Electromagnetic Properties of Light and Heavy Baryons in the Relativistic Quark Model

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Diana Nicmorus Marinescu
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Dekan:	Prof. Dr. Nils Schopohl
1. Berichterstatter:	Prof. Dr. Thomas Gutsche
2. Berichterstatter:	Prof. Dr. Dr. h.c. mult. Amand Fäßler

Suena de capao, corazon cabao ....

Abstract

One of the main challenges of nowadays low-energy physics remains the description of the internal structure of hadrons, strongly connected to the electromagnetic properties of matter.

In this vein, the success of the relativistic quark model in the analysis of the hadron structure constitutes a solid motivation for the study carried out throughout this work. The relativistic quark model is extended to the investigation of static electromagnetic properties of both heavy and light baryons. The bare contributions to the magnetic moments of the single-, double- and triple-heavy baryons are calculated. Moreover, the relativistic quark model allows the study of the electromagnetic properties of the light baryon octet incorporating meson cloud contributions in a perturbative manner. The long disputed values of the multipole ratios $E2/M1$ and $C2/M1$ and the electromagnetic form factors of the $N \to \Delta\gamma$ transition are successfully reproduced.

The relativistic quark model can be viewed as a quantum field theory approach based on a phenomenological Lagrangian coupling light and heavy baryons to their constituent quarks. In our approach the baryon is a composite object of three constituent quarks, at least in leading order. The effective interaction Lagrangian is written in terms of baryon and constituent quark fields. The effective action preserves Lorentz covariance and gauge invariance. The main ingredients of the model are already introduced at the level of the interaction Lagrangian: the three-quark baryon currents, the Gaussian distribution of the constituent quarks inside the baryon and the compositeness condition which sets an upper limit for the baryon-quark vertex. The S-matrix elements are expressed by a set of Feynman quark-diagrams. The model contains only few parameters, namely, the cut-off parameter of the Gaussian quark distribution and the free quark propagator, which are unambiguously determined from the best fit to the data. The heavy quark limit within this model reveals an exact agreement in leading order with the model-independent predictions for the magnetic moments of the heavy baryons. For the light sector, a Lorentz covariant chiral quark Lagrangian is used to dress the constituent quarks by pseudoscalar meson clouds. The main achievement consists in the factorization of the valence quark contributions and the meson cloud contributions in the calculation of electromagnetic properties of light baryons.

Abstrakt

Eine der Hauptherausforderungen der heutigen Niederenergiephysik besteht in der Beschreibung der internen Struktur von Hadronen, welche im engen Zusammenhang mit den elektromagnetischen Eigenschaften der Materie zusammenhängt. Der vorherige Erfolg des relativistischen Quarkmodells in der Analyse der Hadronenstruktur stellt eine solide Grundlage für die Untersuchungen, welche in dieser Arbeit durchgeführt werden, dar. In diesem Zugang wird das Baryon, zumindest in führender Ordnung, durch den Bindungszustand aus Konstituentenquarks beschrieben.

Das relativistische Quarkmodell wird in der vorliegenden Arbeit erweitert, um die statischen, elektromagnetischen Eigenschaften von schweren und leichten Baryonen zu untersuchen. Das relativistische Quarkmodell basiert auf einen quantenfeldtheoretischen Zugang, welcher, in einer phänomenologischen Lagrange-Dichte formuliert, die Kopplung von leichten und schweren Baryonen an die Konstituentenquarks beschreibt. Die entsprechende effektive Wirkung erhält Lorentz-Kovarianz und Eichinvarianz. Die Hauptannahmen des Modells werden auf dem Niveau der Wechselwirkungs Lagrange-Dichte eingeführt: Drei-Quark Baryonen-Ströme, die Gauss-Verteilung der Konstitutentenquarks im Baryon und die sogenannte 'compositeness condition', welche die Kopplung der nackten Baryonen an die Konstituentenquarks bestimmt. Die entsprechenden S-Matrix Elemente werden durch einen Satz von Feynman-Quark-Diagrammen ausgedrückt. Das relativistische Quarkmodell beinhaltet einen Satz von wenigen Parametern: Breite der Gaussverteilung und Konstituentenquarkmassen, welche durch Anpassung an vorherige Daten fixiert werden.

Die Valenzquarkbeiträge zu den magnetischen Mometen der einfach, doppelt und dreifach schweren Baryonen werden berechnet. Im Limes schwerer Quarkmassen kann innerhalb dieses Modells in führender Ordnung exakte Übereinstimmung mit den modell-unabhängigen Vorhersagen zu den magnetischen Momenten schwerer Baryonen erreicht werden. Im leichten Baryonen-Sektor wird zusätzlich ein Lorentz kovariante, chirale Quark-Lagrange-Dichte genutzt, um die See-Quark Beiträge, ausgedrückt durch pseudoskalare Mesonen, in Konsistenz mit chiraler Symmetrie einzubinden. Die chirale Lagrange-Dichte wird zur Ordnung $O(p^4)$ und in der Einschleifen-Näherung ausgewertet, um die durch die chiralen Effekte angezogenen Quarkoperatoren zu bestimmen. Das Hauptergebnis dieser Technik ist, dass die Beiträge der Mesonen und der Valenzquarks in den elektromagnetischen,

baryonischen Amplituden faktorisieren. Anwendungen dieser chiralen Erweiterung des relativistischen Quarkmodell auf elektromagnetischen Eigenschaften des leichten Baryonen-Oktetts werden ausgearbeitet. Zusätzlich studieren wir die elektromagnetischen $N - \Delta$ Übergänge, wo relativistische Effekte eine entscheidende Rolle spielen. Insbesondere können die Multipolverhältnisse $E2/M1$, $C2/M1$ und die elektromagnetischen Formfaktoren des radiativen Übergangs $N \to \Delta\gamma$ erfolgreich reproduziert werden.

Contents

Chapter 1

Introduction

In this work we investigate the structure of baryons applying a relativistic constituent quark model. Properties of both light and heavy baryons are studied in a quantum field theory approach [1, 2].

Baryons are systems governed by the strong interaction. We aim to describe such interactions introducing a phenomenological Lorentz covariant and gauge invariant Lagrangian. Due to the success of the naive quark model predictions for static baryon properties [3, 4, 5, 6] and motivated by the results of deep inelastic lepton scattering, it is reasonable to think of baryons in leading order as composite systems of three quarks. The prototype of a such system is the nucleon. The model we describe here has been already developed and applied to the nucleon, and later on improved for a series of low energy processes obeyed by other heavier states [7, 8, 9, 10, 11, 12].

Among various attempts to describe the low-energy behaviour of nucleons in a relativistic framework, our approach has particular advantages: it is rather intuitive, employs a solid and within quantum field theory consistent calculational technique, has a fixed and small number of parameters. Our results are in good agreement with other models and effective approaches. Moreover, our approach recovers model independent predictions, as the ones offered by chiral perturbation theory and heavy quark effective theory.

Having in mind the importance and success of these approaches we will start by giving a short overview on the naive quark models, on the basics of chiral perturbation theory and heavy quark symmetry. We continue with an overview of the main issues of the relativistic quark model, followed by two applications: the calculation of magnetic moments of single-, double-, triple-heavy baryons and the calculation of magnetic moments of light baryons, taking into consideration meson cloud effects. A challenging aspect is the study of the $N \rightarrow \Delta$ transition, which we will present in the last part of this work.

The nucleon and its observed detailed properties are of great importance to understand and test the theory of strong interaction. A theoretical study of the nucleon collects phenomena associated with QCD (confinement, spontaneous symmetry breaking, and asymptotic freedom) as well as phenomenological approaches, that is models. At low energies it is possible to obtain model independent predictions (from chiral perturbation theory, for instance), whereas at high energies one can make use of a perturbative analysis. Among the most popular non-perturbative methods used to describe low-energy properties of hadrons are lattice gauge theory, chiral perturbation theory (ChPT), various models based on a effective QCD action, heavy quark effective theory (HQET) and QCD sum rules.

1.1 Models of the baryons

Non-perturbative approaches are essential for practical calculations in the low-energy regime. For momentum transfers of ${}^2Q < 1$ GeV the running coupling $\alpha_s(Q^2)$ of QCD is of order 1 such that an expansion in powers of α_s is not useful. Thus, an analysis of the nucleon structure from first principles is difficult. Phenomenological models preserve and use some of the most important aspects of QCD and provide an insight into the structure of hadrons.

Among various approaches, models still remain a powerful tool for practical calculation and are successful especially in the determination of static properties. Understanding the nucleon and its structure has been the task of many years of theoretical work, especially in phenomenological frameworks. There are three classes of models that have dealt with the nucleon and analyzed experimental data [13]: the non-relativistic quark models [3, 4], the bag models [14] and the soliton models [15]. In the non-relativistic quark model, which uses ingredients of the phenomenological shell-model in nuclear physics, baryons are composite objects of three constituent quarks which are treated non-relativistically in a confining potential. The main and original achievement of this model is a good description of the anomalous magnetic moments of the baryon octet, as well as the baryon spectrum. The drawback of this model is that it treats the quarks as virtual particles, massive enough to move non-relativistically, whereas quarks are relativistic particles.

The bag models promote the hadrons as bags where the valence quarks are confined to a spherical cavityand which are singlets under $SU(3)$ color transformation. A particular class among bag models are the chiral bag models which use important aspects of chiral symmetry. As a consequence, the quarks interact with pseudoscalar Goldstone bosons generated by spontaneous chiral symmetry breaking.

In soliton models, the picture of the nucleon emerges from the energy density created by quarks and mesons. The chiral solitons, in particular, are systems formed by the non-linear interaction of quarks and pions. A well known version of the soliton models is the Skyrme model, working with topological solitons [16]. This model is based on the effective Lagrangian of the non-linear sigma model.

Though non-fundamental, the approaches above have proved to be phenomenologically succesful and give a good understanding of the baryon structure.

Zweig [3] and Gell-Mann [4] gave a primary understanding of the nucleon structure, by using a simple non-relativistic algebraic model of quarks. In the constituent quark model frame, the main ingredient is the assumption that quarks exist as quasi-particles inside the baryons. These constituents are not the fundamental QCD quarks, but they preserve their fundamental properties: electric charge, baryon number, color and flavor, they are fermions and their masses are larger than the current-quark masses, due to the dressing by the strong interactions. A constituent quark is often defined in the vast literature as a massive object whose mass arises from the interaction of the "bare" valence quark with the surrounding clouds of quark-antiquark pairs and gluons. The coupling of such massive quasi-particles with the quark quantum numbers to Goldstone bosons, due to spontaneous chiral symmetry breaking, was not historically described in the framework of constituent models.

In the nonrelativistic quark model the baryon wave function is constructed by the three constituents moving in a confining potential, each being a single particle state created by a creation operator acting on the vacuum. Using the anticommutation algebra and assuming that the constituents are in their lowest orbits, that is s-states, one can obtain the color singlet baryon wave function. The input in such models is the effective constituent quark mass. Using the Dirac formula for magnetic moments of fermions one can easily calculate the baryon magnetic moments as matrix elements of the non-relativistic magnetic moment operator. The magnetic moments are in very good agreement with the experimental values, as the nucleon wave functions display the $SU(6)$ symmetry properties, but do not emerge from detailed dynamical reasons. Within the constituent quark model also the $N \to \Delta$ transition magnetic moment has been calculated [17]. It is worth to mention that, unfortunately, in the naive quark picture the electric quadrupole moment $E2$ of the $N \to \Delta$ transition vanishes. A broad introduction to the $N \to \Delta\gamma$ transition properties can be found in Ref. [18].

Georgi, deRujula and Glashow [5] improved the model by introducing further QCD aspects into the dynamics of the constituent quarks. Their model is a non-relativistic $SU(6)$-symmetric model, but the $SU(3)$ symmetry breaking is induced by the quark masses as in QCD. The potential contains a long-range confining force, on one side, and a short-range spin-flavor dependent interaction part, on the other side, emerging from the ideal assumption of one-gluon exchange between the quarks. The potential energy will contain therefore a confining-like form and a more extended form which includes: a Coulomb term, a Darwin term, a spin-spin interaction term, a spin-orbit interaction term and a tensor term. The $SU(3)$ breaking is obtained by conveniently rewriting the Hamiltonian in terms of a $SU(3)$ symmetric piece and a symmetry breaking piece. The eigenstates of the full Hamiltonian are the $SU(6)$ multiplets. Mass formulas can be derived with good accuracy, like the mass splitting between the Δ and the nucleon.

The Isgur-Karl model [6] adds to the harmonic oscilator potential an anharmonic part, responsible for $SU(6)$ breaking, and omits the spin-orbit contributions. In this picture the predictions for the baryon spectra are improved. In the study of the $N \to \Delta$ transition, a new ingredient is considered within this frame: the admixture of d-states into the N and Δ wave functions. This admixture is introduced via a residual tensor force between the quarks, induced by the one gluon exchange (OGE) supplementing potential. Though the magnetic monopole transition $M1$ is dominant, as a consequence of a small d-state admixture, the value of the electric quadrupole transition $E2$ is not zero as predicted by the simple quark models, but has a small non-vanishing value, as also shown in experiments. Various quantities (diagonal transition amplitudes, radiative decay widths, etc) can be calculated with the phenomenologically simple Hamiltonian proposed by Isgur and Karl, the success of the model relying in the combination of the $SU(6)$ group theory and a sufficient dynamical input.

Although the non-relativistic quark models use a long-range confining potential to express the effect of confinement on quarks, they do not represent the physical picture of confinement itself as understood from QCD. In the framework of soliton models, Lee [15] proposed that a quasi-classical theory is a good choice for modelling confinement. In this vein the QCD vacuum is treated as a color dia-electric medium; any color charge introduced in such a medium will produce a vacuum-hole, so that a charge will be surrounded by a perturbative vacuum cavity. This charge will feel repulsion to the charges placed on the surface of the vacuum-cavity and the energy needed to shrink this vacuum-hole will be proportional to a confining-like term. One writes down a Lagrangian density in terms of valence quarks, a scalar field corresponding to the lowest energy state and a term which involves a dielectric function treated perturbatively; when there are no valence quarks (non-perturbative vacuum) the scalar field potential describes the vacuum energy state, and when the quark density is high (perturbative vacuum), the scalar field potential has a small value.

The Bag models, widely used in the late 60's and 70's have as a basis the model of Bogoliubov [14]. A way to make quarks confined was to consider them massive enough and bound in a deep potential so that the hadronic states will still have reasonable masses. Bogoliubov [14] proposed that the quark masses $m \to \infty$ and placed them in a sphere with radius R (put in by hand), within which they felt an attractive scalar field. When QCD arose as a theory, together with the asymptotic freedom at short distances, the bag models tried to incorporate this property as well. The whole philosophy is to devide the space into two regions, one inside the bag, where the quarks had small masses and felt decreased forces, and the exterior of the bag, where quarks were not allowed to propagate. The model of Bogoliubov solves the Dirac equation for a particle of mass m inside a spherical cavity of radius R which feels a constant scalar potential. It was not suitable for a spectroscopical analysis, but the MIT bag model (70's-80's) overcame this problem, by a formulation which leaves the radius to be determined dynamically, giving a relativistic description of the baryon structure [19],[20]. The Lagrangian involves quark fields, a term which distinguishes between the interior (perturbative vacuum) and the exterior (non-perturbative vacuum) of the bag and a bag-surface term essential to confine the quarks.

There are also variations of the bag model which incorporate chiral symmetry, the chiral quark models [21]. The Lagrangian is invariant under the chiral transformations and the surface term is responsibe for chiral symmetry breaking. To overcome the problem of dynamical symmetry breaking leading to massless Goldstone bosons, this Lagrangian includes a pion field which couples to the confined quarks so that, under chiral transformation of these two fields, the Lagrangian is invariant. One example of a famous model in the late 70's is the cloudy bag model [22], which, besides the ingredients of the MIT bag model, introduces pion-quark interaction in the Lagrangian. The pion is treated perturbatively. Other models have been developed, all derived from the original bag model, incorporating chiral symmetry. The merit of all these approaches relies on a better insight into the hadron structure and spectroscopy. Their effective nature is underlined, for example, by the assumption that the pion is an elementary field, in the long wave-length limit.

1.2 Chiral Perturbation Theory

Nowadays, a widely used technique in the study of hadron structure at low energies is Chiral Perturbation Theory (ChPT) [23, 24]. At small momentum transfers the elementary degrees of freedom of QCD (quarks and gluons) are replaced by effective degrees of freedom, namely by composite hadrons. Their dynamics is described by an effective action, instead of the original QCD action. The main reason of such a replacement is founded in a manifest symmetry of the QCD Lagrangian at low energies, the chiral symmetry.

Chiral symmetry is a symmetry of the QCD Lagrangian achieved in the limit where the light quark masses, up, down and strange, vanish, that is $m_{u,d,s} \rightarrow 0$. In reality the quark masses are small, but not zero and Chiral Perturbation Theory provides a tool for treating the quark masses as small perturbations. As elsewhere in field theory one expects that due to a continuous symmetry the corresponding currents are conserved. Chiral symmetry is related to the conserved right- or left-handness of the light quarks which leads to the conservation of the vector and axial vector currents. Though being an approximate symmetry one deals with the concept of sponaneous and explicit breaking of the symmetry.

The concept of spontaneous breaking of a symmetry is translated as follows: the Hamiltonian of the system is invariant under the symmetry transformation laws, whereas the ground state of the theory is not. As consequence we have the appearance of massless modes, called the Goldstone-bosons. Actually, Goldstone's theorem implies the existence of eight pseudoscalar Goldstone bosons in the case of the $SU(3)_L \times SU(3)_R$ invariance of the QCD

Lagrangian for massless light quarks. We take as an example the pion. Admitting that the pion is not massless is to admit that chiral symmetry is not an exact symmetry of the nature, which is perfectly valid. But the pion has a mass, in the sense that is smaller than other hadron masses of the order of 1 GeV.

In ChPT, the masses of the light quarks (5-9 MeV) are typically much smaller than the QCD energy scale Λ_{QCD} ~200 MeV. Thus, one makes use of chiral symmetry in the low energy regime where the processes are dominated by light mesons and where all observables can be expressed as expansions in light meson masses and momenta. This procedure is know as Chiral Perturbation Theory. Of course, heavier states like the vector mesons can also be taken into consideration. There should be no confusion between ChPT as a perturbation theory in terms of Goldstone boson masses and momenta and the perturbation theory in the strong coupling α_s. The range of applicability of ChPT is suited for the investigation of non-perturbative, low-energy physics, thus ChPT is not a perturbation theory in the QCD coupling constant. A perturbative expansion in the strong coupling α_s is not reasonable, as in the low energy regime $\alpha_s \to 1$.

