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The Merger of Populations, the Incidence of Marriages,
and Aggregate Unhappiness

by

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and aggregate unhappiness*

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Abstract

Let a society's unhappiness be measured by the aggregate of the levels of relative deprivation of its members. When two societies of equal size, F and M , merge, unhappiness in the merged society is shown to be higher than the sum of the levels of unhappiness in the constituent societies when apart; merger alone increases unhappiness. But when societies F and M merge and marriages are formed such that the number of households in the merged society is equal to the number of individuals in one of the constituent societies, unhappiness in the merged society is shown to be lower than the aggregate unhappiness in the two constituent societies when apart. This result obtains regardless of which individuals from one society form households with which individuals from the other, and even when the marriages have not (or not yet) led to income gains to the married couples from increased efficiency, scale economies, and the like. While there are various psychological reasons for people to become happier when they get married as opposed to staying single, the very formation of households reduces social distress even before any other happiness-generating factors kick in.

Keywords: Merger of populations; Integration of societies; Unhappiness; Marriages; Relative Deprivation

JEL classification: D0; D10; D31; D63

1. Introduction

In this paper we combine three strands of literature: on the integration of groups (societies, populations, nations); the living arrangements of individuals (single, married); and the overall sense of distress in a community (societal unhappiness).

The integration of groups can be perceived as a merger of populations. In general, when two populations merge, a variety of benefits are anticipated: denser markets, increased efficiency and productivity brought about by scale effects, and the like. Classical trade theory has it that integration liberalizes trade and smoothes labor and financial flows. Denser and larger markets improve resource allocation and the distribution of final products. The welfare of the integrating populations is bound to rise. Rivera-Batiz and Romer (1991) emphasize the influence of integration on the prevailing stock of knowledge and on the speed of technological advancements, and van Elkan (1996) points to the role of integration in narrowing the technological gap between countries, which in turn stimulates growth. (Henrekson et al., 1997, who attend to the long-run growth effect of European integration, point to a particularly beneficial effect of integration.) The picture may not be so bright, however. Convergence in the income levels of the integrating countries or regions is not by any means inevitable. Behrens et al. (2007) show that to secure gains from integration, a significant degree of coordination of policies between countries is required, while Rivera-Batiz and Xie (1993) and Zeng and Zhao (2010) caution that the income inequality repercussions of integration may well depend on the characteristics of the countries or regions involved, which, when unfavorable, can result in increased inequality. Beckfield (2009), who studies European integration and individual levels of income, reports reduced between-country income inequality but increased within-country income inequality. The inconclusiveness of these outcomes also pervades research on firms: whereas Qiu and Zhou (2006) report increased profitability following the international merger of firms, Greenaway et al. (2008) point to a greater likelihood of a closedown when a firm faces tighter competition in a liberalized market. An interesting strand of literature deals with the merger of firms and workplaces, employing “social identity theory” (originally developed by Tajfel and Turner, 1979). A recurrent finding (cf. Terry et al., 2001; Terry and O’Brien, 2001; Fischer et al., 2007) is the contrasting perceptions of different groups of individuals: a merger is viewed most negatively by those of low status, whereas high status people are more at ease with the merged structure. This finding is in line with what we show in the first part of the

current paper where we provide conditions under which belonging to a larger society results in a heightened aggregate level of distress or unhappiness.

When we employ a fairly general measure of societal unhappiness, we find, quite startlingly, that holding incomes constant, the merger of two populations consisting each of n individuals exacerbates unhappiness: the level of happiness of the integrated population is higher than the sum of the levels of unhappiness of the constituent populations when apart. (Our measure of societal unhappiness is superadditive with respect to vector concatenation.) We next ask what happens if upon a merger of two populations, the merged population consists entirely of n couples (regardless of who in one population is matched with whom in the other population). We find that the preceding result is reversed: the level of unhappiness in the integrated population is lower than the sum of the levels of unhappiness in the two constituent populations when apart; the very formation of households raises social contentment even before any of the standard happiness-generating factors kicks in. (Our measure of societal unhappiness is subadditive with respect to vector addition.)¹

Merger of populations and integration of societies occur in all spheres of life, and in all times and places: conquests bring hitherto disparate populations into one, provinces merge into regions, adjacent villages experiencing population growth coalesce into a single town, and European countries have been merging into a union. Mergers arise as a result of administrative considerations or naturally, they are imposed or chosen by election. A merger of societies is a far cry from the merger of production lines. The social environment and the social horizons that the individuals who constitute the merged society face change fundamentally upon a merger: others who were previously outside the individuals' social domain are now within. One consequence of this revision of the social landscape, which hitherto appears not to have received attention, is a "built-in" increase in social distress. Therefore, this process of integration can chip away at the sense of wellbeing in quite unexpected ways.

