NETWORK EFFECTS, COMPATIBILITY DECISIONS, AND HORIZONTAL PRODUCT DIFFERENTIATION

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This paper analyzes decisions on (in)compatibility and product design of two competing hardware suppliers in the presence of network effects. We show that they either establish compatibility and differentiate their variants strongly or maintain incompatibility and locate their variants at the center of the consumer distribution. In the latter case, a 'standards war' takes place. Moreover, we show that a commitment to compatibility becomes more attractive for suppliers when it can be done before product designs are fixed because then, it can significantly soften competition in locations. Considering welfare, it turns out that standards wars can be welfare superior to compatibility.

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I. INTRODUCTION

IN this paper we discuss compatibility decisions and choices of location of two competing hardware suppliers in the presence of network effects, i.e. taking into account that a user's surplus from hardware-software systems and from communications systems positively depends on the total number of users of compatible systems. Whereas the working of these network effects is obvious in the case of communication systems such as facsimile systems and e-mail systems, they arise indirectly in the case of hardware-software systems such as VCR systems, CD systems, and videogame systems. There, usually, software components (pre-played video tapes, CDs and video games) are produced with high fixed costs and low constant marginal costs, so that a rise in the total demand for compatible systems increases the variety of software, which in turn increases the surplus of each user.[1] In the following, we restrict ourselves to analyzing those cases in which a move to compatibility requires the consent of both suppliers. The order of the decisions on (in)compatibility and locations is treated as an endogenous variable; i.e, we will derive the profit-maximizing sequence of these commitments. In the last stage of the game, the duopolists compete in prices, and uniformly distributed consumers form their expectations concerning prospective network sizes and each buy one unit of hardware.

The paper presents three new results. First, we will show that suppliers do not always opt for compatibility and differentiated variants, but that there is the possibility that they maintain incompatibility and locate their variants at the center of the consumer distribution. Then, a standards war takes place. Whether this is possible depends on how expectations concerning future network sizes are formed. If these expectations depend first and foremost on prices, a standards war will never happen, because then the suppliers would be forced to set prices equal to marginal cost in order to have a chance of winning. However, when price commitments are not feasible and switching costs are significant, network-size expectations also depend on factors such as small first-mover advantages and successful marketing campaigns; i.e., they can be more or less 'stubborn' (with regard to prices). Then, the supplier who is favored by expectations can skim off parts of the network-effect rent, so that it might be profitable to wage a standards war. Our second main result concerns the profit-maximizing order of (in)compatibility and product-design commitments. We

[1] See Church and Gandal [1992] for a model which discusses the emergence of network effects.

will show that for a medium range of the general significance of the network effects, suppliers will commit themselves to compatibility before fixing locations whenever such an ex-ante commitment to compatibility is feasible, whereas otherwise, they will be involved in a standards war. Here, a commitment to compatibility becomes more attractive for suppliers when it can be done before product designs are fixed. Finally, considering total welfare, we will confirm the intuition that a social planner who can intervene both in the compatibility decisions and in the choices of location would always enforce compatibility. This result, however, cannot serve as a benchmark for policy recommendations because the idea of interventions in the horizontal differentiation is, obviously, unrealistic. Therefore, we use that welfare level as a benchmark which results when the social planner can intervene only in the compatibility decisions. It turns out that against the background of this second-best welfare optimum, standards wars are always welfare superior to compatibility.

As for the case of duopolistic Nash equilibria, our analysis builds on Anderson, Goeree and Ramer [1997] and on Baake [1995]. The former showed that without network effects, duopolists differentiate their variants excessively. We will see that given compatibility, they choose the same locations as in the absence of network effects. Moreover, Baake proved that this result also holds when incompatibility is given.[2] As for the case of standards wars, our central assumptions with regard to network-size expectations formation draw on Farrell and Katz [1998].

Our paper is organized as follows: after the main assumptions have been presented in Section II, we discuss price competition, expectations formation, and demand for given locations and given (in)compatibility in Section III. In Section IV, we present our basic game where it is assumed that the suppliers can commit themselves to compatibility only before they choose their locations. Then, in Section V, we compare the results of this basic game with those of a game with the reverse exogenous order of commitments; from this comparison, it is straightforward to deduce the profit-maximizing order of compatibility and product design decisions. Finally, in Section VI, we present our welfare analysis and derive policy recommendations.

[2]The pioneering work in the analysis of compatibility decisions in the presence of network effects is Katz and Shapiro [1985] where the case of homogeneous network-effect goods is discussed. For an analysis of compatibility decisions within a Hotelling model but with exogenously given locations, see Farrell and Saloner [1992] and Woeckener [1999a]. The case of a vertical product differentiation is examined in Baake and Boom [1997].

II. THE MODEL

We consider a model with two single-product suppliers, S1 and S2, who produce two substitutive variants of the central hardware component of a hardware-software system or of a communications system, V1 and V2, and sell them at prices p_j $(j = 1, 2)$. We do not impose any a-priori restrictions on the suppliers' decisions on product design, i.e., they can locate their variants anywhere on the real line. Whereas formerly, the design of hardware components was dominated by the criterion of technical functionalism, it is nowadays increasingly subject of strategic considerations concerning horizontal and vertical product differentiation. A clear example for a horizontal differentiation is the size of home audio systems. Here, systems of the same quality are offered in a high number of variants which primarily differ in size. More recent examples are the iMac and the vast differentiation of cellular telephones of a given quality. In the following, when the suppliers choose different locations, the variant to the left is called V1, and its address is $d_1 \in R$. With $d_2 \in R$ as the address of V2 (and $d_2 > d_1$), the distance between both variants, i.e. the extent of horizontal product differentiation, amounts to $d_2 - d_1$. We assume constant and equal marginal costs as well as equal fixed costs and normalize both to zero. Moreover, it is assumed that compatibility can only be established if both suppliers opt for it. This may be due to the fact that intellectual property rights are attached to the interface specifications or that product specifications of the competitor's variant which are necessary for establishing compatibility are unknown. The suppliers can credibly commit themselves to compatibility via an enforceable compatibility contract. In the basic version of our game, they can establish compatibility only before they choose their locations; in the second version, they can establish compatibility only after choice of locations; and finally, in the third version, the suppliers can choose whether they decide on (in)compatibility before committing themselves to locations or afterwards. Compatibility causes no extra costs irrespective of whether it is agreed upon before or after product designs are fixed.[3]

