The Persistence and Asymmetry of Time-Varying Correlations

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Abstract

Existing multivariate GARCH models either impose strong restrictions on the parameters or do not guarantee a well-defined (positive definite) covariance matrix. We focus on the multivariate GARCH model of Baba, Engle, Kraft and Kroner (BEKK) and show that the covariance and correlation is not adequately specified. This implies that any analysis of the persistence and the asymmetry of the correlation is difficult and potentially biased. We illustrate this by the use of Monte-Carlo simulations for different correlation processes and propose a new Bivariate Dynamic Correlation (BDC) model that parameterizes the conditional correlation directly and eliminates the shortcomings of the BEKK model. Empirical results for correlations of the German stock market index with three international stock market indices reveal that correlations exhibit different degrees of persistence and different asymmetric reactions than variances. In addition, we find that correlations do not necessarily increase with variances implying a justification for international portfolio diversification.

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The knowledge of the time-varying behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management. However, correlations are considerably less frequently analyzed than the variances (i.e. volatilities) of asset returns. We mainly explain this with the higher difficulty in estimating such time-varying correlations and the larger number of parameters to estimate. Consequently, studies comparing the existing multivariate GARCH models are rare as opposed by the existing studies that compare univariate time-varying volatility models (see Pagan and Schwert (1990) and Engle and Ng (1993) among others).

For multivariate GARCH models we are only aware of the work of Kroner and Ng (1998), Engle (2000) and Engle and Sheppard (2001). While Kroner and Ng (1998) compare the main existing models by the use of real data, Engle (2000) and Engle and Sheppard (2001) use Monte-Carlo simulations to analyse different models than Kroner and Ng (1998) with a focus on the Dynamic Conditional Correlation (DCC) estimator.

The univariate GARCH model proposed by Bollerslev (1986) was extended by Bollerslev, Engle and Wooldridge (1988). This model uses the vech operator and is thus referred to as VECH-model. It does not guarantee a positive-definite covariance matrix and the number of parameters is relatively large. Baba, Engle, Kroner and Kraft (1991) proposed a multivariate GARCH model, called BEKK (named after the names of the authors), that guarantees the positive definiteness of the covariance matrix. Restricting the BEKK model to be diagonal reduces the number of parameters that must be estimated. The Factor GARCH model also reduces the number of parameters but can be transformed to a BEKK model. Interestingly, it seems that even the restricted BEKK model has too many parameters since commonly bivariate models have been estimated (see Bekaert and Wu, 2000, Engle, 2000, Karolyi, 1995, Kroner and Ng, 1998, Longin and Solnik, 1998 and Ng, 2000). In addition, we are not aware of any multivariate GARCH model that has been estimated with a higher lag order than GARCH(1,1).

The Constant Correlation Model of Bollerslev (1990) does also circumvent the problem of possible non-positive definiteness of the covariance matrix but is very restrictive since it does not allow correlations to be time-varying.

Kroner and Ng (1998) proposed the general asymmetric dynamic covariance (ADC) model that nests the VECH, the Factor GARCH, the BEKK model and the Constant Correlation Model and extended these models to include asymmetries in the reaction to shocks as proposed by Glosten et al. (1993) in a univariate context. However, their nested model requires further restrictions to guarantee a positive-definite covariance matrix.

Recently, Tse and Tsui (2000) proposed a new multivariate GARCH model that parameterizes the conditional correlation directly by using the empirical correlation and Engle (2000) proposed a time-varying correlation model, called Dynamic Conditional Correlations (DCC) that also parameterizes the conditional correlation directly but uses a two-stage estimation strategy. The Bivariate Dynamic Correlations (BDC) estimator proposed in this paper can be assumed to be in the same class as the models by Tse and Tsui (2000) and Engle (2000) but is different in various aspects which we discuss later on.

We briefly review the existing multivariate GARCH models and focus on the BEKK model since it is the only one that guarantees a positive definite covariance matrix. Then we introduce a new bivariate model that parameterizes the conditional correlation directly and guarantees positive definite covariance matrices with fewer parameters than the full BEKK model and more flexibility than the restricted BEKK model. The fact that this model does only exist in bivariate form is theoretically very restrictive. However, we will argue that an adequate bivariate model has not been introduced so far and usually bivariate models are estimated as mentioned above.

We focus on the persistence and the asymmetry of time-varying correlations since these characteristics are important for portfolio diversification, i.e. it is important to know whether correlations are equally persistent as volatilities and whether there is any difference in the reaction of correlations to positive and negative shocks.

The remainder of this paper is as follows: Section 1 discusses existing multivariate GARCH models and focusses on the full and restricted BEKK model and its asymmetric extensions. We also discuss the Constant Correlation Model of Bollerslev (1990) and use this model as a benchmark for volatility estimates. Section 2 introduces a new bivariate dynamic correlations model that has various advantages over existing bivariate GARCH models. Section 3 shows results of Monte-Carlo simulations for all discussed models. Section 4 estimates the BDC model for empirical data and section 5 concludes.

1 Multivariate GARCH Models

We use a simple specification for the mean equation since our interest is the time-varying covariance matrix. Thus, returns are modeled as follows:

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \qquad\qquad \boldsymbol{\epsilon}_t \,|\, \boldsymbol{\Omega}_{t-1} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{H}_t) \tag{1}$$

where $\boldsymbol{r}_t$ is a vector of appropriately defined returns and $\boldsymbol{\mu}$ is a $(N \times 1)$ vector of parameters. The residual vector is $\boldsymbol{\epsilon}_t$ with the corresponding conditional covariance matrix $\boldsymbol{H}_t$ given the available information set $\boldsymbol{\Omega}_{t-1}$.

We focus on the BEKK model since it is the only time-varying covariance model that guarantees a positive-definite covariance matrix. We also discuss the Constant Correlation Model (CCM) of Bollerslev (1990) and a zero correlation model (ZCM) that are used as benchmark models.

1.1 The BEKK Model

The BEKK model was introduced by Baba, Engle, Kraft and Kroner (1991) and can be seen as an improvement to the VECH model (introduced by Bollerslev, Engle and Wooldridge, 1988). First, the number of parameters is reduced and second, the positive-definiteness of the covariance matrix is guaranteed.

We initially present the full (unrestricted) BEKK model and its asymmetric extension and then restrict this model to the diagonal BEKK. The covariance matrix of the unrestricted BEKK model is

$$\boldsymbol{H}_t = \boldsymbol{A}'\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{B} + \boldsymbol{C}'\boldsymbol{H}_{t-1}\boldsymbol{C} \tag{2}$$

$\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are matrices of parameters with appropriate dimensions. It is obvious from the equation above that the covariance matrix is guaranteed to be positive definite as long as $\boldsymbol{A}'\boldsymbol{A}$ is positive definite. Furthermore, the parameters are squared or cross-products of themselves leading to variance and covariance equations without an univariate GARCH counterpart. Note that this is not true for the vech model which is a simple extension of univariate GARCH models to a multivariate form.