When individuals form marriages, scale economies, joint consumption, opportunities to eliminate duplication, and heightened efficiency accrue.² These repercussions facilitate a more productive participation in the labor market which, in turn, results in a higher joint

¹ This result extends quite straightforwardly to the case of a general convex and homogenous index of deprivation.

² Two individuals who live together as a couple can enjoy the same level of consumption as two individuals who live singly for 27 to 41 percent less income (Browning et al., 2006; Fleurbaey and Gaulie, 2009).

income. The possibility that benefits accrue directly from being in a two-member household as opposed to being single even if the joint income remains unchanged and even if the psychological benefits of being married as opposed to being single are not factored in, is a rather novel finding.

For more than a decade now, economists have sought to unravel the sources of societal happiness. As noted by Fleurbaey (2009), surveys consistently find a positive correlation between marriage and reported happiness (Veenhoven, 1989; Clark and Oswald, 2002; Blanchflower and Oswald, 2004), with causality running from the former to the latter. The empirical literature on the topic has not sought to provide a theoretical basis for the causes of this favorable social outcome: it *assumes* that the two types of factors - economic efficiency and the psychological benefits that flow from not being single - are “to blame.” The possibility that if neither holds the positive outcome will still arise is not acknowledged in writings on the topic. Thus, the result reported in this paper - that when societies merge couple formation can reduce societal unhappiness - complements the received literature in a non-trivial way.

In the remainder of this paper we proceed as follows. In section 2, we define a measure of societal unhappiness which is the aggregate of individual levels of unhappiness. In section 2, Case 1, we show (holding incomes constant) that when two populations, F and M , of equal size are merged, unhappiness in the merged population is higher than the sum of the levels of unhappiness in the two constituent populations when apart; in and by itself, merger increases unhappiness. In section 2, Case 2, we show that when societies F and M merge and marriages are formed such that the number of households in the merged population is equal to the number of individuals in a constituent population, unhappiness in the merged population is lower than the sum of the levels of unhappiness in the two constituent populations when apart. This result obtains irrespective of which individual from one population forms a household with which individual from the other population, *and* even when the actual marriages have not led to income gains to the married (in comparison with the sum of the pre-marriage incomes). In section 3 we provide concluding remarks.

2. Why a merger of populations without marriages increases unhappiness, whereas a merger with universal marriages increases happiness

Let the unhappiness of a population F of n individuals whose incomes are $f = (f_1, \dots, f_n)$ be measured by total relative deprivation, TRD . No economist in his right mind will exclude income from the equation of happiness, and this is not what we do here either; by holding (absolute) incomes constant, we are freed to concentrate on the role of relative income considerations as determinants of happiness.

Without loss of generality, we can order the incomes: $f_1 \leq f_2 \leq \dots \leq f_n$.

The relative deprivation of an individual with income f_i , $i = 1, 2, \dots, n$, who is a member of population F is defined as:

$$RD(f_i, f) \equiv \frac{1}{n} \sum_{k=i+1}^n (f_k - f_i) \quad (1)$$

where it is understood that $RD(f_n, f) = 0$.³

Total relative deprivation is defined as the aggregate of the individual relative deprivations:

$$TRD(f) \equiv \sum_{i=1}^{n-1} RD(f_i, f) = \frac{1}{n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (f_k - f_i) \quad (2)$$