As, typically, a hardware variant's inherent product characteristics and the size of its network are poor substitutes, we specify the consumers' surplus as additive in the general willingness to pay for the variant, on the one hand, and that part of the

[3] This assumption is made in order to highlight the strategic reason behind a commitment to compatibility in general and behind the profit-maximizing timing of a commitment to compatibility in the third version of the game.

willingness to pay which is due to the network effects (the 'network-effect rent'), on the other hand. With regard to the consumers' general willingness to pay, we assume that it is uniformly distributed along the unit intervall $]-0.5, 0.5[$. Moreover, it is assumed that the alienation terms ('transportation costs') are quadratic with respect to distance, i.e. that preferences are convex. With $b > 0$ as the basic willingness to pay for a variant, $t > 0$ as a measure of the heterogenity of consumers' preferences, and $-0.5 \leq i \leq 0.5$ as a consumer's address, this general willingness to pay for variant Vj can be formulated as $b - t(i - d_j)^2$. Considering the part of the willingness to pay which is due to the network effects, we assume that it is linear in network size, and that consumers do not differ in their valuation of network size. Let n be the measure of the general significance of the network effects, and let x_j be the demand for Vj, i.e., for given incompatibility, its network size. Then, in the case of incompatibility, the surplus of a consumer with address i when using variant Vj is

$$s_{ij} = b - t(i - d_j)^2 + nx_j - p_j \; . \tag{1}$$

We assume that a sufficiently high basic willingness to pay b guarantees that each consumer buys one (and only one) unit of hardware. Moreover, it is assumed that the constant consumer density amounts to one, so that the total mass of consumers is normalized to one. Then, the absolute demand for a variant (its network size under incompatibility) equals its market share, and $x_2 = 1 - x_1$ holds. Under compatibility, the variants have a joint network of size one, i.e. $s_{ij} = b - t(i - d_j)^2 + n - p_j$ holds.

III. PRICE COMPETITION, EXPECTATIONS FORMATION, AND DEMAND

In this section, we derive the Nash equilibria of the last stage of the game, i.e. for given locations and given (in)compatibility. In the case of given compatibility, the formation of network-size expectations is trivial; then, all consumers will be in a joint network of size one, and everybody knows this fact. In the case of given incompatibility, we restrict ourselves to analyzing Nash equilibria with fulfilled expectations.

The Case of Given Compatibility

As consumers are uniformly distributed along the unit interval $]-0.5, 0.5[$ with a density of one, the demand for variant Vj amounts to $x_j = 0.5 \pm \imath$ where $\imath$ is the address of those consumers who are indifferent between the two variants.[4] Hence, the demand functions can be derived from the condition $s_{i1} = s_{i2}$. Taking into account that $0 \leq x_j \leq 1$ holds, we obtain

$$(2) \quad x_j = \begin{cases} 0 & \text{if} \quad p_j \geq p_k + t(d_2 - d_1)[1 \pm (d_1 + d_2)] \\ 1 & \text{if} \quad p_j \leq p_k - t(d_2 - d_1)[1 \mp (d_1 + d_2)] \\ 0.5 + \dfrac{p_k - p_j}{2t(d_2 - d_1)} \pm \dfrac{d_1 + d_2}{2} & \text{otherwise} \end{cases}$$

with $j, k = 1, 2$ and $j \neq k$. Obviously, the price elasticity of demand is higher (in absolute terms) the lower the extent of product differentiation $d_2 - d_1$ is which results from the suppliers' decisions on locations. Note that $d_1 + d_2 > 0$ (< 0) means that V1 (V2) has a location advantage, because then V1 (V2) is located closer to the center $i = 0$ than V2 (V1), so that its average distance to consumers is lower. In the following, we assume without loss of generality that $d_1 + d_2 \geq 0$ holds, i.e. if a location advantage exists, it is an advantage of V1.[5] With given compatibility, the existence of network effects only leads to a higher address-independent part of the willingnesses to pay ($b + n$ instead of b for both variants). Obviously, such an increase by the same amount cannot affect the competition between the variants. Thus, equilibrium prices and profits are the same as in the Hotelling model without network effects. Maximizing $\Pi_j = p_j x_j$ with x_j according to Equation (2) with respect to p_j leads via the best-response functions $p_j = 0.5\{p_k + t(d_2 - d_1)[1 \pm (d_1 + d_2)]\}$ to[6]

$$(3) \qquad p_j^{c,\ell} = t(d_2 - d_1)\left(1 \pm \frac{d_1 + d_2}{3}\right),$$

where c denotes given compatibility and ℓ denotes given locations. Substituting the price difference $p_k^{c,\ell} - p_j^{c,\ell}$ into Equation (2) leads to

$$(4) \qquad x_j^{c,\ell} = 0.5 \pm \frac{d_1 + d_2}{6},$$

[4] In the following, $\pm$ and $\mp$ means that the upper sign holds for supplier S1 and the lower sign for supplier S2.