To clarify the difficulties in interpreting the parameters of the covariance matrix we consider the general BEKK model in bivariate form. h_{11} and h_{22} denote the variances of the return series and h_{12} their covariance:

$$\begin{aligned}\begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} &= A'A + \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} \begin{pmatrix} \epsilon^2_{1,t-1} & \epsilon_{t-1}\epsilon_{2,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} & \epsilon^2_{2,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} + \\ &+ \begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} \begin{pmatrix} h_{11,t-1} & h_{12_t-1} \\ h_{12,t-1} & h_{22,t-1} \end{pmatrix} \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\end{aligned} \tag{3}$$

Without using matrices, we get the following form:

$$
\begin{aligned}
h_{11,t} &= a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + 2b_{11}b_{21}\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{21}^2 \epsilon_{2,t-1}^2 + c_{11}^2 h_{11,t-1} + 2c_{11}c_{21}h_{12,t-1} + \\
&\quad + c_{21}^2 h_{22,t-1} \\
h_{12,t} &= a_{12}a_{11} + b_{11}b_{12}\epsilon_{1,t-1}^2 + (b_{12}b_{21} + b_{11}b_{22})\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{21}b_{22}\epsilon_{2,t-1}^2 + \\
&\quad + c_{11}c_{12}h_{11,t-1} + (c_{12}c_{21} + c_{11}c_{22})h_{12,t-1} + c_{21}c_{22}h_{22,t-1} = h_{21,t} \\
h_{22,t} &= a_{12}^2 + a_{22}^2 + b_{12}^2 \epsilon_{2,t-1}^2 + 2b_{12}b_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{22}^2 \epsilon_{2,t-1}^2 + c_{12}^2 h_{11,t-1} + \\
&\quad + 2c_{12}c_{22}h_{12,t-1} + c_{22}^2 h_{22,t-1}
\end{aligned}
\tag{4}
$$

The latter formulation clarifies that even for the bivariate model the interpretation of the parameters may be misleading since there is no equation that does possess its own parameters, i.e. parameters that exclusively govern an equation. Hence, it is possible that a parameter is biased by the fact that it influences two equations simultaneously or by the sole number of regressors (see also Tse, 2000), e.g. the regressors $\epsilon_{2,t-1}^2$ and the regressor $h_{22,t-1}$ in the first variance equation ($h_{11,t}$) could both be viewed as a volatility spillover from the second return. In addition, the statistical significance of the parameters is also unclear due to the combinations of different parameters serving as new coefficients for particular regressors.

These critics do not all apply to the diagonal BEKK model where both parameter matrices are diagonal. Thus, the off-diagonal elements are all equal to zero (apart from the constant term $A'A$). The number of parameters to be estimated is significantly lower while maintaining the main advantage of this specification, the positive definiteness of the conditional covariance matrix. Instead of equation (4) we have

$$
\begin{aligned}
h_{11,t} &= a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} \\
h_{22,t} &= a_{11}^2 + a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} \\
h_{12,t} &= h_{21,t} = a_{11} a_{22} + b_{11} b_{22} \epsilon_{1,t-1} \epsilon_{2,t-1} + c_{11} c_{22} h_{12,t-1} \\
h_{21,t} &= h_{12,t}
\end{aligned}
\tag{5}
$$

This model exhibits essentially the same problems as the Full BEKK model since there is no parameter in any equation that exclusively governs a particular covariance equation. Hence, it is not clear whether the parameters for h_{12} are just the result of the parameter estimates for h_{11} and h_{22} or if the covariance equation alters the parameter estimates of the variance equations.

In addition, the model can be severely misspecified. For example, assuming that the persistence of shocks to volatility is relatively high for both return series, say $b_{ii} + c_{ii} = 0.05 + 0.90 = 0.95$ for $i = 1, 2$, then the persistence of the covariance must be almost equally high, $b_{ii}b_{jj} + c_{ii}c_{jj} = 0.05 \cdot 0.05 + 0.9 \cdot 0.9 = 0.0025 + 0.81 = 0.8125$ for $i = 1$ and $j = 2$. Supposed that covariances are less persistent or equally persistent as volatilities it is clear that either volatilities or the covariance is misspecified.

1.2 Constant Correlation Model and Zero Correlation Model

The Constant Correlation Model (CCM) of Bollerslev (1990) does model time-varying covariances more parsimoniously than the models discussed above. The bivariate model is given by

$$
\begin{aligned}
h_{11,t} &= a_{11} + b_{11}\epsilon^2_{1,t-1} + c_{11}h_{11,t-1} \\
h_{22,t} &= a_{22} + b_{22}\epsilon^2_{2,t-1} + c_{22}h_{22,t-1} \\
h_{12,t} &= \rho\sqrt{h_{11,t}h_{22,t}} \\
h_{21,t} &= h_{12,t}
\end{aligned} \tag{6}
$$

where ρ is a parameter that can be estimated almost freely (ρ must be in the range $[-1, 1]$) and is equal to the empirical correlation coefficient (see Bollerslev, 1990). In contrast to the BEKK model there is a parameter in the CCM (ρ) that exclusively governs the covariance equation. Note that the CCM exhibits time-varying covariances but only constant correlations.

To guarantee positive variances we use the variance equations of the diagonal BEKK model for the variance equations of the CCM as suggested by Kroner and Ng (1998).

Setting ρ to zero implies a model that we call Zero Correlation Model (ZCM).

We will use both the CCM and the ZCM to analyse in which respect covariances do affect variance estimates.

1.3 Asymmetric Extensions

While it is straightforward in the diagonal BEKK Model to analyze whether the covariance exhibits the same degree of persistence as the variances, the relevant parameter estimates measuring the persistence of shocks are potentially biased by each other. This is also true for the full BEKK Model and possibly more severe due to the larger number of parameters.

The same problem arises for the asymmetric extensions of the models. To illustrate this, we analyse the asymmetric extensions proposed by Kroner and Ng (1998) and focus on the diagonal BEKK model.

For the bivariate case the asymmetric extension is

$$\begin{aligned} h_{11,t} &= ... + d_{11}^2 \eta_{1,t-1}^2 \\ h_{22,t} &= ... + d_{22}^2 \eta_{2,t-1}^2 \\ h_{12,t} &= ... + d_{11} d_{22} \eta_{1,t-1} \eta_{2,t-1} \end{aligned} \tag{7}$$

where $\eta_{i,t} = min_{i,t}(\epsilon_{i,t}, 0)$ and $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t}, ...)'$.

Here, the covariance reacts to negative shocks $\eta_{i,t}$ as determined by the asymmetry implied by the variance equations or vice versa. For example, assuming that variance one (h_{11}) does not react asymmetrically to positive and negative shocks ($d_{11} = 0$) and variance two (h_{22}) does ($d_{22} = 0.2$), then the asymmetric effect for the covariance would be zero ($d_{11} d_{22} = 0$). Assumed that there is an asymmetric effect of the covariance either the variance equation or the covariance equation will be misspecified. Another example is the case where the asymmetry of the covariance is equal to 0.2. Then, the parameters d_{11} or d_{22} would have to be very large to capture this covariance asymmetry (e.g. $d_{11} = d_{22} = \sqrt{0.2}$) [1].