Following Refs. [23] and [24] we will try to give a short overview on ChPT as a tool. The symmetry property of any Lagrangian under some transformation will lead, via the Noether theorem, to a conserved current and further, to a conserved "charge". The chiral symmetry transformations belong to the class of unitary transformations. Adding a mass term into the Lagrangian, which explicitly breaks the symmetry, leads to the non-conservation of the current, in our case to the breaking of chiral symmetry due to the physical quark masses, finite but not zero. The masses of the light quarks are much smaller than the masses of the lightest hadrons containing light quarks and also much smaller than the energy scale of spontaneous symmetry breaking. Thus, to study spontaneous symmetry breaking and hence the appearance of non-vanishing light scalar condensates, it is customary to omit the heavy degrees of freedom in the QCD Lagrangian. The QCD Lagrangian for massless light quarks reads as:

$$L^0_{QCD} = \sum_{f=u,s,d} \bar{q}_f \, i \, \not{D} q_f - \frac{1}{4} G_{\mu\nu,a} G^{\mu\nu}_a \ . \tag{1.1}$$

The interesting property of this Lagrangian is that it remains invariant under the global $SU(3)_L \times SU(3)_R \times U(1)_V$ transformations. These global symmetries become transparent when we project the quark fields onto their chiral components q_L and q_R. The Noether theorem leads to conserved currents associated with the two independent global trasformations of the left- and right-handed quark fields, $SU(3)_L$ and $SU(3)_R$, respectively, defined as a set of 3×3 unitary matrices:

$$\begin{aligned} &\exp[-i\sum_{a=1}^{8}\theta_a^L\frac{\lambda_a}{2}] \ , \\ &\exp[-i\sum_{a=1}^{8}\theta_a^R\frac{\lambda_a}{2}] \end{aligned} \tag{1.2}$$

where $\theta_a^{L,R}$ are real numbers and λ_a are the Gell-Mann matrices. There are eight conserved currents associated with the transformations of the left-handed quarks and eight conserved currents associated with the transformation of the right-handed quarks. One uses two linear combinations of the chiral "left" and "right" currents which transform under parity as vector and axial-vector quantities, respectively. Equivalently, the Lagrangian is invariant under $SU(3)_V \times SU(3)_A$. The vector currents and the eight axial vector currents are conserved

$$\begin{aligned} V^{\mu,a} &= \bar{q}\gamma^\mu\frac{\lambda^a}{2}q, \qquad \partial_\mu V^{\mu,a} = 0 \ , \\ A^{\mu,a} &= \bar{q}\gamma^\mu\gamma_5\frac{\lambda^a}{2}q, \quad \partial_\mu A^{\mu,a} = 0 \ . \end{aligned} \tag{1.3}$$

A mass term added to the Lagrangian (1.1), namely $-(\bar{q}_f M q_f)$, where M =diag$\{m_u, m_d, m_s\}$ is the current quark mass matrix, is reflected, in general, in non-vanishing values of the divergencies of the symmetry currents. In fact for equal quark masses $m_u = m_d = m_s$ the eight vector currents $V^{\mu,a}$ are conserved, while the eight axial currents have explicit divergencies, that is

$$\begin{aligned} \partial_\mu V^{\mu,a} &= i\bar{q}[M, \frac{\lambda_a}{2}]q \ , \\ \partial_\mu A^{\mu,a} &= i\bar{q}\{\frac{\lambda_a}{2}, M\}\gamma_5 q \ . \end{aligned} \tag{1.4}$$

Hence, the axial symmetry is not a good one for finite quark masses, but an approximate one as long as the quark masses stay small compared to the QCD energy scale $\Lambda_{QCD} \sim 200$ MeV. The approximate symmetry of the axial current results in the partial conservation of this current. The slight breaking of the invariance of the Lagrangian under axial transformations, due to small, but finite masses, of the light quarks, leads to a new concept in the theory: the Partial Conserved Axial Current (PCAC). As long as the symmetry breaking is small, the effect of a partial conservation of the axial current can be treated using the perturbation theory tools.

It is known that a good way to illustrate PCAC is to study the weak decay of the pion $\pi^+ \rightarrow \mu^+\nu_\mu$. Together with the πN strong interaction analysis this will lead to the famous Goldberger-Treiman relation. The PCAC relation relates the matrix element of the divergence of the axial-vector current in the pion decay to the mass of the pion. As long as the mass of the pion can be considered small with respect to hadronic scales, the divergence of the axial current is nonzero, but very small and the current is partially conserved

$$< 0|\partial_\mu A^{\mu,a}(x)|\pi(q)^b >= -iF_\pi m_\pi^2 \delta^{ab} e^{-iqx} \tag{1.5}$$

where $F_\pi = 93MeV$ is the decay constant, q is pion momentum and π^b is the pion field. From here one can define the pion field as the divergence of the axial-vector current

$$A^{\mu,a}_{pion} = F_\pi \partial^\mu \pi^a(x) \ . \tag{1.6}$$

The Goldberger-Treiman relation comes as more evidence for the conservation of the axial current. To check this, one can write down the axial current for the nucleon

$$A^{\mu,a}_{nucleon} = g_A \bar{\Psi}_N \gamma^\mu \gamma_5 \lambda^a/2\Psi_N \tag{1.7}$$

and observe that the divergence of this current is proportional to the mass of the nucleon, which by no means can be considered smaller than the QCD scale

$$\partial_\mu A^{\mu,a}_{nucleon} = ig_A M_N \bar{\Psi}_N \gamma_5 \lambda^a \Psi_N \neq 0 \ . \tag{1.8}$$

Thus, we do not expect the axial current of the nucleon to be conserved. Writing the total axial current for the pion-nucleon interaction as a sum of the pion and nucleon contributions and requiring that the total current is conserved one ends up with a Klein-Gordon equation for a massless pion coupled to the nucleon field. So, the requirement of a conserved axial current leads to a massless pion. However, when a pion finite mass term is added, that is when requiring the PCAC relation to be valid, one can relate the pion-nucleon couplig constant to the weak pion decay constant and obtain the Goldberger-Treiman relation

$$g_{\pi NN} = g_A \frac{M_N}{F_\pi} \ . \tag{1.9}$$

The relation is remarkably well fulfilled when inserting the experimental values.

To illustrate the necessity of spontaneous breaking of chiral symmetry, one also looks at the hadronic mass spectrum, for example at the mass difference between the pion and the $\sigma-$meson, as well as the one between the $\rho-$meson and the a_1. If the axial transformation, which rotates the π into a σ and the ρ into a a_1, were an exact symmetry then these particles should have the same mass eigenvalues. The small difference between u and d current quark masses cannot explain a difference of nearly 600 MeV between the ρ and the a_1 meson masses. In addition, the apparent paradox of a partially conserved axial current, but a non-representation of the axial symmetry in the mesonic spectrum can be also explained in the framework of spontaneous symmetry breaking. The axial-vector symmetry is spontaneously broken, that is the ground state does not preserve this symmetry whereas the Hamiltonian does. Due to spontaneous symmetry breaking the mass of the pion is zero, but via an axial transformation one expects a finite scalar quark condensate $\langle \bar{q}q \rangle \neq 0$. This is consistent with the PCAC relation. A pedagogical overview is given in [23], where a classical mechanics analogue is offered to understand spontaneous symmetry breaking.

In literature the concepts of spontaneous symmetry breaking and explicit symmetry breaking are realized in the simple frame of toy models: the linear sigma-model and the improved version, the non-linear sigma-model.

Introduced by Gell-Mann and Levy [25], the linear sigma-model is a chiral invariant model involving pions and nucleons. The Lagrangian of the model is a Lorentz scalar and invariant under vector and axial-vector transformations. The meson-nucleon interaction is given by two terms. One piece is a pseudoscalar combination of the nucleon field multiplied by the pion field. The other piece is a scalar conbination of the nucleon field multiplied by the sigma-meson field. This term has the structure of a nucleon mass term and possesses the advantage of introducing the mass of the nucleon without explicitly breaking the chiral symmetry. To mimic the spontaneous breaking of chiral symmetry a potential term is added to this Lagrangian. This potential generates the vacuum expectation value F_π of the σ field and remains invariant under vector and axial-vector transformations, thus it is a function of the invariant structure $(\pi^2 + \sigma^2)$, like below:

$$V(\pi^2 + \sigma^2) \sim (\pi^2 + \sigma^2 - F_\pi^2)^2 \ . \tag{1.10}$$

Due to its shape this potential is often called the "Mexican-hat" potential. The total invariant Lagrangian will also involve the kinetic terms for the nucleons, pions and the $\sigma-$mesons. In the linear sigma-model the axial vector is calculated and shown to fulfill the PCAC relation. To explicitly break the chiral symmetry one should reproduce the effect of a mass term in the Lagrangian ($\sim -m\bar{q}q$) of the sigma-model. This translates into adding a parameter ϵ in the "Mexican-hat" potential, which gives a measure of the symmetry breaking. The explicit symmetry breaking is illustrated by slightly tilting the "Mexican-hat". In fact, allowing for a small breaking one allows the spontaneous symmetry breaking effect to be larger than the effect of the explicit symmetry breaking (the light quark masses are much smaller than the QCD energy scale). In the linear sigma-model the mass generated by explicit breaking is the small pion mass (that is the axial-vector symmetry is still partially fulfiled), whereas the spontaneous breaking generates the mass of the nucleon. The mass of the pion is proportional to the breaking parameter. Due to explicit symmetry breaking a small change in the mass of the nucleon is produced. This small change can be experimentally deduced and is know under the name of the pion-nucleon term $\sigma_{\pi N}$. The pion-nucleon sigma term gives a relation between this change and the symmetry breaking (SB) parameter, as follows:

$$\sigma_{\pi N} = \delta M_N^{\chi SB} \simeq \epsilon \ .$$

In literature one can find other well-known relations that can be derived in the context of chiral symmetry constraints, which mainly link the scalar quark condensate to measurable quantities like $f_\pi, m_\pi, \sigma_{\pi,N}$. For example, the pion-nucleon sigma term is the expectation value of the explicit symmetry breaking piece in the Lagrangian, taken between the nucleon states:

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} < N|\bar{u}u + \bar{d}d|N > \ .$$

The famous Gell-Mann-Oakes-Renner formula relates the mass of the pion to the quark condensate:

$$m_\pi^2 F_\pi^2 = -\frac{m_u + m_d}{2} < 0|\bar{u}u + \bar{d}d|0 > \ .$$

The other toy-model, the non-linear sigma model, has been developed from the simple idea that the σ field has not been identified with any physical particle. The $\sigma-$meson field is removed from the above Lagrangian by sending its mass to infinity. The consequence is that the dynamics is expressed now only with pion and nucleon fields and the coupling between pions and nucleons is expressed via derivatives (momenta) of the pion field. The effect on the "Mexican-hat" potential is that it becomes infinitely steep in the sigma-direction and vanishes on the circle defined by its minimum. The explicit breaking of chiral symmetry in the non-linear sigma-model is introduced via an explicit pion mass term.

Chiral perturbation theory is a systematic tool for the investigation of the low-energy properties of QCD. At very low energies the effective Lagrangian of ChPT is expressed in terms of the members of the pseudoscalar octet (π, K, η) degrees of freedom which are the Goldstone bosons of the spontaneous breaking of the $SU(3)_L \times SU(3)_R$ chiral symmetry down to $SU(3)_V$. The light pseudoscalar mesons achieve masses in the explicit symmetry breaking process. The non-vanishing scalar quark condensate in the chiral limit is a sufficient condition for the spontaneous symmetry breaking of QCD. The construction of the effective Lagrangian which describes the dynamics of the Goldstone bosons and, implicitly, the concept of spontaneous symmetry breaking, is based on the following conditions: firstly, the effective Lagrangian is invariant under $SU(3)_L \times SU(3)_R \times U(1)_V$ in the chiral limit. Secondly, it should contain eight pseudoscalar degrees of freedom which transform as an octet under $SU(3)_V$. Also, the ground state should be invariant under $SU(3)_V \times U(1)_V$, which is the symmetry group of the effective Lagrangian after the spontaneous breaking. The notation commonly used for the fields of the effective Lagrangian [24] is a 3×3 unitary matrix $U(x)$ in its exponential representation:

$$U(x) = \exp\left(i\frac{\Phi(x)}{F_\pi}\right) \,, \tag{1.11}$$

where the fields $\Phi(x)$ are given by

$$\Phi(x) = \sum_{a=1}^{8} \lambda_a \Phi_a(x) \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} \,. \tag{1.12}$$

The $SU(3)$ matrix $U(x)$ transforms under $SU(3)_L \times SU(3)_R$ as:

$$U(x) \to RU(x)L^\dagger \ . \tag{1.13}$$

The most general effective Lagrangian with the above properties reads as:

$$L_{eff} = \frac{F_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) \ , \tag{1.14}$$

where the free parameter $F_\pi \approx 93$ MeV is the pion decay constant. The $\Phi_a(x)$ fields describe the eight independent massless Goldstone bosons. The expansion of the exponential (1.11)

$$\begin{aligned} U &= 1 + \frac{i\Phi}{F_\pi} + ... \\ \partial_\mu U &= \frac{i\partial_\mu \Phi}{F_\pi} + ... \end{aligned} \tag{1.15}$$

generates the kinetic part and meson-meson interaction part in the effective Lagrangian, respectively:

$$L_{eff} = \frac{1}{2} \partial_\mu \Phi_a \partial^\mu \Phi_a + L_{int} \ . \tag{1.16}$$

The chiral "left" and "right" currents associated with the invariance of this Lagrangian are conserved, as well as their vector and axial vector combinations. The finite masses of the Goldstone bosons are achieved by adding a mass term in the above Lagrangian, which explicitly breaks the symmetry. Though the mass matrix M is just a constant matrix, M should transform under $SU(3)_L \times SU(3)_R$ as the $U(x)$ matrix:

$$M \to RML^\dagger \ , \tag{1.17}$$

such that the full Lagrangian is invariant under the transformations (1.13) and (1.17). The explicit symmetry breaking term at lowest order in powers of M is given by:

$$L_{sym.break} = \frac{F_\pi^2 B_0}{2} Tr(MU^\dagger + UM^\dagger) \ , \tag{1.18}$$

where B_0 is related to the scalar quark condensate as:

$$3F_\pi^2 B_0 = -\langle \bar{q} q \rangle \ . \tag{1.19}$$

The masses of the Goldstone bosons are identified with the terms of second order in the fields Φ_a in the symmetry breaking part (1.18):

$$L_{sym.break} = -\frac{B_0^2}{2} Tr(\Phi^2 M) + ... \tag{1.20}$$

Using the explicit form (1.12) for the fields Φ_a one obtains the Goldstone boson masses to the lowest order in the quark masses $m_u = m_d = m$ and m_s:

$$M_\pi^2 = 2B_0 m, \quad M_K^2 = B_0(m + m_s), \quad M_\eta^2 = \frac{2}{3} B_0(m + 2m_s) \ . \tag{1.21}$$

These relations together with the relation (1.19) for the B_0 parameter are also referred in the literature as Gell-Mann, Oakes, Renner formulas. The calculation of S-matrix elements with the above Lagrangian is strongly connected to a theorem originally introduced by Weinberg. A consequence of the theorem of Weinberg for the perturbative description in terms of a general effective Lagrangian, uniquely derived from the underlying symmetry principles, is that the effective ChPT Lagrangian contains an infinite number of terms involving an infinite number of free parameters. Therefore one needs a systematic method to organize the Lagrangian and to count the relevant diagrams generated by the interaction part of the Lagrangian to the physical matrix elements. This method is known in the literature as the Weinberg power counting scheme. The basic idea is to relate the chiral dimension of a given diagram to the number of vertices and the number of independent loops of the diagram. According to this scheme the effective Lagrangian describing the dynamics of the Goldstone bosons, for example, is a summation of terms with increasing number of derivatives and quark mass terms and, consequently, the chiral orders are always even:

$$L_{eff} = L_2 + L_4 + L_6 + ... \ . \tag{1.22}$$

In the above expression the subscripts indicate the order of the momentum and quark mass expansion: two derivatives or one mass term, respectively, are of order $O(p^2)$, as prescribed by the Feynman rules and the on-shell condition for the Goldstone bosons $M^2 = p^2$. Each term includes tree-level diagrams and loop diagrams, as dictated by the expansion in powers of the field operator (1.15).

In fact, the appropriate effective chiral Lagrangian which describes the real physical processes at low energies involves also couplings to external fields. The coupling to external fields is realized in an elegant fashion through functional methods. One can obtain in principle the chiral Green functions through a functional derivative with respect to the external fields of an effective generating functional involving the coupling to external fields. Moreover, one can obtain the relations among the chiral Green functions known as the Ward identities. We refer the reader to the procedure of Gasser and Leutwyler [26] of introducing external fields into the Lagrangian. However, such an approach requires a locally invariant chiral Lagrangian under $SU(3)_L \times SU(3)_R$.

A series of processes can be investigated at lowest order: the weak pion decay, pion-pion scattering. The masses of the Goldstone bosons can be derived in ChPT at $O(p^4)$, by means of the construction of the most general, locally chiral invariant Lagrangian containing four derivatives of the meson fields. Chiral perturbation theory is suitable to the study of baryons as well, especially for the description of the nucleon-meson interactions. At lowest order, the locally invariant effective Lagrangian under $SU(3)_L \times SU(3)_R$ symmetry group is now written in terms of meson fields, nucleon (baryon) fields (using the convenient notation of (1.11)) and the coupling to external fields. A famous application at the tree level is the calculation of the matrix element of the axial vector between nucleon states and the derivation of the Goldberger-Treiman relation.

Chiral perturbation theory is a powerful consistent calculational technique that provides model independent predictions for a series of low-energy hadronic interactions: pion photoproduction, pion electroproduction, nucleon electromagnetic interaction, etc.

1.3 Heavy Quark Effective Theory

In the past decades, the experimental progress in heavy flavor physics has lead to improvements in the theoretical approaches as well. The discovery and formulation of heavy quark symmetry became the basis for constructing a new effective theory, the heavy quark effective theory (HQET). The HQET analyzes properties of heavy hadrons with one heavy quark using as technique an expansion in the inverse of the heavy quark mass. In the following we will try to explain the main ingredients of HQET and its achievements.

The heavy quark symmetry is exhibited in single heavy-quark systems when the mass of the heavy quark is larger than the QCD scale ($\sim$ 200

MeV). The running coupling constant of the heavy quark $\alpha_s(m_Q)$ becomes smaller, which implies that on length scales comparable to the Compton wave-length $\lambda_Q \sim 1/m_Q$ one can make use of perturbative QCD to describe strong interactions. The picture that emerges in systems of the type Qqq, one heavy quark and two light flavors, is that of a heavy quark surrounded by a cloud composed of light quarks, antiquarks and gluons. The Compton wavelength at the QCD scale is proportional to the inverse of the heavy quark mass, therefore small enough, and smaller than the hadron's size $\lambda_Q \ll R_{had}$, to be considered for a soft gluon exchange between the heavy quark and the light quarks. The main conclusion is that the light degrees of freedom become blind to the flavor and the spin orientation of the heavy quark. The only field they feel is the color field (a long range confining force) of the heavy quark, so that the whole composite object is colorless. Since the heavy quark spin participates in the interaction only through relativistic effects, in the rest frame of the heavy quark, when we attain the limit $m_Q \to \infty$, this limit provides for the decoupling of the heavy-quark spin. The conclusion is that in the limit $m_Q \to \infty$, single-heavy systems will have the same configuration of their light degrees of freedom, though the spin and flavor quanum numbers of the heavy constituent are different. From this last statement, some approximate symmetries of the strong interactions of heavy quarks can be derived. If the heavy quark is assigned the velocity v, the configuration of the light degrees of freedom does not change when replacing the heavy quark with another heavy quark of the same velocity, but different spin-flavor quantum numbers. The color field generated by each of the two heavy quarks is similar. Thus there is a $SU(2) \times SU(2)$ spin-flavor symmetry under which the strong interaction Lagrangian is invariant. In contradistinction to chiral symmetry displayed by the QCD Lagrangian, in the limit of light-quark masses $m_q \to 0$, the heavy-quark symmetry , in the limit of the heavy-quark masses $m_Q \to \infty$ is not a symmetry of the Lagrangian. The heavy-quark symmetry is a symmetry of an effective theory that is a good approximation to QCD in the low energy regime. The heavy-quark symmetry manifests itself in a certain kinematic region: when heavy quarks exchange only soft gluons. This effective low energy theory preserves Lorentz invariance [27]. The heavy quark carries most of the energy and momentum of the baryon, thus moves with the velocity of the baryon. If the whole system has the 4-velocity v^μ, the 4-momentum is $p^\mu = m_Q v^\mu_Q$, and when the mass of the heavy quark is large compared to the QCD energy scale, Λ_{QCD}, then one can approximate $m_B \sim m_Q$.

If an interaction takes place at a fixed momentum transfer value, then in the limit $m_Q \to \infty$, the new velocity of the system is $v_Q^{'\mu} \sim v_Q^\mu$. Therefore the velocity is a constant and defines the quark field of a certain heavy-quark. This is the starting point for constructing the HQET Lagrangian from the QCD Lagrangian; thus for a heavy quark we have

$$L_Q = \bar{Q}(i \not{D} - m_Q)Q \ , \tag{1.23}$$

where $\not{D} = \gamma^\mu D_\mu$ is the covariant derivative involving the gluon field. As an infinite heavy quark is irrelevant in the low energy regime, the next step would be to remove this degree of freedom. The procedure of removing the heavy quark degree of freedom consists of three steps: first, the heavy quark field is "integrated out" in the generating functional of the Green functions of the theory. Then, the resulting non-local action is rewriten as an infinite series of local terms using the Operator Product Expansion. Such an expansion corresponds to an expansion in powers of $1/m_Q$. It is this step that makes a separation between the long-distance physics (interaction below the QCD scale Λ_{QCD}) and the short-distance physics (interactions above the scale imposed by m_Q). The low energy interactions are included in the local effective theory generated, while the large virtual momenta interactions arise from quantum corrections. Such short-distance interactions are not contained in the effective theory, since the heavy quark field has been integrated out, therefore a third step is necessary: the addition of short-distance effects in a perturbative manner, using renormalization group techniques.