[5] Then, for $d_1 + d_2 > 0$, there are two cases: either V1 lies to the left and V2 to the right of the center and $-d_1 < d_2$ holds, or both variants lie to the right of the center and $d_1 < d_2$ holds. Of course, the natural candidates for an equilibrium of our symmetric model are locations which are symmetric relative to the center, so that $d_1 + d_2 = 0$ holds (i.e. $-d_1 = d_2$ with $d_2 > 0$ for differentiated variants and $d_j = 0$ for homogeneous variants).

[6] The second-order condition reads $-1/[t(d_2 - d_1)] < 0$, and this is always fulfilled.

and by multiplication with equilibrium prices, we obtain equilibrium profits as

$$(5) \qquad \Pi_j^{c,\ell} = 0.5t(d_2 - d_1)\left(1 \pm \frac{d_1 + d_2}{3}\right)^2 .$$

From Equations (3) and (4), it becomes clear that if S1 could gain a location advantage, he would subsequently have both the higher price and the higher market share. Hence, it would appear that S2 will never accept such a disadvantage, so that we can presume that only Nash equilibria which are symmetric in locations will be of relevance in the overall game.[7] From Equation (3), it also becomes clear that the existence of a Nash equilibrium requires $d_1 + d_2 < 3$ to hold. This can be taken for granted, because otherwise S2 could always realize strictly positive profits by re-locating his variant into the support of the consumer distribution.

The Case of Given Incompatibility

For given incompatibility, there are two basic cases; while in the case of a dominating product differentiation $d_2 - d_1 > n/t$, both variants have positive market shares, dominating network effects $d_2 - d_1 < n/t$ turn the market into a natural monopoly.

• *In the case of a dominating product differentiation* $d_2 - d_1 > n/t$, equating s_{i1} with s_{i2}, assuming fulfilled expectations and using $x_j = 0.5 \pm \imath$ results in

$$(6) \quad x_j = \begin{cases} 0 & \text{if} \quad p_j \geq p_k + t(d_2 - d_1)[1 \pm (d_1 + d_2)] - n \\ 1 & \text{if} \quad p_j \leq p_k - t(d_2 - d_1)[1 \mp (d_1 + d_2)] + n \\ 0.5 + \dfrac{p_k - p_j \pm t(d_2 - d_1)(d_1 + d_2)}{2[\,t(d_2 - d_1) - n\,]} & \text{otherwise} . \end{cases}$$

By comparing the denominators, it becomes clear that the price elasticity of demand is higher under incompatibility than under compatibility. This is due to bandwagon effects, which are induced by the network effects whenever systems are incompatible. Hence, price competition is more intensive under incompatibility, so that duopoly prices and profits are lower than in the case of given compatibility. It is straightforward to prove the following lemma:

[7] However, we need the general formulas for some out-of-equilibrium considerations later on.

Lemma 1a. With given locations and given incompatibility (*in*), a dominating product differentiation $d_2 - d_1 > n/t$ leads to an incompatible duopoly with equilibrium prices, market shares, and individual profits

$$p_j^{in,\ell} = t(d_2 - d_1)\left(1 \pm \frac{d_1 + d_2}{3}\right) - n\,, \tag{7}$$

$$x_j^{in,\ell} = 0.5 \pm \frac{t(d_2 - d_1)(d_1 + d_2)}{6[\,t(d_2 - d_1) - n\,]} = \frac{p_j^{in,\ell}}{2[\,t(d_2 - d_1) - n\,]} \quad \text{and} \tag{8}$$

$$\Pi_j^{in,\ell} = \frac{\left[t(d_2 - d_1)\left(1 \pm \dfrac{d_1 + d_2}{3}\right) - n\right]^2}{2[\,t(d_2 - d_1) - n\,]}\,, \tag{9}$$

respectively.

Proof. Maximizing $\Pi_j = p_j x_j$ with x_j according to Equation (6) with respect to p_j leads via the best-response functions $p_j = 0.5\{p_k + t(d_2 - d_1)[1 \pm (d_1 + d_2)] - n\}$ to equilibrium prices according to Equation (7). The second-order condition reads $-1/[t(d_2 - d_1) - n] < 0$, and this is (only) fulfilled when the product differentiation dominates. Substituting $p_k^{in,\ell} - p_j^{in,\ell}$ into Equation (6) leads to equilibrium market shares, and subsequent multiplication results in equilibrium profits. The existence of a duopolistic Nash equilibrium requires $(d_2 - d_1)[1 - (d_1 + d_2)/3] > n/t$ to hold. Insofar as $d_2 - d_1 > n/t$ holds, this (again) can be taken for granted, because otherwise S2 could always realize strictly positive profits by re-locating his variant. □

As in the case of given compatibility, we can presume that only Nash equilibria with $d_1 + d_2 = 0$ are of relevance in the overall game.

• *In the case of dominating network effects* $d_2 - d_1 < n/t$, the inner branch of Equation (6) is upward sloping and irrelevant because there, the second-order condition of profit maximization is not fulfilled (see the above proof). In this case, the relatively high general significance of the network effects turns the market into a natural monopoly. Given fulfilled expectations, only $x_1 = 1$ and $x_1 = 0$ can be (stable) Nash equilibria.[8] As this is (assumed to be) common knowledge, the only locations of relevance are

$$d_j^{in,*} = 0\,. \tag{10}$$

[8] With symmetric locations, the splitting of consumers into two networks ($x_j = 0.5$) is an equilibrium with fulfilled expectations, too; but it is unstable under restricted best-response dynamics.