The asymmetric extension of the CCM introduced by Kroner and Ng (1998) has

[1] Ang and Chen (2001) report misspecifications of an asymmetric GARCH-M model by the interpretation of the resulting asymmetry.

the variance equations of the diagonal BEKK model and the covariance equation as given in the original model (see equation 6). Again, we could use this model to analyze how variance estimates change when correlations are modeled time-varying. This question is further examined in the simulation study in section 3.

2 Bivariate Dynamic Correlations (BDC)

We propose a new bivariate model that is more flexible than the discussed models and parameterizes the conditional correlations directly. In contrast, the conditional correlations in the BEKK model are derived from the ratio of the covariance with the product of the roots of the conditional variances (see equations (3) and (5)).

We write the covariance matrix $\boldsymbol{H}_t$ in the following form:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{8}$$

where $\boldsymbol{D}_t$ is a diagonal matrix with the roots of the variances on the main diagonal and $\boldsymbol{R}_t$ is a correlation matrix. In a bivariate form $\boldsymbol{R}_t$ is

$$\boldsymbol{R}_t = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \tag{9}$$

with ρ_t denoting the correlation between two series. $\boldsymbol{H}_t$ is positive definite if $\boldsymbol{R}_t$ is positive definite. This is guaranteed as long as $|\rho_t| < 1$. Thus we restrict $|\rho_t|$ to be smaller than one by using the following transformation:

$$\rho_t^\star = \frac{\rho_t}{\sqrt{1+\rho_t^2}} \tag{10}$$

where $\rho_t^\star$ is the correlation restricted to be in the interval $(-1;1)$. This restriction allows to use own parameters for the correlation (covariance) equation and to include additional regressors without risking semi-definite or indefinite covariance matrices.

Hence, the Bivariate Dynamic Correlations model (BDC) is specified by the following equations:

$$\begin{aligned} h_{11,t} &= a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} \\ h_{22,t} &= a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} \\ \rho_t &= a_{12} + b_{12} \epsilon_{1,t-1} \epsilon_{2,t-1} + c_{12} \rho_{t-1} \\ \rho_t^\star &= \frac{\rho_t}{\sqrt{1+(\rho_t)^2}} \\ h_{12,t} &= \rho_t^\star \cdot \sqrt{h_{11,t} h_{22,t}} \end{aligned} \tag{11}$$

The BDC Model is a dynamic correlation model since $\rho_t^\star$ and thus $h_{12,t}$ are time-varying. The covariance does possess its own parameters and the covariance matrix is always guaranteed to be positive-definite. The specification allows to compare the parameter estimates and therefore the degree of persistence for the variance equations and the correlation equation.

We use the cross product $\epsilon_{1,t-1}\epsilon_{2,t-1}$ to model the correlation equation and additionally the cross product of the standardized residuals $z_{1,t-1}z_{2,t-1}$ to analyze the different behavior of the correlation process. Tse (2000) points out that there is no a priori reason to expect the standardized residuals to be a better specification. Contrary to this statement we expect the results to be different due to the fact that z_t is corrected for volatility movements. In addition, the use of z_t is a more natural specification for the conditional correlations (see Engle, 2000 and Tse, 2000).

We refer to the model using the raw residuals ϵ_t as BDC_ϵ and to the model using the standardized residuals z_t as BDC_z. The correlation equation for the BDC_z model is given by:

$$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} \tag{12}$$

The next subsection introduces the asymmetric extension of the BDC model.

2.1 Asymmetric BDC Model

An extension of the presented BDC models can also capture asymmetric effects of the time-varying correlation. Thus, h_{11} and h_{22} and ρ_t are specified as follows:

$$\begin{aligned} h_{11,t} &= a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} + d_{11}^2 \eta_{1,t-1}^2 \\ h_{22,t} &= a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} + d_{22}^2 \eta_{2,t-1}^2 \\ \rho_t &= a_{12} + b_{12} \epsilon_{1,t-1} \epsilon_{2,t-1} + c_{12} \rho_{t-1} + d_{12} \eta_{1,t-1} \eta_{2,t-1} \end{aligned} \tag{13}$$

Again, $\eta_{i,t} = min_{i,t}(\epsilon_{i,t}, 0)$ with $\boldsymbol{\eta}_t$ containing only the negative shocks of the returns at t. Note, that this specification does not require an asymmetric extension for the variance equations which is necessary for all other multivariate GARCH models. Hence, this feature can also be used to include additional regressors in the correlation equation without risking an indefinite covariance matrix, e.g. thresholds or spillover effects are easily includable

Note that the model is different to the Dynamic Conditional Correlation (DCC) Estimator of Engle (2000) in various respects. First, we estimate all variance and covariance equations simultaneously. Second, the BDC model can differentiate between the use of the raw residuals $\boldsymbol{\epsilon}$ and the standardized residuals $\boldsymbol{z}$ and third, the BDC model is flexibly extendable, e.g. asymmetric extensions as presented above or a threshold as suggested by Longin and Solnik (1998) can be included.

2.2 Estimation

The estimation of the models based on a sample of T observations of the returns vector $\boldsymbol{r}_t$ is done through numerical maximization of a likelihood function assuming normally distributed returns:

$$\log L(\boldsymbol{\theta}; \boldsymbol{r}_1, \ldots, \boldsymbol{r}_T) = -T/2 \, \log(2\pi) - 1/2 \, \log(|\boldsymbol{H}_t|) - 1/2 \boldsymbol{\epsilon}_t' \boldsymbol{H}_t^{-1} \boldsymbol{\epsilon}_t \, . \quad (14)$$

The standard errors and associated t-values reported in this article are calculated using the quasi-maximum likelihood methods of Bollerslev and Wooldridge (1992), i.e. the standard errors are robust to the density function underlying the residuals.

The next section introduces a new bivariate model that reduces the number of parameters compared to the full BEKK model and extends the flexibility compared to the restricted BEKK model.

3 Simulations

In this section, we compare the covariance estimates of the full BEKK, the diagonal BEKK, the BDC model, the CCM and Zero Correlation Model (ZCM). We use the CCM and the ZCM to compare the variance estimates and to analyse the impact of the covariance specification on the variance estimates (Tse (2000) suggested such an analysis [2]).

The simulations and tests are partially similar to the ones undertaken by Engle (2000)[3].

We simulate bivariate GARCH models 200 times with 1000 observations with different time-varying correlations. We first generate 1000 Gaussian random numbers ϵ_i for $i = 1, 2$ with mean zero and variance one and a given (time-varying) correlation. Then we generate the variance equations:

$$\begin{aligned} h_{11,t} &= 0.1 + 0.04\epsilon_{1t-1}^2 + 0.95h_{11,t-1} \\ h_{22,t} &= 0.1 + 0.20\epsilon_{2t-1}^2 + 0.50h_{22,t-1} \end{aligned} \tag{15}$$

h_{1t} is highly persistent and h_{2t} is less persistent. Given these variances we simulate different correlation processes:

[2]In other words, Are the estimates of the parameters in the conditional-variance estimates robust with respect to the constant correlation assumption ? (page 109)

[3]Engle did compare the DCC model with the scalar BEKK, the diagonal BEKK, a moving average process, an exponential smoother and a principle components GARCH. He did not include the full BEKK model, the Factor GARCH model and any asymmetric extensions.