As mentioned, the heavy quark moves with the same velocity as the hadron and is almost on-shell

$$p_Q^\mu = p_B^\mu = m_Q v_Q^\mu + k^\mu \ , \tag{1.24}$$

where k^μ is called residual momentum exchanged by the heavy quark with the light quarks. If an interaction takes place on the hadron, its momentum will be conserved, the residual momentum will change as well as the heavy quark momentum. But, as long as the variation $\Delta k^\mu \sim \Lambda_{QCD}$ with $\Lambda_{QCD} \ll m_Q$ (that is the heavy quark exchanges only soft gluons with the light quarks and, hence, its momentum fluctuates around the mass shell by an amount of order Λ_{QCD}), then the fluctuation of the heavy quark velocity is $\Delta v^\mu \sim \frac{\Delta k^\mu}{m_Q} \to 0$ in the limit $m_Q \to \infty$.

Based on this property one can split the heavy quark field into "smaller" and "larger" components, rewrite the Lagrangian, observe that the large component does not introduce mass terms, therefore describes massless particles. The small component introduces terms $\sim 2m_Q$ and one tries to eliminate these heavy degrees of freedom. Various ways are illustrated in Ref. [28], the most elegant one being the derivation of the effective Lagrangian by integrating out the heavy degrees of freedom in the functional integral framework . One will end up with an effective Lagrangian written in terms of large components only. Following [28] we briefly show the construction of the low-energy effective theory. The heavy quark field is written in terms of "large" and "small" components:

$$Q(x) = e^{-im_Q v\cdot x}[h_v(x) + H_v(x)] \ , \tag{1.25}$$

where the "large" component field h_v and "small" component H_v field are given by:

$$\begin{aligned} h_v(x) &= e^{im_Q v\cdot x} P_+ Q(x) \\ H_v(x) &= e^{im_Q v\cdot x} P_- Q(x) \end{aligned} \tag{1.26}$$

and $P_\pm = \frac{1\pm \not{v}}{2}$ are projection operators. The new fields satisfy the following equations:

$$\begin{aligned} \not{v} h_v &= h_v \\ \not{v} H_v &= -H_v \ , \end{aligned} \tag{1.27}$$

which means that the h_v field operator anihilates a heavy quark with velocity v, whereas the H_v field operator creates a heavy antiquark with velocity v. Substituting in the QCD Lagrangian (1.23) for the heavy quark we have:

$$L_Q = \bar{h}_v iv\cdot D\, h_v - \bar{H}_v(iv\cdot D + 2m_Q)H_v + \bar{h}_v i \not{D}_\perp H_v + \bar{H}_v i \not{D}_\perp h_v \ , \tag{1.28}$$

where $\not{D}_\perp^\mu = D^\mu - v^\mu v\cdot D$ is the orthogonal part of the covariant derivative and thus $v\cdot D_\perp = 0$. It is obvious that the heavy degree of freedom which must be removed is the small component, since it introduces fluctuations with twice m_Q. The third and fourth terms in the Lagrangian mix the two components and describe the pair creation and anihilation of heavy quarks and antiquarks. In fact there are two types of virtual processes involving pair creation: the propagation of a heavy quark forward and backward in time and the heavy quark loops. Both phenomena are short-distance effects, they are excluded from the effective theory, provided that the small components are integrated out. The short-distance processes are proportional to the small coupling $\alpha_s(m_Q)$ and are calculated in perturbation theory; their effects are added to the new constructed effective theory in the renormalization procedure. The heavy degrees of freedom H_v are eliminated in a classical way, using the equation of motion. The variation of (1.28) with respect to $\bar{H}_v$ leads to

$$(iv \cdot D + 2m_Q)H_v = i \not{D}_\perp h_v \ , \tag{1.29}$$

which leads to the formal solution

$$H_v = \frac{1}{2m_Q + iv \cdot D} i \not{D}_\perp h_v \ . \tag{1.30}$$

The solution above indicates clearly that the small component is of order $1/m_Q$ and inserting this solution into (1.28) the non-local effective Lagrangian of the theory is obtained:

$$L_{eff} = \bar{h}_v iv \cdot D \, h_v + \bar{h}_v i \not{D}_\perp \frac{1}{2m_Q + iv \cdot D} i \not{D}_\perp h_v \ . \tag{1.31}$$

The second term is responsible for the first type of virtual processes mentioned above and can be equivalently written as:

$$\begin{aligned} \bar{h}_v i \not{D}_\perp \frac{1}{2m_Q + iv \cdot D} i \not{D}_\perp h_v &= \bar{h}_v i \not{D}_\perp \frac{2m_Q - iv \cdot D}{4m_Q^2} i \not{D}_\perp h_v \\ &= \bar{h}_v i \not{D}_\perp \left(\frac{1}{2m_Q} - \frac{iv \cdot D}{4m_Q^2}\right) i \not{D}_\perp h_v \ , \end{aligned} \tag{1.32}$$

where $v^2 = 1$. In the heavy quark limit $m_Q \to \infty$ this term vanishes and the effective Lagrangian of the HQET is obtained:

$$L_\infty = \bar{h}_v iv \cdot D \, h_v \ , \tag{1.33}$$

leading to the appropriate Feynman rules. The derivation of this Lagrangian can be also done by employing the generating functional for the QCD Green functions containing heavy quark fields as shown in [28]. Before ending this short description of the basic ingredients of HQET we remind that the only term (1.33) which survives in the heavy quark limit provides for the $SU(2N_f)$ symmetry of the Lagrangian for N_f-flavors. In the expression (1.33) no Dirac structures appear and consequently the interaction of the heavy quark with the gluons does not change its spin. Thus, the Lagrangian L_∞ is invariant under the $SU(2)$ symmetry. Also, since the heavy quark mass does not appear in (1.33), for two different heavy quarks moving with the same velocity, the effective Lagrangian is invariant under $SU(2)$ flavor transformations. The parameter of HQET is the heavy quark mass, used in the phase redefinition of the heavy quark field Eq. (1.25). Using the perturbation theory technique one can perform an expansion of the new Lagrangian in terms of $1/m_Q$. The definition of the mass of the heavy quark has to do with some subleties of the theory, like the residual mass term (provided by quantum correction) and the binding energy, which are non-perturbative parameters of HQET. In fact a term containing the residual mass must be included in the Lagrangian (1.33). To understand how these parameters affect the theory and how one can proceed further to include the short-distance effects using the perturbative QCD prescription, the problem of renormalization and how to calculate observables, we recommend [28]. We mention only a few of the applications of HQET : spectroscopy of the heavy mesons B and D, analysis of various decay processes and calculation of the Isgur-Wise function.

After this compact introduction of the main concepts both relevant in modelling baryons and linked to the underlying QCD, we turn to the specific model considered in the present work. First we offer a comprehensive overview of the Relativistic Three-Quark Model (RTQM) and elaborate in detail on the main ingredients of the model: the three-quark currents and the compositeness condition. We also present the procedure of gauging the non-local interaction of baryons, constituent quarks and the electromagnetic field. In the third chapter we focus on the calculation of magnetic moments of heavy baryons. In the fourth and fifth chapter we extend the applicability of the RTQM and give results for the magnetic moments of light baryons and the $N \to \Delta$ transition properties. Moreover, in the study of the light baryon sector, we take into consideration meson cloud corrections. Using a Lorentz covariant chiral quark Lagrangian we perform the dressing of the constituents by light pseudoscalar mesons. We collect the main conclusions of this work in the Summary.

Chapter 2

Relativistic Three Quark Model

2.1 General overview

In view of the difficulties of describing baryons as relativistic three-quark systems many methods and models have been developed. Of particular importance are the electromagnetic properties of baryons, which provide a great deal of information on their internal structure. In this chapter we present the main ingredients of the relativistic quark model, which later will be applied to study magnetic moments of the ground state heavy baryons, of light baryons and to investigate an old problem in hadron physics, the electromagnetic properties of $N \to \Delta$ transition.

The model has been developed and applied already for a series of processes: a study of the electromagnetic form factors of the nucleon, an analysis of the semileptonic decays of the double heavy baryons and of the semileptonic decays of Λ_b and Λ_c baryons as well as in the investigation of the strong and radiative decays of heavy baryons [7, 8, 9, 10, 11, 29, 30, 31, 32]. Ref. [7] contains a study of the electromagnetic properties of the nucleon: magnetic moments, electric and magnetic charge radii.

In Refs. [8, 9, 29] the exclusive semileptonic, nonleptonic strong and electromagnetic decays of single heavy baryons have been calculated in the heavy quark limit; [11] constitutes a first attempt to extend the model to double heavy baryons. In Ref. [30] the model was extended to the study of heavy-baryon transitions at finite values of the heavy-quark mass with no explicit expansion in terms of the inverse heavy quark mass m_Q .

In our approach the baryon is a composite system of three constituent quarks. Motivated by quantum field theory, we write a phenomenological Lagrangian, which models the interaction of the light and heavy baryons with their own constituents. The effective action preserves gauge invariance and Lorentz covariance.

The main ansaetze of the model are already made in the definition of the effective Lagrangian. The coupling of the baryon to its own constituents can be determined numerically (though not necessarily) by the compositeness condition (see Section 2.3). Starting with the Lagrangian we are able to write the S-matrix elements of baryon-baryon interaction. Further we derive the Feynman rules related to a set of Feynman quark-diagrams which are to be to computed for a given process. The model contains a few parameters: the constituent quark masses and the parameters related to the size of the distribution of the quarks inside the baryon. In all references already given above the same set of model parameters has been consistenly employed.

The coupling of a baryon B to its own constituents q_1, q_2, q_3 is described by the strong interaction Lagrangian

$$L_{int}^{str}(x) = g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 F_B(x; y_1, y_2, y_3) J_B(y_1, y_2, y_3) + h.c. \quad (2.1)$$

where $J_B(y_1, y_2, y_3)$ is the three-quark current with the quantum numbers of the baryon under consideration and $F_B(x; y_1, y_2, y_3)$ is the vertex function. For the beginning we display the general form of a three-quark current, later on we give a detailed discussion on various aspects regarding the currents, as well as the choice of specific currents. In general the three-quark current is given by:

$$J_B(y_1, y_2, y_3) = \epsilon^{abc} \Gamma_1 q_1^{Ta}(y_1) q_2^b(y_2) C \Gamma_2 q_3^c(y_3) \; , \quad (2.2)$$

where $\Gamma_{1,2}$ are Dirac structures and $C = \gamma^0 \gamma^2$ is the charge conjugation matrix; a, b, c are the color indices.

The vertex function $F_B(x; y_1, y_2, y_3)$ can be related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon [33, 34]. In principle a solution for the Bethe-Salpeter (or Fadeev-Popov) equation for baryons as bound states of three quarks could be obtained. As by now, a form for this function has not been given, the shape of our vertex function is established based on intuitive and phenomenological reasons.
The function $F_B(x; y_1, y_2, y_3)$ must be invariant under translation

$$F_B(x+a; y_1+a, y_2+a, y_3+a) = F_B(x; y_1, y_2, y_3) \ , \tag{2.3}$$

where a is any four-vector.

This function should be chosen in such manner that, at the level of computing Feynman graphs, the loop- integrals will converge in the ultraviolet region. Therefore, any shape for $F_B(x; y_1, y_2, y_3)$ is appropriate as long as it falls off sufficiently fast in this region. From a phenomenological point of view , as $F_B(x; y_1, y_2, y_3)$ is responsible for the finite size of the baryon, such a requirement is reasonable enough to set the appropriate shape of this function, though there is some freedom in the choice of the vertex function. The function $F_B(x; y_1, y_2, y_3)$ depends on the baryon and quark coordinates as:

$$F_B(x; y_1, y_2, y_3) = \delta^4(x - \sum_{i=1}^{3} \omega_i y_i) \ \Phi_B[\sum_{i<j} (y_i - y_j)^2] \ . \tag{2.4}$$

The correlation function of the three constituent quarks, Φ_B, depends on the sum of the three-quark four-space coordinates; the delta functions assures the center-of-mass system frame, the baryon has the four-coordinate x and sits at the center-of-mass (CM). The variable $\omega_i = \frac{m_i}{m_1+m_2+m_3}$, where m_1, m_2, m_3 are the constituent quark masses, depends only on the relative Jacobi coordinates (ξ_1, ξ_2). Among the possible choices to relate Jacobi coordinates to the four-space coordinates, we choose

$$\begin{aligned} \xi_1 &= \sqrt{\frac{1}{2}}(y_2 + y_3 - 2y_1) \ , \\ \xi_2 &= \sqrt{\frac{3}{2}}(y_3 - y_2) \ . \end{aligned} \tag{2.5}$$

From this definition it follows that

$$\begin{aligned} y_1 &= x - \frac{\xi_1}{\sqrt{2}}(\omega_2 + \omega_3) + \frac{\xi_2}{\sqrt{6}}(\omega_2 - \omega_3) \ , \\ y_2 &= x + \frac{\xi_1}{\sqrt{2}}\omega_1 - \frac{\xi_2}{\sqrt{6}}(\omega_1 + 2\omega_3) \ , \\ y_3 &= x + \frac{\xi_1}{\sqrt{2}}\omega_1 + \frac{\xi_2}{\sqrt{6}}(\omega_1 + 2\omega_2) \ , \end{aligned} \tag{2.6}$$

where $x = \sum_{i=1}^{3} \omega_i y_i$ is the center-of-mass coordinate.

Now we express the three-quark correlation function Φ_B in Jacobi coordinates

$$\Phi_B[\sum_{i<j}(y_i - y_j)^2] = \Phi_B[\xi_1^2 + \xi_2^2] \ . \tag{2.7}$$

The Fourier transform reads

$$\Phi_B[\xi_1^2 + \xi_2^2] = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \times e^{-ik_1\xi_1 - ik_2\xi_2} \tilde{\Phi}_B(-k_1^2 - k_2^2) \ . \tag{2.8}$$

We continue with the discussion about the specific form of the correlation function Φ_B. As already mentioned this is a priori free function , except that it should deliver Feynman graphs UV finite. In the previous work of Ref. [35] related to the pion-nucleon form factors, various shapes of this function have been tested: monople, dipole, Gaussian and Coulomb form. The conclusion is that basic observables in low-energy physics (pion weak decay constant, pion charge form factor, etc.) depend weakly on the specific choice of the vertex function.

In this work we choose an universal Gaussian function, which in Minkowski space becomes

$$\tilde{\Phi}_B(-k_1^2 - k_2^2) = \exp(\frac{k_1^2 + k_2^2}{\Lambda_B^2}) \ . \tag{2.9}$$

A Gaussian shape has the advantage to be easily used in momentum space. We call it universal, because we will use the same shape for all baryons. However, we will allow for a dependency on the flavor, in the sense that the cut off parameter Λ_B in the denominator of the exponential varies with the flavor: this parameter, related to the size of the baryon, will have a larger value as the baryon is heavier, and a smaller one for a lighter baryon. Still, within the heavy sector, the cut-off parameter will have different values, either for single, double or triple heavy baryons. The range parameter enters actively the analytical expressions of the matrix elements and at the end is replaced with numerical values obtained from the best fit to data of the observables of interest. The Feynman diagrams are calculated in Euclidean space where the above vertex function decreases rapidly. That is, with $k_{1,2}^2 \to -k_{1,2E}$ we have

$$\tilde{\Phi}_B(k_{1E}^2 + k_{2E}^2) \doteq exp(-\frac{k_{1E}^2 + k_{2E}^2}{\Lambda_B^2}) \ . \tag{2.10}$$

2.2 Three-quark baryon currents

An important issue of our Lagrangian is related to the three-quark currents. In general, hadronic currents are constructed as products of quark fields, normal ordered with respect to the perturbative vacuum and singlets under color SU(3) transformation:

$$\Psi^{ABC}_{\alpha\beta\gamma}(x,y,z) =: \Psi^{Aa}_{\alpha}(x)\Psi^{Bb}_{\beta}(y)\Psi^{Cc}_{\gamma}(z) : \epsilon_{abc} \ , \tag{2.11}$$

where Ψ^{Aa}_{α} is a Dirac spinor with flavour A and color a and ”::” denotes the normal-ordered product with respect to the perturbative vacuum.

Following Ref. [36] we will list briefly the properties of the linear combinations of the operators $\Psi^{ABC}_{\alpha\beta\gamma}(x,y,z)$ under Lorentz transformations and SU(3) flavour transformations. We only consider operators which do not contain derivatives of the quark fields, thus we describe the baryon ground state. If α,β,γ are the Dirac indices, μ,ν,λ,ρ the Lorentz indices and C the charge conjugation matrix, then for u,d,s flavours the three-quark product : $u_\alpha d_\beta s_\gamma$: can be decomposed into the irreducible representations (see Appendix A) $D_{(1/2,0)}, D_{(1,1/2)}, D_{(3/2,0)}$ of the Lorentz group.
In the $D_{(1/2,0)}$ representation there are 5 different operators :

$$\begin{gathered}(u^T C d)s_\gamma \ , \\ (u^T C\gamma^5 d)s_\gamma \ , \\ (u^T C\gamma^5\gamma^\mu d)(\gamma^5\gamma_\mu s)_\gamma \ , \\ (u^T C\gamma^\mu d)(\gamma_\mu)s_\gamma \ , \\ (u^T C\sigma^{\mu\nu} d)(\sigma_{\mu\nu} s)_\gamma \ .\end{gathered} \tag{2.12}$$

In the $D_{(1,1/2)}$ representation there are 3 different operators :

$$\begin{gathered}(u^T C\gamma_\rho d)[(g^{\mu\rho} - \tfrac{1}{4}\gamma^\mu\gamma^\rho)s]_\gamma \ , \\ (u^T C\gamma^5\gamma_\rho d)[(g^{\mu\rho} - \tfrac{1}{4}\gamma^\mu\gamma^\rho)s]_\gamma \ , \\ (u^T C\sigma_{\rho\lambda} d)[(g^{\mu\rho} - \tfrac{1}{4}\gamma^\mu\gamma^\rho\gamma^\lambda)s]_\gamma \ .\end{gathered} \tag{2.13}$$

The spin-3/2 components of these operators must satisfy the Rarita-Schwinger constraint $\gamma_\mu\Psi^\mu = 0$.
In the $D_{(3/2,0)}$ representation there is only one operator :

$$(u^T C\sigma_{\rho\lambda} d)[(g^{\mu\rho}g^{\nu\lambda} - \frac{1}{2}g^{\nu\lambda}\gamma^\mu\gamma^\rho + \frac{1}{2}g^{\mu\lambda}\gamma^\nu\gamma^\rho\frac{1}{2}\sigma^{\mu\nu}\sigma^{\rho\lambda})s]_\gamma \ , \tag{2.14}$$

which satisfies the constraints $\Psi^{\mu\nu} = -\Psi^{\nu\mu}$ and $\gamma_\mu \Psi^{\mu\nu} = 0$.

