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Generation Adjusted Discounting in Long-term Decision-making

Dr. Stefan Bayer, Tübingen

1 Introduction

The usage of benefit-cost-analyses in long-term decision-making is common sense in neoclassical models. It is assumed that there is no difference between short-term and long-term projects. Therefore, no special adjustments have to be made in the analytical framework. This is especially true for the choice of the discount rate, the question of what is to be discounted, and the discounting procedure. Most commonly, economic theory analyzes long-term costs and benefits in the framework of optimal growth models in tradition of Frank Ramsey:[1] A representative agent is supposed to maximize its lifetime-utility subject to a given budget constraint. The agent lives as long as the planning horizon of a specific project wants him to live. Taking natural resources theory as example, *Solow* (1974) as well as *Stiglitz* (1974) assume, that the planning horizon is infinitely long which means that the agent lives infinitely as well.[2] However, individual lifetimes are, of course, limited. This implies that we should use models with finite lifetimes of all agents for more realistic findings, OLG-models for instance.[3] But these models all assume the existence of long-term *optimal* growth paths.[4] Ad-hoc policy - for example to prevent further anthropogenic climate change due to carbon emissions - is not necessary because of perfect foresight and all-time rational behavior. Certainly, reality cannot be depicted as assumed in these models. New insights in climatic interdependencies for instance force policy-makers to react, especially when there exist (intertemporal) externalities.

Therefore, our analysis goes as follows: We want to concentrate on discounting single projects. We do not investigate the discounting process within a first-best world and the necessity of all time optimality. Our discountiung considerations relate to calculations of societal present values to determine whether a long-term, market failure correcting project is favorable or not.

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[1] See *Ramsey* (1928), extended and applied in *Barro/Sala-i-Martin* (1995). However, most applications do not exactly refer to Ramsey's original work because he strictly refuses utility discounting as "ethically indefensible".

[2] Ironically, in an answer to critical remarks made by *Daly* (1997), both *Solow* (1997) and *Stiglitz* (1997) stress that the planning horizon in fact is not infinitely long. They only wanted to get approximately "good" results for a planning horizon of 60 to 70 years. The long-held discussion about the convergence of the utility integral with conventional assumptions, especially positive utility discount rates until forever, could have been avoided if they had made this point clearer in their 1974 articles or in following publications.

[3] See *Blanchard/Fischer* (1989), *Howarth* (1996), or *Marini/Scaramozzino* (1995).

[4] See *Bayer/Cansier* (1999), *Bayer* (2000).

2 Basic Considerations of Economic Theory concerning Discounting

Conventionally, benefit-cost-analyses are carried out using constant discount rates throughout the whole planning horizon in an exponential form. The effect of constant exponential discounting is shown in figure 1. Therefore, we take two five-period projects with equal costs in the planning period, but different benefit-streams throughout the planning horizon as an example (in US-$):

Project A	-3,000	-300	-100	0	500	5000
Project B	-3,000	900	900	900	900	900

These data lead to the following figure:

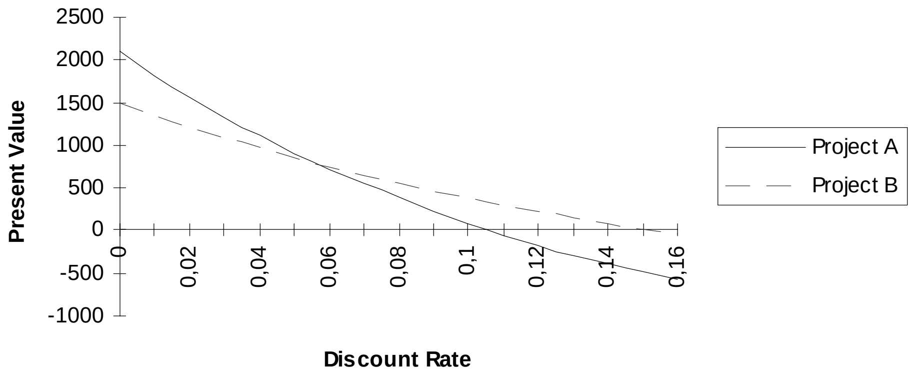

Figure 1: Efficiency of two projects A and B depending on the level of the discount rate

Using a discount rate of 5.6368 %, both projects have the same present value. Taking lower ones, project A is efficient, whereas a discount rate larger than 5.6368 % characterizes project B as efficient. Our small example shows the importance of the discount rate in project evaluation. The efficiency of projects depends heavily on the level of a fixed, exponential discount rate. Using slightly different discount rates, 5.6% or 5.65% for instance, two different projects are indicated as efficient ones.

We want to place our basic considerations into a more theoretical framework, using an optimal growth theory approach. A representative individual maximizes the following utility integral subject to a given budget constraint:

$$(1) \qquad \max \int_{t=0}^{\infty} U(C_t) \cdot e^{-\rho t} dt \qquad \text{s.t. } \dot{K}_t = Y_t - C_t .$$

This leads to the well known Ramsey-rule as first-order condition:[5]

[5] See *Ramsey* (1928).

$$\underbrace{r=\frac{\partial Y}{\partial K}}_{OCR}=\delta=\underbrace{\underbrace{\rho}_{PTPR}+\underbrace{\varepsilon\cdot g}_{GTPR}}_{TPR}. \qquad (2)$$

The marginal opportunity cost rate (OCR) is given on the left-hand side. It is equal to the marginal productivity of capital. In first-best worlds, i.e. without distorting taxes, uncertainties or intertemporal externalities, the marginal OCR equals the marginal time preference rate δ (TPR), which is given on the right-hand side of equation (2). The total marginal TPR is given as sum of two components: The pure or myopic time preference rate ρ (PTPR) and the growth time preference rate $\varepsilon\cdot g$ (GTPR). In first-best economies either the OCR or the TPR can be used as relevant discount rates. Both rates lead to identical results.

