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A Concern about Low Relative Income, and the Alignment of
Utilitarianism with Egalitarianism

by

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Abstract

A utilitarian social planner who maximizes social welfare assigns the available income to those who are most efficient in converting income into utility. However, when individuals are concerned about their income falling behind the incomes of others, the optimal income distribution under utilitarianism is equality of incomes.

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1. Introduction

In this short paper we present the first result that we obtained when we studied the tension between utilitarianism (conceptualizing social welfare as the sum of the individuals' utilities) and egalitarianism (cherishing equality between individuals). In contrast with the received literature that pits the two as competing social objectives, we show that when the maximization of social welfare takes into account individuals' concern about low relative income, there is no difference between a utilitarian income allocation distribution and an egalitarian income distribution; the two align.

For a good many years now, an effort has been made to season utilitarianism with egalitarian gravy. Prominent economists as early as Marshall (1823) and Pigou (1920) defended utilitarianism as a guide to the maximization of social welfare. The argument made was that the maximization of the sum of individual utilities requires equalization of marginal utilities. However, equating marginal utilities is equivalent to equating incomes only under a very special assumption of identical utility functions. In general, a utilitarian social planner will not choose to distribute incomes equally. Still, utilitarianism was applied in evaluating income inequality (Dalton, 1920; Tinbergen, 1970). In other words, utilitarianism was the launch pad for assessing inequality from a welfarist standpoint. This stand was criticized by Sen (1973, p. 18): "It seems fairly clear that fundamentally utilitarianism is very far from an egalitarian approach." Pattanaik (2009) voiced a similar criticism. In what follows we show that once individuals' concern for low relative income is factored in, the utilitarian rule and the egalitarian approach are fundamentally the same.

Evidence from econometric studies, experimental economics, social psychology, and neuroscience indicates that humans routinely engage in inter-personal comparisons, and that the outcome of that engagement impinges on their sense of wellbeing. People are dismayed when their consumption, income, or social standing fall below those of others with whom they naturally compare themselves (those who constitute their "comparison group"). Examples of responses to such dismay include Stark and Taylor (1991), Zizzo and Oswald, (2001), Luttmer (2005), Fliessbach et al. (2007), Blanchflower and Oswald

(2008), Takahashi et al. (2009), Stark and Fan (2011), Stark and Hyll (2011), Fan and Stark (2011), and Stark, Hyll, and Wang (2012).

Taking total income as given, we show that when individuals care only about their absolute income, the maximization of a social welfare function that sums up the individuals' utilities mandates allocating the available income such that the individual who values income more ends up receiving more income than the individual who values income less. This result is trivial, and, of course, is well-known. However, when individuals care also about trailing behind others in the income hierarchy (exhibit a concern for relative deprivation), the maximization of a social welfare function that sums up the individuals' utilities (with these utilities incorporating the said concern) mandates income equalization. This is anything but trivial. Apparently, relative income concerns elevate equalization of incomes to the "status" of the optimal societal scheme.

In the next section we present our core argument for the case of two individuals. In our more comprehensive paper (Stark, Kobus, and Jakubek, 2011) we prove the robustness of the result reported here along several dimensions: we provide an extension of our argument to the case of any $n \geq 2$ individuals; we revert to a more general specification of the weights of absolute income and relative deprivation in the individuals' utility functions; and we show that our result is not confined to a particular utility specification in which the preference concerning absolute income is characterized by a linear function. In section 3 we offer our conclusion.

2. The tension between utilitarianism and income equality forgone: the case of two individuals

Let there be a society that consists of two individuals: "I1" with income x_1 , and "I2" with income x_2 , such that $x_1 + x_2 = 1$ and $x_1, x_2 \geq 0$. The utility function of "I1" is $u_1 = \alpha_1 x_1$, $\alpha_1 > 0$, and the utility function of "I2" is $u_2 = \alpha_2 x_2$, $\alpha_2 > 0$.