All other operators without derivatives , like $(d^T C s)u$ can always be reduced to the ones listed above using Fierz transformations, which will be discussed later. For now we observe that it is possible to decompose all these operators into irreducible representations of flavour SU(3). In $D_{(1/2,0)}$ there are four octets and one singlet to form a basis for this representation. The following form is suitable for the singlet:

$$(u^T C d) s_\gamma \epsilon_{uds} \ . \tag{2.15}$$

For the octet there are two types of operators, the $\Lambda-$type

$$\begin{aligned} &\sqrt{\frac{1}{6}}\{2(u^T C\gamma^5 d)s_\gamma + (u^T C\gamma^5 s)d_\gamma - (d^T C\gamma^5 s)u_\gamma\} \ , \\ &\sqrt{\frac{1}{6}}\{(u^T C d)\gamma^5 s_\gamma + (u^T C s)\gamma^5 d_\gamma - (d^T C s)\gamma^5 u_\gamma\} \end{aligned} \tag{2.16}$$

and $\Sigma-$type

$$\begin{aligned} &\sqrt{\frac{1}{2}}\{(u^T C\gamma^5 s)d_\gamma + (d^T C\gamma^5 s)u_\gamma\} \ , \\ &\sqrt{\frac{1}{2}}\{(u^T C s)d_\gamma + (d^T C s)u_\gamma\} \ . \end{aligned} \tag{2.17}$$

These operators contain spin-1/2 components, therefore they are suitable to describe the spin-1/2 baryons. The number of operators reduces if two flavours are identical, like in the case of the nucleon, when only one most general operator can be written in $D_{(1/2,0)}$

$$a(u^T C\gamma^5 d)u_\gamma + b(u^T C d)\gamma^5 u_\gamma \tag{2.18}$$

where a, b are arbitrary constants.
For the $D_{(1,1/2)}$ there is one decuplet operator

$$\sqrt{\frac{1}{3}}(u^T C\gamma^\mu d)s_\gamma + (d^T C\gamma^\mu s)u_\gamma + (s^T C\gamma^\mu u)d_\gamma \tag{2.19}$$

and two octet operators, one $\Lambda-$type

$$\sqrt{\frac{1}{6}}(g^{\mu\rho} - \frac{1}{4}\gamma^\mu\gamma^\rho)_{\beta\gamma}[2(u^T C\gamma^5\gamma_\rho d)s_\gamma + (u^T C\gamma^5\gamma_\rho s)d_\gamma - (d^T C\gamma^5\gamma_\rho s)u_\gamma] \tag{2.20}$$

and one $\Sigma-$type

$$\sqrt{\frac{1}{2}}(g^{\mu\rho} - \frac{1}{4}\gamma^{\mu}\gamma^{\rho})_{\beta\gamma}[(u^T C\gamma^5\gamma_{\rho}s)d_{\gamma} + (d^T C\gamma^5\gamma_{\rho}s)u_{\gamma}] \ . \qquad (2.21)$$

In this representation there are contributions from both the spin-1/2 and spin-3/2 components. If two flavours are chosen equal, there is only one operator left

$$(u^T C\gamma^5 d)[(g^{\mu\rho} - \frac{1}{4}\gamma^{\mu}\gamma^{\rho})u]_{\gamma} \ . \qquad (2.22)$$

The $D_{(3/2,0)}$ representation contains only one decuplet, containing only the spin-3/2 components.

An important property of the interpolating fields is the parity transformation. If the quark field Ψ_{α} is assigned internal positive parity

$$P\Psi_{\alpha}(x) = \gamma^0_{\alpha\alpha'}\Psi_{\alpha'}(x') \ , \qquad (2.23)$$

then the parity transformation properties are easily obtained for the composite fields. For example, a structure like $\bar{\Psi}\gamma^5\Psi$ transforms like a pseudoscalar under parity transformation, but $(\bar{\Psi}_{\alpha}C\gamma^5\Psi_{\beta})\Psi_{\gamma}$, where $C = \gamma^0\gamma^2$, is positive under parity. The parity of any of the operators listed above can be reversed by simply multiplying with γ^5. The role played by the charge conjugation matrix C is to preserve the non-relativistic limit of these currents, by an appropriate mixing of the upper and lower components.

Following the literature provided by Ref. [36] one can choose the optimal interpolating field as constrained by a set of inequalities for the baryonic masses. In fact, based on the approach suggested by Shifman, Vainshtein and Zakharov [37], a baryon sum rule investigation is carried out in [36], taking into consideration also non-perturbative terms, due to the non-vanishing value of quark and gluon condensates. The inclusion of non-perturbative effects plays an important role for the selection of the appropriate baryonic currents. In the language of baryon QCD sum rules, the inclusion of non-perturbative effects resumes to using the full propagator, perturbative and non-perturbative piece. For example, in the case of spin$-(1/2)^+$ baryons, the calculation of the two-point correlation function for a specific baryon is given by the time-order product of the above relevant currents, on one side, and can be expressed by the dispersion relation (Lehmann representation), on the other side. A Borel tranformation of this sum rule sets two independent inequalities for the baryon masses. The experimental measurement of the spectral function rules out certain structures for the currents and sets an upper limit for the baryon masses. Also, the non-perturbative piece in the propagator indicates that the baryon mass receives an important contribution from the scalar condensate. The calculation of the two-point correlation function at high momenta is done within perturbative QCD, while the non-perturbative parameters are obtained from the fit to experimental values of the spectral function. In principle, more complicated operators can be used, including fields with more than three quarks or with derivatives of the quark fields.

We will retain from all this additional extension only the basic principle of constructing three-quark currents. To strengthen the main ideas on the choice of the currents, we offer another example, given in Ref. [38]. For the Δ^{++} isobar, the physical state consists of s-wave quarks so that the introduction of derivatives of the quark fields is not desired. Ioffe constructs currents from current quarks and underlines the simplicity of using three-quark currents as products of only three quark fields, and not to use p-wave current quarks.

In this vein, the form of the current with the necessary quantum numbers of Δ^{++} (isospin $T = \frac{3}{2}$ and $J^P = \frac{3^+}{2}$) reads

$$\Psi = (u^{Ta} C \gamma_\mu u^b) u^c \epsilon^{abc} \ . \tag{2.24}$$

The dependence on the coordinate has been omitted and for simplicity it has been assumed to be a local operator. Futhermore, it can be proven [38] that all other non-zero currents can be expressed via this relation. All possible currents can be written in the most general form

$$\Psi = (u^{Ta} C O_k u^b) \gamma_\mu O_k u^c \epsilon^{abc} \tag{2.25}$$

with $O_k = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$. If we perform transposition of the spinors u^a and u^b and use $C^T = -C$ then we have

$$u^{Ta} C u^b \epsilon^{abc} = u^{Ta} C \gamma^5 u^b \epsilon^{abc} = u^{Ta} C \gamma_\nu \gamma_5 u^b \epsilon^{abc} = 0 \ . \tag{2.26}$$

The transposition of the spinors u^b and u^c and the use of Fierz transformation give

$$(u^{Ta} C \sigma_{\lambda\nu} u^b) \gamma_\mu \sigma_{\lambda\nu} u^c \epsilon^{abc} = -\frac{1}{2} (u^{Ta} C \sigma_{\lambda\nu}) \gamma_\mu \sigma_{\lambda\nu} u^c \epsilon^{abc} \ . \tag{2.27}$$

With the help of the Fierz transformation and the relation (2.26) we obtain

$$(u^{Ta}C\gamma_\lambda u^b)\sigma_{\lambda\mu}u^c\epsilon^{abc} = i(u^{Ta}C\gamma_\mu u^b)u^c\epsilon^{abc} \ . \tag{2.28}$$

It can be seen that the form proposed a priori (2.24) is a unique current for the Δ^{++} isobar. There is a non-relativistic limit for this current, which is desirable.

In the case of the nucleon one proceedes similarly. Using the above equalities, one employes the following three-quark current with quantum numbers of the nucleon and without quark field derivatives:

$$\Psi = (u^{Ta}C\gamma_\mu u^b)\gamma_5\gamma^\mu d^c\epsilon^{abc} \ . \tag{2.29}$$

There is one more current without derivatives and with the desired properties, the tensor current

$$\Psi_1 = (u^{Ta}C\sigma_{\mu\nu}u^b)\sigma^{\mu\nu}\gamma_5 d^c\epsilon^{abc} \ . \tag{2.30}$$

However, in the sum rule approach this current is discarded in practical calculations, since the contribution of the amplitudes of the baryon transition into the tensor current are small [38].

To convince the reader of the uniqueness of the three-quark current for the Δ^{++} state, we refer to a detailed discussion of the QCD equalities for baryon matrix elements in [39]. First of all, Ref. [39] gives a justification of the basic ideas of writing such general QCD motivated expression for three-quark currents. This work is focused on the fundamental QCD equalities based directly on the symmetries of the QCD path integral. Furthermore, such equalities must be met by any model which aims to describe nonperturbative QCD. A good approximation and the only one used in this work is the isospin limit, $m_u = m_d$, as $\Lambda_{QCD} \gg (m_d - m_u)$. Important aspects in this work related to the construction of three-quark currents may be found: a review of the path integral formalism and a discussion of the operators used to calculate current matrix elements for low-lying baryons.

It is well known that in field theories the calculation of hadronic properties is centered around the QCD vacuum expectation values of the appropriate operators. In the path-integral formalism the expectation values are calculated using the functional integral. The exponent contains the gauge action and a mass term depending on the gauge field. One can perform the functional integration and observe that every pair of fermion-antifermion fields generates the well-known free fermion propagator.

The essence of QCD equalities lies in those terms that contain the fermion propagators multiplied by various Dirac structures, thus the physical difference between various hadrons. The rest of the terms (the exponential of the gauge action and the determinant of the mass) are common to any hadron considered. To calculate the current matrix elements one uses the time-ordered product of three operators: an operator exciting the hadron from the vacuum, followed by a current operator, the one suitable to the calculation of the desired properties, followed by an operator which anihilates the hadron to the vacuum.

Ref. [39] also contains a discussion of the simplest case of Δ^{++} and the unique choice of the three-quark current. Also it is shown that operators which couple to the other Δ's are obtained from the one for Δ^{++} by using the isospin-lowering operator. Other decuplet baryon fields are obtained by a proper substitutions among the quark fields. In the case of the nucleon, there are two local fields of minimal dimension, being commonly used a linear combination of the two forms.

To calculate observables, one needs to calculate the two-point correlation function, thus the quark field operator pairs have to be contracted, which leads to the quark propagators. For the baryonic currents we therefore consider non-local operators as product of three constituent quark fields linearly combined in such way that the full expression has the quantum numbers of the baryon under consideration.

Since the mathematical technique for writing all possible three-quark baryon currents is the same, with disregard whether one uses current quarks or constitutent quarks, we rewrite the most general form of a nonlocal baryonic current with no quark field derivatives as

$$J_B(y_1, y_2, y_3) = \Gamma_1 q_1^{Ta}(y_1)[q_2^b(y_2) C\Gamma_2 q_3^c(y_3)]\epsilon^{abc} \;, \tag{2.31}$$

which we employ for the description of the properties of the ground state baryons all way through this work. In the above expression the antisymmetric tensor ϵ^{abc} preserves the baryon as a color singlet and y_i, $i = 1, 2, 3$ are the four-space coordinates of the three quarks; the flavour matrix is omitted. The Γ matrices provide the right spin quantum numbers of a given baryon with $\Gamma_{1,2} = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$.

One can construct five bilinear covariants under Lorentz transformations

$$
\begin{array}{lc}
Scalar & J^S = \gamma_5 \; q_1^{Ta} \; q_2^b \; C \; q_3^c \; \epsilon^{abc} \; , \\
Pseudoscalar & J^P = q_1^{Ta} \; q_2^b \; C \; \gamma_5 \; q_3^c \; \epsilon^{abc} \; , \\
Vector & J^V = \gamma_\mu \; \gamma_5 \; q_1^{Ta} \; q_2^b \; C \; \gamma^\mu \; q_3^c \; \epsilon^{abc} \; , \\
Axial-vector & J^{AV} = \gamma_\mu \; q_1^{Ta} \; q_2^b \; C \; \gamma^\mu \; \gamma^5 \; q_3^c \; \epsilon^{abc} \; , \\
Tensor & J^T = \sigma_{\mu\nu} \; q_1^{Ta} \; q_2^b \; C \; \sigma^{\mu\nu} \; q_3^c \; \epsilon^{abc} \; .
\end{array} \tag{2.32}
$$

In the following we will discuss the non-relativistic limit of these currents and the Fierz transformation. In principle, for a given baryon one should take the linear combination of all the five forms, but due to these transformations one can restrict their number. Later on, we will see that a study of heavy baryons helps in choosing a unique form, based on symmetry considerations.

To illustrate in an intuitive fashion the usefulness of the Fierz transformations, let us make the following notations for various Dirac matrix products entering the currents

$$
\begin{aligned}
P &= I \otimes C\gamma_5 \; , \\
S &= \gamma_5 \otimes C \; , \\
A &= \gamma^\mu \otimes C\gamma_\mu\gamma_5 \; , \\
V &= \gamma^\mu\gamma_5 \otimes C\gamma_\mu \; , \\
T &= \frac{1}{2}\sigma^{\mu\nu}\gamma_5 \otimes C\sigma_{\mu\nu} \; .
\end{aligned} \tag{2.33}
$$

The factor $\frac{1}{2}$ in front of the tensor form provides for a proper normalization. The Fierz identities allow to rewrite the product of two spinor bilinears as a linear combination of products of two spinor bilinears [40].

Most generally, the Fierz transformations for the 16 Dirac structures read:

$$
\Gamma_1^{\alpha\beta} \otimes \Gamma_2^{\rho\sigma} = \Gamma_1^{\alpha\mu} \otimes \Gamma_2^{\rho\nu} \delta^{\mu\beta} \delta^{\nu\sigma} \tag{2.34}
$$

with the completeness condition

$$
\delta^{\mu\beta} \delta^{\nu\sigma} = \frac{1}{4} \sum_l O_l^{\mu\sigma} O_l^{\nu\beta} \; . \tag{2.35}
$$

It follows that

$$
\begin{aligned}
\Gamma_1^{\alpha\beta} \otimes \Gamma_2^{\rho\sigma} &= \frac{1}{4} \Gamma_1^{\alpha\mu} \otimes \Gamma_2^{\rho\nu} \sum_l O_l^{\mu\sigma} O_l^{\nu\beta} \\
&= \frac{1}{4} \sum_l (\Gamma_1 O_l)^{\alpha\sigma} \otimes (\Gamma_2 O_l)^{\rho\beta} \; .
\end{aligned} \tag{2.36}
$$

So, the various Dirac matrices can be easily related to other Dirac matrices via

$$\Gamma_1 \otimes \Gamma_2 = \frac{1}{4}\sum_l (\widetilde{\Gamma_1 O_l}) \otimes (\widetilde{\Gamma_2 O_l}) \ . \tag{2.37}$$

In our case (see the notations (2.33)), Fierz transformations relate the following structures

$$\begin{aligned}
S &= \frac{1}{4}[\tilde{S} + \tilde{P} - \tilde{V} + \tilde{A} + \frac{\tilde{T}}{2}] \ , \\
P &= \frac{1}{4}[\tilde{S} + \tilde{P} + \tilde{V} - \tilde{A} + \frac{\tilde{T}}{2}] \ , \\
V &= \tilde{P} - \tilde{S} - \frac{1}{2}[\tilde{V} + \tilde{A}] \ , \\
A &= \tilde{S} - \tilde{P} - \frac{1}{2}[\tilde{V} + \tilde{A}] \ , \\
T &= 3(\tilde{S} + \tilde{P} - \frac{\tilde{T}}{2}) \ .
\end{aligned} \tag{2.38}$$

From here one can extract the relations

$$\begin{aligned}
P + S &= \frac{1}{2}[\tilde{P} + \tilde{S} + \frac{\tilde{T}}{2}] \ , \\
P - S &= \frac{1}{2}[\tilde{V} - \tilde{A}] \ .
\end{aligned} \tag{2.39}$$

To show the behaviour in the non-relativistic limit we choose as an example the nucleon currents. There are two possibilities for the nucleon: vector and tensor current with

$$\begin{aligned}
J_p^V &= \gamma^\mu \gamma^5 d^{Ta} u^b C \gamma_\mu u^c \epsilon^{abc} \ , \\
J_p^T &= \frac{1}{2}\sigma^{\mu\nu}\gamma^5 d^{Ta} u^b C \sigma_{\mu\nu} u^c \epsilon^{abc} \ .
\end{aligned} \tag{2.40}$$

Keeping in mind that $C = \gamma_0\gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$ and $C\gamma^5 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}$, we list below the non-relativistic limits:

$Pseudoscalar:$ $\quad u^{Ta} u^b C \gamma^5 d^c \epsilon^{abc}$

$$
\begin{aligned}
&= \epsilon^{abc}\chi_u^{fl}\chi_a^{color}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_u^{fl}\chi_b^{color}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_2}\left(\begin{array}{cc}0 & \sigma_2\\ \sigma_2 & 0\end{array}\right)\\
&\times \chi_d^{fl}\chi_c^{color}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_3}\\
&= ...\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\chi_{s_3}\\
Scalar: \quad & \gamma^5 u^{Ta}u^bCd^c\epsilon^{abc} \to 0\\
Axial-Vector: \quad & \gamma^\mu u^{Ta}u^bC\gamma_\mu\gamma_5 d^c\epsilon^{abc}\\
&= \gamma^0 u^{Ta}u^bC\gamma_0\gamma_5 d^c\epsilon^{abc} - \underbrace{\vec{\gamma}u^{Ta}u^bC\vec{\gamma}\gamma_5 d^c\epsilon^{abc}}_{=0}\\
&= ...\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\chi_{s_3}\\
Vector: \quad & \gamma^\mu\gamma^5 d^{Ta}u^bC\gamma_\mu u^c\epsilon^{abc}\\
&= \gamma^0\gamma^5 d^{Ta}u^bC\gamma_0 u^c\epsilon^{abc} - \vec{\gamma}\gamma^5 d^{Ta}u^bC\vec{\gamma}u^c\epsilon^{abc}\\
&= ...\vec{\sigma}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\vec{\sigma}\chi_{s_3}\\
Tensor: \quad & \frac{1}{2}\sigma^{\mu\nu}\gamma^5 d^{Ta}u^bC\sigma_{\mu\nu}u^c\epsilon^{abc}\\
&= \sigma^{0i}\gamma^5 d^{Ta}u^bC\sigma_{0i}u^c\epsilon^{abc}\\
&= ...\vec{\sigma}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\vec{\sigma}\chi_{s_3}
\end{aligned}
\tag{2.41}
$$

Here, σ_i stands for the Pauli matrices and χ_{s_i} stands for the quark spin wave functions. In each expression above the dots stand for the spin-wave function of the baryon $\chi_{s_B}^+(1\ \ 0)$. Hence, in the non-relativistic limit, both

$$\text{pseudoscalar and axial-vector} \quad \Longrightarrow \left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\chi_{s_3}\ ,$$

$$\text{vector and tensor} \Longrightarrow \vec{\sigma}\left(\begin{array}{c}1\\0\end{array}\right)\chi_{s_1}\chi_{s_2}\sigma_2\vec{\sigma}\chi_{s_3}\ , \tag{2.42}$$

become degenerate. In a more transparent notation, the pseudoscalar current (PS), for example, can be written in its non-relativistic (NR) form as:

$$PS_{NR} \to 1_{2\times 2}\otimes\sigma_2\times(u\,u\,d)\times(\uparrow\uparrow\downarrow)\ , \tag{2.43}$$

where $\uparrow$ and $\downarrow$ represent the quark spinors $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively, and u, d are the quark flavours.

As shown it is suitable to choose two nucleon currents, vector (V) and tensor (T), as the scalar current vanishes in the non-relativistic limit and, due to the Fierz transformations, the other two currents can be easily related to V and T.

Now we will discuss the heavy baryon currents. In our approach magnetic moments are calculated for single-, double- and triple heavy baryons. The three-quark currents with correct quantum numbers are of two types, according to the symmetry/antisymmetry of the baryon wave function under permutation of flavour indices of the second and third quark, respectively. Therefore, we have $\Lambda-$type baryons, bound states of a quark and a diquark system (with spin 0), and $\Sigma-$type baryons, bound states of a quark and diquark system (with spin 1). The $\Lambda-$type non-derivative currents that in general can be constructed are pseudoscalar, scalar and axial-vector. These currents are antisymmetric under permutation of flavour indices of quark 2 and 3 as indicated by the symbol [...]

$$\begin{aligned} J^P_{\Lambda_{q_1[q_2q_3]}} &= \epsilon^{abc} q_1^{Ta} q_2^b C\gamma_5 q_3^c \ , \\ J^S_{\Lambda_{q_1[q_2q_3]}} &= \epsilon^{abc} \gamma^5 q_1^{Ta} q_2^b C q_3^c \ , \\ J^A_{\Lambda_{q_1[q_2q_3]}} &= \epsilon^{abc} \gamma^\mu q_1^{Ta} q_2^b C\gamma_\mu\gamma_5 q_3^c \ . \end{aligned} \tag{2.44}$$

The $\Sigma-$type non-derivatives currents that can be constructed are vector and tensor. These currents are symmetric in flavour indices of quark 2 and 3 as indicated now by the symbol {...}

$$\begin{aligned} J^V_{\Sigma_{q_1\{q_2q_3\}}} &= \epsilon^{abc} \gamma^\mu\gamma^5 q_1^{Ta} q_2^b C\gamma_\mu q_3^c \ , \\ J^T_{\Sigma_{q_1\{q_2q_3\}}} &= \epsilon^{abc} \sigma^{\mu\nu}\gamma^5 q_1^{Ta} q_2^b C\sigma_{\mu\nu} q_3^c \ . \end{aligned} \tag{2.45}$$

q_i stands for a light or heavy quark, so that q_iq_j, $i,j = 2,3$ can be a light-diquark system or a heavy-diquark system. In each case we will restrict to one form: the pseudoscalar form for $\Lambda-$type baryons and the vector form for $\Sigma-$type baryons. As we saw these currents have the right non-relativistic limit. The scalar current goes to zero in the non-relativistic limit, the pseudoscalar and the axial-vector become degenerate; the naive quark model spin-flavor wave function matching these currents reads:

$$|\Lambda_{q_1[q_2q_3]}> = \frac{1}{2}|q_1(q_2q_3 - q_3q_2)> | \uparrow (\uparrow\downarrow - \downarrow\uparrow)> \ . \tag{2.46}$$

The same degeneracy in the non-relativistic limit is encountered by the vector and the tensor currents; the naive quark model spin-flavor wave function reads:

$$|\Sigma_{q_1q_2q_3}>= \frac{1}{2\sqrt{3}}|q_1(q_2q_3+q_3q_2)>|\uparrow(\uparrow\downarrow+\downarrow\uparrow)-2\downarrow\uparrow\uparrow> \quad . \tag{2.47}$$

With these identities, one can safely employ one of the forms in each case. We restrict to spin-$1/2^+$ baryons.