³ This measure of relative deprivation is explained further in Stark and Hyll (2011). The measure is based on the seminal work of Runciman (1966), on a proposal made by Yitzhaki (1979), and on axiomatization by Ebert and Moyes (2000) and Bossert and D'Ambrosio (2006). However, in the Appendix we show that the results derived in the body of the paper are robust with respect to two other measures of relative deprivation: the aggregate of the excesses of incomes, and the distance from the highest income. It is worth noting that since the 1960s, a considerable body of research evolved, demonstrating empirically that interpersonal comparisons of income (that is, comparisons of the income of an individual with the incomes of higher income members of his reference group) bear significantly on the perception of well-being, and on behavior. (For a recent review see Clark et al., 2008.) One branch of this body of research has dealt with migration. Several studies have shown empirically that a concern for relative deprivation impacts significantly on migration outcomes (Stark and Taylor, 1989; Stark and Taylor, 1991; Quinn, 2006; Stark et al., 2009). Theoretical expositions have shown how the very decision to resort to migration and the choice of migration destination (Stark, 1984; Stark and Yitzhaki, 1988; Stark and Wang, 2007; Stark and Fan, 2011; Fan and Stark, 2011), as well as the assimilation behavior of migrants (Fan and Stark, 2007), are modified by a distaste for relative deprivation.

To ease the analysis that follows, it is convenient to express TRD in a different form.

Lemma 1: $TRD(\mathbf{f}) = \frac{1}{2n} \sum_{i=1}^n \sum_{k=1}^n |f_k - f_i|.$

Proof: For all $i, k = 1, \dots, n$ either $f_k - f_i \geq 0$, or $f_i - f_k \geq 0$. TRD in (2) includes only non-negative differences between incomes in a distribution. Since the TRD in Lemma 1 includes the absolute values of *all* the differences between incomes, it counts a difference between a pair of given incomes twice.

Thus, we have that

$$\frac{1}{n} \sum_{i=1}^n \sum_{k=1}^n |f_k - f_i| = 2 \frac{1}{n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (f_k - f_i). \quad (3)$$

By inserting (3) into (2) we obtain the expression in the Lemma. \square

From the expression of TRD in Lemma 1 we can see that as a function of \mathbf{f} , for a fixed population size n , and as a sum of continuous and convex functions, TRD is continuous and convex. The TRD function is also homogenous of degree 1, and its value can be calculated even when the incomes that constitute vector \mathbf{f} are not ordered.

Let us consider a second population, M , also of size n , with incomes $\mathbf{m} = (m_1, \dots, m_n)$.

From Lemma 1 we know that

$$TRD(\mathbf{m}) = \frac{1}{2n} \sum_{i=1}^n \sum_{k=1}^n |m_k - m_i|.$$

Case 1: A merger of populations without marriages

Suppose that the two populations F and M merge into a new population, $N = F \cup M$, which now has incomes $\mathbf{f} \cup \mathbf{m} = (f_1, \dots, f_n, m_1, \dots, m_n)$, where the order of f_i in relation to m_k can be any. For example, think of F as populating one village, of M as populating another, and now the two villages merge into a new, integrated village.

Claim 1: $TRD(\mathbf{f} \cup \mathbf{m}) \geq TRD(\mathbf{f}) + TRD(\mathbf{m}).$

This claim states that unhappiness in the integrated village is higher than the sum of the levels of unhappiness in the two villages F and M ; in and by itself merger increases unhappiness.⁴

Proof: We first state and prove the following Lemmas.

Lemma 2: Let $u \leq v$ and $r \leq s$ be real numbers. Then

$$|u - s| + |v - r| \geq (v - u) + (s - r). \quad (4)$$

Proof: Given that $u \leq v$ and $r \leq s$, there are six possible orderings of these numbers. We consider each case separately.

1. $u \leq v \leq r \leq s$. Then, $|u - s| + |v - r| \geq (s - u) = (s - r) + (r - v) + (v - u) \geq (v - u) + (s - r)$.

The case where $r \leq s \leq u \leq v$ follows by symmetry.

2. $u \leq r \leq v \leq s$. Then, $|u - s| + |v - r| = (s - u) + (v - r) = (v - u) + (s - r)$. The case where $r \leq u \leq s \leq v$ follows by symmetry.