This changes when we investigate more realistic economies. Usually, the OCR is larger than the TPR and there exists a problem, which rate should be used as social discount rate. The choice of one specific discount rate now shows fundamental attitudes towards future effects. Using the higher OCR underevaluates ceteris paribus effects in the remote future. This is avoided employing TPRs instead. Applying the OCR as discount rate can be interpreted as an attempt to prejudice the inefficiency of projects with long-term benefits. Of course, the argument changes when projects with high future costs and "low" current benefits should be evaluated: The usage of the OCR prejudices the efficiency of such kind of projects. However, employing the lower TPR instead tends to avoid prejudications. Thus, a societal decision-maker has to take care of unreflected usages of discount rates which result according to the Ramsey-rule.

2.1 Principles using Discount Rates

Our considerations get clearer when we have a closer look at the economical aggregates that relate to the two different concepts. With respect to the OCR, we argue in marginal capital units which are increasing or decreasing. This leads to additional and/or decreasing future consumption units. Talking about TPRs, we investigate variations of consumption units today. We want to start our analysis with the latter case.

2.1.1 Time Preference Rates ("Prescriptive Approach")

(a) Assumptions using the Time Preference Approach

The time preference approach relates to consumption earlier or later in time. Therefore, we assume that benefit-cost-analyses are carried out in consumption units: Positive consumption effects are taken into consideration as benefits and negative ones as costs, respectively. Environmental improvements for instance are assumed to increase the consumption basis, environmental deterioration decreases it. For correct benefit-cost-analyses, all project-induced effects until the end of the planning horizon have to be identified and to be evaluated. If there is lack of information - as is usual - the societal decision-maker has to try to get better knowledge of these information deficiencies.[6] Present values can be calculated according to the following formula:

$$(3) \qquad PV = -C_0 + \sum_{t=1}^{\infty} \frac{C_t}{(1+\delta)^t},$$

where C_t is given as net consumption in each period t. δ represents the TPR, the planning horizon is infinitely-long, and δ can be split up into the PTPR ρ and the GTPR $\varepsilon \cdot g$.
We conclude that if a project is only connected with consumption variations, we have to use the time preference approach. The usage of the OCR distorts our decision-problem. Projects with positive consumption effects mainly in the remote future are discounted to heavily. However, the TPR consists of two components and we want to have a closer look at the appropriateness of theses components in the intergenerational framework.

(b) Pure Time Preference Rate (PTPR)

The most common theoretical assumption is that an individual values consumptive goods the less, the further in the future consumption takes place. Prospective needs are valued less highly solely because they occur in the future (PTPR ρ). This phenomenon is due to individual "myopia", impatience and other influences. Some authors utilize this rate not only because of myopia and impatience, but also in order to depict remaining life expectancy of individuals.[7]
In economic theory, especially growth and natural resources theory, the PTPR is applied as utility discount rate as well.[8] The main purpose is to ensure the convergence of the utility integral. Usually, it is modeled as an exponential utility discount rate. Each (representative) individual maximizes the sum of the weighted utilities of consumption units during the planning horizon (infinity) with reference to the planning time-period in Ramsey-models. In essence, the PTPR compares the relative importance of consumption units at different time-periods of individuals. Their application as utility discount rate has logically to be separated from the "conventional" TPR-approach, especially because of the subjectivity of utilities. In our further analysis this kind of differentiation does not have to be done because of depicting consumption increases as benefits (utilities) and decreases as costs (disutilities).
Focusing on different individuals, the utility of a special good which is available for future generations is worth less than the same good is worth for today's generations in utility units. Discounting utility now implies an ethical judgment about the position of generations.[9] Future generations are worth less the later they are born. This implicit setting of a norm is inconsistent

[6] We do not concentrate on evaluation mechanisms and problems. These problems are assumed to be solved.
[7] See e.g. *Pearce/Ulph* (1995).
[8] See the usage of ρ in equation (1) and (2).
[9] See e.g. *Solow* (1974).

with the neoclassical efficiency criteria. These criteria guarantee that individuals are ranked equally, because no individual is allowed to be disadvantaged; respectively the sum of the utilities of all individuals has to be maximized. Looking at the Kaldor-Hicks criterion, this valuation is ethically justified in utilitarianism, which is not interested in improving the welfare of special groups, but rather of all affected persons. All human beings are ranked equally: "... utilitari[ani]sm attaches exactly the same importance to the utilities of all people in the objective function, and that feature ... guarantees that everyone's utility gets the same weight in the maximizing exercise."[10]

Valuation of future generations implies that economic theory gives up its neutrality regarding distributional aspects. Economic theory favors today's generations and discriminates against future ones because of distributional reasons. Judgments regarding long-term projects are distorted. There is an innate bias against long-term projects where short-term costs appear and where utilities are feasible mainly in the remote future, for example climate change policy. Benefit-cost-analyses arbitrarily mix statements concerning efficiency and distributional aspects.