(i) constant correlations: $\rho_t = 0.5$, (ii) highly persistent time-varying correlations ($\rho_t = \alpha + \beta sin(t/50 * f)$ with a fast sine function given by $\alpha = 0$, $\beta = 0.5$, $f = 1$ and a slow sine given by $\alpha = 0$, $\beta = 0.9$, $f = 5$) and (iii) a step function ($\rho_t = 0.5 * round(t/900)$). For the asymmetric extensions of the models we use the following variance equations:

$$\begin{aligned} h_{11,t} &= 0.1 + 0.04\epsilon_{1t-1}^2 + 0.85h_{11,t-1} + 0.1\eta_{1,t-1}^2 \\ h_{22,t} &= 0.1 + 0.10\epsilon_{2t-1}^2 + 0.50h_{22,t-1} + 0.2\eta_{2,t-1}^2 \end{aligned} \tag{16}$$

The correlation process is assumed to be the same as for the non-asymmetric models.

We compare the estimates of $h_{11,t}$, $h_{22,t}$ and ρ_t with the true variance and covariance series by (i) the mean absolute deviation (MAD), (ii) the means of the correlations of the true covariance series ($h_{ij,t}$ for $i, j = 1, 2$) with the estimated covariance series and (iii) a F-test of a regression of the squared standardized residuals on the first four lagged values and a constant to detect remaining autocorrelation.

The means of the correlations are computed since they provide a measure of the fit of the estimated model compared to the simulated one.

3.1 Simulation Results

Table 1 present the results for the six different models (full BEKK, diagonal BEKK, BDC_ϵ, BDC_z, CCM and ZCM) in its non-asymmetric specification. The table contains the results for the mean absolute deviation (MAD), the mean of the correlation of the estimated process (variances and correlations) with the true simulated series and the number of rejections of F-tests. The values denoted with a star do indicate the minimum value among the estimated models and among the different correlation processes (constant correlations, fast sine function, sine function and step function).

Constant correlations are best estimated by the CCM and time-varying correlations are best estimated by the BDC Model. The fast sine function is an exception. Here, the diagonal BEKK model performs best. However, the difference to the BDC Model is rather negligible. In addition, we believe that the fast sine is economically not a relevant case.

The results for the variances show that the CCM, the ZCM and the diagonal BEKK perform best for the variance of series 1. For the variance of series 2 the BEKK model and the ZCM perform best. All models do provide rather poor estimates for h_{22} which is mostly due to the different variance process generated with more noise than the first variance series.

The table where the mean of correlations is tabulated shows that the general results do not change considerably. For the variance 1 the CCM and the Diagonal BEKK model perform best and for the variance 2 the Diagonal BEKK model and the BDC model have the highest values.

The lower part of the table presents the number of cases where F-tests pointed to remaining autocorrelations of the squared standardized residuals. Engle (2000) showed for a bivariate model that only z_2^2 is dependent on the estimated time-varying correlations while z_1^2 is not. Panel 1 tabulates the results for z_1^2 and z_2^2. Stars denote the minimum value of the number of remaining autocorrelations. Here, the CCM and the ZCM perform best.

For the asymmetric models results do not change considerably. Consequently, we do not report our results due to space considerations.

We conclude from our simulation results that correlation estimates are closest to the true values in the BDC model for time-varying correlations and the CCM is best for constant correlations. The BDC_z model does perform better than the BDC_ϵ model which we attribute to the variance correction that potentially leads to less noise in the correlation process.

The relatively good performance of the CCM and the ZCM with respect to the variances indicates that the correlation specification does not have any effect on the variance estimates.

4 Empirical Results

We estimate the simple (non-asymmetric) and the asymmetric versions of the BDC and the asymmetric diagonal BEKK model.

We use daily (close-to-close) continuously compounded returns of the German DAX stock index, the DOW Jones Industrial Average, the FTSE100 and the NIKKEI225. The indices span a time-period of 10 years from January, 1990 until December, 2000 with $T = 2580$ observations for each stock index. Non-trading days are included to synchronize the data. Dummy variables to account for these days are not used since this would further augment the number of parameters and not change the estimation results.

Tables 2 and 3 present the results for the correlation estimates for the BDC_ϵ model and the BDC_z model in its non-asymmetric specification, respectively. Comparing the values $b_{12} + c_{12}$ indicating persistence shows that the use of the standardized residuals implies higher persistence for the (DAX, NIKKEI) and the (DAX, FTSE) but lower persistence for the (DAX, DOW) correlations.

Table 4 gives results for the asymmetric BDC_ϵ Model of the variance and correlation estimates of the DAX with the NIKKEI (second column), the FTSE (third column) and the DOW (fourth column), respectively. The estimated volatilities are highly persistent indicated by the sums of the parameters $b_{11}^2 + c_{11}^2$ and $b_{22}^2 + c_{22}^2$. These volatilities do all react asymmetrically to positive and negative shocks, i.e. negative shocks do augment volatilities by more than positive shocks.

The correlation estimates do not give such a homogeneous picture: The correlation between the DAX and the NIKKEI is constant and shocks are not persistent. The correlation between the DAX and the FTSE is time-varying and shocks are highly persistent. Finally, the correlation between the DAX and the DOW appears to be constant and shocks are not persistent. The graphs of these estimated correlations are plotted in figure 2.

The use of the standardized residuals z_t for the correlation process in the BDC model leads to different results. Table 5 shows that the parameter values are considerably different to the ones obtained from the estimation of the BDC model with the raw residuals ϵ_t: all correlations are highly persistent and jointly negative shocks do increase conditional correlations in all cases. We attribute this difference to the fact that the shocks z_t are corrected for the conditional variance.

Summarizing the results for the BDC model in its different specifications, we find that correlations do exhibit a lower persistence when the raw residuals are used and approximately the same persistence as volatilities when the standardized residuals are used. The asymmetric effect of the time-varying correlation is smaller than for volatilities and even negative for the (DAX, DOW) correlations. The lower asymmetry is independent of the type of shock (raw or standardized) that is used.

Table 6 presents results for the diagonal BEKK model estimated with the same return pairs as above. Comparing the parameter estimates for the variance equations of the DAX (a_{11}, b_{11}, c_{11}) among the three estimations shows that parameters vary substantially for the same return series (DAX) which clarifies that the variance estimates are influenced by the second return series and by the estimated covariance. This is clear evidence that parameter estimates are biased since this variation is not apparent in the BDC model. However, it seems that the bias is mainly in the variance parameters.

Since the BEKK model does not estimate the correlation process directly but is the ratio of the covariance and the squared root of the product of the variances, we can only analyze the persistence and asymmetry of the variances and the covariance and conclude that there is covariance asymmetry as indicated by the product of the parameters d_{11} and d_{22}. A direct comparison of parameters with the BDC model is not possible.