3. $u \leq r \leq s \leq v$. Then, $|u - s| + |v - r| = (s - u) + (v - r) = (v - u) + (s - r)$. The case where $r \leq u \leq v \leq s$ follows by symmetry. \square

$$\mathbf{Lemma\ 3:} \quad TRD(\mathbf{f} \cup \mathbf{m}) = \frac{nTRD(\mathbf{f})}{n+n} + \frac{\sum_{i=1}^n \sum_{k=1}^n |f_i - m_k|}{n+n} + \frac{nTRD(\mathbf{m})}{n+n}.$$

Proof: Using Lemma 1, we have that

$$TRD(\mathbf{f} \cup \mathbf{m}) = \frac{1}{2(n+n)} \left[\sum_{i=1}^n \sum_{j=1}^n |f_j - f_i| + \sum_{k=1}^n \sum_{l=1}^n |m_l - m_k| + 2 \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k| \right]. \quad (5)$$

The first two double sums in (5) are clearly $2nTRD(\mathbf{f})$ and $2nTRD(\mathbf{m})$, respectively. We therefore have that

⁴ This is not a result that informed intuition yields. Consider, for example, the most intuitive measure of heterogeneity - the variance. The merger of two populations can result in a *decrease* of the variance to a level below the sum of the variances of the constituent populations. For example, consider the merger of two populations with incomes $\mathbf{f} = (1,9)$ and $\mathbf{m} = (2,10)$. Prior to the merger, $Var(\mathbf{f}) + Var(\mathbf{m}) = 16 + 16 = 32$.

But after the merger, $Var(\mathbf{f} \cup \mathbf{m}) = 16 \frac{1}{4} < Var(\mathbf{f}) + Var(\mathbf{m})$, the very opposite of the result rendered by resorting to TRD .

$$TRD(\mathbf{f} \cup \mathbf{m}) = \frac{1}{n+n} [nTRD(\mathbf{f}) + nTRD(\mathbf{m})] + \frac{1}{n+n} \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k|. \quad \square$$

We are now in a position to prove Claim 1. For the two populations, F with incomes $f_1 \leq \dots \leq f_n$, and M with incomes $m_1 \leq \dots \leq m_n$, we know that

$$TRD(\mathbf{f}) = \frac{1}{n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (f_k - f_i)$$

and similarly, that

$$TRD(\mathbf{m}) = \frac{1}{n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (m_k - m_i).$$

From Lemma 3, we get that

$$TRD(\mathbf{f} \cup \mathbf{m}) = \frac{1}{2} (TRD(\mathbf{f}) + TRD(\mathbf{m})) + \frac{1}{2n} \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k|. \quad (6)$$

Drawing on Lemma 2 with $f_i \leq f_k$ and $m_i \leq m_k$, and leaving out the terms with $i = k$, we have that

$$\frac{1}{2n} \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k| \geq \frac{1}{2n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (|f_i - m_k| + |f_k - m_i|) \geq \frac{1}{2n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n [(f_k - f)_i + (m_k - m_i)]. \quad (7)$$

The most right hand side term in (7) is equal to $\frac{1}{2} (TRD(\mathbf{f}) + TRD(\mathbf{m}))$. Thus, we have that

$$\frac{1}{2n} \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k| \geq \frac{1}{2} (TRD(\mathbf{f}) + TRD(\mathbf{m})). \quad (8)$$

Upon inserting (8) into (6) we get that

$$TRD(\mathbf{f} \cup \mathbf{m}) \geq TRD(\mathbf{f}) + TRD(\mathbf{m}). \quad \square$$

To appreciate that Claim 1 is anything but an intuitive prediction, we can consider, for example, the merger of a relatively poor population M with a relatively rich population F , such that $m_i < f_j \quad \forall i, j = 1, \dots, n$. Upon integration, all members of the richer population except

the richest are subjected to less relative deprivation, whereas all members of the poorer population are subjected to more relative deprivation. Claim 1 asserts that the gain of the richer population in terms of the decreased relative deprivation of its members will never be large enough to offset the loss of the poorer population in terms of the increased relative deprivation of its members; the TRD of the merged population will always be higher than the sum of the TRD s of the constituent populations.⁵

