financial Research.

Bates, D.: 1991, The crash of 87: Was it expected? The evidence from options markets, *Journal of Finance* **46(3)**, 1009–1044.

Bates, D.: 2000, Post-87 crash fears in the S&P 500 futures option market, *Journal of Econometrics* **94(1/2)**, 181–238.

Bessembinder, H. and Lemmon, M. L.: 2002, Equilibrium pricing and optimal hedging in electricity forward markets, *Journal of Finance* **57(3)**, 1347–1382.

Bhanot, K.: 2000, Behaviour of power prices: implications for the valuation and hedging of financial contracts, *Journal of Risk* **2**, 43–62.

Borovkova, S.: 2004, Modelling seasonalities and spikes in electricity prices, International Conference on Stochastic Finance 2004, Lisbon, Portugal.

Box, G. E. P. and Jenkins, G. M.: 1976, *Time Series Analysis: Forecasting and Control*, Holden-Day, San Francisco.

Brennan, M. J. and Schwartz, E. S.: 1985, Evaluating natural resource investments, *Journal of Business* **58**, 135–157.

Bühler, W., Korn, O. and Schöbel, R.: 2000, Pricing and hedging of oil futures — a unifying approach —, Forthcoming in *Review of Derivatives Research* .

Chacko, G. and Viceira, L. M.: 2003, Spectral gmm estimation of continuous-time processes, *Journal of Econometrics* **116**, 259–292.

Cheng, P. and Scaillet, O.: 2002, Linear-quadratic jump-diffusion modelling with application to stochastic volatility. FAME Working Paper, University of Geneva, Switzerland.

Cox, J. C., J. E. Ingersoll, J. and Ross, S. A.: 1985, A theory of the term structure of interest rates, *Econometrica* **53**, 385–407.

Das, S. R.: 2002, The surprise element: Jumps in interest rates, *Journal of Econometrics* **106**, 27–65.

De Jong, C. and Huisman, R.: 2002, Option formulas for mean-reverting power prices with spikes. Working Paper, Rotterdam School of Management at Erasmus University Rotterdam.

Deng, S.: 2000, Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. Working Paper, University of California Energy Institute.

Duffie, D.: 1996, *Dynamic Asset Pricing Theory*, second edn, Princeton University Press, Princeton, New Jersey.

Duffie, D., Pan, J. and Singleton, K. J.: 2000, Transform analysis and option pricing for affine jump-diffusions, *Econometrica* **68**, 1343–1376.

Elliott, R., Sick, G. and Stein, M.: 2003, Modelling electricity price risk. Working Paper, University of Calgary.

Energy Indicators per Country: 2000,
`http://www.iea.org/stats/files/selstats/keyindic/country/denmark.htm`,
`http://www.iea.org/stats/files/selstats/keyindic/country/norway.htm`,
`http://www.iea.org/stats/files/selstats/keyindic/country/sweden.`

htm,

http://www.iea.org/stats/files/selstats/keyindic/country/finland.htm.

Escribano, A., Peña, J. I. and Villaplana, P.: 2002, Modelling electricity prices: international evidence. Working Paper, Universidad Carlos III de Madrid, 02–27 (08).

Eviews 3.1 Help: 1999, Part of Eviews 3.1.

Eydeland, A. and Geman, H.: 1998, Pricing power derivatives, *Risk* **11** (October), 71–73.

Feller, W.: 1951, Two singular diffusion problems, *The Annals of Mathematics* **54(1)**, 173–182.

Fleten, S.-E. and Lemming, J.: 2001, Constructing forward price curves in electricity markets. Working Paper, Risø National Laboratory, Denmark.

Geman, H. and Roncoroni, A.: 2003, A class of marked point processes for modeling electricity prices, European Financial Management Association Meeting Helsinki 2003.

Gibson, R. and Schwartz, E. S.: 1990, Stochastic convenience yield and the pricing of oil contingent claims, *Journal of Finance* **45(3)**, 959–967.

Guthrie, G. and Videbeck, S.: 2002, High frequency electricity spot price dynamics: an intra-day markets approach. Working Paper, New Zealand Institute for the Study of Competition and Regulation.

Heston, S.: 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* **6**, 327–343.

Hilliard, J. E. and Reis, J.: 1998, Valuation of commodity futures and options under stochastic convenience yields, interest rates, and jump diffusions in the spot, *Journal of Financial and Quantitative Analysis* **33(1)**, 61–86.

Huisman, R. and Mahieu, R.: 2001, Regime jumps in electricity prices. Working Paper, Rotterdam School of Management at Erasmus University Rotterdam.

Hull, J. C.: 1989, *Options, Futures & Other Derivatives*, fourth edn, Prentice Hall, NJ.

Jäckel, P.: 2002, *Monte Carlo Methods in Finance*, Wiley.

Jaillet, P., Ronn, E. I. and Tompaidis, S.: 2001, Valuation of commodity-based swing options. Working Paper, University of Austin, Texas.

Kanamura, T. and Ohashi, K.: 2004, A structural model for electricity prices with spikes – measurement of jump risk and optimal policies for hydropower plant operation. Hitotsunbashi University, ICS, Finance Working Paper No. FS-2004-E-02. `http://ssrn.com/abstract=560603`.

Kåresen, K. F. and Husby, E.: 2000, A joint state-space model for electricity spot and futures prices. Norsk Regnesentral, Norway.

Kellerhals, B. P.: 2001, *Financial Pricing Models in Continous Time and Kalman Filtering*, Springer, Berlin.

Knittel, C. R. and Roberts, M. R.: 2001, An empirical examination of deregulated electricity prices. Working Paper University of California Energy Institute.

Koekebakker, S. and Ollmar, F.: 2001, Forward curve dynamics in the Nordic electricity market. Working Paper, Norwegian School of Economics and Business Administration.