Now we will discuss the choice of the three-quark currents used to study the $N-\Delta$ transition. In the calculation of the baryon matrix elements of the valence quarks we deal only with light flavours and display the isospin limit explicitly. As we know already from the discussion of Ioffe [38], the $\Delta^+(1232)$ has the most simple choice for the current. One can write a nonlocal three-quark vector-type current, with no derivatives, to meet the correct quantum numbers of a Rarita-Schwinger spinor, $J^P=\frac{3}{2}^+$

$$J^\mu_\Delta = \epsilon^{abc}\Gamma_1 d^{Ta}(y_1)u^b(y_2)C\Gamma_2 u^c(y_3)\ , \tag{2.48}$$

with $\Gamma_1 = 1$ and $\Gamma_2=\gamma^\mu$. For the sake of completeness we will return to Ref. [39] to illustrate how the tensor and the vector form can be expressed using the above form. One can start with a vector current for Δ^+, as in Ref. [39], with

$$\begin{aligned}
J^\mu_{\Delta^+} &= \frac{\epsilon^{abc}}{2\sqrt{3}}[d^{Ta}u^bC\gamma^\mu u^c + 2u^{Ta}u^bC\gamma^\mu d^c] \\
&= \frac{\epsilon^{abc}}{2\sqrt{3}}[d^{Ta}u^bC\gamma^\mu u^c + d^{Ta}u^bC\gamma^\mu u^c + \gamma_5 d^{Ta}u^bC\gamma^\mu\gamma^5 u^c - i\gamma_\nu d^{Ta}u^bC\sigma^{\mu\nu}u^c] \\
&= \frac{\epsilon^{abc}}{2\sqrt{3}}[2d^{Ta}u^bC\gamma^\mu u^c - i\gamma_\nu d^{Ta}u^bC\sigma^{\mu\nu}u^c]\ ,
\end{aligned} \tag{2.49}$$

where we made use of the following Fierz transformations in the last two lines

$$\begin{aligned}
V_\mu &= 1\otimes C\gamma_\mu\ , \\
T_\mu &= i\gamma^\nu\otimes C\sigma_{\mu\nu}\ ,
\end{aligned}
\qquad
\begin{aligned}
V_\mu &= \frac{1}{2}[\tilde{V}_\mu+\tilde{A}_\mu-\tilde{T}_\mu]\ , \\
A_\mu &= \frac{1}{2}[\tilde{V}_\mu+\tilde{A}_\mu+\tilde{T}_\mu]\ , \\
T_\mu &= -\tilde{V}_\mu+\tilde{A}_\mu\ .
\end{aligned} \tag{2.50}$$

The general current is then given by

$$\alpha V_\mu + \beta A_\mu + \gamma T_\mu = \tilde{V}_\mu[\frac{\alpha}{2} + \frac{\beta}{2} - \gamma] + \tilde{A}_\mu[\frac{\alpha}{2} + \frac{\beta}{2} + \gamma] + \tilde{T}_\mu[-\frac{\alpha}{2} + \frac{\beta}{2}] \ , \tag{2.51}$$

where the structures enter with various weights α, β and γ. Redefining

$$\begin{aligned} &\beta = 0 \\ &\frac{\alpha+\beta}{2} + \gamma = 0 \\ &\alpha = \frac{\alpha}{2} + \frac{\beta}{2} - \gamma \\ &\gamma = -\frac{\alpha - \beta}{2} \ , \end{aligned}$$

we obtain

$$\begin{aligned} &\beta = 0 \\ &\frac{\alpha}{2} = -\gamma \ , \end{aligned}$$

and the left-hand side of Eq. (2.51) becomes

$$\alpha[V_\mu - \frac{T_\mu}{2}] \ .$$

In this way we obtain the terms in the last line of Eq. (2.49). In fact, it is common to absorbe the factor $\frac{1}{2}$ in Eq. (2.49) in the redefinition of the tensor current (see also Eq. (2.33)).

As already mentioned, for the nucleon we will choose the vector and tensor currents, testing also the sensitivity of the observables on such a choice. In a most general notation we have for the non-local current

$$J_p^i = \epsilon^{abc}\Gamma_1 d^{Ta}(y_1) u^b(y_2) C\Gamma_2 u^c(y_3) \ , \tag{2.52}$$

where

$$\begin{aligned} i = V \quad &\to \quad \Gamma_1 = \gamma^\mu\gamma^5 \ , \ \Gamma_2 = \gamma_\mu \ , \\ i = T \quad &\to \quad \Gamma_1 = \sigma^{\mu\nu}\gamma^5 \ , \ \Gamma_2 = \sigma_{\mu\nu} \ . \end{aligned} \tag{2.53}$$

A complete list of the three-quark currents for all the baryon states can be found in Appendix B.

2.3 Compositeness condition

In the interaction Lagrangian (2.1), both hadron and quark degrees of freedom enter. To make sure that there is no double counting, in our approach we will use the concept provided by the compositeness condition.

The coupling constants g_B in the Lagrangian can be determined by this condition. The compositeness condition implies that the renormalization constant of the hadron wave function is set to zero:

$$Z_B = 1 - \Sigma'_B(m_B) = 1 - g_B^2 \Pi'_B(m_B) = 0, \tag{2.54}$$

where Σ'_B is the derivative of the baryon mass operator.

Before explaining the consequences and the physical meaning of this condition, we give a brief overview of the historical hypothesis and the necessity of the compositeness condition.

A first formulation of the compositeness condition has been given by Weinberg in 1962 [41]. He attempts to rewrite a non-relativistic theory by introducing fictitious elementary particles. In the Hamilton formalism it is proven that the physical observables are not affected by this procedure, as long as the interaction part of the Hamiltonian is properly modified (it is sufficiently weak to allow the use of perturbation theory). The basic idea is to represent real composite particles, such as baryons, by fictitious elementary particles.

In this vein, a fictitious elementary particle is in fact a real elementary particle with infinite unrenormalized bare mass, therefore with a wave function renormalization constant Z equal to zero.

Weinberg further discusses the consequences of introducing quasi-elementary particles in a theory in such way that the physical predictions are not affected. The equivalence with the original theory is given by imposing that bare energy of the quasi-particles is much larger than any energy explored by the experiment.

The main achievement emerging from this condition is the calculation of the coupling of a real composite particle to its own constituents. The compositeness condition distinguishes between bound states and elementary particles, providing a sum rule for the coupling of the bound state to its presumed constituents. For a full understanding of the mathematical method used to introduce quasi-particles, we recommend the reading of Ref. [41]. Here we retain only that $Z^{1/2}$ is the matrix element between the physical state, the baryon for example, and the bare state, the fictious elementary baryon:

$$< B_0|B >= Z^{1/2} . \tag{2.55}$$

One can see that the emerging inequality $0 < Z < 1$ sets an upper limit for the coupling constant of the physical particle to its constituents, the limit attended at $Z = 0$, equivalent to stating that the physical particle ($|B >$ one-particle state) is not elementary at all.

Further insight in the compositeness criteria of particles has been realized in Ref. [42]. In a Lagrangian approach, a composite particle in Quantum Field Theory is a composite one provided that it is not included in the original Lagrangian and can be eliminated from the Lagrangian by setting its wave function renormalization constant to zero.

The work of Ref. [42] illustrates the case of a Yukawa interaction between a scalar field and a Dirac field, i.e. between nucleons and pions. It is proven that in the picture of a pion as a composite of nucleon-antinucleon, the unrenormalized Yukawa coupling tends to the Fermi coupling. Several equivalence proofs between compositeness condtion in Quantum Field Theory and the S-matrix theory may be found.

The condition $Z_B = 0$ imposes that the physical state does not contain the bare one, therefore the physical baryon is described as a bound state.

The full Lagrangian, the interaction part and the free part, contains both the constituent degrees of freedom (quarks) and the physical particles (baryons) which are taken to be bound states of the constituents. As a result of the interaction of the bound state with its own constituents and the consequences of the compositeness condition, it comes out that the physical state can exist only as a dressed one. That is, its mass and wave function have to be renormalized. Thereby the $Z_B = 0$ condition effectively excludes the constituent degrees of freedom from the physical space. The picture which emerges from this condition is that constituents can exist only as virtual states.

As a corrolary, the compositeness condition provides for the absence of a direct interaction of the dressed charged particle (the physical baryon) with the electromagnetic field. The interaction with external fields holds only at the level of the constituent quark.

Taking into account both the tree-level diagram and the diagrams with self-energy insertions into the external legs (by means, the tree-level diagram times $(Z_B - 1)$), a common factor Z_B is obtained and this one is equal zero.

To describe hadron observables a model has been proposed and developed by the Dubna group [7, 31, 32, 43]. The subject of the compositeness condition and the basic ideas are strongly connected to confinement and QCD-hadronization, two main ingredients which are included in the Quark Confinement Model, a QCD-motivated model. A complete mathematical procedure with respect to the link between Quantum Field Theory and the compositeness condition is given in Ref. [43].

2.4 Electromagnetic interaction

In the following we will show how to introduce the electromagnetic interaction in our non-local baryon-quark Lagrangian. First of all, we use the standard free Lagrangians for quarks and baryons

$$\mathcal{L}_{\rm free}(x) = \bar{B}(x)(i\not{\partial} - m_B)B(x) + \sum_q \bar{q}(x)(i\not{\partial} - m_q)q(x) \; . \tag{2.56}$$

When introducing the interaction with the electromagnetic field, the free Lagrangians are gauged in the standard manner using minimal substitution

$$\begin{aligned} \partial^\mu B \to (\partial^\mu - ie_B A^\mu)B \,, &\quad \partial^\mu \bar{B} \to (\partial^\mu + ie_B A^\mu)\bar{B} \,, \\ \partial^\mu q_i \to (\partial^\mu - ie_{q_i} A^\mu)q_i \,, &\quad \partial^\mu \bar{q}_i \to (\partial^\mu + ie_{q_i} A^\mu)\bar{q}_i \,, \end{aligned} \tag{2.57}$$

where e_B is the electric charge of the baryon B and e_{q_i} the electric charge of the quark of flavor q_i. The resulting Lagrangian reads as

$$L^{em(1)}_{int}(x) = e_B \bar{B}(x)\not{A}B(x) + \sum_q e_q \bar{q}(x)\not{A}q(x) \; . \tag{2.58}$$

As we stated before, due to the compositeness condition, the electromagnetic field does not couple directly to the charged physical field, but it couples to the virtual particles, the constituents.

To introduce the electromagnetic interaction in the nonlocal baryon-quark Lagrangian in such a fashion that gauge invariance is preserved, we will follow the prescription of Ref. [44] and already successfully used in Ref. [7].

We will explain in brief the main ideas used in Ref. [44]. An easy example is the gauging of a local theory which couples quarks to photons, a case in which one makes use of the standard minimal substitution

$$L = \bar{q}(x)[\gamma^\mu \partial_\mu - ieQA_\mu(x)]q(x) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.59}$$

with Q being the charge matrix of the quarks. This Lagrangian preserves gauge invariance as can be easily checked when performing the infinitesimal transformations

$$\begin{aligned} q(x) &\to [1 + i\alpha(x)Q]q^{'}(x) \ , \\ \bar{q}(x) &\to \bar{q}^{'}(x)[1 - i\alpha(x)Q] \ , \\ A_\mu(x) &= A^{'}(x) + \frac{1}{e}\partial_\mu\alpha(x) \ . \end{aligned} \tag{2.60}$$

This is a well-known result; we show a useful method employing a similar procedure for gauging a non-local interaction, which couples quarks, baryons and photons. Based on a path-exponential formalism introduced by Bloch and a path-ordered formalism used by Wilson, Ref. [44] develops a method of gauging nonlocal Lagrangians. The path-ordering of Wilson is useful for introducing the weak interaction, while for Abelian groups, like the $U(1)$ group of the electromagnetic gauge field, the path-exponential formalism is used.

A free-quark action

$$\begin{aligned} S &= \int dx^4 \int dy^4 \bar{q}(x)\delta(x-y) \not{\partial} q(y) \\ &= \int dx^4 \int dy^4 \bar{q}(x)[-\not{\partial}_y \delta(x-y)]q(y) \end{aligned} \tag{2.61}$$

is gauged by introducing an exponential of a line integral of the gauge field. The same procedure is used in the path-exponential formalism for nonlocal actions, the path exponential has certain properties under the infinitesimal gauge transformations above

$$e^{-ieQ\int_x^y A_\mu dz^\mu} = [1 + i\alpha(x)Q]\, e^{-ieQ\int_x^y A^{'}_\mu dz^\mu}[1 - i\alpha(y)Q] \ . \tag{2.62}$$

The simple properties of the path exponential are preserved in the path-ordering for non-Abelian groups. Using this property, the action becomes ($F_{\mu\nu}F^{\mu\nu}$ is ignored) gauge invariant

$$\begin{aligned} S &= \int dx^4 \int dy^4 \bar{q}(x)[-\not{\partial}_y \delta(x-y)]e^{-ieQ\int_x^y A_\mu dz^\mu} q(y) \\ &= \int dx^4 \int dy^4 \bar{q}(x)\delta(x-y) \not{\partial}_y e^{-ieQ\int_x^y A_\mu dz^\mu} q(y) \ . \end{aligned} \tag{2.63}$$

When expanding the exponential in the last line up to the desired order, the derivative ∂_y will act on the line integral of the electromagnetic field.

Mandelstam [45] solved this ambiguity by introducing the notation

$$I(x,y,P) \equiv \int_x^y A_\mu dz^\mu \tag{2.64}$$

to show the explicit dependence of the line integral on the path P from x to y. The derivative of $I(x,y,P)$ is defined as

$$\begin{aligned} & \lim_{dy_\mu \to 0} dy_\mu \frac{\partial}{\partial y^\mu} I(x,y,P) \ , \\ = & \lim_{dy_\mu \to 0} I(x,y+dy,P^{'}) - I(x,y,P) \ . \end{aligned} \tag{2.65}$$

with $P^{'}$ being the path obtained by shifting the end-point y of the path P by the quantity dy_μ. With the definition above we obtain

$$\frac{\partial}{\partial y^\mu} I(x,y,P) = A_\mu(y) \tag{2.66}$$

and the derivative of the path integral is independent of the path. In the expression of the gauge invariant action, if the path P shrinks to a point (that is the action becomes local), not to a closed loop, that is, if $x \to y$, then $\delta(x-y)I(x,y,P) = 0$ and

$$S = \int dx^4 \int dy^4 \bar{q}(x)\delta(x-y)(\gamma^\mu \partial_\mu - ieQA_\mu)q(y) \ , \tag{2.67}$$

where the standard minimal substitution is recovered.

We return to our action and following all the steps above, we will multiply each quark field $q(y_i)$ in the strong interaction part Lagrangian L_{int}^{str} of (2.1) with an exponential of the gauge field A_μ.
One obtains

$$\begin{aligned} L_{int}^{str+em(2)}(x) &= g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 F_B(x;y_1,y_2,y_3) \\ &\times \epsilon^{abc}\Gamma_1 e^{-ie_{q_1} I(y_1,x,P)} q_1^a(y_1) e^{-ie_{q_2} I(y_2,x,P)} q_2^b(y_2) \\ &\times C\Gamma_2 e^{-ie_{q_3} I(y_3,x,P)} q_3^c(y_3) \ , \end{aligned} \tag{2.68}$$

where

$$I(y_i,x,P) = \int_x^{y_i} dz_\mu A^\mu(z) \ . \tag{2.69}$$

The full Lagrangian suitable to calculate electromagnetic properties of any baryon is given now by

$$L_{full}(x) = L_{free}(x) + L_{int}^{em(1)}(x) + L_{int}^{str+em(2)}(x) \; . \tag{2.70}$$

If one performs the gauge transformations

$$\begin{aligned} q_i(x) &\to e^{ie_{q_i} f(x)} q_i(x), \\ \bar{q}_i(x) &\to \bar{q}_i(x) e^{-ie_{q_i} f(x)}, \end{aligned} \qquad \begin{aligned} A^\mu(x) &\to A^\mu(x) + \partial^\mu f(x), \\ B(x) &\to e^{ie_B f(x)}\, B(x)\,, \\ \bar{B}(x) &\to \bar{B}(x)\, e^{-ie_B f(x)}\;, \end{aligned} \tag{2.71}$$

the full Lagrangian remains invariant. The total baryon charge is $e_B \sum_{i=1}^3 e_{q_i}$. The next step is to expand each gauge exponential up to a certain power of A_μ relevant to the desired order of perturbation theory in a given process. That is how the second term of the electromagnetic interaction $L_{int}^{em(2)}$ arises. Actually, in calculating static properties, such as magnetic moments, one will retain only terms proportional to A_μ. To obtain such terms it is sufficient to multiply with the gauge exponential only one quark field.

The photon couples equally to any of the three constituents and at the end we will sum up all contributions. Though one may think that the observables will depend on the path P, which connects the end points in the path integral, that is x and y_i, the results will obviously only depend on the derivatives of the path integral. Using the formalism suggested by Terning [44], the derivatives of the path-integral $I(y, x, P)$ are defined as path-independent (see Eqs. (2.65) and (2.66)).

In Appendix C we explicitly give the derivation of Feynman rules for a non-local Lagrangian coupling hadrons to photons and quarks.

Chapter 3

Magnetic moments of heavy baryons

In the past decades great amount of information has been provided by experiments in the heavy hadron sector. The spectra of single charm and single bottom baryons were confirmed by numerous Fermilab in the early 80's and 90's [46, 47, 48]. Information on double heavy flavored hadrons came later with the first observation of the B_c meson in an experiment performed by the CDF Collaboration in 1998, $B_c^+ \to J/\Psi l^+ \nu_l$, [49]. The Selex-Fermilab Collaboration [50] reported three candidates for the doubly charmed baryon Ξ_{cc}^+ with masses around 3520 MeV/c^2 in $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$. The measured static properties were in good agreement with the prescriptions of the naive quark model. Though there is still not enough information listed in the PDG for static properties of double heavy baryons, using various theoretical approaches theorists can give predictions which hopefully will be confirmed by future experiments.

The study of the electromagnetic interaction is relevant for gaining insight to the hadronic structure. The success of the naive quark model in describing baryon spectra, magnetic moments and other static observables, together with the indications from deep inelastic lepton scattering form a solid basis for the consideration that, a baryon is described by a three quark bound state at least in the leading order.

In the literature one can find different approaches to calculate magnetic moments of mainly single heavy baryons. In Refs. [51, 52, 53, 54, 55, 56] the naive quark model was extensively used based on different realizations of spin-flavor symmetry. In Refs. [57, 58, 59, 60] the magnetic moments of charmed and bottom hyperons have been calculated using advanced quark models which incorporate the ideas of hadronization, confinement, chiral symmetry and Poincare covariance. Soliton-type approaches have been used in Refs. [61, 62, 63], while QCD sum rules in the presence of an external electromagnetic field have been used to calculate the magnetic moments of the Σ_c and Λ_c baryons [64] and the magnetic moments of Λ_b and Λ_c [65].