Ethical aspects cannot be used to legitimatize intergenerational discounting either. It is neither possible to fall back upon the theory of utilitarianism nor upon the Rawlsian fairness theory in order to justify intergenerational discounting. In the various contract-theoretical concepts following Rawls in environmental ethics - environment as a fundamental liberty[11] or as an economic good, where the difference principle could be applied as a fairness norm[12] - equal treatment of generations is stressed explicitly. Causes of pure time preference are attributed to human impatience and myopia. These phenomena are connected with weakness of will, weakness of imagination, defective telescopic faculty, etc., all of which cannot be ethically accepted as reasons for intergenerational discounting. Well-known authors such as *Hume*, *Ramsey*, *Pigou*, *Harrod*, and *Georgescu-Roegen*[13] reject pure time discounting of future utilities because they regard it as irrational and immoral.

Nobody knows if a PTPR really exists and if so, how high it might be. Various attempts to estimate the PTPR have produced different results and are not comparable with each other because they merge different influences, for instance individual versus societal rates, short-term versus long-term rates, utility- and consumption-oriented rates, PTPRs of industrialized and developing countries etc.[14] In economic models of climate protection, a standard rate of 3% is

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[10] *Sen* (1992), see *Broome* (1992) as well.

[11] See *Singer* (1988).

[12] See *Pearce* (1988).

[13] See e.g. *Price* (1993), pp. 100, and *Georgescu-Roegen* (1979), p. 101.

[14] See for comprehensive overall views *Pearce/Ulph* (1995) and *Price* (1993).

applied.[15] *Pearce/Ulph* (1995) mention further studies in which the PTPR tends to be around 1.5%. However, experimental behavior-theoretical studies partially result in negative PTPRs.[16] In summary, no convincing reasons exist for discounting utilities of human beings only because they are living in the future. The ethical basis, the methodology of neoclassical models and the inherent rationality assumptions forbid the application of an individual PTPR where future generations are concerned. However, if generations are acting myopically for themselves, this has to be taken into account in intergenerational decision-making. Myopic discounting could be necessary for intragenerational present value calculations, but has to be neglected whenever intergenerational comparisons have to be carried out.

(c) Growth Time Preference Rate (GTPR)

An individual growth discount rate can be determined when we have simplifying assumptions about the utility function and the growth of consumption. We want to work with a CES-utility function (constant elasticity of substitution) which is characterized by:

$$(4) \qquad U[C_t] = \frac{C_t^{\,1-\varepsilon}}{1-\varepsilon} \;.$$

C_t symbolizes consumption in period t, and ε is the elasticity of marginal utility with respect to consumption.[17] The discount factor is given by $(1+g)^{\varepsilon \cdot t}$ for constant consumption (real) growth rates g. The term $\varepsilon \cdot g$ is a good approximation for this expression for plausibly small values of g. This shows the equivalence to one component of the Ramsey-rule given in equation (2).

Growth discounting can be utilized in intergenerational comparisons as well. However, three assumptions have to be fulfilled: diminishing marginal utility with respect to consumption when consumption increases, similar (theoretically: identical) utility or welfare functions, and long-term growth. If there is negative growth, we have to discount negatively. Even authors who are critical of intergenerational discounting acknowledge this argument. Discounting now means that a future individual values an extra unit of consumption with a lower marginal utility than a present one only because the future individual is wealthier. The utility function is the same for both of them. If we accept this idea, then growth discounting is only a necessary condition for maximizing utilities intertemporally in neoclassical models. Equal levels of utility are given equal weights, thus, no differences exist between generations. The utilitarian requirement for justice is actually fulfilled, but only in this case. If we did not discount in this situation, we would rank future generations higher than today's generation in case that there is a positive growth rate in the economy. Projects appear to be too beneficial. However, if we carry out benefit-cost-analyses in utility units, consumption discounting is impermissible be-

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[15] See as most prominent and controversial analysis *Nordhaus* (1994).

[16] See *Loewenstein/Prelec* (1991) and the critical remarks in *Bayer* (2000).

[17] Assuming ε=1 the utility function is given as the "logarithmus naturalis" of periodical consumption amounts.

cause all effects of diminishing marginal utility are taken into account in the utility function itself.

We should keep in mind that individual welfare is influenced by both consumption and environmental resources. Despite positive per capita consumption growth rates it is possible that future individuals' welfare is not significantly higher than the present's because the environmental conditions have (drastically) deteriorated. Increases in individual welfare are possibly quite modest or even negative. The development of the growth rate in the very long-term is most uncertain. Neoclassical growth theorists stress unlimited technological progress which guarantees a positive long-term growth rate of per capita consumption. On the other hand, ecological economists are critical of future development because of limits of natural resources, limited substitutions between natural and anthropogenic capital, and the possible endangering of the natural existential basis.

Even reasonable predictions of the growth rate cannot conceal that methodological problems with respect to how to determine utility and how to specify the utility function still exist. Total welfare levels of individuals are not measurable in cardinal units. This is the most important critical point of view concerning the scientific utilization of the growth discount rate. Even attempts to estimate the elasticity of marginal utility of consumption cannot deny the fact that utility is not objectively ascertainable in reality. All of the statements are speculative. It is unknown if and how rapidly utility does increase with rising consumption. Knowing that marginal utility is decreasing is insufficient. It is also impossible for politicians to have information about the utility functions of the citizens and, therefore, they are unable to control the assumptions of the benefit-cost-calculation. Each prediction of consumption growth rates is simultaneously used as reference for determining the level of the growth discount rate. This implies that it is useless to undertake sensitivity calculations with alternatively higher or lower rates. Nobody knows which assumptions are meaningful. If there are no clues about the rate of decrease of the marginal utility, then there hardly is another possibility for researchers other than to ignore the phenomenon of diminishing marginal utility as a source of legitimization for discounting.[18]

In summary, neglecting methodological problems of cardinal utility measurement, a positive time discount rate can be founded on a positive consumption growth rate. Inevitably, this makes it an approximative and subjective, and, therefore, political procedure: The societal decision-maker is most important in these circumstances. He/She has to judge which level of consumption growth rate should be used in intergenerational project evaluations. However, there is hardly another possibility to take into account diminishing marginal utility.