Kubilius, K. and Platen, E.: 2002, Rate of weak convergence of the Euler approximation for diffusion processes with jumps, *Monte Carlo Methods and Applications* **8(1)**, 83–96.

Lien, G. and Strøm, Ø.: 1999, Modelling jumps in commodity prices, European Finance Association 26th Annual Meeting.

Litzenberger, R. H. and Rabinowitz, N.: 1995, Backwardation in oil futures markets: Theory and empirical evidence, *Journal of Finance* **50(5)**, 1517–1545.

London Metal Exchange: n.d., `http://www.lme.co.uk/data_prices/daily_prices.asp`.

Lucia, J. J. and Schwartz, E. S.: 2002, Electricity prices and power derivatives: Evidence from the Nordic power exchange, *Review of Derivatives Research* **5(1)**, 5–50.

Lukacs, E.: 1970, *Characteristic functions*, second edn, Griffin, London.

Miltersen, K. R. and Schwartz, E. S.: 1998, Pricing options on commodity futures with stochastic term structures of convenience yields and interest rates, *Journal of Financial and Quantitative Analysis* **33(1)**, 33–59.

Musiela, M. and Rutkowski, M.: 1997, *Martingale Methods in Financial Modelling*, Springer, Berlin.

Nord Pool ASA: n.d., `http://www.nordpool.com,ftp://ftp.nordpool.com`.

Nord Pool ASA – Market Information: n.d., `http://www.nordpool.com/marketinfo/index.cgi?url=exchange/area/exchange.cgi?interval=last8\&ccurrency=nok\&type=html\&usecookie=true\&toarea=AREA`.

Nord Pool Information: n.d., `http://www.nordpool.com/information`.

Nord Pool Options: 2002, `http://www.nordpool.com/products/financial/Electricpoweroptions.pdf`.

Nord Pool Press Release 02/2003: 2003, `http://194.19.110.70/information/press_releases/2003-002.html`.

Nord Pool Product Calendar: n.d., `http://www.nordpool.com/products/financial/prod_kalender/Prod.kalender.htm`.

Nord Pool Product Reports: 2002, `http://www.nordpool.com/information/reports/introduction.html`.

Nord Pool Products: Version of December 2002 and former versions, `http://www.nordpool.com/products`.

Nord Pool Rulebook for The Financial Electricity Market: 2002, `http://necservice.nordpool.com/Agreement/2002/StandardTermspr130502_revkap7.pdf`.

NZEM Overview: n.d., `http://www.nzelectricity.co.nz/C2Overview.htm`.

Øksendal, B.: 2000, *Stochastic Differential Equations*, fifth edn, Springer, Berlin.

Pilipovic, D.: 1998, *Energy Risk: Valuing and managing energy derivatives*, McGraw-Hill, New York.

Pirrong, C. and Jermakyan, M.: 1999, Valuing power and weather derivatives on a mesh using finite difference methods, *Energy Modelling and the Management of Uncertainty*, Risk Books.

PJM Timeline: 2002, http://www.pjm.com/about/glance.html, http://www.pjm.com/contributions/news-releases/2002/20020926-pjm-history-timeline.pdf.

Reform of the Electricity Industry in Australia: 2001, http://www.efa.com.au/reform.html.

Resa Corporation and Licensors: 2000–2003, Statistical distributions, http://www.xycoon.com.

Ross, S. A.: 1997, Hedging long run commitments: Exercises in incomplete market pricing, *Economic Notes by Banca Monte dei Paschi di Siena SpA* **26**, 385–420.

Schöbel, R.: 1995, *Kapitalmarkt und zeitkontinuierliche Bewertung*, Physica Verlag, Heidelberger betriebswirtschaftliche Studien. Heidelberg.

Schöbel, R. and Zhu, J.: 1999, Stochastic volatility with an Ornstein–Uhlenbeck process: An extension, *European Finance Review* **3**, 23–46.

Schönbucher, P. J.: 2003, *Credit Derivatives Pricing Models*, Wiley, Chichester, England.

Schwartz, E. S.: 1997, The stochastic behaviour of commodity prices: Implications for valuation and hedging, *Journal of Finance* **52(3)**, 923–973.

Schwartz, E. S. and Smith, J. E.: 2000, Short-term variations and long-term dynamics in commodity prices, *Management Science* **46(7)**, 893–911.

Scott, L.: 1997, Pricing stock options in a diffusion model with stochastic volatility and interest rates: Applications of Fourier inversion methods, *Mathematical Finance* **7(4)**, 413–424.

Skantze, P. and Ilic, M.: 2000, The joint dynamics of electricity spot and forward markets: Implications on formulating dynamic hedging strategies. Working Paper, Energy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Standard Terms for Trading And Clearing in Nord Pool Spot AS' Physical Markets: n.d., http://www.nordpool.com/products/elspot/avtal/Spot-englishversion.pdf.

Stuart, A. and Ord, J. K.: 1987, *Kendall's advanced theory of statistics*, Vol. 2, fifth edn, Griffin & Co., London.

Vasicek, O.: 1977, An equilibrium characterization of the term structure, *Journal of Financial Economics* **5**, 177–188.

Villaplana, P.: 2003, Pricing power derivatives: A two-factor jump-diffusion approach. EFMA 2004 Basel Meetings Paper. http://ssrn.com/abstract=493943.

Weron, R., Simonsen, I. and Wilman, P.: 2003, Modeling high volitile and seasonal markets: evidence from the Nord Pool electricity market. Working Paper, Wroclaw University of Technology, Poland.

Zhang, P. G.: 1998, *Exotic Options*, second edn, World Scientific Publishing, Singapore.

Zhu, J.: 2000, *Modular Pricing of Options*, Springer, Berlin.