The constancy and non-persistence of shocks for the correlation process in two cases estimated with the BDC Model is unique since it could not be revealed by the single use of any standard multivariate GARCH model.

4.1 The Asymmetry of Correlations

Asymmetric effects of volatilities to positive and negative shocks are well documented in the literature and explained with the leverage effect (Black, 1976 and Christie, 1982) and the volatility feedback effect (Campbell and Hentschel, 1992). However, little is known about the temporal behaviour of stock return correlations (see Andersen et al., 2000 and Andersen et. al., 2001) and even less of the potential asymmetric effects of positive and negative shocks.

Theoretically, correlations should increase if shocks of two time series have the same sign and decrease if shocks have opposite signs. In addition, correlations should increase by the same value for jointly positive and jointly negative shocks, i.e. there is no asymmetric effect.

In contrast, the estimation results of the asymmetric BDC model show that there is an asymmetric effect of correlations and that this asymmetry is not similar to the one observed for volatilities.

Focussing on the parameters b_{12} and d_{12} in Table 4 reveals that correlations increase with jointly positive shocks for the (DAX, FTSE) and (DAX, DOW) returns and slightly decrease for the (DAX, NIKKEI) correlation. Correlations increase with negative shocks for the (DAX, NIKKEI) and (DAX, FTSE) series and decrease for the (DAX, DOW) return series.

To clarify these findings we plot news-impact surfaces for the correlations given by Figure 3 and by figure 4 (frontal views). These functions show how correlations react to different combinations of shocks of two different time-series. We set the range of positive and negative shocks to $[-5, +5]$.

Due to the construction of the correlation process, all news-impact surfaces exhibit a symmetric behaviour for the residuals of opposite signs, i.e. negative shocks of one stock index do not have a larger influence on the correlation than negative shocks of the other stock index. The same is true for positive shocks. Such a symmetric picture is not existent if both shocks do have the same sign since we account for such differences in the correlation equation (equation (13)).

The asymmetry of correlations is closely related to the empirical finding that correlations increase with volatility. More precisely, it was often stated that correlations increase in bear markets thus calling into question the desirability of international portfolio diversification (see De Santis and Gerard, 1997, Longin and Solnik, 1998, Longin and Solnik, 2001, Ng, 2000, Ramchand and Susmel, 1998 and Susmel and Engle, 1993).

Interpreting simultaneously high positive and negative values of ϵ_1 and ϵ_2 as a high-volatility state, we can answer this question.

The results for the correlation of the DAX with the NIKKEI and the FTSE do replicate the findings in the literature, i.e. international portfolio diversification is not effective whenever it is needed most. The result for the correlation of the DAX and the DOW is counter to the findings in the literature and further encourages international portfolio diversification between Germany and the US.

5 Conclusions

We have shown that the existing multivariate GARCH models do not adequately model constant or time-varying correlations. The same is true for the asymmetric extensions of these models. The Bivariate Dynamic Correlations Model introduced here performs clearly better in this regard. Taking into account that usually multivariate GARCH models are estimated only for two asset returns, the BDC model is empirically not so restrictive.

We have estimated the BDC model for the DAX with three international stock market indices and found that correlations do exhibit a different temporal behaviour as volatilities, i.e. correlations are less persistent than volatilities and the asymmetry of shocks on volatility is more pronounced than the asymmetric effects of jointly positive or negative shocks on correlations.

Furthermore, we find that correlations do not always increase with volatilities. It is necessary to differentiate between jointly positive and negative shocks. This reveals that correlations increase with jointly negative shocks for the (DAX, NIKKEI) and (DAX, FTSE) portfolio but not for the (DAX, DOW) portfolio where correlations decrease with jointly negative shocks. This result shows that international portfolio diversification is at least effective for an equally weighted portfolio composed of the DAX and the DOW.

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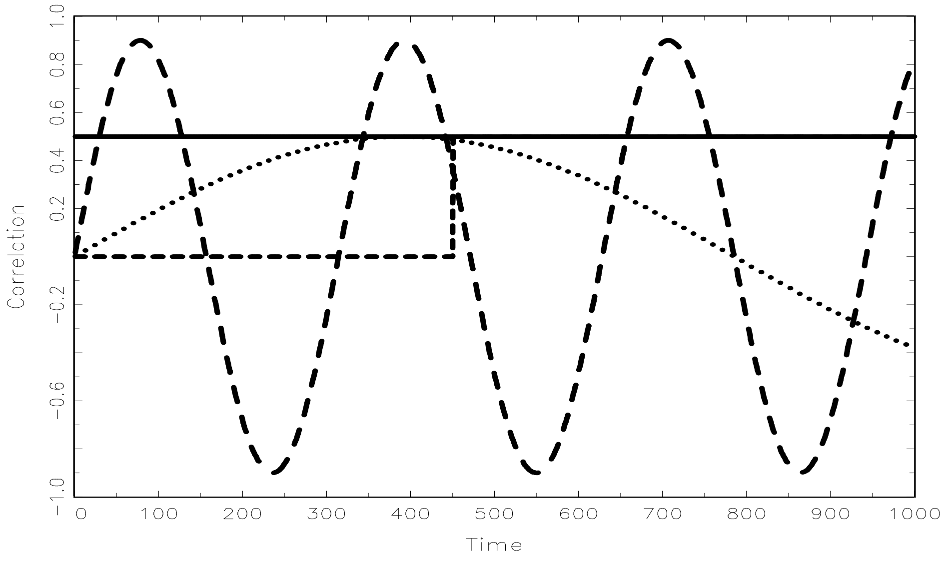

Figure 1: *Simulated Correlation processes*

Table 1: **Simulation Results**

Mean of MAD

	F.BEKK	D.BEKK	BDC_ϵ	BDC_z	CCM	ZCM
correlations						
constant	0,0629	0,1945	0,0290	0,0319	0,0176*	0,5000
fast sine	0,1646	0,1600*	0,2061	0,1771	0,5655	0,5662
sine	0,1658	0,1950	0,1287	0,1232*	0,2184	0,2931
step	0,1650	0,2017	0,1446	0,1300*	0,2473	0,2745
variance 1						
constant	0,8508	1,0084	0,6328*	0,6922	0,7100	0,6572
fast sine	1,7981	1,7833	0,8517	1,0445	0,7087*	0,7193
sine	0,8904	0,7062	0,7471	0,7007	0,6531	0,6247*
step	0,8526	0,8326	0,6754*	0,7760	0,7300	0,6896
variance 2						
constant	0,0841	0,0596*	0,0739	0,0714	0,0736	0,0746
fast sine	0,0974	0,0901	0,0735	0,0774	0,0725	0,0716*
sine	0,0643*	0,0696	0,0703	0,0718	0,0741	0,0741
step	0,0652*	0,0698	0,0701	0,0713	0,0714	0,0736