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[18] In this case the requirement for not discounting at all can politically be justified in intergenerational project evaluations.

2.1.2 Opportunity-Cost Rates ("Descriptive Approach")

In contrast to time preference approaches, choosing discount rates according to opportunity-cost-methods concentrates on investment and capital units, respectively. Its implicit assumption is that the project displaces only investment units. All project-induced returns are used to increase investments. Therefore, the relevant discount rate is the rate of return of displaced investments and an equal rate of return is available for all new project-induced additional investment units throughout the whole planning horizon. The formula for present value calculations is then given as

$$(5) \qquad PV = -I_0 + \sum_{t=1}^{\infty} \frac{I_t}{(1+r)^t},$$

where I_t is given as net investment in each period *t*. *r* represents the OCR which is used as discount rate in this approach. The OCR *r* has to be calculated as internal rate of return of the investment. Therefore, a lot of restrictive assumptions have to be employed.[19] However, - according to the case of the growth discount rate - it is better to have rough estimates of the level of the internal rate of return than the complete lack of its value.

Opportunity costs have to be taken into account in all economical approaches. This is valid in intergenerational comparisons as well, of course. But taking opportunity costs into account by discounting is not the best solution. The approach in equation (5) assumes that during the whole planning horizon all investment effects induce the same opportunity costs according to the fixed internal rate of return *r*. It is completely unrealistic that this is the case within a planning horizon of 200 or more years. Another shortcoming is obvious: We cannot assume that only investment effects occur during the planning horizon. In the planning time period there are only investment losses (with a fixed internal rate of return) and during the planning period additional investment opportunities guarantee exactly the same internal rate of return as at the beginning.

A more meaningful approach is to calculate investment effects according to their shadow prices of capital.[20] Investments are undertaken to increase the consumption oriented welfare basis in the future. Investment effects can, therefore, be calculated as amounts which increase the future consumption basis. Investment units have to be translated in consumption ones to make comparisons feasible. This is done by calculating shadow prices of capital or consumption equivalents. They show how much one (displaced or additional) investment unit today is worth in consumption units. A shadow price of capital of 2, for instance, means that one displaced investment unit today is of equal present value as the displacement of two units of consumption. After identifying all investment effects and calculating their respective shadow prices we are able to simply add these values to the periodical consumption units and discount the

[19] See e.g. *Bayer* (2000) and *Price* (1993).

[20] See for some examples *Bayer* (2000), *Bayer/Cansier* (1999), and *Cline* (1992).

resulting aggregate to the planning horizon using the consumption oriented TPR. A great deal of different methods to calculate shadow prices of capital of (displaced/additional) investment units exists. The resulting shadow prices range from very small values (Bradford-approach) to very high or even negative ones (Lind-approach). They are most importantly dependent on the reinvestment requirements during the planning horizon.[21]

2.1.3 Realistic Circumstances

In reality almost every project displaces consumption as well as investment units. Therefore, the pure application of the time preference approach and the opportunity cost approach describes reality insufficiently. Both pure concepts are not realistic. Projects displace consumption as well as investment units in the planning period, (net) induced effects increase/decrease the consumption as well as the investment basis of future periods. The benefit-cost-approach modifies to

$$(6) \qquad PV = -v_0 \cdot I_0 - C_0 + \sum_{t=1}^{\infty} \frac{v_t \cdot I_t + C_t}{(1+\delta_t)^t} ,$$

where δ_t is given as time preference rate and v_t is the shadow price of capital which calculates the consumption equivalent according to their respective internal rate of return (r). Each investment unit increases consumption possibilities in the future (during the planning horizon). Opportunity costs are taken into account by calculating shadow prices of capital of each displaced/additional investment unit. All economical costs and benefits are taken into account, thus, benefit-cost-analyses according to equation (6) depict reality comprehensively. Additionally, the TPR can be used as social discount rate because our approach fully concentrates on consumption units.

2.2 Summary

Neither the "descriptive" nor the "prescreptive" approach could be used in pure form for determining an intergenerational discount rate. Concentrating on consumption effects, a discount rate could be based on the sum of the pure and the growth time preference rate. However, the ethical basis of neoclassical theory demands for equal treatment of each affected individual as well as generation, which leads to the inapplicability of the PTPR when effects of different individuals and generations, respectively, have to be discounted. Intergenerational comparisons require, however, the GTPR to take into account varying project-induced effects between different generations because of diminishing marginal utility. Concentrating on capital effects, each displaced as well as additional investment unit has to be accounted for by using shadow price of capital methods. The resulting shadow prices of capital can then be added to all con-

[21] Further and more detailed discussion and analysis can be found in *Bayer* (2000), *Bayer/Cansier* (1999), *Cline* (1992), and *Lind* (1982).

sumption effects which are directly given in each period. It is then easy to apply our considerations in project appraisal taking the TPR as a systematic discount rate. However, the methodology of the discounting process has not yet been investigated which is done in the following section.