Case 2: A merger of populations with universal marriages and with income pooling

Imagine that all the individuals in F happen to be females, that all the individuals in M happen to be males, and that upon the integration of the two villages, individuals from F set up households with individuals from M . For the argument that follows, it makes no difference which individual from F , earning f_i , forms a household with which individual from M , earning m_j ; we can have a “marriage pairing” or couple formation of any individual from population F with any individual from population M . To keep the vector notation below consistent, we denote by \mathbf{m}' some permutation of the vector \mathbf{m} which arranges the “marriage pairing” across the populations. The income of the new household is $h_i = f_i + m'_i$. From Lemma 1, we know that the value of $TRD(\mathbf{x})$, for some vector of incomes $\mathbf{x} = (x_1, \dots, x_n)$, does not depend on the ordering of the \mathbf{x} vector. Therefore, $TRD(\mathbf{m}) = TRD(\mathbf{m}')$. We refer to the resulting population as H , with incomes $\mathbf{h} = (h_1, \dots, h_n) = \mathbf{f} + \mathbf{m}'$. How does $TRD(\mathbf{h})$ relate to $TRD(\mathbf{f})$ and $TRD(\mathbf{m})$? In this section we assume that once married, the joint income of the household is the income that the household compares with the incomes of other households; there is a *complete income pooling* but *no income sharing*.

Claim 2: $TRD(\mathbf{h}) \leq TRD(\mathbf{f}) + TRD(\mathbf{m})$.

This claim states that unhappiness in the population of n households, that is, the unhappiness experienced following the formation of marriages that retains the total number of households

⁵ To see this vividly, let $\mathbf{m} = (1, 2)$ and $\mathbf{f} = (3, 4)$. Prior to a merger, the levels of relative deprivations are, respectively, $(1/2, 0)$ for the members of the M population, and $(1/2, 0)$ for the members of the F population, yielding $TRD(\mathbf{m}) = 1/2$ and $TRD(\mathbf{f}) = 1/2$. Upon a merger, the RD of the individual earning 3 declines from $1/2$ to $1/4$, whereas the RD 's of the members of the poorer population rise to, respectively, $(3/2, 3/4)$. The TRD of the merged population then registers an increase: $TRD(\mathbf{f} \cup \mathbf{m}) = 5/2 \geq TRD(\mathbf{f}) + TRD(\mathbf{m})$.

the same as in any of the constituent populations, is lower than the sum of the levels of unhappiness in the two populations, even though in and by themselves marriages have not (or have not as yet) led to income gains on account of efficiency, scale economies, and the like.⁶

Proof: The convexity of the TRD function implies that

$$\alpha TRD(\mathbf{f}) + (1 - \alpha) TRD(\mathbf{m}') \geq TRD(\alpha \mathbf{f} + (1 - \alpha) \mathbf{m}') \quad \forall \alpha \in (0, 1).$$

With $\alpha = 1/2$ we get that

$$TRD(\mathbf{f}) + TRD(\mathbf{m}') \geq 2 \cdot TRD\left(\frac{\mathbf{f} + \mathbf{m}'}{2}\right).$$

The homogeneity of degree 1 of the TRD function implies that

$$TRD\left(\frac{1}{2} \cdot \mathbf{h}\right) = \frac{1}{2} TRD(\mathbf{h})$$

or that

$$2 \cdot TRD\left(\frac{\mathbf{f} + \mathbf{m}'}{2}\right) = 2 \cdot \frac{1}{2} TRD(\mathbf{f} + \mathbf{m}') = TRD(\mathbf{f} + \mathbf{m}').$$

Therefore,

$$TRD(\mathbf{f}) + TRD(\mathbf{m}) = TRD(\mathbf{f}) + TRD(\mathbf{m}') \geq TRD(\mathbf{f} + \mathbf{m}') = TRD(\mathbf{h}). \quad \square$$

Corollary 1: It is possible for merger cum marriages to result in unhappiness in the integrated population that is even lower than the unhappiness in any of the constituent populations prior to the merger.