Any other location can be ruled out a priori, because not locating one's variant at the center of the consumer distribution means accepting a product disadvantage and, thus, offering the competitor the opportunity of monopolizing the market via limit pricing. Hence, given incompatibility and dominating network effects, a standards war between homogeneous variants occurs. Whether the suppliers wage such a standards war when they have the alternative of establishing compatibility decisively depends on the process of expectations formation.[9] With regard to this process (and with homogeneous variants), the conventional hypothesis probably is that network-size expectations exclusively depend on prices. In this case, suppliers would be forced to set prices equal to marginal costs in order to have a chance of winning. Then, obviously, given the alternative of a move to compatibility, the suppliers will never wage a standards war. This hypothesis, however, ignores the fact that in reality, a standards war is a dynamic process where price commitments are rare and the switch to a competing system usually leads to considerable tangible and intangible switching costs. Taking into account this fact, it seems obvious that in the case of a standards war, network-size expectations can be more or less stubborn (with regard to prices) and are influenced by a lot of other factors such as marketing efforts and product preannouncements. In the following, these factors are modelled as exogenous shocks which come into play in stage three, i.e. when (in)compatibility and locations are fixed. Whereas these shocks cancel out during a duopolistic competition, they are decisive for the outcome of a standards war. This is due to the fact that in the latter case (and given that a price commitment is not feasible), an exogenous shock can break the symmetry (in locations) and make one of the two pareto-equivalent equilibria focal.[10] We assume, for example, that during the market introduction one of the suppliers can achieve a small (but decisive) first-mover advantage or that he can convince consumers that important software producers will develop software only for his variant. Moreover, we assume that a priori nothing can be said about which supplier will be the lucky one who gets the decisive lead. Therefore, when a supplier opts for a standards war, he knows that he has a fifty-per-cent chance of winning.

The most important implication of the fact that expectations are not influenced by prices alone is that the supplier of the (more or less stubbornly) favored variant

[9] For the following, see Farrell and Katz [1998], pp. 616ff.

[10] As for the duopoly case (where Nash equilibria are always unique), note that it would be more precise to denote the equilibrium values as expected values. However, in order to avoid notational clutter, we have refrained from doing so.

can skim off parts of the network-effect rent. This is due to the fact that then an expected advantage in network size is equivalent to a quality advantage and enables the supplier who is favored by (more or less) stubborn expectations to monopolize the market with a strictly positive limit price. A clear example is the case of totally stubborn expectations. Assume, for example, that during the market introduction supplier S2 initially is so unlucky with getting software support that everybody is convinced that his competitor will be the winner. Then, consumers compare $s_{i1} = b - t(i - d_1)^2 + n - p_1$ with $s_{i2} = b - t(i - d_2)^2 - p_2$, so that S1 can monopolize the market via a limit price of n. In reality, how much of the network-effect rent n the winner can appropriate depends on the concrete circumstances. For the following, we assume that he can set a limit price of qn where q is a random variable which is uniformly distributed along the unit interval $]0, 1[$ and reflects the degree of stubborness of expectations. Hence, the profits which a supplier expects given that he will be the winner amount to $0.5n$, whereas the profits which a supplier can a priori expect when he wages a standards war amount to $0.25n$. In the following, the former are denoted as $\Pi_j^{in,*,e}$ and the latter as $\Pi_j^{in,*,exp}$. To sum up, we can state

Lemma 1b. With given incompatibility, dominating network effects lead to a standards war with (a priori) expected individual profits of

$$\Pi_j^{in,*,exp} = 0.25n \; . \tag{11}$$

In Lemma 1b, we have anticipated that given a natural monopoly, always $d_j = 0$ holds. However, in order to examine for which parameter constellations a standards war is a Nash equilibrium of the overall game later on, we have to make an assumption about what will happen in the case of a location advantage of supplier S1. Here, there are two cases; with a significant product advantage, $x_1 = 1$ is the unique equilibrium, whereas with a small product advantage, $x_1 = 0$ is an equilibrium, too. As for the latter case, it seems natural to assume that a product advantage has a dominating influence on expectations so that S1 is the winner with certainty (given that he sets a limit price), because we see product-design decisions as credible commitments. This assumption is supported by the fact that in this case, $x_1 = 1$ is the pareto-superior equilibrium (cumulated alienation effects are lower than in $x_1 = 0$).[11] From the surplus equations, we obtain the limit price and the expected

[11] Moreover, note that the importance of the exogenous shocks in symmetric cases (and when a credible price commitment is not feasible) stems from the fact that consumers use them as a

profits of the supplier with the product advantage as

$$\Pi_1^{in,\ell,e} \;=\; 0.5n \;+\; t(d_2 - d_1)(d_1 + d_2 - 1)\ . \tag{12}$$

IV. COMPATIBILITY DECISION BEFORE CHOICE OF LOCATION

In this section, we assume that suppliers can establish compatibility only before they noncooperatively and simultaneously choose their locations. If compatibility is not established, this is a commitment to incompatibility. The analysis in this section is not only a preliminary step for the derivation of the profit-maximizing order of the decisions on (in)compatibility and product design in Section V but, in addition, it is for some markets of direct relevance. In the case of home entertainment systems such as VCR systems, CD systems, and videogame systems, for example, compatibility cannot be achieved with reasonable costs once the hardware components have been designed, i.e. by means of an adapter or converter. This is due to the fact that in these cases, even small differences in the chosen technology lead to interfaces which are too variant-specific.[12]

Choice of Location with Given Compatibility

From the standard Hotelling model, it is well known that there are two opposing effects of location on individual profits. On the one hand, for given prices, a move towards the center results in higher profits because it reduces the average distance to consumers. On the other hand, such a move leads to more intensive price competition, lowering profits. Anderson, Goeree and Ramer [1997] showed that without network effects, these two effects are of equal amount for $d_1 = -0.75$ and $d_2 = 0.75$ (see p. 125). These profit-maximizing locations are outside the support of the consumer distribution. In the case of compatibility, this result holds in the presence of

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coordination device, whereas with asymmetric locations, the product advantage is a by far more reliable coordination device. Of course, there might be cases where a big shock overcompensates a small product advantage. Nevertheless, in the sequel, we do not allow for this case. Allowing for it would only change the concrete value of the borderline between duopolistic Nash equilibria and standards wars (see the second step of the proof of Lemma 2 and the first step of the proof of Lemma 3).