Let there be a social planner who, by means of allocating a unit of income between the two individuals, seeks to maximize social welfare, SWF, where social welfare is the sum of utilities: $SWF(x_1, x_2) = u_1(x_1) + u_2(x_2)$. Using a star to indicate optimal values, if $\alpha_1 > \alpha_2$ then $x_1^* = 1$ and $x_2^* = 0$; and if $\alpha_2 > \alpha_1$ then $x_1^* = 0$ and $x_2^* = 1$: the individual who is more “productive” in converting income into utility receives the entire available income.¹ Put differently, regardless of the magnitudes of the weights that the two individuals attach to (absolute) income and as long as those magnitudes differ one from the other, social welfare maximization is orthogonal to income equality. In sum: when for all levels of the available income I2 is more “productive” in converting income to utility than I1 then, regardless of the initial distribution of income, $x_2^* = 1$ and $x_1^* = 0$.

Consider an alternative setting in which individuals I1 and I2 have, respectively, the following utility functions:

$$u_1(x) = \alpha_1 x_1 - (1 - \alpha_1)RD_1(x) = \alpha_1 x_1 - (1 - \alpha_1) \frac{\max(x_2 - x_1, 0)}{2}$$

and

$$u_2(x) = \alpha_2 x_2 - (1 - \alpha_2)RD_2(x) = \alpha_2 x_2 - (1 - \alpha_2) \frac{\max(x_1 - x_2, 0)}{2},$$

where $x = (x_1, x_2)$, $\alpha_1, \alpha_2 \in (0, 1)$, and the measure of the concern for relative income, $RD_i(x)$ for $i \in \{1, 2\}$, is the index of “relative deprivation,” based on the seminal work of Runciman (1966), and proposed by Yitzhaki (1979).² The $RD_i(x)$ index can be shown (see, for example, Stark, 2010) to be equal to the fraction of the individuals in the population whose incomes are higher than the income of the individual, times their mean excess income.

¹ If $\alpha_1 = \alpha_2$, then any distribution is optimal.

² Below we show, however, that our argument does not hinge on measuring the concern for relative income by this particular index.

The social planner thus maximizes

$$\begin{aligned} & \max_{x_1, x_2} \left[\alpha_1 x_1 - (1 - \alpha_1) \frac{\max(x_2 - x_1, 0)}{2} + \alpha_2 x_2 - (1 - \alpha_2) \frac{\max(x_1 - x_2, 0)}{2} \right] \\ & \text{s.t. } x_1 + x_2 = 1, x_1, x_2 \geq 0 \end{aligned}$$

or, since $x_2 = 1 - x_1$,

$$\begin{aligned} & \max_{x_1} \left[\alpha_1 x_1 - (1 - \alpha_1) \frac{\max(1 - x_1 - x_1, 0)}{2} + \alpha_2 (1 - x_1) - (1 - \alpha_2) \frac{\max(x_1 - 1 + x_1, 0)}{2} \right] \\ & \text{s.t. } 0 \leq x_1 \leq 1 \end{aligned}$$

or, equivalently

$$\begin{aligned} & \max_{x_1} \left[x_1 (\alpha_1 - \alpha_2) - (1 - \alpha_1) \frac{\max(1 - 2x_1, 0)}{2} + \alpha_2 - (1 - \alpha_2) \frac{\max(2x_1 - 1, 0)}{2} \right] \\ & \text{s.t. } 0 \leq x_1 \leq 1. \end{aligned}$$

Constrain first the range of x_1 , such that $x_1 \leq 1/2$. Then, the problem simplifies to

$$\begin{aligned} & \max_{x_1} \left[\alpha_1 x_1 - (1 - \alpha_1) \left(\frac{1 - 2x_1}{2} \right) + \alpha_2 (1 - x_1) \right] \\ & \text{s.t. } 0 \leq x_1 \leq 1/2 \end{aligned}$$

or, equivalently, to

$$\begin{aligned} & \max_{x_1} \left[x_1 (1 - \alpha_2) - (1 - \alpha_1) \frac{1}{2} + \alpha_2 \right] \\ & \text{s.t. } 0 \leq x_1 \leq 1/2. \end{aligned}$$

Because the function to be maximized is linear with respect to x_1 and has a positive slope

$1 - \alpha_2$, the solution is $x_1^* = 1/2$ which, together with $x_2 = 1 - x_1$, implies that $x_2^* = 1/2$.