In a series of papers [66, 67, 68, 69, 70] the heavy quark symmetry has been combined with elements of chiral symmetry, a theory known under the generic name of heavy hadron chiral perturbation theory (HHChPT), and used to calculate magnetic moments of single heavy baryons. The general Lagrangian describing the soft interactions of single heavy systems contains heavy hadrons, light mesons and external fields, the photons. As explained in the introductory part, in the heavy quark limit, $m_Q \to \infty$, single heavy baryons can be classified according to the spin of the light diquark system. The $\Lambda-$type baryons are antisymmetric states, $\bar{3}$ states with the total spin of the light diquark system $s_l = 0$ (isosinglet Λ_Q with quark content $Q[ud]$ and isodoublet Ξ_Q with quark content $Q[us]$ and $Q[ds]$). The $\Sigma-$type baryons are symmetric states, 6 states with the total spin of the light diquark system $s_l = 1$ (isotriplet Σ_Q with quark content Quu, Qud, Qdd and isodoublet $\Xi_Q^{'}$ with quark content Qus and Qds). In the first case the total spin of the heavy baryon is made up by the spin of the heavy quark, so that in the heavy quark limit the magnetic moment of such baryons vanishes. In the second case the magnetic moments in the heavy quark limit are nonzero, as the light degrees of freedom give contributions. The leading order $O[1/m_Q\Lambda_\chi]$ long-distance contributions to $\Lambda-$type baryon magnetic moments come from spin symmetry breaking, which leads also to the mass splitting $\Sigma_Q^* - \Sigma_Q$ [68]. The next-to-leading order corrections are of order $O[1/m_Q\Lambda_\chi^2]$. The scale parameter $\Lambda_\chi \simeq 1.2\ GeV$ is related to the spontaneous breaking of chiral symmetry. The next-to-leading order corrections are of order $O[1/m_Q\Lambda_\chi^2]$ and have been calculated in Ref. [69]. For the $\Sigma-$type baryons the leading order corrections coming from the light- and heavy-quark magnetic interactions are of order $O[1/\Lambda_\chi]$ and $O[1/m_Q]$, respectively [67]. The next-to leading order contributions are of order $O[1/\Lambda_\chi^2]$ [69].

All the approaches listed above have both advantages and drawbacks. As the sector of double flavored baryons is barely discussed, we find it useful to apply the Relativistic Three-Quark Model (RTQM) to the computation of magnetic moments of single-, double- and triple-heavy baryons.

As in Eq. (2.1) the nonlocal coupling of the three constituent quarks to any baryon is given by the effective interaction Lagrangian

$$L_{int}^{str}(x) = g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 F_B(x; y_1, y_2, y_3) J_B(y_1, y_2, y_3) + h.c. \tag{3.1}$$

The most general form of a three quark current is given in Eq. (2.2), whereas the vertex function F_B of (2.4) and the definition of the four-space quark and baryon coordinates in terms of Jacobi coordinates have been discussed in Eq. (2.6). Of course, the calculation will be carried in momentum space, so we will use the Fourier transform of the vertex function (three-quark correlation function) as given in Eq. (2.8). We also motivated the choice of a universal Gaussian shape of the vertex function (see Eq. (2.9)).

In our study of heavy baryon magnetic moments we will split the model into three parts: (1) an exact calculation with no approximation, using a "full" baryonic correlation function and a free quark propagator, indicated as BCF (Baryonic Correlation Function). We furthermore consider two specific limits for the correlation functions and heavy quark propagators: (2) the heavy quark limit for the heavy-quark masses in the expression of the BCF will be called HQL BCF approximation. For single heavy baryons the heavy-quark limit means that they are treated as bound states of a heavy quark and a light diquark system. In addition, the heavy quark is placed in the center of the system and is surrounded by the light degrees of freedom. This limit will eliminate the heavy quark degree of freedom $m_1 = m_Q \to \infty$ which, replaced in the definition for the kinematical quantity ω_i, translates into

$$\begin{aligned} \omega_1 &\to 1 \\ \omega_2 &\to 0 \\ \omega_3 &\to 0 \end{aligned}$$

and therefore

$$y_1 \to x \ .$$

Double heavy baryons are considered as bound states of a light quark and a heavy diquark system, which sits in the center of the double heavy baryon system. This corresponds to the limit: $m_2 = m_Q \to \infty, m_3 = m_{Q'} \to \infty$

which in turn corresponds to

$$\begin{aligned}
\omega_1 &\to 0 \\
\omega_2 &\to 1/2 \\
\omega_3 &\to 1/2 \ .
\end{aligned}$$

With respect to the triple heavy baryons we point out two examples : the Ω^+_{ccb} is a bound state of a heavy bottom quark, while the double charm system is considered relatively "light"; so $m_1 = m_b \to \infty$ and therefore $\omega_1 \to 1, \omega_2 \to 0, \omega_3 \to 0$ as in the case of a single heavy baryon; the Ω^0_{cbb} baryon is a bound state of a relatively "light" charm quark and a heavy diquark system composed of the bottom flavours, so that $m_2 = m_3 = m_b \to \infty$ and $\omega_1 \to 0, \omega_2 \to 1/2, \omega_3 \to 1/2$, similarly to the structure of a double heavy baryon in the heavy quark limit. Following this philosophy one can easily simplify the correlation functions expressed through Jacobi coordinates. As a last approximation (3), we take the heavy-quark limit both for the baryonic correlation function and the heavy-quark propagator (only for single-heavy baryons), refered as HQL BCF + HQP.

With respect to the three-quark currents, in the previous chapter we gave a list of possibilities for the $\Sigma-$type and $\Lambda-$type and explained our choice based on the previous study of Chung and Ioffe [36, 38]. We stress that for the $\Sigma-$type baryons we choose to work with vector currents and for the $\Lambda-$type we choose to work with pseudoscalar currents.

Also, a useful classification of the heavy-baryon states (spin-parity, flavor content and quantum numbers, mass spectrum) is given in Table 1 for the single charm baryons, in Table 2 for the single bottom baryons and in Table 3 for double and triple heavy baryons. All data are taken from Refs. [71, 72, 73] and apply to the $J^P = \frac{1^+}{2}$ states.

Table 1. Single charm $1/2^+$ baryons

Notation	Content	J^P	SU(3)	I_3	S	C	Mass (GeV)
Λ_c^+	$c[ud]$	$1/2^+$	$\bar{3}$	0	0	1	2.286
Ξ_c^+	$c[su]$	$1/2^+$	$\bar{3}$	1/2	-1	1	2.466
Ξ_c^0	$c[sd]$	$1/2^+$	$\bar{3}$	-1/2	-1	1	2.472
Σ_c^{++}	cuu	$1/2^+$	6	1	0	1	2.453
Σ_c^+	$c\{ud\}$	$1/2^+$	6	0	0	1	2.451
Σ_c^0	cdd	$1/2^+$	6	-1	0	1	2.452
$\Xi_c^{\prime +}$	$c\{su\}$	$1/2^+$	6	1/2	-1	1	2.574
$\Xi_c^{\prime 0}$	$c\{sd\}$	$1/2^+$	6	-1/2	-1	1	2.579
Ω_c^0	css	$1/2^+$	6	0	-2	1	2.698

Table 2. Single bottom $1/2^+$ baryons

Notation	Content	J^P	SU(3)	I_3	S	B	Mass (GeV)
Λ_b	$b[ud]$	$1/2^+$	$\bar{3}$	0	0	1	5.624
Ξ_b^0	$b[su]$	$1/2^+$	$\bar{3}$	1/2	-1	1	5.80
Ξ_b^-	$b[sd]$	$1/2^+$	$\bar{3}$	-1/2	-1	1	5.80
Σ_b^+	buu	$1/2^+$	6	1	0	1	5.82
Σ_b^0	$b\{ud\}$	$1/2^+$	6	0	0	1	5.82
Σ_b^-	bdd	$1/2^+$	6	-1	0	1	5.82
$\Xi_b^{\prime 0}$	$b\{su\}$	$1/2^+$	6	1/2	-1	1	5.94
$\Xi_b^{\prime -}$	$b\{sd\}$	$1/2^+$	6	-1/2	-1	1	5.94
Ω_b^-	bss	$1/2^+$	6	0	-2	1	6.04

Table 3. Double and triple heavy $1/2^+$ baryons

Notation	Content	J^P	I_3	S	C	B	Mass (GeV)
Ξ_{cc}^{++}	$u\{cc\}$	$1/2^+$	1/2	0	2	0	3.519
Ξ_{cc}^{+}	$d\{cc\}$	$1/2^+$	-1/2	0	2	0	3.519
Ω_{cc}^{+}	$s\{cc\}$	$1/2^+$	0	-1	2	0	3.59
Ξ_{bb}^{0}	$u\{bb\}$	$1/2^+$	1/2	0	0	2	10.09
Ξ_{bb}^{-}	$d\{bb\}$	$1/2^+$	-1/2	0	0	2	10.09
Ω_{bb}^{-}	$s\{bb\}$	$1/2^+$	0	-1	0	2	10.18
Ξ_{cb}^{+}	$u[cb]$	$1/2^+$	1/2	0	1	1	6.82
Ξ_{cb}^{0}	$d[cb]$	$1/2^+$	-1/2	0	1	1	6.82
Ω_{cb}^{0}	$s[cb]$	$1/2^+$	0	-1	1	1	6.91
$\Xi_{cb}^{\prime\,+}$	$u\{cb\}$	$1/2^+$	1/2	0	1	1	6.85
$\Xi_{cb}^{\prime\,0}$	$d\{cb\}$	$1/2^+$	-1/2	0	1	1	6.85
$\Omega_{cb}^{\prime 0}$	$s\{cb\}$	$1/2^+$	0	-1	1	1	6.93
Ω_{ccb}^{+}	bcc	$1/2^+$	0	0	2	1	8.0
Ω_{cbb}^{0}	cbb	$1/2^+$	0	0	1	2	11.5

Having all the ingredients, we discuss now the full Lagrangian, which is given by the standard free fermion Lagrangian for baryons and quarks and the two interaction terms: one coming from the coupling of the photon to the free fields (using the minimal standard substitution) and one coming from the coupling of the photon to the baryon-quark interaction vertex (using the path exponential formalism for gauging).

We list the complete Lagrangian and refer to the previous chapter where we explained the technique of introducing external fields in the interaction Lagrangian:

$$
\begin{aligned}
L_{full}(x) &= \bar{B}(x)(i\,\not\partial - m_B)B(x) + \sum_q \bar{q}(x)(i\,\not\partial - m_q)q(x) \\
&+ e_B\bar{B}(x)\not A\,B(x) + \sum_q e_q\,\bar{q}(x)\not A\,q(x) \\
&+ g_B\bar{B}(x)\int\!dy_1\!\int\!dy_2\!\int\!dy_3\,F_B(x,y_1,y_2,y_3)\,\epsilon^{a_1a_2a_3}\,\Gamma_1 e^{-ie_{q_1}I(y_1,x,P)} \\
&\times\; q_1^{a_1}(y_1)\,e^{-ie_{q_2}I(y_2,x,P)}\,q_2^{a_2}(y_2)C\,\Gamma_2\;e^{-ie_{q_3}I(y_3,x,P)}\,q_3^{a_3}(y_3)\;+\;\text{h.c.}
\end{aligned}
$$

(3.2)

In Fig. 1 we give a diagramatic representation of the interaction vertices arising from the Lagrangian of Eq. (3.2). Fig. 1(a) shows the non-local vertex coupling involving the constituent quarks (thin line), the photon (wiggly line) and the baryon (thick line). Fig. 1 (b) represents the interaction vertex arising from the coupling of the photon to the elementary baryon, while Fig. 1 (c) shows the interaction vertex of the constituent quark with the photon. Fig. 1 (a) corresponds to the last term of the full Lagrangian (3.2), Fig. 1 (b) and (c) correspond to the second and the third term in (3.2), respectively.

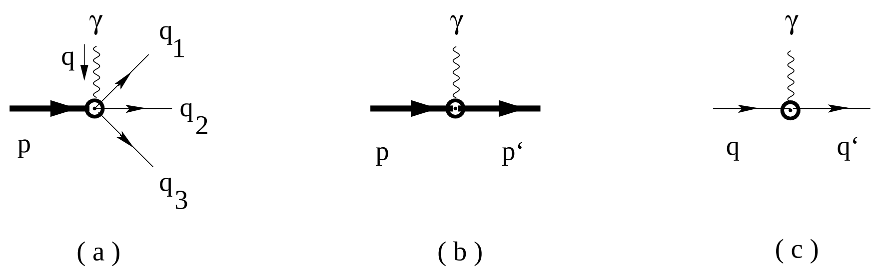

Fig. 1 (a) Coupling vertex of baryon (thick line), photon (wiggly line) and three quarks (thin line); (b) Coupling vertex of baryon and photon; (c) Coupling vertex of constituent quark and photon.

The free part in the Lagrangian leads to the free fermion propagator for the constituent quark of the form:

$$i\, S_q(x-y) = \langle 0|T\, q(x)\, \bar{q}(y)|0\rangle \;=\; \int \frac{d^4k}{(2\pi)^4 i}\, e^{-ik(x-y)}\; \tilde{S}_q(k)\;, \tag{3.3}$$

where

$$\tilde{S}_q(k) = \frac{1}{m_q - \not{k} - i\epsilon} \tag{3.4}$$

is the usual free fermion propagator in momentum space. We shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by postulating the condition that the baryon mass must be less than the sum of the constituent quark masses $M_B < \sum_i m_{q_i}$.

We will discuss now the set of parameters used in the calculation of the heavy baryon magnetic moments. As already mentioned, the constituent quark masses are parameters in our model. We will use the following set taken from Refs. [11, 30]:

$m_{u(d)}$	m_s	m_c	m_b	
0.42	0.57	1.7	5.2	GeV

(3.5)

Although the vertex function has the same shape for all baryons considered, the size of the quark distribution is different,varying from state to state. However, for the study of heavy baryons we can limit the number of these cutoff parameters to three: $\Lambda_{B_{Qqq}}$ is the size parameter of the quark distribution for single heavy baryons, $\Lambda_{B_{qQQ}}$ the one for the double heavy baryons and $\Lambda_{B_{QQQ}}$ refers to the triple heavy baryons. We give a list below:

$\Lambda_{B_{qqq}}$	$\Lambda_{B_{Qqq}}$	$\Lambda_{B_{qQQ}}$	$\Lambda_{B_{QQQ}}$	
1.25	1.8	2.5	5	GeV

(3.6)

The numerical values for these parameters as well as for the constituent quark masses $m_u = m_d, m_s, m_c$ and m_b are motivated by fits made in previous analyses on the properties of single and double heavy baryons [11, 30]. The size parameter for the triple-heavy quark system arises some questions, since it was not constrained previously by phenomenological considerations. Its value is fixed at 5 GeV on the basis of the naive relation: $\Lambda_{B_{Qqq}} : \Lambda_{B_{qQQ}} : \Lambda_{B_{QQQ}} \simeq m_{B_{Qqq}} : m_{B_{qQQ}} : m_{B_{QQQ}}$, where the masses have typical values for the single-, double- and triple-heavy baryons, respectively. The value chosen for $\Lambda_{B_{QQQ}}$ is approximate, but later on we will perform an analysis displaying the sensitivity of the magnetic moments of triple-heavy baryons on the variation of this parameter in the range of 3-7 GeV.

3.1 Baryon mass operator and matrix elements

To calculate magnetic moments we need to compute the baryon matrix elements of the electromagnetic current. As usual, the matrix elements can be expressed in terms of Pauli and Dirac form factors as

$$
\begin{aligned}
M_\mu &= \bar{u}_B(p')\,\Lambda_\mu(p,p')u_B(p)\ , \\
\Lambda_\mu(p,p') &= \gamma_\mu F_D(q^2) + \frac{i}{2m_B}\sigma_{\mu\nu}q^\nu F_P(q^2)\ , \qquad (3.7)
\end{aligned}
$$

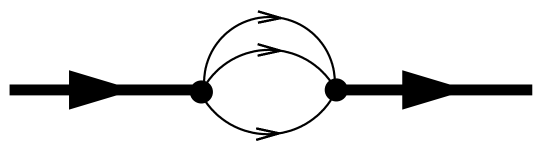

Fig. 2 Baryon mass operator

where $u_B(p)$ is the baryon spinor with the normalization condition $\bar{u}_B(p)u_B(p) = 2m_B$. The 4-momenta $p, p^{'}$ refer to the incoming and outgoing baryon respectively, and $q = p^{'} - p$ is the photon momentum. The definition of the magnetic moment is then:

$$\mu_B = \ [\, F_D(0) + F_P(0)\,]\ \frac{m_p}{m_B} \qquad \text{(in units of n.m.)} \tag{3.8}$$

where m_p is the proton mass. The nuclear magneton (n.m) is defined as $\mu_N = \frac{e}{2m_p}$ with $\hbar = 1$.

The most easy diagram to compute is the baryon mass operator indicated in Fig. 2.

The vertex function $\Lambda_\mu(p, p^{'})$ is actually obtained by computing the three diagrams relevant for the calculation of magnetic moments, the so-called triangle diagrams and "bubble" diagrams given in Fig. 3.

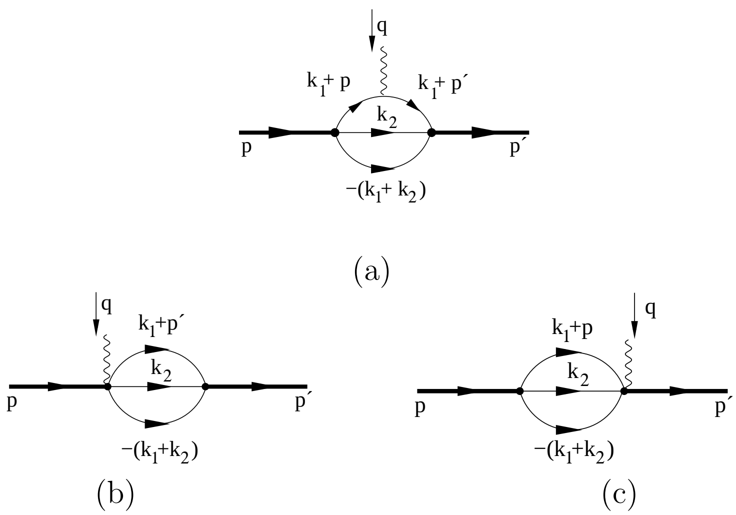

Fig. 3 Diagrams contributing to the baryon electromagnetic vertex function: triangle (a), bubble (b) and (c) diagrams.

These diagrams arise from the derivation of the Feynman rules, as shown in Appendix C. The electromagnetic field couples to each quark, hence a sum over all contributions is needed (there is great deal of simplification when two of the quarks are identical, for example quark 2 and quark 3: once we know how to calculate the diagram for the coupling to quark 2, we obtain the diagram for the quark 3 by the simple replacement $m_2 \leftrightarrow m_3$ we obtain the diagram for the quark 3). The photon does not couple to the physical particles, the baryons, as it is forbidden by the compositeness condition. The "bubbles" represent the coupling of the photon to the nonlocal baryon-quark vertex, as contained in the last term of the Lagrangian (3.2).

For off-shell baryons the electromagnetic vertex function can be separated conveniently into two terms: an "orthogonal" and a "parallel" term. To clarify the terminology: orthogonal and parallel refer to the photon momentum transfer; the separation can be done, since we are able to use for the four-vectors the following representation :

$$\gamma_\mu \, = \, \gamma_\mu^\perp \, + \, q_\mu \, \frac{\not{q}}{q^2} \, , \qquad k_\mu \, = \, k_\mu^\perp \, + \, q_\mu \, \frac{kq}{q^2} \tag{3.9}$$

such that $\gamma_\mu^\perp \, q^\mu \; = \; 0$ and $k_\mu^\perp \, q^\mu \, = \, 0$. Then, for instance, the orthogonal vertex function $\Lambda_\mu^\perp(p, p')$ is expressed in terms of $\gamma_\mu^\perp$ and $k_\mu^\perp$.