[12] The competition between VHS and Betamax VCR systems as well as the competition between Nintendo 64 and Sony's Playstation are clear examples.

network effects. Differentiating profits according to Equation (5) with respect to d_j leads to the best-response functions $d_j = d_k/3 \mp 1$, and these have a unique point of intersection:[13]

$$d_j^{c,*} = \mp 0.75 \; . \tag{13}$$

Substituting this result into Equations (3) to (5), we obtain $x_j^{c,*} = 0.5$ and

$$\Pi_j^{c,*} = 0.5 p_j^{c,*} = 0.75t \; . \tag{14}$$

As for realized welfare, the individual equilibrium surplus from Vj amounts to $b - t(i \pm 0.75)^2 + n - 1.5t$. Via integration, we obtain the cumulated equilibrium surplus as

$$S^{c,*} = b + n - \frac{85t}{48} \; . \tag{15}$$

Finally, adding equilibrium profits results in a total realized welfare of

$$W^{c,*} = b + n - \frac{13t}{48} \; . \tag{16}$$

Choice of Location with Given Incompatibility

As for the case of given incompatibility, we have seen in the previous section that the location decisions of suppliers determine whether the market is a duopoly or a natural monopoly. Baake [1995] showed that given a duopoly, the profit-maximizing locations are the same as under compatibility (pp. 9f). Moreover, we know that given a natural monopoly, the suppliers are forced to locate their variants at the center of the consumer distribution in order to prevent exclusion. The main question is under what circumstances the suppliers get involved in a standards war and under what circumstances they differentiate their variants so strongly that they can coexist. We can prove the following lemma:

Lemma 2. Given that the suppliers have maintained incompatibility in the first stage of the game,

- they differentiate their variants strongly in the case of $n/t < 0.6875$; then

$$d_j^{in,*} = \mp 0.75 \tag{17}$$

holds. This leads to a duopoly with $x_j^{in,*} = 0.5$ and

$$\Pi_j^{in,*} = 0.5 p_j^{in,*} = 0.75t - 0.5n \; . \tag{18}$$

[13] The second-order conditions read $-(t/9)[6 \pm (3d_j + d_k)] < 0$, and this is fulfilled for $d_j = \mp 0.75$.

• they locate their variants at the center of the consumer distribution in the case of $n/t > 0.6875$; i.e. then $d_j^{in,*} = 0$ holds, and a standards war with expected profits according to Equation (11) takes place.

The proof consists of three steps. In the first step, we give a more direct proof of Equation (17) than is given in Baake [1995]. In the second step, we show that this strong product differentiation is a Nash equilibrium for $n/t < 0.6875$. Finally, the third step proves that locating the variants at the center is a Nash equilibrium for $n/t > 0.34$. Hence, for $0.34 < n/t < 0.6875$, both Nash equilibria coexist. In this case, it seems natural to assume that the Nash equilibrium which leads to higher profits is focal. Comparing duopoly profits according to Equation (18) with the profits a supplier expects from a standards war ($0.25n$), it is obvious that the duopoly equilibrium is always focal.

Proof. (a) Differentiating Equation (9) with respect to d_j leads to the first-order conditions

$$\frac{\partial \Pi_j^{in,\ell}}{\partial d_j} = \mp t\left(1 \pm \frac{2d_j}{3}\right) x_j^{in,\ell} + p_j^{in,\ell}\, \frac{-2td_j[\,t(d_2-d_1)-n\,] + t^2(d_2^2-d_1^2)}{6[\,t(d_2-d_1)-n\,]^2} = 0\ .$$

Taking into account that $\partial x_2^{in,\ell}/\partial d_2 = -\partial x_1^{in,\ell}/\partial d_2$ as well as $p_j^{in,\ell} = 2[t(d_2-d_1) - n]x_j^{in,\ell}$ hold, we can formulate these conditions as

$$\frac{\partial \Pi_j^{in,\ell}}{\partial d_j} = \mp t\left(1 \pm \frac{2d_j}{3}\right) x_j^{in,\ell} \pm 2[\,t(d_2-d_1)-n\,]\, x_j^{in,\ell}\, \frac{\partial x_1^{in,\ell}}{\partial d_j} = 0\ .$$

Dividing by $x_j^{in,\ell}$ and using $\partial x_1^{in,\ell}/\partial d_1 = \partial x_1^{in,\ell}/\partial d_2$, it becomes clear that there is a unique solution with $\partial p_1^{in,\ell}/\partial d_1 = \partial p_2^{in,\ell}/\partial d_2$, i.e. with $-d_1 = d_2$. Substituting this back into the first-order conditions leads to $d_j = \mp 0.75$. Evaluated at this point, the second-order conditions reduce to $-2t(1.5t - n) < 0$. This is always fulfilled insofar as the product differentiation dominates. Finally, substituting $d_j = \mp 0.75$ into the profit function (9) leads to Equation (18).