⁶ Once again, this is not a result that informed intuition yields. The perception that upon a decrease of the number of income units by half we should *unreservedly* expect a decrease in inequality is incorrect. To see this, we resort again to the variance as a measure of income differences in a population and consider the merger of populations with incomes $\mathbf{f} = (1, 9)$ and $\mathbf{m} = (2, 10)$. Then, as noted in footnote 3, prior to the merger $Var(\mathbf{f}) + Var(\mathbf{m}) = 16 + 16 = 32$. However, following marriages that result in $(h_1, h_2) = (3, 19)$, the variance rises to $Var(\mathbf{h}) = 64$.

Proof: Let $(f_1, f_2) = (1, 9)$, and let $(m_1, m_2) = (2, 10)$. Possible are marriage pairings that result in an incomes vector $(h_1, h_2) = (11, 11)$. Clearly, prior to merger and marriage $TRD(1, 9) = TRD(2, 10) = 4$, but in their wake, $TRD(11, 11) = 0$. \square

Comment: The Case 2 scenario presented hitherto is of a “pure” type in which each and every individual from one population matches up with a spouse in the other population. If, instead, we allow the merger to facilitate some marriages rather than universal marriages, and if we reinterpret marriage as a procedure for some sharing of incomes between the spouses such that marriage is conceived as the equivalent of a spread-reducing income transfer from the higher-income spouse to the lower-income spouse, then the following result obtains. Let the Gini coefficient of a population of size k with incomes vector \mathbf{x} be defined as

$$G(\mathbf{x}) = \frac{\sum_{i=1}^k \sum_{j=1}^k |x_i - x_j|}{2k \sum_{i=1}^k x_i}. \text{ Upon the said income transfer, } G(\mathbf{x}) \text{ must decline because some}$$

income differences are narrowed. From a comparison with (3), it follows that

$$TRD(\mathbf{x}) = G(\mathbf{x}) \sum_{i=1}^k x_i. \text{ When } G(\mathbf{x}) \text{ declines and } \sum_{i=1}^k x_i \text{ is held constant, } TRD \text{ must decline. In}$$

such a setting then, marriages reduce the otherwise merger-triggered increase in unhappiness, although the extent to which this effect can mute or reverse the increase is unclear.

3. Conclusions

We provided a simple proof that it is the very formation of households rather than the quality of the matches that makes a population of married people happier than populations of singles.

Of course, and as already noted, there are various psychological reasons for people to become happier when they get married than they were when single. But what we have seen is that the very formation of two-member households raises social contentment even before any of the other happiness-generating factors kicks in.

A tentative implication is that when, for example, nations in Europe integrate into a union, the merger *as such* will increase social distress (Claim 1). *To the extent* that integration facilitates or fosters marriages, the increase in social distress will be reversed (Claim 2) or mitigated (Comment).

Our results were derived for a specific measure of unhappiness, and for a specific measure of a population's total relative deprivation for that (equation (2)). In line with the first sentence of footnote 3, the appeal of this measure is that it emanates from a solid social-psychological foundation, it rests on a sound axiomatic basis, and it has been shown to be empirically significant. Nonetheless, unhappiness and, in particular, a population's total relative deprivation can be measured in a variety of ways and by different indices. In the Appendix we show that our main claim concerning aggregate unhappiness is robust to the use of alternative measures of individuals' relative deprivation.

Appendix

The results derived in the body of the paper are not dented when alternative measures of relative deprivation are employed. We consider two such measures: the aggregate of the excesses of incomes, and the distance from the highest income. We attend to these two measures in turn.

In population F , let the relative deprivation of an individual with income f_i , $i = 1, 2, \dots, n$, be defined as

$$RD'(f_i, \mathbf{f}) \equiv \sum_{k=i+1}^n (f_k - f_i).$$

Then, the total relative deprivation of population F , as the sum of the excesses of incomes, is

$$TRD'(\mathbf{f}) \equiv \sum_{i=1}^{n-1} RD'(f_i, \mathbf{f}) = \sum_{i=1}^{n-1} \sum_{k=i+1}^n (f_k - f_i).$$

A property that is analogous to Lemma 1 characterizes this measure:

$$TRD'(\mathbf{f}) = \sum_{i=1}^{n-1} \sum_{k=i+1}^n (f_k - f_i) = \frac{1}{2} \sum_{i=1}^n \sum_{k=1}^n |f_k - f_i|.$$

When a second population, M , merges with population F , we derive the superadditivity result once again.