Mean of correlations

	F.BEKK	D.BEKK	BDC_ϵ	BDC_z	CCM	ZCM
correlations						
constant	0,1971	0,0757	0,4426*	0,4381	-0,0010	
fast sine	0,9433	0,9448*	0,9142	0,9337	-0,0199	
sine	0,6054	0,5000	0,7715	0,7815*	0,0715	
step	0,6233	0,4800	0,6857	0,7626*	-0,0286	
variance 1						
constant	0,9356	0,9674	0,9676	0,9652	0,9722*	0,9717
fast sine	0,8929	0,9181	0,9567	0,9372	0,9671*	0,9663
sine	0,9401	0,9764*	0,9550	0,9607	0,9730	0,9713
step	0,9400	0,9718	0,9685	0,9729	0,9643	0,9735*
variance 2						
constant	0,5434	0,7779*	0,6673	0,6806	0,6452	0,6627
fast sine	0,7476	0,7776*	0,7398	0,7311	0,6690	0,6754
sine	0,6920	0,6895	0,7011*	0,6960	0,6483	0,6500
step	0,6821	0,7092	0,6961	0,7243*	0,6682	0,6731

F-tests

	F.BEKK	D.BEKK	BDC_ϵ	BDC_z	CCM	ZCM
variance 1						
constant	14	8	6*	8	6*	6*
fast sine	44	30	12	16	6*	6*
sine	18	16	6	8	4*	4*
step	14	14	1*	4	8	1*
variance 2						
constant	14	24	2	2	0*	0*
fast sine	52	42	132	76	6*	6*
sine	18	6*	10	6*	6*	6*
step	8	22	4	8	14	1*

a star denotes the minimum or maximum value in a row

Table 2: BDC_{ϵ} **MODEL Estimation Results**

Parameters	DAX, NIKKEI	DAX, FTSE	DAX, DOW
a_{12}	0,0720*	0,0609***	0,0018
	(0,0498)	(0,0234)	(0,0018)
b_{12}	0,0125**	0,0208***	0,0017**
	(0,0061)	(0,0070)	(0,0010)
c_{12}	0,7230***	0,8846***	0,9918***
	(0,1758)	(0,0403)	(0,0067)

Correlation equation:

$\rho_t = a_{12} + b_{12}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{12}\rho_{t-1}$

Table 3: BDC_{z} **MODEL Estimation Results**

Parameters	DAX, NIKKEI	DAX, FTSE	DAX, DOW
a_{12}	0,0247*	0,0331**	0,0089
	(0,0189)	(0,0160)	(0,0197)
b_{12}	0,0191***	0,0318***	0,0061
	(0,0072)	(0,0078)	(0,0050)
c_{12}	0,8943***	0,9228***	0,9626***
	(0,0656)	(0,0282)	(0,0704)

Correlation equation:

$\rho_t = a_{12} + b_{12}z_{1,t-1}z_{2,t-1} + c_{12}\rho_{t-1}$

Table 4: BDC_ϵ **MODEL Estimation Results**

Parameters	DAX, NIKKEI	DAX, FTSE	DAX, DOW
μ_1	0,0430 **	0,0507 **	0,0440 **
	(0,0236)	(0.0238)	(0,0231)
μ_2	-0,0426 **	0,0357 **	0,0432 ***
	(0,0245)	(0,0163)	(0,0147)
a_{11}	0,3344 ***	0,3331 ***	0,3514 ***
	(0,0831)	(0,0802)	(0,0831)
a_{22}	0,2642 ***	0,1170 ***	0,1131 ***
	(0,0491)	(0,0181)	(0,0252)
b_{11}	0,2764 ***	0,2868 ***	0,2727 ***
	(0,0436)	(0,0410)	(0,0454)
b_{22}	0,1681 ***	0,1424 ***	0,1332 ***
	(0,0371)	(0,0324)	(0,0338)
c_{11}	0,8937 ***	0,8971 ***	0,8875 ***
	(0,0302)	(0,0288)	(0,0308)
c_{22}	0,9344 ***	0,9649 ***	0,9661 ***
	(0,0144)	(0,0065)	(0,0105)
d_{11}	0,3047 ***	0,2558 ***	0,3179 ***
	(0,0867)	(0,0835)	(0,0834)
d_{22}	0,3655 ***	0,2527 ***	0,2483 ***
	(0,0436)	(0,0287)	(0,0537)
a_{12}	0,1697	0,0665 **	0,2565 ***
	(0,1958)	(0,0395)	(0,0644)
b_{12}	-0,0090	0,0222	0,0131 *
	(0,0198)	(0,0248)	(0,0097)
c_{12}	0,3301	0,8726 ***	0,1606
	(0,7458)	(0,0694)	(0,2114)
d_{12}	0,0444	0,0022 *	-0,0456 ***
	(0,0431)	(0,0269)	(0,0115)
F-stat. z_1^2	0.1383	0.1611	0.1623
F-stat. z_2^2	0.5080	1.4352	0.5647
mean ρ	0.2599	0.5289	0.2831

***, **, * denote significance at the 1, 5, 10 percent level, respectively
standard errors in parenthesis

Covariance Equations:

$h_{11,t} = a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} + d_{11}^2\eta_{1,t-1}^2$

$h_{22,t} = a_{22}^2 + b_{22}^2\epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} + d_{22}^2\eta_{2,t-1}^2$

$\rho_t = a_{12} + b_{12}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{12}\rho_{t-1} + d_{12}\eta_{1,t-1}\eta_{2,t-1}$

Table 5: BDC_z **MODEL Estimation Results**

Parameters	DAX, NIKKEI	DAX, FTSE	DAX, DOW
μ_1	0,0409 **	0,0490 **	0,0677 ***
	(0,0241)	(0.0237)	(0,0237)
μ_2	-0,0425 **	0,0369 **	0,0489 ***
	(0,0257)	(0,0164)	(0,0147)
a_{11}	0,3290 ***	0,3396 ***	0,3424 ***
	(0,0813)	(0,0804)	(0,0811)
a_{22}	0,2657 ***	0,1207 ***	0,1172 ***
	(0,0489)	(0,0175)	(0,0274)
b_{11}	0,2873 ***	0,2762 ***	0,2778 ***
	(0,0423)	(0,0407)	(0,0470)
b_{22}	0,1705 ***	0,1369 ***	0,1331 ***
	(0,0371)	(0,0329)	(0,0374)
c_{11}	0,8946 ***	0,8959 ***	0,8892 ***
	(0,0294)	(0,0297)	(0,0302)
c_{22}	0,9336 ***	0,9644 ***	0,9633 ***
	(0,0142)	(0,0065)	(0,0121)
d_{11}	0,2914 ***	0,2707 ***	0,3116 ***
	(0,0871)	(0,0836)	(0,0853)
d_{22}	0,3676 ***	0,2589 ***	0,2644 ***
	(0,0427)	(0,0280)	(0,0544)
a_{12}	0,0174 *	0,0268 **	0,0002
	(0,0119)	(0,0142)	(0,0011)
b_{12}	0,0106	0,0128	-0,0031
	(0,0113)	(0,0125)	(0,0027)
c_{12}	0,9174 ***	0,9346 ***	0,9949 ***
	(0,0439)	(0,0266)	(0,0039)
d_{12}	0,0125	0,0260 **	0,0090 **
	(0,0146)	(0,0145)	(0,0048)
F-stat. z_1^2	0.3309	0.3809	0.3519
F-stat. z_2^2	0.6944	1.3277	1.4649
mean ρ	0.2650	0.5307	0.2631