3 Generation Adjusted Discounting (GAD)

Our basic considerations now have to be applied to the intergenerational framework. Therefore, we further want to abstract from neoclassical first-best models where everything is determined on efficient (consumption) paths. We want to concentrate on single projects with intergenerational impacts, for example climate change, storage of radioactive waste, etc. This cannot be planned in a consistent policy due to limited foresight and knowledge.

3.1 Assumptions and Methodology of GAD

Developing the GAD it is reasonable to simplify our basic considerations for its derivation. First of all, we want to concentrate on consumption effects. All investment effects are given in consumption equivalents. The relevant consumption effects result due to the realization of the project. They can be positive or negative. In a first step, we introduce the GAD within a framework of four overlapping generations (G=4), all living for four periods (L=4). We want to assume that each generation can be represented by a representative individual.[22] At each new period the oldest generation dies, all still living generations are getting one year older and a new youngest generation is born. There are constantly four generations alive. We further assume that all consumption effects are distributed equally at each point in time between all living generations. The planning horizon lasts until the period PH. At last, a societal decision-maker has to be installed: He has to account for all consumption effects throughout the whole planning horizon and relate them to generations which are then alive. There is no need for the societal decision-maker to give instructions or suggestions how specific generations should behave. He is not a social planner, but furthermore a societal accounter for project-induced effects.

Our considerations in section two have shown, that - according to the Ramsey-rule - each generation can utilize the total TPR as generation-specific discount rate. All consumption effects within the generation-specific lifetime are then discounted to the beginning of each generations' life. This can be labeled "*intragenerational discounting*" to determine intragenerational project-induced present values of (increased/decreased) consumption. Each generation is free to choose the discount rate according to their preferences. The last column in table 1 ("sum")

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[22] Although it is a restrictive assumption because it refuses intragenerational differences in marginal utilities, this approach is meaningful. Our purpose is to concentrate on intergenerational aspects. For critical remarks to the generation-representative approach see Price in this volume.

shows all generation-specific (intragenerational) project-induced consumption present values. This is the first step in the GAD and is illustrated in the following table 1:

Generation	t_0	t_1	t_2	t_3	t_4	t_5	⋯	t_n	Sum
A	c_0								c_0
B	c_0	$c_1 \cdot \theta^{-1}$							$c_0 + c_1 \cdot \theta^{-1}$
C	c_0	$c_1 \cdot \theta^{-1}$	$c_2 \cdot \theta^{-2}$						$c_0 + c_1 \cdot \theta^{-1} + c_2 \cdot \theta^{-2}$
D	c_0	$c_1 \cdot \theta^{-1}$	$c_2 \cdot \theta^{-2}$	$c_3 \cdot \theta^{-3}$					$c_0 + c_1 \cdot \theta^{-1} + c_2 \cdot \theta^{-2} + c_3 \cdot \theta^{-3}$
E		c_1	$c_2 \cdot \theta^{-1}$	$c_3 \cdot \theta^{-2}$	$c_4 \cdot \theta^{-3}$				$c_1 + c_2 \cdot \theta^{-1} + c_3 \cdot \theta^{-2} + c_4 \cdot \theta^{-3}$
F			c_2	$c_3 \cdot \theta^{-1}$	$c_4 \cdot \theta^{-2}$	$c_5 \cdot \theta^{-3}$	⋯	⋯	$c_2 + c_3 \cdot \theta^{-1} + c_4 \cdot \theta^{-2} + c_5 \cdot \theta^{-3}$
G				c_3	$c_4 \cdot \theta^{-1}$	$c_5 \cdot \theta^{-2}$	⋯	⋯	$c_3 + c_4 \cdot \theta^{-1} + c_5 \cdot \theta^{-2} + c_6 \cdot \theta^{-3}$
H					c_4	$c_5 \cdot \theta^{-1}$	⋯	⋯	$c_4 + c_5 \cdot \theta^{-1} + c_6 \cdot \theta^{-2} + c_7 \cdot \theta^{-3}$
⋮						⋮	⋯	⋯	⋮
Sum		$c_1 \cdot (1+3 \cdot \theta^{-1})$	$c_2 \cdot (1+\theta^{-1}+2 \cdot \theta^{-2})$	$c_3 \cdot (1+\theta^{-1}+\theta^{-2}+\theta^{-3})$	$c_4 \cdot (1+\theta^{-1}+\theta^{-2}+\theta^{-3})$	$c_5 \cdot (1+\theta^{-1}+\theta^{-2}+\theta^{-3})$	⋯	⋯	Σ

Table 1: Intragenerational project-induced consumption effects, $\theta \equiv (1+\delta)$, $\delta \equiv \rho + \varepsilon \cdot g$. All consumption effects are registered with their present values within each generations' life.

In the second step we have to discount *intergenerationally* to calculate present values at the beginning of the planning horizon. Up to generation D solely intragenerational effects appear. We only have to sum up the present values of rows 2 to 5 in the last column. Starting with generation E, which is born after the beginning of the project, we have to discount intergenerationally. Therefore, the intragenerational present value has to be discounted once again to the planning time period t_0. This holds for all later born generations as well. The time periods which have to be discounted intergenerationally are illustrated by the dark shaded area in table 1.