(b) In this step, let us assume that initially, the variants are located at $d_1 = -0.75$ and $d_2 = 0.75$, respectively, and that one of the two suppliers, say S1, can deviate from his location in order to monopolize the market. Maximizing his expected monopoly profits according to Equation (12) with respect to d_1 (for given $d_2 = 0.75$) leads to $d_1 = 0.5$; i.e., S1 would locate his variant at the right boundary of the consumer distribution. Then, his expected profits would amount to $0.5n + 0.0625t$. Comparing these profits with the duopoly profits of $0.75t - 0.5n$ makes clear that

a deviation from $d_1 = -0.75$ in order to monopolize the market pays off for $n/t > 0.6875$, whereas for $n/t < 0.6875$, choosing $d_j = \mp 0.75$ is a Nash equilibrium.
(c) In this step, let us assume that initially, the variants are located at $d_j = 0$, and that one of the two suppliers, say S2, can deviate from there in order to prevent a standards war. From Equation (9), we obtain the profit-maximizing location

$$d_2 = 0.5 + \frac{2n}{3t} + \sqrt{\left(0.5 + \frac{2n}{3t}\right)^2 - \frac{n}{t}} \, .$$

Here, for $n/t < 0.75$, the discriminant is always positive and $\partial d_2/\partial n > 0$ holds (for $n/t > 0.75$, duopoly profits cannot be positive). A numerical evaluation of Equation (9) with $d_1 = 0$ and d_2 as noted above makes clear that deviating in order to prevent a standards war pays off for $n/t < 0.34$. Hence, for $n/t > 0.34$, waging a standards war is a Nash equilibrium. (However, as shown above, it is neither unique nor focal for $n/t < 0.6875$.) □

Considering realized welfare, the calculation of total welfare is straightforward in the case of a duopoly; it consists of the cumulated basic willingness to pay b, cumulated network effects of $0.5n$, and the cumulated alienation effects. The latter are equal to those under compatibility, i.e. they amount to $-13t/48$. Hence, we obtain

(19) $$W^{in,*} = b + 0.5n - \frac{13t}{48} \, .$$

Substracting total profits of $1.5t - n$ results in a cumulated consumers' surplus of

(20) $$S^{in,*} = b + 1.5n - \frac{85t}{48} \, .$$

In the case of a standards war, cumulated alienation effects amount to $-2t \int_0^{0.5} i^2 di = -t/12$; i.e., they are lower (in absolute terms) than in a duopoly. Moreover, now the cumulated network effects amount to n, so that we obtain

(21) $$W^{in,*} = b + n - \frac{t}{12} \, .$$

The distribution of the network-effect rent n depends on the realization of the random variable q; the expected value of the monopoly profit reads $\Pi_j^{in,*,e} = 0.5n$ (with $j = 1$ *or* $j = 2$) and, thus, we obtain for the expected value of consumers' surplus

(22) $$S^{in,*,e} = b + 0.5n - \frac{t}{12} \, .$$

Compatibility Decision

The decision on (in)compatibility is made by comparing profits under compatibility $\Pi_j^{c,*} = 0.75t$ (Equation [14]) with profits under incompatibility $\Pi_j^{in,*} = 0.75t - 0.5n$ (Equation [18]) for $n/t < 0.6875$ and with the expected profits of a standards war $\Pi_j^{in,*,exp} = 0.25n$ (Equation [11]) for $n/t > 0.6875$. For $n/t < 0.6875$, the decision on (in)compatibility is a decision on whether to compete within a compatible duopoly or within an incompatible duopoly (both with $d_j = \mp 0.75$). Obviously, the former is always profit maximizing. For $n/t > 0.6875$, the decision on (in)compatibility is a decision on whether to compete within a compatible duopoly (with $d_j = \mp 0.75$) or for the market (with $d_j = 0$). Here, the former is profit maximizing for $n/t < 3$, whereas the latter leads to higher expected profits whenever the general significance of the network effects is (relatively) high, i.e. for $n/t > 3$. Thus, for $n/t < 3$, profits, consumers' surplus, and total welfare are according to Equations (14) to (16), whereas in the case of a high general significance of the network effects, expected profits, total welfare, and expected consumers' surplus are according to Equations (11), (21), and (22), respectively. Hence, to sum up, we can state:

Proposition 1. Given that the suppliers can commit themselves to compatibility only before they choose their locations,

• they establish compatibility whenever the general significance of the network effects is not high ($n/t < 3$). Then, they differentiate their variants strongly ($d_j^{*,*} = \mp 0.75$), and this leads to a symmetric compatible duopoly with individual profits $\Pi_j^{*,*} = 0.75t$, consumers' surplus $S^{*,*} = b + n - 85t/48$, and total welfare $W^{*,*} = b + n - 13t/48$.

• they maintain incompatibility whenever the general significance of the network effects is high ($n/t > 3$). Then, they locate their variants at the center ($d_j^{*,*} = 0$), i.e. a standards war takes place with expected individual profits $\Pi_j^{*,*,exp} = 0.25n$, expected consumers' surplus $S^{*,*,e} = b + 0.5n - t/12$, and total welfare $W^{*,*} = b + n - t/12$.

For $n/t < 0.6875$, compatibility leads to higher profits because the price elasticity of demand is higher in an incompatible duopoly than in a compatible duopoly, so that the move to compatibility softens price competition and results in higher prices. For $0.6875 < n/t < 3$, the duopoly price under compatibility amounts to $1.5t$ and, thus, is always higher than the expected limit price in a standards war (which amounts to $0.5n$). In a standards war, the monopolist has double the market share than in a duopoly, but this is offset by the uncertainty about who will be the monopolist. Hence, in the decision on whether to compete within a compatible duopoly or wage a standards war, only prices matter, so that $n/t < 3$ leads to the former and $n/t > 3$ to the latter.