This result obtains regardless of the specific magnitudes of $\alpha_1, \alpha_2 \in (0, 1)$.

Constrain next the range of x_1 such that $x_1 \geq 1/2$. Then, the problem simplifies to

$$\begin{aligned} \max_{x_1} & \left[x_1 \alpha_1 - \alpha_2 + \alpha_2 - (1 - \alpha_2) \frac{2x_1 - 1}{2} \right] \\ \text{s.t.} & \quad 1/2 \leq x_1 \leq 1 \end{aligned}$$

or, equivalently, to

$$\begin{aligned} \max_{x_1} & \left[x_1 \alpha_1 - 1 + (1 + \alpha_2) \frac{1}{2} \right] \\ \text{s.t.} & \quad 1/2 \leq x_1 \leq 1. \end{aligned}$$

Since the maximized function is linear with a negative slope $\alpha_1 - 1$, it attains its maximum for the smallest possible value of the argument, that is, at $x_1^* = 1/2$.

Summing up: in both cases we have that the solution is $x_1^* = 1/2$ which, together with $x_2 = 1 - x_1$, implies that $x_2^* = 1/2$; namely, optimal social welfare is achieved when incomes are equal. Again, this result obtains regardless of the specific magnitudes of $\alpha_1, \alpha_2 \in (0, 1)$.

Moreover, the result of an equal division of income is robust to alternative specifications of the dismay that I1 senses on account of his income falling below the income of I2. To see this, suppose that rather than being attached to $\frac{x_2 - x_1}{2}$, the disutility weight $(1 - \alpha_1)$ is attached to $(x_2 - x_1)$ for $x_1 \leq x_2$; it is merely the excess income, not the fraction of those in the population whose income is higher times the mean excess income, that measures the dismay. Then, the social planner maximizes

$$\begin{aligned} \max_{x_1, x_2} & \left[\alpha_1 x_1 - (1 - \alpha_1) (x_2 - x_1) + \alpha_2 x_2 \right] \\ \text{s.t.} & \quad x_1 + x_2 = 1; \quad 0 \leq x_1 \leq x_2 \end{aligned}$$

or, since $x_2 = 1 - x_1$,

$$\begin{aligned} & \max_{x_1} [\alpha_1 x_1 - (1 - \alpha_1) (1 - x_1 - x_1) + \alpha_2 (1 - x_1)] \\ & \text{s.t. } 0 \leq x_1 \leq 1 - x_1 \end{aligned}$$

or, equivalently

$$\begin{aligned} & \max_{x_1} [x_1 (2 - \alpha_1 - \alpha_2) - (1 - \alpha_1) + \alpha_2] \\ & \text{s.t. } 0 \leq x_1 \leq 1/2. \end{aligned}$$

Since $2 - \alpha_1 - \alpha_2 > 0$, the maximizing value is $x_1^* = 1/2$. Because this result is independent of the specific magnitudes of $\alpha_1, \alpha_2 \in (0, 1)$, the case $x_2 \leq x_1$ is symmetric to the case discussed above.

Alternatively, let the coefficient $(1 - \alpha_1)$ be attached to the distance from mean income; that is, to $x_2 - \frac{x_1 + x_2}{2}$. Since $x_2 - \frac{x_1 + x_2}{2} = \frac{x_2 - x_1}{2}$, we get the same representation as the one that we started with.