Intragenerationally, it has been stated that each generation is allowed to discount according to the Ramsey-rule due to (expected) consumption growth (and decreasing marginal utility) *and* myopia or impatience. These reasons cannot be applied for intergenerational comparisons. Therefore, we have to discount intergenerationally only with the growth discount rate. The present value share of generation E is given by $(c_1+...+c_4 \cdot \theta^{-3})/(1+\varepsilon \cdot g)^1$. Looking at generation F, the intergenerational discount factor's exponent is two. This holds on into the future until the end of the planning horizon. Using the GAD, the intergenerational discount factor rises exponentially as well, but not as fast as in conventional neoclassical theory because of a smaller intergenerational discount rate.[23]

[23] If benefit-cost-analyses are carried out in utility units, only intragenerational discounting with the pure time preference rate is allowed. A societal planner simply has to sum up all intragenerational present values to determine the efficiency of a specific project.

3.2 General Formula of the GAD

Our basic considerations concerning the GAD assume - for simplification - a constant TPR and, therefore, a constant PTPR and GTPR. Relaxing all restrictive basic assumptions of the GAD, a general formula for present value calculations using the GAD can then be given as follows:

$$PV_{OLG} = \sum_{j=0}^{L-1}(L-j)\cdot\frac{c_j / G_j}{\left(1+\rho_j+\varepsilon_j\cdot g_j\right)^j} + \sum_{\ell=1}^{PH}\frac{\sum_{i=\ell}^{\ell+(L-1)}\frac{c_i / G_i}{\left(1+\rho_i+\varepsilon_i\cdot g_i\right)^{i-\ell}}}{\left(1+\varepsilon_\ell\cdot g_\ell\right)^\ell}, \quad \text{where} \quad c_i, c_j = 0 \quad \text{for all} \quad i, j > PH. \tag{7}$$

PH symbolizes the planning horizon of the analyzed project and *L* represents the life expectancy of each generation. *G* is the number of generations which live simultaneously. All periodical project-induced consumption effects are assumed to be equally distributed between all then living generations. The variables *j*, *i*, and ℓ are used as time indices.

The first term of the sum considers all intragenerational consumption effects which appear in the planning period for all presently-living generations. In analogy to our table 1, we want to assume that climate protection policy cannot be anticipated by the individuals. Therefore, the living generations will value the project differently from those born after the planning period t_0. The longer the planning horizon is assumed, the less important is the first term in equation (7).

The fracture in the numerator (right-hand term of the sum in equation (7)) expresses the intergenerational consumption effects - as seen by the societal decision-maker - of all generations born after the planning period t_0. As these effects are discounted to the beginning of the lives of the respective generation only (intragenerational present value calculations), the intragenerational present value has to be related to the planning period as well in order to evaluate the social profitability as perceived in the planning time-period. This is done by discounting with the GTPR in the denominator of the fraction on the right-hand side of equation (7).

We still have to consider the fact that intergenerational, as well as intragenerational effects which become relevant after the end of the planning horizon, cannot be taken into account in our calculations. Therefore consumption effects c_i and c_j (where $i, j > PH$) have to be explicitly set to zero.

3.3 Numerical Examples in more Realistic Circumstances and Conclusions

We want to assume a lifetime of each generation of forty years (*L*=40). The number of simultaneously living generations is assumed to be 40 (*G*=40) as well. This will sufficiently express the maximum remaining lifetime expectancy of the youngest adult generation world-wide. The generations living in the periods 0 to 39 discount their investment-induced consumption effects directly to the planning time-period t_0. This is amended from generation 40 on. Consumption effects belonging to this generation are discounted to the beginning of their lives (period t_1). In order to correctly analyze benefits and costs, we have to discount the present value at period t_1 to the planning time-period t_0 again by using the growth discount rate. The further generations live in the future, the more distinct is the distance between the birth of any future generation and the planning time-period t_0.

We assume one consumption effect with an amount of 400 in 200 years from now. In the *Ramsey* model, the present value (PV_R) - discounted using a PTPR ρ=3% and a constant growth rate of per capita consumption g=3% (ε=1) - is given by:

$$(8) \qquad PV_R = \frac{c_{200}}{(1+\rho+g)^{200}} = 3.47 \cdot 10^{-3}.$$

Using GAD the present value changes. The consumption amount of 400 in period t_{200} is distributed equally amongst all 40 living generations in period t_{200}. Each generation living in t_{200} receives an amount of 10 consumption units. The effects occurring in the periods exceeding the maximum life expectancy are discounted by just using the growth discount rate. Effects within the individual lifetimes are assumed to be discounted using the growth discount rate as well as the PTPR.

Looking at our example, a PV_{GAD} results as 0.6532. This is about 188 times larger than the Ramsey one. If the consumption effect takes place in t_{300}, the GAD-present value is about 3.319 times larger than the Ramsey one (PV_{GAD}=0.03399; PV_R=1.024·10^{-5}). The difference diminishes if the consumption effect occurs in t_{100}. However, the GAD present value is still about 10.6 times larger than the Ramsey one (PV_{GAD}=12.554; PV_R=1.179).

We want to give another example demonstrating the inadequacy of constant discounting in Ramsey-models. In contrast, the GAD leads to different results using a constant total TPR when the components vary. Therefore, a total TPR of 5% is assumed. Ramsey-models are independent of the structure of the total TPR. It does not matter whether the GTPR is 4% and the PTPR is 1% or vice versa. This is not the case in the GAD as can be seen in the following tables 2 to 4. For simplicity's sake, we assume one consumption effect of 400 at the end of the planning horizon and a constant elasticity of marginal utility with respect to consumption (ε=1).[24]

[24] The differences will multiply when we investigate sequences of consumption effects as it is the case in reality. Tables 2 to 4 only present rough (underestimated) ideas of the discrepancy using different discounting methods.