***, **, * denote significance at the 1, 5, 10 percent level, respectively
standard errors in parenthesis

Covariance Equations:

$h_{11,t} = a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} + d_{11}^2 \eta_{1,t-1}^2$

$h_{22,t} = a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} + d_{22}^2 \eta_{2,t-1}^2$

$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} + d_{12} \eta(z)_{1,t-1} \eta(z)_{2,t-1}$

Table 6: **Diagonal BEKK MODEL Estimation Results**

Parameters	DAX, NIKKEI	DAX, FTSE	DAX, DOW
μ_1	0.0559 ***	0.0534 ***	0.0576 ***
	(0.0201)	(0.0205)	(0.0197)
μ_2	-0.0458 **	0.0327 ***	0.0401 ***
	(0.0242)	(0.0154)	(0.0143)
a_{11}	0.2479 ***	0.1767 ***	0.1644 ***
	(0.0418)	(0.0375)	(0.0325)
a_{22}	0.0598 ***	0.0860 ***	0.0588 ***
	(0.0277)	(0.0298)	(0.0233)
b_{11}	0.2796 ***	0.2173 ***	0.2328 ***
	(0.0302)	(0.0414)	(0.0469)
b_{22}	0.1097 ***	0.1836 ***	0.1217 ***
	(0.0490)	(0.0421)	(0.0586)
c_{11}	0.9294 ***	0.9550 ***	0.9605 ***
	(0.0156)	(0.0122)	(0.0123)
c_{22}	0.9445 ***	0.9342 ***	0.9405 ***
	(0.0111)	(0.0209)	(0.0182)
d_{11}	-0.1693 **	0.1809 ***	-0.0817
	(0.0849)	(0.0410)	(0.0752)
d_{22}	-0.3589 ***	0.3068 ***	-0.3522 ***
	(0.0375)	(0.0392)	(0.0554)
$(d_{11}d_{22})$	0.0608	0.0555	0.0288
F-stat. z_1^2	0.2806	0.2721	0.2545
F-stat. z_2^2	2.3377 ***	4.6050 ***	1.6418 *
mean ρ	0.3365	0.4199	0.2420

***, **, * denote significance at the 1, 5, 10 percent level, respectively
standard errors in parenthesis

Covariance Equations:

$h_{11,t} = a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} + d_{11}^2\eta_{1,t-1}^2$

$h_{22,t} = a_{11}^2 + a_{22}^2 + b_{22}^2\epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} + d_{22}^2\eta_{2,t-1}^2$

$h_{12,t} = a_{11}a_{22} + b_{11}b_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{11}c_{22}h_{12,t-1} + d_{11}d_{22}\eta_{1,t-1}\eta_{2,t-1}$

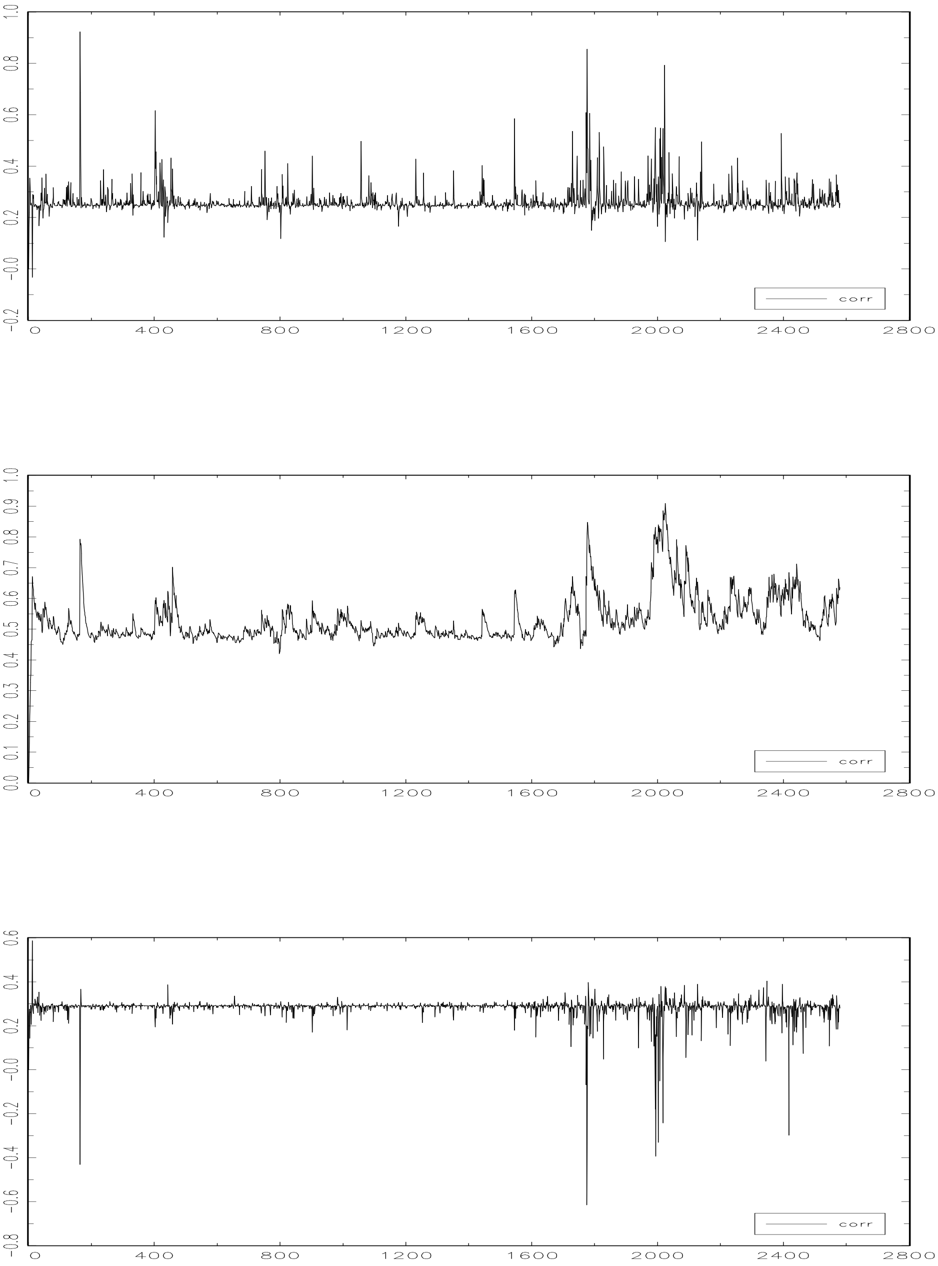

Figure 2: *Estimated Time-varying Correlations (BDC_ϵ) for (DAX, NIKKEI), (DAX, FTSE), (DAX, DOW)*