As for total welfare, with $n/t < 0.6875$, the move to compatibility has the advantage of realizing additional network effects, whereas for $n/t > 0.6875$, consumers are in a joint network both under compatibility and under incompatibility. Moreover, in the case of a standards war, cumulated alienation effects are lower (in absolute terms) than under compatibility due to the fact that the monopolist locates his variant at the center. Hence, realized welfare is always higher in a standards war.

Consumers (as a whole) always suffer from the move to compatibility for $n/t < 3$ and almost always suffer from the monopolization for $n/t > 3$. With regard to the case of $n/t < 0.6875$, comparing cumulated consumers' surplus under compatibility (Equation [15]) with cumulated consumers' surplus in an incompatible duopoly (Equation [20]) makes clear that the compatibility advantage of having a joint network is overcompensated by higher prices. For $0.6875 < n/t < 3$, compatibility has no network-size advantage but both the disadvantage of a price which is higher than the expected value of the limit price in a standards war and the disadvantage of higher cumulated alienation effects. Finally, for $n/t > 3$, the price under compatibility would be lower than the expected value of the limit price, but cumulated alienation effects are lower (in absolute terms) in a standards war. Comparing $S^{c,*}$ with $S^{in,*,e}$ according to Equation (22) makes clear that compatibility would lead to a higher cumulated consumers' surplus for $n/t > 3.375$. Hence, except for the parameter range $3 < n/t < 3.375$, the interests of suppliers and consumers (as a whole) with regard to (in)compatibility are always conflicting.

V. ENDOGENOUS ORDER OF THE DECISIONS ON COMPATIBILITY AND LOCATIONS

In this section, we derive the profit-maximizing order of the commitments to (in)compatibility and locations. In the first subsection, we assume that suppliers can commit themselves to compatibility only after they have chosen locations. Comparing the results of this game with those of the previous section, it is straightforward to deduce the outcome of the game where suppliers can choose whether they decide on (in)compatibility before or after they choose locations (in the second subsection).

Obviously, the fact that $d_j = \mp 0.75$ are the optimal locations for given compatibility and that either $d_j = \mp 0.75$ or $d_j = 0$ are the optimal locations for given incompatibility is independent of the sequence of decisions. Moreover, we have seen that for $d_j = \mp 0.75$, compatibility leads to higher profits than incompatibility. Hence, as for the second stage of this game, if suppliers have chosen $d_j = \mp 0.75$ in the first stage, they opt for compatibility, whereas if they have chosen $d_j = 0$ in the first stage, they maintain incompatibility.[14] However, as becomes clear from the analysis of the first stage, the borderline between the parameter regimes of these two cases *is* affected by the order of commitments. We can prove the following lemma:

Lemma 3. Given that the suppliers can commit themselves to compatibility only after they have chosen their locations,

- they differentiate their variants strongly ($d_j^{*,*} = \mp 0.75$) and establish compatibility for $n/t < 1.375$, i.e. whenever the general significance of the network effects is (relatively) low. In this case, a symmetric compatible duopoly comes about with individual profits, consumers' surplus, and total welfare as stated in the first part of Proposition 1.
- they locate their variants at the center ($d_j^{*,*} = 0$) and maintain incompatibility for $n/t > 1.375$. This is the case of a standards war with expected individual profits, expected consumers' surplus, and total welfare as stated in the second part of Proposition 1.

The proof consists of two steps which are analogous to the second and third step of the proof of Lemma 2. In the first step, we prove that choosing $d_j = \mp 0.75$ (and subsequently opting for compatibility) is a Nash equilibrium whenever the general significance of the network effects is low, i.e. for $n/t < 1.375$. In the second step, it is shown that locating the variants at the center of the consumer distribution (and subsequently opting for incompatibility) is a Nash equilibrium for $n/t > 0.\bar{8}$. Thus, for $0.\bar{8} < n/t < 1.375$, both kinds of equilibria coexist. Here, again, we assume that the equilibrium is focal which leads to higher profits. Comparing duopoly profits with the expected profits from a standards war, it is obvious that the duopoly equilibrium is always focal.

[14] In particular, they never opt for incompatibility in the second stage when they have chosen $d_j = \mp 0.75$ in the first stage. All asymmetric locations can be ruled out by analogous reasoning as in the previous sections; see Woeckener [1999b] for a more detailed proof.

Proof. (a) In this step, let us assume that initially, the variants are located at $d_1 = -0.75$ and $d_2 = 0.75$, respectively, and that supplier S1 can deviate from his location in order to monopolize the market. Maximizing expected monopoly profits with respect to d_1 shows that if he deviated from $d_1 = -0.75$, S1 would locate his variant at the right boundary of the consumer distribution and would expect profits of $0.5n + 0.0625t$: see Step (b) of the proof of Lemma 2. Now, however, he compares these expected profits with profits in a compatible duopoly ($0.75t$). Hence, a deviation from $d_1 = -0.75$ (and subsequent monopolization) pays off for $n/t > 1.375$, whereas for a low general significance of the network effects, choosing $d_j = \mp 0.75$ is a Nash equilibrium.
(b) In this step, let us assume that initially, the variants are located at $d_j = 0$, and that S2 can deviate from there in order to enforce a duopoly. If he deviated, he would always prefer a compatible to an incompatible duopoly because in the latter case, both his price and his market share would be lower. This becomes clear from comparing Equations (3) and (4) with Equations (7) and (8), respectively. In case of deviating and enforcing a compatible duopoly, S2 would make profits of $\Pi_2^{c,\ell} = 0.5td_2(1 - d_2/3)^2$ (see Equation [5] with $d_1 = 0$). Maximizing these profits with respect to d_2 shows that S2 would choose $d_2 = 1$ and would make profits of $0.\bar{2}t$ (instead of $\Pi_2^{in,*,exp} = 0.25n$). Thus, deviating from the center and enforcing a compatible duopoly pays off for $n/t < 0.\bar{8}$. It is straightforward to show that for this parameter regime (and given $d_1 = 0$ and $d_2 = 1$), S1 would not block compatibility, because profits under incompatibility would be lower (see Equations [5], [9], and [12]). Hence, for $n/t < 0.\bar{8}$, S2 indeed deviates and enforces a compatible duopoly, whereas for $n/t > 0.\bar{8}$, choosing $d_j = 0$ (and subsequently opting for incompatibility and waging a standards war) is a Nash equilibrium. (However, as shown above, it is neither unique nor focal for $n/t < 1.375$.) □