	δ=0.05; ρ=0.05; *g*=0	δ=0.05; ρ=0.04; *g*=0.01	δ=0.05; ρ=0.03; *g*=0.02	δ=0.05; ρ=0.02; *g*=0.03	δ=0.05; ρ=0.01; *g*=0.04	δ=0.05; ρ=0; *g*=0.05
PV_R	0.0231	0.0231	0.0231	0.0231	0.0231	0.0231
PV_{GAD}	180.17	28.29	4.58	0.763	0.131	0.0231
Difference	7,800	1,225	198	33	5.7	0

Table 2: Comparisons of present values: Constant total TPR and varying components (Planning horizon 200 years, one single effect in the year 200).

	δ=0.05; ρ=0.05; *g*=0	δ=0.05; ρ=0.04; *g*=0.01	δ=0.05; ρ=0.03; *g*=0.02	δ=0.05; ρ=0.02; *g*=0.03	δ=0.05; ρ=0.01; *g*=0.04	δ=0.05; ρ=0; *g*=0.05
PV_R	3.042	3.042	3.042	3.042	3.042	3.042
PV_{GAD}	180.17	76.52	33.16	14.66	6.612	3.042
Difference	59	25	10.9	4.8	2.17	0

Table 3: Comparisons of present values: Constant total TPR and varying components (Planning horizon 100 years, one single effect in the year 100).

	δ=0.05; ρ=0.05; *g*=0	δ=0.05; ρ=0.04; *g*=0.01	δ=0.05; ρ=0.03; *g*=0.02	δ=0.05; ρ=0.02; *g*=0.03	δ=0.05; ρ=0.01; *g*=0.04	δ=0.05; ρ=0; *g*=0.05
PV_R	$1.759 \cdot 10^{-4}$	$1.759 \cdot 10^{-4}$	$1.759 \cdot 10^{-4}$	$1.759 \cdot 10^{-4}$	$1.759 \cdot 10^{-4}$	$1.759 \cdot 10^{-4}$
PV_{GAD}	180.17	10.46	0.63	0.0397	0.00259	$1.759 \cdot 10^{-4}$
Difference	1,024,275	59,466	3,582	226	14.7	0

Table 4: Comparisons of present values: Constant total TPR and varying components (Planning horizon 300 years, one single effect in the year 300).

The GAD-approach takes varying components of the total TPR into account in a more accurate way than in conventional Ramsey approaches. Although the Ramsey present value is the same in all six cases (independent of the length of the planning horizon), the profitability of the project can vary in accordance with the relationship between the GTPR and the PTPR. This is taken into account in the GAD approach more accurately.

Our numerical examples enable us to draw the following conclusions:

1. The difference between GAD- and Ramsey-present values are getting more distinct, the further the planning horizon is extended into the future and the further it exceeds the life-expectancy of the youngest adult generation at the time-period of the realization of the project.
2. The range of differences and the differences themselves are getting larger with increasing planning horizons and smaller with decreasing time distances.

3. The higher the pure time preference ρ is - assuming constant total time preference δ and, therefore, decreasing growth time preference $\varepsilon \cdot g$ -, the more distinct is the difference between the present values in the two concepts.
4. Ramsey-present values can be approached using the GAD (assuming a constant total TPR), when the PTPR decreases towards zero.[25]
5. Ramsey-present value calculations can be utilized as good proxy when, firstly, short-term planning horizons without significant intergenerational effects occur and, secondly, the growth discount rate is "low".

Our considerations will become more realistic if we refuse the constancy of the elasticity of marginal utility with respect to consumption, the constant life-expectancies, and the constant numbers of simultaneously living generations. These assumptions are, therefore, not necessary for utilizing the GAD. We could easily reject them. Once again, this shows that the GAD is more flexible than conventional discounting techniques, especially those in Ramsey-models.

In comparison to the conventional neoclassical growth models (Ramsey- and/or OLG-models) the discounting process is more explicit using the GAD. We do not have to fulfill the optimality conditions which are assumed in neoclassical models. Additionally, we are able to distinguish intergenerational (societal) and intragenerational (individual) discounting. Each generation is allowed to maximize its respective utility throughout their lives according to their preferences. Afterwards, their "welfare" has to be related to the beginning of the planning horizon to judge whether or not a project should be realized. Therefore, the societal decision-maker has to take care that each generation will be ranked equally. This is done by using the growth discount rate when consumption effects are discounted.

The GAD can be used in a more general way than conventional consumption discounting in optimal growth models (Ramsey- as well as OLG-models) based on utilitarian welfare functions. The discounting process employed in these kind of models can be modeled as special cases of the GAD. However, approaching the conventional Ramsey-discounting technique, we have to reject our utilitarian welfare base. This is not the case in OLG-models when we take into account that present values of lifetime utilities of future generations are not discounted once again when present values at the beginning of the planning horizon are calculated. Furthermore we are able to neglect myopic reasons simply by setting the PTPR to zero exogenously.

Thus, the GAD can be interpreted in another way: The first step (intragenerational discounting) describes each generations' individual valuation of project-induced consumption effects.

[25] This is not more than a trend. If we analyze a sequence of consumption effects including effects which are available for generations living at the realization time-period, the Ramsey-present value is always smaller than the GAD one. But the statement still is correct: The difference becomes less distinct for smaller pure time preference rates ρ - assuming constant total time preference rates δ.