The full electromagnetic vertex function is:

$$\Lambda_\mu(p,p') = \frac{q_\mu}{q^2}\, e_B \left[\Sigma_B(p') - \Sigma_B(p)\right] \,+\, \Lambda^\perp_\mu(p,p')\ . \tag{3.10}$$

Each term in the full electromagnetic vertex function, the parallel and the perpendicular, contains contributions from the triangle ($\circ$) and bubble (Δ) diagrams. For the othogonal vertex function we have

$$\Lambda^\perp_\mu(p,p') = \Lambda^\perp_{\mu,\,\Delta}(p,p') + \Lambda^\perp_{\mu,\,\circ}(p,p')\ . \tag{3.11}$$

The importance of such a separation is clear: it helps to verify the Ward-Takahashi identities, thus the gauge invariance of the theory (though the Lagrangian is gauge invariant by construction). The Ward-Takahashi identity relates the baryon vertex function to the mass operator :

$$q^\mu \Lambda_\mu(p,p') = e_B \left[\Sigma_B(p') - \Sigma_B(p)\right]. \tag{3.12}$$

For on mass-shell baryons $\not p = m_B$ and $q = 0$ the Ward identity emerges as

$$\Lambda_\mu(p,p) = e_B\, \frac{\partial}{\partial p_\mu} \Sigma_B(p)\,. \tag{3.13}$$

In the following we offer a list of the analytical expressions for the mass operator, the triangle and the bubble diagrams, both when the photon couples to the left and to the right side of the diagram (that is Fig. 3 (b) and (c)). The expression for the mass operator is

$$\Sigma_B(p) \;\;=\;\; \alpha_B \int dk_{123}\; \tilde\Phi^2(z_0)\, R_\Sigma(k_1^+, k_2^+, k_3^+) \tag{3.14}$$

We denote $\alpha_B = 6 g_B^2$, where the coupling constant g_B is defined by the compositeness condition (2.54), and $dk_{123} = \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^8 i^2}\, \delta^4(k_1+k_2+k_3)$; $\tilde\Phi^2$ is the square of the Fourier transform of the correlation function and $z_0 = -6(k_1^2+k_2^2+k_3^2)$. The $R_\Sigma(r_1,r_2,r_3) = \Gamma_{1f} S_{q_1}(r_1) \Gamma_{1i}\, \mathrm{tr}\left[\Gamma_{2f} S_{q_2}(r_2) \Gamma_{2i} S_{q_3}(-r_3)\right]$ represents the trace over the quark momenta. The explicit evaluation of the mass operator of Eq. (3.14) resumes to the calculation of three integrals and to performing the trace algebra. Introducing the following notations :

$$\begin{aligned} k_i^+ &= k_i + p\omega_i \\ k_i'^{\,+} &= k_i + p'\omega_i\,, \end{aligned}$$

$$
\begin{aligned}
z_1(q) &= -12q^2(\omega_2^2 + \omega_2\omega_3 + \omega_3^2) - L_1 q\,,\\
z_2(q) &= -12q^2(\omega_1^2 + \omega_1\omega_3 + \omega_3^2) - L_2 q\,,\\
z_3(q) &= -12q^2(\omega_1^2 + \omega_1\omega_2 + \omega_2^2) - L_3 q\,,\\
L_i &= 12(k_i - \sum_{j=1}^{3} k_j\omega_j)
\end{aligned}
\tag{3.15}
$$

and

$$
\begin{aligned}
R^{\perp}_{\mu\,,\Delta_1}(r_1,r_2,r_3,q) &= \Gamma_{1f}S_{q_1}(r_1+q)\gamma^{\perp}_{\mu}S_{q_1}(r_1)\Gamma_{1i}\mathrm{tr}\left[\Gamma_{2f}S_{q_2}(r_2)\Gamma_{2i}S_{q_3}(-r_3)\right]\,,\\
R^{\perp}_{\mu\,,\Delta_2}(r_1,r_2,r_3,q) &= \Gamma_{1f}S_{q_1}(r_1)\Gamma_{1i}\,\mathrm{tr}\left[\Gamma_{2f}S_{q_2}(r_2+q)\gamma^{\perp}_{\mu}S_{q_2}(r_2)\Gamma_{2i}S_{q_3}(-r_3)\right]\,,\\
R^{\perp}_{\mu\,,\Delta_3}(r_1,r_2,r_3,q) &= -\Gamma_{1f}S_{q_1}(r_1)\Gamma_{1i}\,\mathrm{tr}\left[\Gamma_{2f}S_{q_2}(r_2)\Gamma_{2i}S_{q_3}(-r_3)\gamma^{\perp}_{\mu}S_{q_3}(-r_3-q)\right]\,.
\end{aligned}
\tag{3.16}
$$

we display the analytical form of the vertex function corresponding to the bubble and triangle diagrams:

$$
\begin{aligned}
\Lambda^{\perp}_{\mu,\,\Delta}(p,p') &= \alpha_B\int dk_{123}\,\sum_{i=1}^{3} e_{q_i}\,\tilde\Phi(z_0)\,\tilde\Phi[z_0+z_i(q)]\\
&\times\ R^{\perp}_{\mu,\Delta_i}(k_1^+,k_2^+,k_3^+,q)\,,\\
\Lambda^{\perp}_{\mu,\,\circ_L}(p,p') &= -\alpha_B\int dk_{123}\,\sum_{i=1}^{3} e_{q_i}\,L^{\perp}_{i\mu}\,\tilde\Phi(z_0)\\
&\times\ \int_0^1 dt\,\tilde\Phi'[z_0+tz_i(-q)]\,R_\Sigma(k_1'^+,k_2'^+,k_3'^+)\,,\\
\Lambda^{\perp}_{\mu,\,\circ_R}(p,p') &= -\alpha_B\int dk_{123}\,\sum_{i=1}^{3} e_{q_i}\,L^{\perp}_{i\mu}\,\tilde\Phi(z_0)\\
&\times\ \int_0^1 dt\,\tilde\Phi'[z_0+tz_i(q)]\,R_\Sigma(k_1^+,k_2^+,k_3^+)\,.
\end{aligned}
\tag{3.17}
$$

The prime with $\tilde\Phi'$ denotes the derivative:

$$
\tilde\Phi'(s) = \frac{d\tilde\Phi(s)}{ds}\,. \tag{3.18}
$$

The two bubble diagrams can be related to each other by simply exchanging the initial with the final external momentum.

$$\Lambda^{\perp}_{\mu,\circ_L}(p,p') \equiv \Lambda^{\perp}_{\mu,\circ_R}(p',p)\,. \tag{3.19}$$

Details for the further explicit evaluation of the mass operator and the vertex functions are given in Appendix D.

3.2 Heavy-quark limit and matching to Heavy Hadron ChPT

In the first chapter we introduced the notion of the heavy quark limit with the corresponding symmetry generating an effective theory, the heavy quark effective theory (HQET). In this subsection we will discuss important features of heavy hadron chiral perturbation theory (HHChPT), a theory that has been successfully used in the calculation of magnetic moments of single heavy baryons [69]. We will check the consistency of our results with such model-independent predictions. HHChPT is a combination of chiral perturbation theory (ChPT) and heavy-quark effective theory, which is suitable for the description of the soft interactions of hadrons containing one heavy quark with light pseudoscalar mesons and photons. It is well known that in the limit $m_u, m_d, m_s \to 0$, the QCD Lagrangian for light quarks exhibits a $SU(3)_L \times SU(3)_R \times U(1)_V$ symmetry, which is spontaneously broken down to $SU(3)_V \times U(1)_V$. The low-energy interactions of the eight pseudoscalar mesons (Golstone bosons) can be analyzed using the calculational technique of perturbation theory. Thus, ChPT is a systematic and rigorous expansion of the chiral Lagrangian in terms of momenta and masses of the Goldstone bosons. The pseudoscalar octet is organized in the chiral Lagrangian as an exponential representation which contains as an argument a parametrization of the Goldstone bosons. Making use of the chiral transformations of such an exponential, the effective theory is based on the most general Lagrangian consistent with chiral symmetry, involving this exponential (implicitly, the meson fields) and its derivatives. When we add the quark masses, we explicitly break chiral symmetry. This breaking is included in the Lagrangian by a light mass matrix χ which in turn reveals quadratic pseudoscalar mass terms. With this Lagrangian one proceeds further to expand matrix elements in powers of (p/Λ_χ) and (m_q/Λ_χ), where p is the low meson momentum and

$\Lambda_\chi \sim 1GeV$ the chiral symmetry scale. This scales gives the suppression of higher-order terms in the theory. In the same vein one takes the advantages of another effective theory that can be constructed in the limit $m_{c,b} \to \infty$, the HQET discussed in the introductory part. In some kinematical region, not far from the chiral and heavy quark limit one can make use of both approaches ChPT and HQET to study the low-energy properties of heavy hadrons [69]. One constructs a Lagrangian in terms of meson fields and heavy hadrons which can be extended to incorporate the electromagnetic field [57]. The interaction picture is that of a heavy hadron interacting with a light pseudoscalar meson, which does not change the velocity of the heavy hadron. Thus, in the frame of HQET, mesonic loops can dress the hadrons, in a chiral fashion and the photon field can be included. Further information about obtaining calculable expressions for the magnetic moments of single heavy baryons can be found in Ref. [69]. They predict the folloing expressions for the Λ_Q and Σ_Q baryons in the leading ($\sim 1/m_Q$) and next-to-leading ($\sim 1/\Lambda_\chi$) order:

$$\begin{aligned} \mu_{\Lambda_{Q[q_2q_3]}} &= \frac{e_Q}{2m_Q} + \frac{c_{\Lambda_{Qq_2q_3}}}{m_Q\,\Lambda_\chi} + \frac{d_{\Lambda_{Qq_2q_3}}}{m_Q\,\Lambda_\chi^2} + \dots\,, \\ \mu_{\Sigma_{Q\{q_2q_3\}}} &= -\frac{e_Q}{6m_Q} + \frac{c_{\Sigma_{Qq_2q_3}}}{\Lambda_\chi} + \frac{d_{\Sigma_{Qq_2q_3}}}{\Lambda_\chi^2} + \dots \end{aligned} \tag{3.20}$$

Here e_Q is the heavy quark charge and $c_{B_{Qq_2q_3}}$ and $d_{B_{Qq_2q_3}}$ are unknown coupling factors of HHChPT. The values of these couplings can be predicted in our approach [1]. The spontaneous symmetry breaking scale is taken as $\Lambda_\chi = 4\pi F_\pi \sim 1.2$ GeV, with F_π the pion decay constant, which fixes the normalization of the unknown couplings.

Looking at the magnetic moment of the $\Lambda-$type baryons we see that the leading contribution, coming from the coupling of the photon to the heavy quark, should vanish as $m_Q \to \infty$. The leading contribution ($\sim 1/\Lambda_\chi$) for the $\Sigma-$type baryons survives in the heavy quark limit and it comes from the coupling of the photon to the light system, though the leading contribution coming from the coupling to the heavy quark of the order ($\sim 1/m_Q$) vanishes as well. All these qualitative predictions are model-independent and should be satisfied by any model approach. The other terms in the expansions, proportional to the c and d couplings represent chiral corrections, thus mesonic cloud contributions. Such corrections can be computed using the formalism developed in Ref. [74] which is consistent with ChPT as one performs a matching at the level of matrix elements: the physical amplitudes calculated in both approaches are matched at the baryonic level. In the next chapters we will show how one can employ our model and include chiral symmetry ingredients in the calculation of the magnetic moments of the light baryons and other observables, as the $N \to \Delta$ transition, taking into consideration the meson cloud.

For now we discuss how the leading contributions to the Λ_Q and Σ_Q baryons arise in our model, taking into account only the valence quark degrees of freedom. Details and results for the unkown couplings are found in Ref. [1].

The leading contributions are obtain by performing an expansion of the heavy quark free propagator in powers of $1/m_Q$, while keeping just the leading term

$$\tilde{S}_Q(k+p) = \frac{1}{m_Q - \not{k} - \not{p}} = -\frac{1 + \not{v}}{2\,(kv + \bar{\Lambda}_{q_2q_3})} + O(1/m_Q) \;. \tag{3.21}$$

Here $v = p/m_B$ is the four-velocity of the single heavy baryon and the parameter $\bar{\Lambda}_{q_2q_3}$ is the mass difference between the heavy baryon mass and the heavy quark mass in the heavy quark limit:

$$m_{B_{Qq_2q_3}} = m_Q + \bar{\Lambda}_{q_2q_3} + O(1/m_Q) \;. \tag{3.22}$$

This quantity $\bar{\Lambda}_{q_2q_3}$, related to the distribution of the light degrees of freedom in the heavy baryon system, was fixed previously [8, 9, 10, 11, 29, 30]. Its values are usually extracted from an analysis performed on the exclusive semileptonic decays of bottom and charmed baryons and on the strong and radiative decays of heavy baryons, resulting in:

$$\begin{aligned}
&\bar{\Lambda}_{uu} = \bar{\Lambda}_{ud} = \bar{\Lambda}_{dd} = 600 \text{ MeV}\,, \\
&\bar{\Lambda}_{us} = \bar{\Lambda}_{ds} = 750 \text{ MeV}\,, \\
&\bar{\Lambda}_{ss} = 900 \text{ MeV}\,.
\end{aligned} \tag{3.23}$$

The procedure in evaluating the magnetic moments with the expansion (3.21) is just the same as with the full propagator. We refer to Appendix D and to the previous sections, as well. Carrying out the trace algebra and computing the four-dimensional integrals we can extract the magnetic moments of the single heavy baryons in the heavy-quark limit, which are in agreement with

HHChPT

$$\begin{aligned}\mu^{\text{heavy}}_{\Lambda_{Q[q_2q_3]}} &= \frac{e_Q}{2m_Q}\,,\\ \mu^{\text{heavy}}_{\Sigma_{Q\{q_2q_3\}}} &= -\frac{e_Q}{6m_Q}\,.\end{aligned}$$

In general both the triangle and the bubble diagrams contribute to the magnetic moments. When the photon couples to the heavy quark and one calculates the baryon magnetic moment in the HQL BCF+HQL, the leading order term is only given by the triangle diagram. The bubble diagrams do not contribute at this order. For the $\Lambda-$type baryons, we obtain agreement with the HHChPT, as at the order $1/m_Q$ the $\gamma-$heavy-quark coupling contribution vanishes. For the $\Sigma-$type baryons the $\gamma-$light-diquark system contribution still survives in the leading order and the calculated values are given in the third column of Table 4.

The leading contribution of the light system is proportional to the ratio of two integrals dependent on the light constituent masses and other model parameters [1]. Furthermore, it is inversely proportional to the size parameter which enters the single-heavy baryon vertex function. The model is also able to give predictions for the unknown HHChPT couplings, when taking the heavy-quark limit both in the baryonic correlation function (BCF) and in the heavy quark propagator [1]. Following the notation of Ref. [69], the c_S coupling depends on the light-quark content of the heavy baryon. This coupling also depends on flavour because we break $SU(3)$ symmetry. Therefore, the coupling's values depend on whether the heavy baryons are non-strange, single-strange or double-strange. Even in the case of single-strange heavy baryons, so for the cascade states $\Xi^{'}_{Qus}$ and $\Xi^{'}_{Qds}$, we get different predictions for c_S as the contributions of u and $d-$quarks enter with different coefficients. Once again, this coupling does not depend on the heavy flavour, but on the light flavour content [1]. The c_S notation of Ref. [69] refers to the coupling of the photon to single c or b baryons members of a 6 multiplet, organized each in a 3×3 matrix. We do not refer to higher order contributions and do not consider Goldstone boson loop corrections.

3.3 Numerical results and discussion

In the following, in Table 4 we display the results for the magnetic moments of the single-heavy baryons. The first column contains the exact results derived from the full theory, with no approximation in the correlation function and in the propagator. The second column shows the results obtained when using the heavy-quark limit approximation in the baryonic correlation function (HQL BCF). In the third column the results displayed are for the heavy-quark limit taken both in the correlation function and the propagator (HQL BCF+HQP). The two values displayed in the round parenthesis represent the contribution to the magnetic moment coming from the coupling of the heavy quark (c or b) to the photon and the contribution coming from the light degrees of freedom coupled to the photon, respectively. In the last column we present for comparison the predictions of the NRQM calculated with the non-relativistic spin-flavor wave functions listed in Table 5. A complete description of the non-relativistic spin-flavor wave functions is found in Appendix E.

Table 4. Magnetic moments of single heavy baryons (in units of μ_N)

Baryon	RQM			NRQM
	full	HQL BCF	HQL BCF+HQP	
Λ_c^+	0.42 (0.41; 0.01)	0.38 (0.38; 0.003)	0.37 (0.37; 0)	0.37 (0.37; 0)
Λ_b^0	-0.06 (-0.06; 0.002)	-0.06 (-0.06; 0.001)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)
Ξ_c^+	0.41 (0.40; 0.01)	0.37 (0.37; 0.01)	0.37 (0.37, 0)	0.37 (0.37; 0)
Ξ_c^0	0.39 (0.40; -0.01)	0.37 (0.37; -0.004)	0.37 (0.37; 0)	0.37 (0.37; 0)
Ξ_b^0	-0.06 (-0.06; 0.002)	-0.06 (-0.06; 0.001)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)
Ξ_b^-	-0.06 (-0.06; -0.003)	-0.06 (-0.06: -0.001)	-0.06 (-0.06; 0)	-0.06 (-0.06; 0)
$\Xi_c^{\prime +}$	0.47 (-0.11; 0.58)	0.10 (-0.11; 0.21)	0.08 (-0.12; 0.20)	0.51 (-0.12; 0.63)
$\Xi_c^{\prime 0}$	-0.95 (-0.11; -0.84)	-0.38 (-0.11; -0.27)	-0.37 (-0.12; -0.25)	-0.98 (-0.12; -0.86)
$\Xi_b^{\prime 0}$	0.66 (0.02; 0.64)	0.22 (0,02; 0.20)	0.22 (0.02; 0.20)	0.65 (0.02; 0.63)
$\Xi_b^{\prime -}$	-0.91 (0.02; -0.93)	-0.23 (0.02; -0.25)	-0.23 (0.02; -0.25)	-0.84 (0.02; -0.86)
Σ_c^{++}	1.76 (-0.11; 1.87)	0.58 (-0.11; 0.69)	0.53 (-0.12; 0.65)	1.86 (-0.12; 1.98)
Σ_c^+	0.36 (-0.11; 0.47)	0.06 (-0.11; 0.17)	0.04 (-0.12; 0.16)	0.37 (-0.12; 0.49)
Σ_c^0	-1.04 (-0.11; -0.93)	-0.46 (-0.11; -0.35)	-0.44 (-0.12; -0.32)	-1.11 (-0.12; -0.99)
Σ_b^+	2.07 (0.02; 2.05)	0.68 (0.02; 0.66)	0.67 (0.02; 0.65)	2.01 (0.02; 1.99)
Σ_b^0	0.53 (0.02; 0.51)	0.18 (0.02; 0.16)	0.18 (0.02; 0.16)	0.52 (0.02; 0.50)
Σ_b^-	-1.01 (0.02; -1.03)	-0.31 (0.02; -0.33)	-0.30 (0.02; -0.32)	-0.97 (0.02; -0.99)
Ω_c^0	-0.85 (-0.11; -0.74)	-0.32 (-0.11; -0.21)	-0.31 (-0.12; -0.19)	-0.85 (-0.12; -0.73)
Ω_b^-	-0.82 (0.02; -0.84)	-0.17 (0.02; -0.19)	-0.17 (0.02; -0.19)	-0.71 (0.02; -0.73)

In Table 6 we show the results for the magnetic moments of the double- and triple-heavy baryons. In these two cases it is possible to use the heavy-quark limit in the baryonic correlation function, but not in the propagator. The results are organized as follows: the first column contains the results from the full approach, the second column contains the results in the HQL BCF approximation and the third column shows the NRQM predictions. Again, for all results we show the contributions coming from the coupling of γ to the heavy-quark system and, separately, to the light degrees of freedom (where it is the case).

Table 5. Heavy baryon wave functions and magnetic moments in the non-relativistic quark model, where $q, q' = u$ or d and $Q, Q' = c$ or b.