Furthermore, it so happens that even when I2 derives positive utility from being better off than I1, the concern of I1 for relative income renders equality the best social outcome if $\alpha_2 > \alpha_1$. Imagine then the following utility function of I2:

$$u_2(x) = \alpha_2 x_2 + (1 - \alpha_2)(x_2 - x_1),$$

while, as before, the utility function of I1 is

$$u_1(x) = \alpha_1 x_1 - (1 - \alpha_1)(x_2 - x_1).$$

Then, the maximization problem is³

³ In comparison with the first configuration where we obtain the same result when $0 \leq x_1 \leq 1/2$ as when $1/2 \leq x_1 \leq 1$ (that is, when $x_1 \leq x_2$ as when $x_1 \geq x_2$) here, without loss of generality, we additionally assume that $x_1 \leq x_2$.

$$\begin{aligned} & \max_{x_1, x_2} [\alpha_1 x_1 - (1 - \alpha_1)(x_2 - x_1) + \alpha_2 x_2 + (1 - \alpha_2)(x_2 - x_1)] \\ & \text{s.t. } x_1 + x_2 = 1; 0 \leq x_1 \leq x_2 \end{aligned}$$

or, since $x_2 = 1 - x_1$,

$$\begin{aligned} & \max_{x_1} [\alpha_1 x_1 - (1 - \alpha_1)(1 - x_1 - x_1) + \alpha_2(1 - x_1) + (1 - \alpha_2)(1 - x_1 - x_1)] \\ & \text{s.t. } 0 \leq x_1 \leq 1 - x_1 \end{aligned}$$

or, equivalently,

$$\begin{aligned} & \max_{x_1} [x_1(\alpha_1 + 2(1 - \alpha_1) - \alpha_2 - 2(1 - \alpha_2)) - (1 - \alpha_1) + \alpha_2 + (1 - \alpha_2)] \\ & \text{s.t. } 0 \leq x_1 \leq 1/2, \end{aligned}$$

which simplifies to

$$\begin{aligned} & \max_{x_1} [x_1(\alpha_2 - \alpha_1) + \alpha_1] \\ & \text{s.t. } 0 \leq x_1 \leq 1/2. \end{aligned}$$

Thus, if I1 cares more about relative income than I2, namely if $1 - \alpha_1 > 1 - \alpha_2$ which is the same as having $\alpha_2 - \alpha_1 > 0$, equality once again is the socially optimal outcome.

Comment: can it be that our result is simply a consequence of us assuming that “beginning from an egalitarian outcome, the marginal gain to a richer person is higher than the marginal loss felt by a poorer person?” Not so, and for the simple reason that increasing the income of the individual who is the most “efficient” in terms of converting income to utility yields only a seemingly higher “marginal gain.” To see this most vividly, let us indeed begin from “an egalitarian outcome” in a population in which each of two individuals receives the same income. We now take the “most efficient” individual - the one who has the highest coefficient, denoted by α_1 , next to his income - and give him marginally more income. He obtains a “boost” of utility (in marginal terms) of $\alpha_1 x$, since the coefficient next to income in his utility function is the highest in the population. To keep our “budget” balanced, we must take away this small portion of income x from

the other individual. Since the latter becomes relatively deprived and gets less income, his marginal loss is intuitively larger than the gain of the “richer” individual. If we increase the income of the more efficient individual (the one with the higher α coefficient next to his income) by x then, as just noted, he gets a marginal boost of utility of $\alpha_1 x$. However, the individual who has the lower coefficient, denoted by α_2 , experiences a loss in terms of income equal to $\alpha_2 x$, plus a loss caused by an increased relative deprivation that is equal to $(1 - \alpha_2)RD_2(x) = (1 - \alpha_2)\frac{1}{2}2x = (1 - \alpha_2)x$. In sum, we have a gain to the “richer” that is equal to $\alpha_1 x < x$ and a loss to the “poorer” that is equal to the full x . Therefore, our result is not due to us somehow assuming that “the marginal gain to a richer person is higher than the marginal loss felt by a poorer person.”

3. Conclusion

A concern for low relative income (relative deprivation) suffices to eliminate the discord between the stands of two schools of thought: utilitarianism, and egalitarianism; at the very same time, both get their exact way. Given the increasing recognition that unfavorable income comparisons impinge on individuals’ sense of wellbeing, a utility representation that admits this consideration suggests that a long-prevailing tension in social choice and welfare economics is resolved. Our more comprehensive paper (Stark, Kobus, and Jakubek, 2011) reinforces this suggestion by expanding the setting presented in the current paper along several dimensions, according the result with a considerable degree of robustness.

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