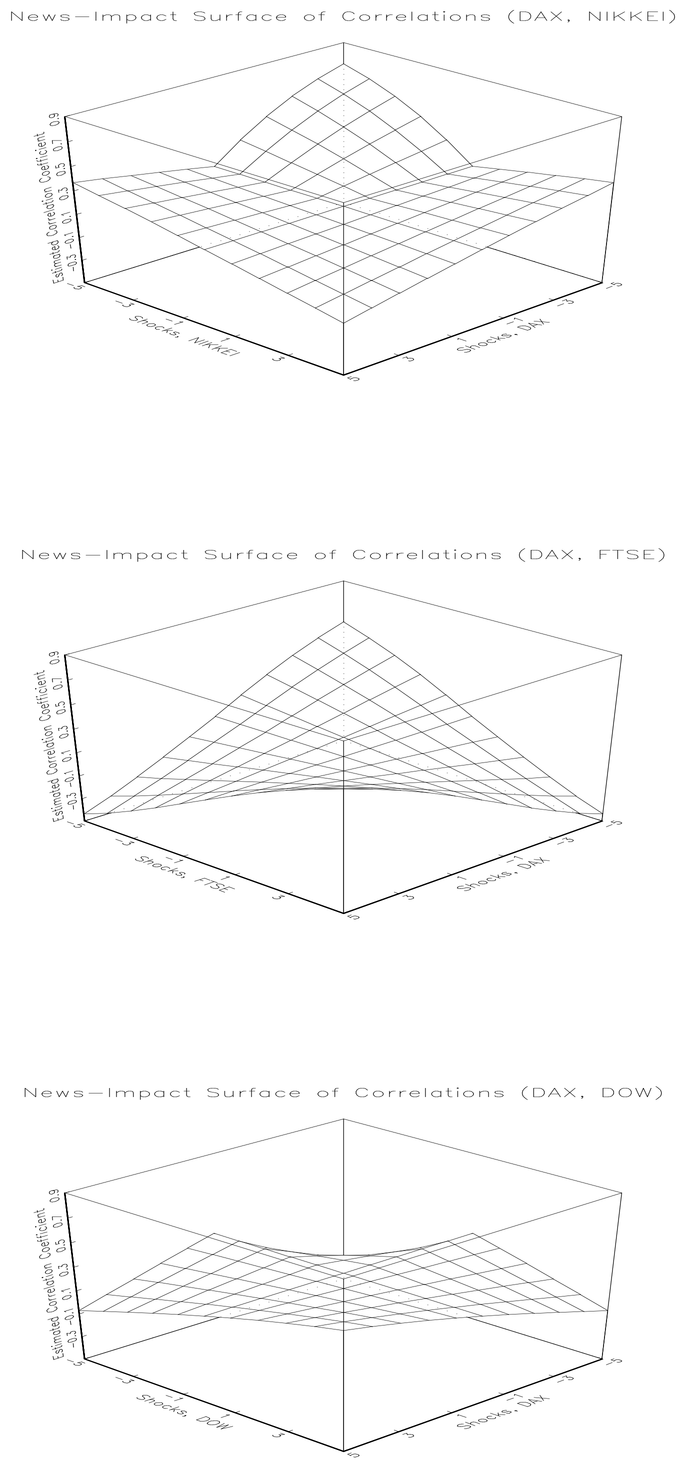

Figure 3: *News-Impact Surfaces*

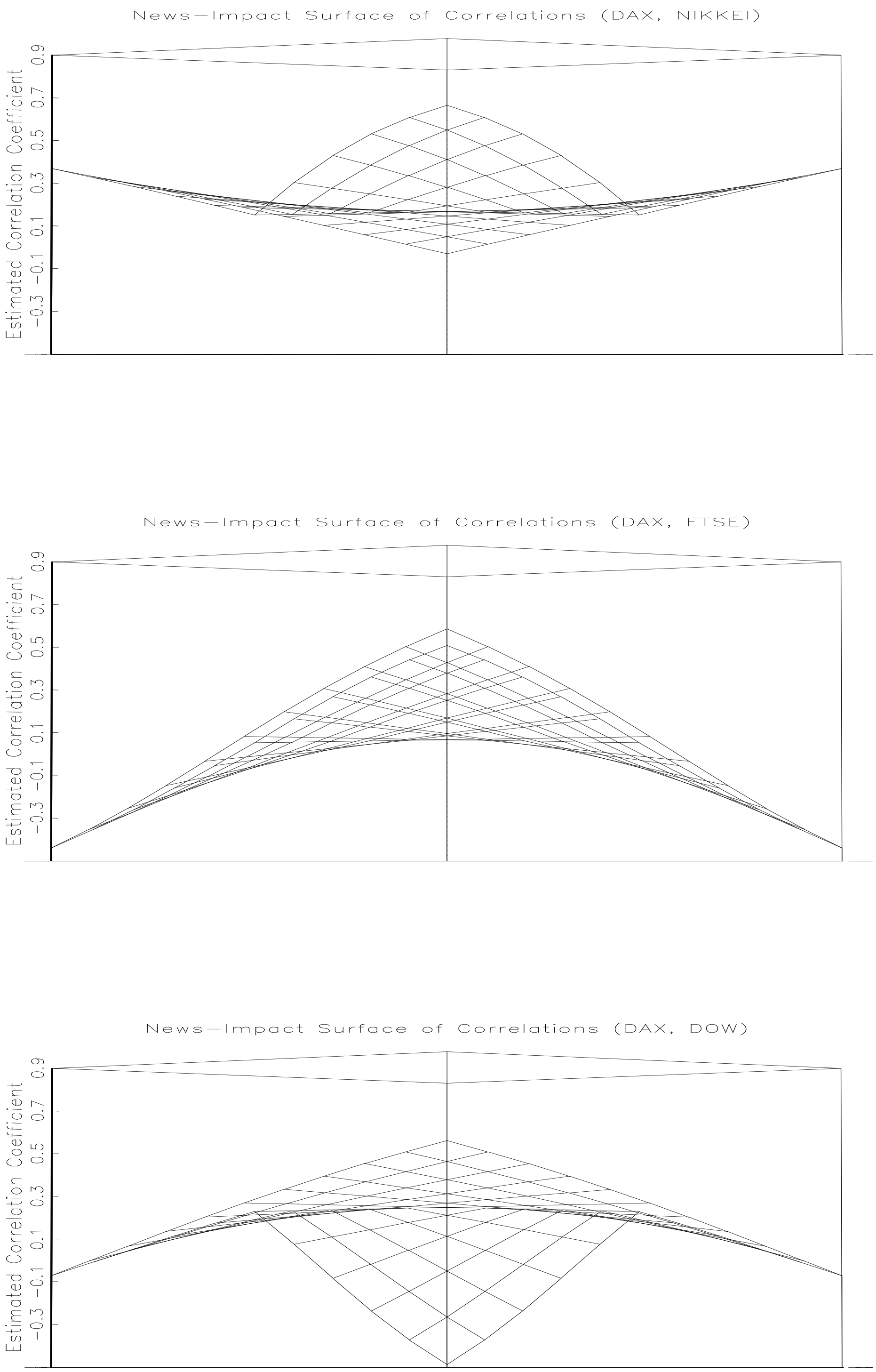

Figure 4: *News-Impact Surfaces (frontal views)*