Compatibility Decision before or after Choice of Location

From Proposition 1 and Lemma 3, it becomes clear that the market outcome does not depend on the sequence of commitments for $n/t < 1.375$ (in either case compatibility and strong product differentiation) and for $n/t > 3$ (in either case incompatibility and homogeneous variants). However, if the general significance of the network effects is neither low nor high, suppliers will establish compatibility whenever a commitment to compatibility is feasible only before the choice of location, but will maintain incompatibility and wage a standards war whenever a commitment to compatibility is feasible only after the choice of location. In the latter case, they expect profits of $0.25n$, whereas under compatibility, profits amount to $0.75t$. Hence, if the suppliers can decide on the order of commitments, they prefer to commit themselves to compatibility before the choice of location for $1.375 < n/t < 3$ in order to prevent a standards war. Obviously, if the general significance of the network effects is neither low nor high, the ability of suppliers to commit themselves to compatibility ex ante significantly softens competition in locations. To sum up, we can state:

Proposition 2. If the general significance of the network effects is neither low nor high ($1.375 < n/t < 3$), the market outcome depends on the order of the commitments on (in)compatibility and product design. Suppliers who can choose whether they decide on (in)compatibility before they choose their locations or afterwards establish compatibility before fixing product designs and, in this way, prevent a standards war.

Or in other words: a commitment to compatibility becomes more attractive for suppliers when it can be done before product designs are fixed. Considering the general robustness of the market outcomes towards more realistic consumer distributions, it is straightforward to show that for symmetric unimodal densities on $]-0.5, 0.5[$, a standards war becomes more probable the more concentrated the consumer distribution is. Whereas expected profits from a standards war do not depend on the shape of the consumer density function (but only on its width), profits under compatibility are lower the higher the density is at its median $i = 0$ (see Anderson, Goeree and Ramer [1997], p. 116).

VI. WELFARE ANALYSIS AND POLICY IMPLICATIONS

A social planner who could effectively intervene in both the choices of locations and the compatibility decisions would always enforce compatibility and locations $d_j = \mp 0.25$, because the former maximizes cumulated network effects and the latter minimizes cumulated alienation effects.[15] However, an intervention in the horizontal differentiation of hardware components is a quite unrealistic idea, and we are not aware of any law or regulation that could enable such an intervention. Hence, considering policy recommendations, this welfare-theoretical first-best optimum is

[15] As for price formation, there is no reason to intervene. Duopoly prices are of equal amount, and this is – due to the symmetry of the model's set-up – welfare optimal.

irrelevant. In order to derive practicable policy recommendations, we have to take into consideration only interventions in the compatibility decisions.[16] That means that we have to compare welfare in Nash equilibria for given compatibility with welfare in Nash equilibria for given incompatibility and use the higher welfare as a reasonable welfare-theoretical second-best standard. From Equations (16), (19), and (21), it becomes clear that the following proposition holds:

Proposition 3. Compatibility is welfare superior to incompatibility only if the alternative is an incompatible duopoly, whereas a standards war always leads to a higher welfare than compatibility.

This result is due to the fact that in a competition for the market, the monopolist is forced to offer a variant which matches consumers' preferences as far as possible, whereas a move to compatibility softens competition over product designs significantly. Hence, in particular policy recommendations concerning standards wars depend on which welfare-theoretical benchmark is chosen. Whereas against the background of the welfare-theoretical first-best optimum, standards wars are always a market failure, they are welfare optimal against the background of a reasonable second-best welfare-theoretical benchmark. Note that the result stated in Proposition 3 is of considerable robustness towards more general consumer distributions. In particular, it holds for the triangular distribution and, thus, for all concave distributions.[17]

Whenever a standards war takes place, calls for an intervention in favor of compatibility are a very common reaction. A conventional policy recommendation is the compulsory licensing of the intellectual property rights attached to the interface specifications. If the suppliers knew ex ante that the winner of the standards war would be forced to disclose his interface specification and license the attached intellectual property rights, they would have no reason to opt for a standards war. Our analysis shows that such a policy recommendation can be misguided. If it is deduced against the background of the fact that compatibility is always first-best op-

[16]In Europe, for example, compatibility arrangements can be prohibited based on Article 85 EEC Treaty, and compatibility can be enforced based on Article 86 EEC Treaty.

[17]However, comparing cumulated alienation effects in a compatible duopoly with cumulated alienation effects in a standards war, the difference only amounts to $t/64$ in the case of the triangular distribution, whereas it amounts to $12t/64$ in the case of the uniform distribution. Hence, probably Proposition 3 does not hold for (very) sharply peaked logconcave densities.

timal, it is misguided because it does not take into account that under compatibility the horizontal hardware differentiation is excessive (and that this fact, realistically, cannot be remedied by policy interventions). Of course, things might be different if policy interventions do not aim at the maximization of total welfare but at the maximization of expected consumers' surplus. In this case, calls for an intervention in favor of compatibility are understandable insofar as the general significance of the network effects is high, because then consumers are almost always better off under compatibility (see our analysis at the end of Section IV).

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