They value additional project-induced consumption effects according to their specific welfare situations and are allowed to use high myopic discount rates if they want to. After determining present values of project-induced effects of each generation, they have to be evaluated in an intergenerational context. This has to be done by a societal decision-maker. He has to take into account all generation-specific (intragenerational) present values and discount them back to the planning horizon. The usage of the growth discount rate to calculate present values at the beginning of the project takes into consideration that all generations have to be ranked equally due to the assumed utilitarian welfare function. If there are richer (poorer) generations they have to be made comparable to each other. Assuming positive residual consumption growth[26] for all (most) of the involved generations means that future generations value additional project-induced consumption effects less highly than today's generations would do. Discounting now implies introducing welfare weights according to each generations' welfare level to get a reference basis where comparisons could be made without distortions in favor of any affected generation.

At the end of our considerations concerning the GAD we should have some remarks on the OCR. As mentioned above, opportunity costs are taken into account by calculating shadow prices of capital. The resulting consumption equivalents are subtracted (added) to the consumption basis at the time-period where investments are displaced (enforced). This allows to solely use the TPR-approach for discounting. It does not matter in the GAD whether the internal rate of return of investments is larger than the TPR. The relative importance of productive investments is taken into consideration by determining shadow prices of capital and integrate them into the present value calculations. However, when calculating shadow prices of capital for investments with intergenerational effects, it could be useful to employ the GAD as well. Shadow prices of capital are getting larger in comparison to conventional calculations with constant TPRs due to the differentiation between intra- and intergenerational discounting.[27]

4 Summary and Conclusions

The main purpose of the GAD as a method of intergenerational discounting is to fulfill the inherent equity-conditions using a utilitarian welfare function: Equal treatment of all affected individuals intergenerationally. However, if individuals wish to act in a nonneutral way during their respective lives, they are allowed to discount in an "unfair" way - due to myopia and/or impatience - according to their preferences. Therefore, it is necessary to differentiate between intra- and intergenerational discounting. For intergenerational comparisons, we have to rank

[26] Residual consumption growth means consumption growth taking place in an economy in the absence of our project.

[27] For a more detailed analysis of this aspect see *Bayer* (2000).

each generation equally. This implies refusing discounting due to myopia or impatience. In neoclassical growth models this simple utilitarian-based condition is usually neglected.[28]

The most important further characteristics of the GAD can be summarized as follows:

- Each generations' representative discounts lifetime consumption effects intragenerationally to the beginning of his/her respective life. If the planning time period is earlier in time than the birthdate of generations, a decision-maker has to discount intergenerationally to the beginning of the planning horizon. Our approach differentiates between generation specific intragenerational discounting and equal treatment between different generations as a consequence of the application of neoclassical theory.
- Our approach does not require the general equilibrium assumption like neoclassical OLG-models. We are able to judge projects as they are, without referring to lifetime consumption planning in the past. Therefore, the assumption of perfect foresight is not necessary, which makes our approach more realistic.
- Societal decision-makers are able to take varying growth rates into account when utilizing our approach. The usage of a single discount rate can easily be avoided. This makes our discounting model more powerful in empirical studies than the conventional neoclassical models.
- Our approach is more explicit with respect to discounting than the neoclassical one, where the discounting process is a consequence of the assumed behavior of all affected generations. In particular, market failures in the long-term can be analyzed in a more correct way using our approach than the neoclassical one. The whole discounting process itself is more transparent for intertemporal decision-makers with our approach than with the implicit adaptation mechanism in neoclassical models.
- The "traditional" method of employing a constant discount rate can only be maintained if there is constant (real) growth in all investigated economies throughout the planning horizon. This seems to be very unrealistic and, therefore, it should be possible to relax this assumption. In our model, we have to take predicted (real) growth rates for all economies into account. Thus, the analyses using our discounting model get much better results than traditional benefit-cost-analyses using a constant discount rate in intergenerational comparisons in terms of efficiency as well as in distributional ones.
- Employing the GTPR can be interpreted as introducing distributional weights in the utilitarian framework. Its purpose is to allow comparisons of different effects between different generations: The higher (lower) the welfare level of a respective generation is, the less (more) important are additional consumption units. Reference point is the planning time-period.

[28] See *Barro/Sala-i-Martin* (1995), *Marini/Scaramozzino* (1995).

It is clear that the correct choice of the discount rate in the intergenerational context is most important. The respective rates have to be carefully investigated before being introduced in the impending benefit-cost-analysis. The usage of unreflected (constant) discount rates and discounting procedures represents reality insufficiently. Sensitivity calculations with higher or lower discount rates cannot overcome this shortcoming either: the cardinal problem, namely the choice of the correct discount rate(s) for the project to be realized, cannot be solved by doing so. If the level of the discount rate is contestable, it is impossible to judge whether a measure is (in)efficient. Fixing a discount rate simply to make calculations feasible has to be rejected due to the same reasons, especially if intergenerational effects are to be evaluated.

The GAD fits best in neoclassical benefit-cost-analyses. Intergenerational distributional aspects are taken into consideration, as well as the complete inclusion of all relevant intragenerational utility effects. It is not necessary to perfectly apply this method in reality. Our simple model using the assumption of a finite lifetime of equally concerned generations provides much better results than conventional neoclassical models and is sufficient for empirical benefit-cost-analyses.

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