Claim A1: $TRD'(\mathbf{f} \cup \mathbf{m}) \geq TRD'(\mathbf{f}) + TRD'(\mathbf{m})$.

Proof: We have that

$$TRD'(\mathbf{f} \cup \mathbf{m}) = \frac{1}{2} \left[\sum_{i=1}^n \sum_{j=1}^n |f_j - f_i| + \sum_{k=1}^n \sum_{l=1}^n |m_l - m_k| \right] + \sum_{i=1}^n \sum_{k=1}^n |f_i - m_k|.$$

Noting that $\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n |f_j - f_i| = TRD'(\mathbf{f})$, that $\frac{1}{2} \sum_{k=1}^n \sum_{l=1}^n |m_l - m_k| = TRD'(\mathbf{m})$, and that

$$\sum_{i=1}^n \sum_{k=1}^n |f_i - m_k| \geq 0, \text{ completes the proof. } \square$$

For the merged population H , introduced in the same manner as in Case 2 in the body of the paper, a claim akin to Claim 2 is now stated and proved.

Claim A2: $TRD'(\mathbf{h}) \leq TRD'(\mathbf{f}) + TRD'(\mathbf{m})$.

Proof: Since TRD' differs from TRD only by a scaling factor (the size of the population), TRD' too is convex and of homogeneity of degree 1. Therefore, the proof of Claim A2 tracks the same steps as those undertaken in proving Claim 2. \square

We next consider measuring relative deprivation as the distance from the highest income.

In population F , let the relative deprivation of an individual with income f_i , $i = 1, 2, \dots, n$, be defined as

$$RD''(f_i, f) \equiv f^* - f_i,$$

where $f^* = \max\{f_1, \dots, f_n\}$.

The total relative deprivation of population F , measured as the aggregate of the distances from the highest income, is

$$TRD''(f) \equiv \sum_{i=1}^n (f^* - f_i).$$

When a second population, M , the total relative deprivation of which is measured in the same way as that of population F , merges with population F , we obtain the superadditivity result once again. The reasoning is straightforward. Unless the two populations have each the same highest income, a merger results in exposure of members of one of the populations to a higher post-merger highest income, while members of the other population continue to be exposed to the same highest income as prior to the merger. Thus, the aggregated distances from the highest income in the merged population will not be less than the sum of the corresponding distances in the constituent populations when apart. For the sake of completeness, we state this result in the following claim.

Claim A3: $TRD''(f \cup m) \geq TRD''(f) + TRD''(m)$.

Proof: The proof is contained in the preceding discussion. \square

For the merged population H , introduced in the same manner as in Case 2 in the body of the paper, a claim akin to Claim 2 is now stated and proved.

Claim A4: $TRD''(h) \leq TRD''(f) + TRD''(m)$.

Proof. For the highest income in population H , h^* , we have

$$h^* = \max \{f_1 + m'_1, \dots, f_n + m'_n\} \leq \max \{f_1, \dots, f_n\} + \max \{m'_1, \dots, m'_n\} = f^* + m^*,$$

where m'_1, \dots, m'_n are the elements of the permuted \mathbf{m}' vector introduced in Case 2 of section 2, and where $m^* = \max \{m'_1, \dots, m'_n\}$, as the vectors \mathbf{m}' and \mathbf{m} differ only in the order of their elements. Then,

$$\begin{aligned} TRD''(\mathbf{h}) &= \sum_{i=1}^n (h^* - h_i) = \sum_{i=1}^n [h^* - (f_i + m'_i)] \leq \sum_{i=1}^n [(f^* + m^*) - (f_i + m'_i)] \\ &= \sum_{i=1}^n [(f^* - f_i) + (m^* - m'_i)] = \sum_{i=1}^n (f^* - f_i) + \sum_{i=1}^n (m^* - m'_i) = TRD''(\mathbf{f}) + TRD''(\mathbf{m}'). \end{aligned}$$

Noting that, obviously, $TRD''(\mathbf{m}') = TRD''(\mathbf{m})$ completes the proof. \square

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