Baryon	Wave function	Magnetic moment
$\Lambda_{Q[ud]}$	$\frac{1}{\sqrt{2}}\,Q(ud-du)\ \ \chi_A$	$\frac{e_Q}{2m_Q}$
$\Xi_{Q[qs]}$	$\frac{1}{\sqrt{2}}\,Q(qs-sq)\ \ \chi_A$	$\frac{e_Q}{2m_Q}$
$\Sigma_{Q\{qq'\}}$	$\frac{1}{\sqrt{2}}\,Q(qq'+q'q)\ \ \chi_S$	$-\frac{e_Q}{6m_Q}+\frac{e_q}{3m_q}+\frac{e_{q'}}{3m_{q'}}$
$\Omega_{Q\{ss\}}$	$Qss\ \ \chi_S$	$-\frac{e_Q}{6m_Q}+\frac{2e_s}{3m_s}$
$\Xi_{q\{QQ'\}}$	$\frac{1}{\sqrt{2}}q(QQ'+Q'Q)\ \ \chi_S$	$-\frac{e_q}{6m_q}+\frac{e_Q}{3m_Q}+\frac{e'_Q}{3m'_Q}$
$\Omega_{s\{QQ\}}$	$sQQ\ \ \chi_S$	$-\frac{e_s}{6m_s}+\frac{2e_Q}{3m_Q}$
$\Xi_{q[cb]}$	$\frac{1}{\sqrt{2}}\,q(cb-bc)\ \ \chi_A$	$\frac{e_q}{2m_q}$
$\Xi_{q\{cb\}}$	$\frac{1}{\sqrt{2}}\,q(cb+bc)\ \ \chi_S$	$-\frac{e_q}{6m_q}+\frac{e_c}{3m_c}+\frac{e_b}{3m_b}$
$\Omega_{s[cb]}$	$\frac{1}{\sqrt{2}}\,s(cb-bc)\ \ \chi_A$	$\frac{e_s}{2m_s}$
$\Omega_{s\{cb\}}$	$\frac{1}{\sqrt{2}}\,s(cb+bc)\ \ \chi_S$	$-\frac{e_s}{6m_s}+\frac{e_c}{3m_c}+\frac{e_b}{3m_b}$
$\Omega_{b\{cc\}}$	$bcc\ \ \chi_S$	$-\frac{e_b}{6m_b}+\frac{2e_c}{3m_c}$
$\Omega_{c\{bb\}}$	$cbb\ \ \chi_S$	$-\frac{e_c}{6m_c}+\frac{2e_b}{3m_b}$

Table 6. Magnetic moments of double and triple heavy baryons (in units of μ_N)

Baryon	RQM		NRQM
	full	HQL BCF	
Ξ_{cc}^{++}	0.13 (0.52; -0.38)	0.25 (0.51; -0.26)	-0.01 (0.49; -0.50)
Ξ_{cc}^{+}	0.72 (0.52; 0.20)	0.64 (0.51; 0.13)	0.74 (0.49; 0.25)
Ξ_{bb}^{0}	-0.53 (-0.06; -0.47)	-0.42 (-0.08; -0.34)	-0.58 (-0.08; -0.50)
Ξ_{bb}^{-}	0.18 (-0.06; 0.24)	0.09 (-0.08; 0.17)	0.17 (-0.08; 0.25)
Ω_{cc}^{+}	0.67 (0.53; 0.14)	0.60 (0.50; 0.10)	0.67 (0.49; 0.18)
Ω_{bb}^{-}	0.04 (-0.08; 0.12)	0.14 (-0.06; 0.20)	0.10 (-0.08; 0.18)
Ξ_{cb}^{+}	1.52 (0.002; 1.52)	0.75 (0.001; 0.75)	1.49 (0; 1.49)
Ξ_{cb}^{0}	-0.76 (0.002, -0.76)	-0.38 (0.001; -0.38)	-0.74 (0; -0.74)
$\Xi_{cb}^{\prime +}$	-0.12 (0.24; -0.36)	0.18 (0.42; -0.24)	-0.29 (0.21; -0.50)
$\Xi_{cb}^{\prime 0}$	0.42 (0.24; 0.18)	0.54 (0.42; 0.12)	0.46 (0.21; 0.25)
Ω_{cb}^{0}	-0.61 (0.002; -0.61)	-0.26 (0.001; -0.26)	-0.55 (0; -0.55)
$\Omega_{cb}^{\prime 0}$	0.45 (0.25; 0.20)	0.50 (0.42; 0.08)	0.39 (0.21; 0.18)
Ω_{ccb}^{+}	0.53 (0.02; 0.51)	0.14 (0.02; 0.12)	0.51 (0.02; 0.49)
Ω_{cbb}^{0}	-0.20 (-0.08; -0.12)	-0.13 (-0.05; -0.08)	-0.20 (-0.08; -0.12)

Next we will indicate a list of the most recent analyses (QCD sum rules [64, 65]; soliton approaches [61]-[63]; other quark models [57]-[60] for the direct study of the magnetic moments of heavy baryons. In each case we compare the previous results to our predictions. The general conclusion is that we agree in the single heavy quark sector with the model independent approaches, like HHCPT [69], as any Lorentz covariant model should. With our approach we reproduce the leading term in the expansion of the magnetic moments in powers of $1/m_Q$ and $1/\Lambda_\chi$. It is also possible to give predictions for the HHCHPT coupling factors of value $c_S = 0.26 - 0.55$. The results of the full approach in the relativistic three-quark model are close to the ones obtained in Ref. [58]:

$$\begin{array}{l}
\mu_{\Lambda_c^+} = \mu_{\Xi_c^+} = \mu_{\Xi_c^0} = 0.35 \\
\mu_{\Sigma_c^{++}} = 2.37 - 2.45\,, \quad \mu_{\Sigma_c^+} = 0.50 - 0.52\,, \quad \mu_{\Sigma_c^0} = -(1.36 - 1.40) \\
\mu_{\Xi_c^{\prime +}} = 0.75 - 0.78\,, \quad \mu_{\Xi_c^{\prime 0}} = -(1.12 - 1.15)\,, \quad \mu_{\Omega_c^0} = -(0.88 - 0.89) \\
\mu_{\Sigma_b^+} = 2.50 - 2.59\,, \quad \mu_{\Sigma_b^0} = 0.64 - 0.66\,, \quad \mu_{\Sigma_b^-} = -(1.22 - 1.26) \\
\mu_{\Xi_b^{\prime 0}} = 0.88 - 0.92\,, \quad \mu_{\Xi_b^{\prime -}} = -(0.98 - 1.01)\,, \quad \mu_{\Omega_b^-} = -(0.74 - 0.75)
\end{array} \tag{3.24}$$

and in Ref. [59]:

$$\begin{array}{l}
\mu_{\Lambda_c^+} = \mu_{\Xi_c^+} = \mu_{\Xi_c^0} = 0.38\,, \\
\mu_{\Sigma_c^{++}} = 2.33\,, \quad \mu_{\Sigma_c^+} = 0.49\,, \quad \mu_{\Sigma_c^0} = -1.35\,, \\
\mu_{\Xi_c^{\prime +}} = 0.65\,, \quad \mu_{\Xi_c^{\prime 0}} = -1.18\,, \quad \mu_{\Omega_c^0} = -1.02\,.
\end{array} \tag{3.25}$$

The limited results of the QCD sum-rule approach are [64]:

$$\mu_{\Sigma_c^{++}} = 2.1 \pm 0.3\,, \quad \mu_{\Sigma_c^+} = 0.6 \pm 0.1\,, \tag{3.26}$$

$$\mu_{\Sigma_c^0} = -(1.6 \pm 0.2)\,, \quad \mu_{\Lambda_c^+} = 0.15 \pm 0.05\,, \tag{3.27}$$

and

$$\mu_{\Lambda_c^+} = 0.40 \pm 0.05\,, \qquad \mu_{\Lambda_b^0} = -(0.18 \pm 0.05)\,. \tag{3.28}$$

as taken from [65]. The QCD sum-rule results are very similar to our predictions in the full approach.

Solitonic approaches (Skyrme model) of Refs. [61]-[63] were used for a detailed analysis of the magnetic moments of single heavy baryons. The values for the Λ baryons are smaller [63]:

$$\mu_{\Lambda_c^+} = 0.12 - 0.13\,, \qquad \mu_{\Lambda_b^0} = -0.02\,, \tag{3.29}$$

while the magnetic moments obtained for the Σ baryons are larger [63]:

$$\mu_{\Sigma_c^{++}} = 2.45 - 2.46\,, \quad \mu_{\Sigma_c^0} = -1.96\,, \tag{3.30}$$
$$\mu_{\Sigma_b^+} = 2.52\,, \quad \mu_{\Sigma_b^-} = -(1.93 - 1.94)\,, \tag{3.31}$$

than in our full approach. A detailed reason for this discrepancy cannot be given, since our approach and the Skyrme model are based on different model assumptions. A recent work [60] using a relativistic quark model involving three different forms of relativistic kinematics, determines the magnetic moments of single-charm baryons. It was shown that for single heavy $\Lambda-$type baryons the dependence on the specific kinematics is small with the results:

$$\mu_{\Lambda_c^+} = 0.39 - 0.52\,, \quad \mu_{\Xi_c^+} = 0.39 - 0.47\,, \quad \mu_{\Xi_c^0} = 0.39 - 0.47\,. \tag{3.32}$$

In contrary, for the $\Sigma-$type baryons the results of

$$\mu_{\Sigma_c^{++}} = 0.90 - 3.07\,, \quad \mu_{\Sigma_c^0} = -(0.74 - 1.78)\,, \quad \mu_{\Omega_c^0} = -(0.67 - 1.03) \tag{3.33}$$

strongly depend on the relativistic kinematics. As for the results obtained in both of our heavy-quark limit approximations, they are close to the predictions given in the context of the MIT bag model [57]:

$$\mu_{\Sigma_c^{++}} = 0.70\,, \quad \mu_{\Sigma_c^0} = -0.44\,, \quad \mu_{\Omega_c^0} = -0.35\,, \tag{3.34}$$
$$\mu_{\Sigma_b^+} = 0.83\,, \quad \mu_{\Sigma_b^-} = -0.40\,, \quad \mu_{\Omega_b^-} = -0.30\,. \tag{3.35}$$

Further we discuss the magnetic moments of double and triple heavy baryons in comparison to various frameworks within they have been calculated. For example, Ref. [60] using the so-called "point form" of the relativistic kinematics obtains:

$$\mu_{\Xi_{cc}^{++}} = 0.29 - 0.30\,, \quad \mu_{\Xi_{cc}^+} = 0.68 - 0.69\,, \quad \mu_{\Omega_{cc}^+} = 0.66\,, \tag{3.36}$$

which is close to our prediction in the HQL BCF approximation.

Some agreement is found when comparing to the predictions given by the relativistic quark potential model of Ref. [58]:

$$\mu_{\Xi^+_{cc}} = 0.78 - 0.79\,, \quad \mu_{\Omega^+_{cc}} = 0.66\,, \quad \mu_{\Xi^0_{bb}} = -(0.71 - 0.73)\,, \tag{3.37}$$
$$\mu_{\Xi^-_{bb}} = 0.23 - 0.24\,, \quad \mu_{\Omega^-_{bb}} = 0.11\,, \quad \mu_{\Xi^+_{cb}} = 1.50 - 1.54\,, \tag{3.38}$$

as well as for the triple heavy baryons:

$$\mu_{\Omega^+_{ccb}} = 0.49\,, \qquad \mu_{\Omega^0_{cbb}} = -0.20\,. \tag{3.39}$$

Though not evaluated, predictions could also be naturally obtained for the double and triple sector using an extended NRQCD framework [75, 76].

We close this discussion by pointing out the agreement with the NRQM values. First of all, these values are obtained by employing specific spin-flavor wave functions of baryons which correspond to our choice for the relativistic baryonic currents in the non-relativistic limit. Using other spin-flavour structures will have as consequence other results. With our particular choice, the NRQM gives results for the double and triple heavy baryons close to those obtained with the full model.

As mentioned in the introductory part of this chapter, the choice of the cut-off parameter for triple-heavy baryons is fixed by intuitive reasons, varying in the range of 3-7 GeV. Therefore, we test the sensitivity of the magnetic moments on the variation of $\Lambda_{B_{QQQ}}$. For the Ω^+_{ccb} and Ω^0_{cbb} baryons the corresponding magnetic moments are:

$$\mu_{\Omega^+_{ccb}} = 0.58 - 0.50\,, \qquad \mu_{\Omega^0_{cbb}} = -(0.21 - 0.20) \tag{3.40}$$

in the “full” model and

$$\mu_{\Omega^+_{ccb}} = 0.09 - 0.16\,, \qquad \mu_{\Omega^0_{cbb}} = -(0.11 - 0.14) \tag{3.41}$$

in the HQL BCF scheme, where the range of indicated values corresponds to the variation in $\Lambda_{B_{QQQ}}$.

Chapter 4

Magnetic moments of light baryons

The magnetic moments of light baryons can be calculated using the relativistic three-quark model as illustrated in the previous chapter. However, in this chapter we discuss an improved method by implementing pseudoscalar mesonic corrections. The formalism, based on a Lorentz covariant chiral quark Lagrangian, has been proposed and developed in Ref. [12]. This chapter offers an overview of the method of dressing constituent quarks by light mesonic clouds in a consistent chiral fashion. We will point out the most important features of the Lorentz covariant chiral quark model, but we will focus on the application itself: the calculation of static properties of light baryons. We stress that the calculational technique for the S-matrix elements is adressed to the relativistic three-quark model. The main achievement is the determination of the bare constituent quark distribution in the nucleon, as calculated in the relativistic three-quark model. In practice, as will be outlined in the following, meson contributions are factorized and give multiplicative corrections to the bare matrix elements.

The general belief that both valence and sea-quarks contribute to the electromagnetic properties of light baryons is the starting point for the derivation the Lorentz covariant chiral quark model of Ref. [12]. The approach is based on a non-linear chiral Lagrangian written in terms of constituent quarks degrees of freedom and pseudoscalar mesonic fields. An initial and earlier attempt to introduce chiral quarks has been made by Manohar and Georgi in Ref. [77]. They argue that the confinement scale $\Lambda_{QCD} \simeq 100 - 300$ MeV and the sponaneous breaking scale $\Lambda_{\chi SB} \simeq 1$GeV are not the same.

To describe the physics relevant for the energy region between the two scales, Georgi and Manohar pioneered an effective field theory which is able to explain qualitatively the success of the non-relativistic quark model. An effective Lagrangian between the two scales will contain as fundamental fields the quark fields, the Goldstone boson fields and the gluon fields. The quark-gluon interaction is described still by a $SU(3)_{color}$ gauge theory. However, spontanesous chiral symmetry breaking of $SU(3)_L \times SU(3)_R$ at the scale $\Lambda_{\chi SB}$ implies the existence of an octet of Goldstone bosons, which enter the theory as fundamental fields.

A consistent inclusion of higher-order terms has been proposed by Becher and Leutwyler in Ref. [78]. They formulate an effective chiral Lagrangian written in terms of the meson field and the nucleon Dirac spinors split into two parts: a mesonic part and a baryonic part. The mesonic part is the well known meson Lagrangian containing an even number of derivatives of meson fields, while the nucleon part is bilinear in the Dirac spinors $\bar{\Psi}, \Psi$ and contains both an even and odd number of derivatives in the expansion [78]. Each term with derivatives of order n in the nucleon part represents a term of order n in powers of the nucleon momentum p. This expansion is carried out up to power p^4. For a certain quantity, as for the scalar nucleon form factor, the total contribution is given by summing up certain graphs of the perturbation series. A general method is developed to keep track of the relevant graphs. This method is called "infrared regularization" and is based on dimensional regularization. Reliable conclusions are drawn based on the analysis of the infrared singularities of loop integrals for any dimension d. For a simple example one can refer to the self-energy graph discussed in Ref. [78]. Compared to heavy hadron chiral perturbation theory, the method proposed in this paper provides manifest Lorentz invariance of the Lagrangian.

All terms in the Lagrangian obey chiral symmetry. The dimensional regularization preserves the chiral symmetry invariance of the Lagrangian at every order of the perturbation theory. The concept of renormalizability, characteristic for the high-energy region, is irrelevant in the context of an effective low energy theory. The leading infrared singularity lies in the region of small loop momenta. In a low energy expansion of graph with l loops, m mesonic fields and n nucleon propagators, the infrared singularity is of order $p^{dl-2m-n}$, that is counting the powers is similar to the heavy baryon chiral perturbation theory.

Our phenomenological constituent quark-light meson interaction inspired by the original work of [77] is used to dress the constituent quarks by a cloud of light mesons. Then, within a proper chiral expansion, we calculate the dressed transition operators, relevant for the interaction of the dressed quarks with external fields, taken between baryonic states. The calculational technique of dressing involves infrared dimensional regularization of loop diagrams (see appendix Ref. [12]).

The chiral quark Lagrangian as based on [78], used to perform the dressing of the constituents by mesonic fields, is formulated in terms of quarks and meson degrees of freedom. Up to order four in the external momentum, such Lagrangian contains five terms, four terms coming from the quark-meson interaction and the fifth from the meson-meson interaction. We organize the Lagrangian into two pieces:

$$\mathcal{L}_{qU} \,=\, \mathcal{L}_q + \mathcal{L}_U \tag{4.1}$$

where

$$\mathcal{L}_q \,=\, \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \cdots \,, \mathcal{L}_U \,=\, \mathcal{L}_U^{(2)} + \cdots \tag{4.2}$$

The superscript (i) denotes the low-energy dimension of the Lagrangian. Explicitly, each term is given by:

$$\begin{aligned}
\mathcal{L}_U^{(2)} &= \frac{F^2}{4} < u_\mu u^\mu + \chi_+ > , \\
\mathcal{L}_q^{(1)} &= \bar{q}\left[i\not{D} - m + \frac{1}{2}\, g \not{u}\, \gamma^5 \right] q \,, \\
\mathcal{L}_q^{(2)} &= -\frac{c_2}{4m^2} < u_\mu u_\nu > \, (\bar{q}\, D^\mu\, D^\nu\, q \,+\, \text{h.c.}\,) \,+\, \frac{c_4}{4}\, \bar{q}\, i\, \sigma^{\mu\nu}\, [u_\mu, u_\nu]\, q \\
&+ \frac{c_6}{8m}\, \bar{q}\, \sigma^{\mu\nu}\, F^+_{\mu\nu}\, q \,+\, \cdots , \\
\mathcal{L}_q^{(3)} &= \frac{i d_{10}}{2m}\, \bar{q}\, [D^\mu, F^+_{\mu\nu}]\, D^\nu\, q \,+\, \text{h.c.} \,+ \cdots \,, \\
\mathcal{L}_q^{(4)} &= \frac{e_6}{2} < \chi_+ > \, \bar{q}\, \sigma^{\mu\nu}\, F^+_{\mu\nu}\, q \,+\, \frac{e_7}{4}\, \bar{q}\, \sigma^{\mu\nu}\, \{F^+_{\mu\nu}, \hat{\chi}_+\}\, q \\
&+ \frac{e_8}{2}\, \bar{q}\, \sigma^{\mu\nu} < F^+_{\mu\nu} \hat{\chi}_+ > \, q \,-\, \frac{e_{10}}{2}\, \bar{q}\, [D^\alpha, [D_\alpha, F^+_{\mu\nu}]] \sigma^{\mu\nu}\, q \,+ \cdots ,
\end{aligned} \tag{4.3}$$

Here we used the notations: $\hat{\chi}_+ = \chi_+ - \frac{1}{3} < \chi_+ >$, with $< >$ indicating the trace over flavor indices.

The full form of the chiral Lagrangian is found in Ref. [12]. The form displayed above contains only the terms relevant for the derivation of the electromagnetic dressed quark current operator.

In the above expressions m stands for the quark mass, g for the axial charge in the chiral limit, whereas c_i, d_i and e_i denote the second-, the third-, and the fourth-order low-energy couplings, respectively.

As suggested [77], the scale parameter of spontaneous chiral symmetry breaking $\Lambda_\chi \simeq 4\pi F \sim 1$ GeV (F being the octet decay constant) can be used as a dimensional parameter in the higer-order terms, instead of using the constituent mass. This comes just to a redefinition of the low-energy coupling constants; c_2 is replaced by $c_2(m/\Lambda_\chi)^2$ and d_{10} by $d_{10}(m/\Lambda_\chi)$. The usual scale parameter, m, and the new one Λ_χ contribute to the same order in the chiral expansion, by means of order $O(1)$. The octet of pseudoscalar fields is denoted by the usual 3×3 unitary matrix $U = u^2 = exp(i\Phi/F)$ with:

$$\phi = \sum_{i=1}^{8} \phi_i \lambda_i = \sqrt{2} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} . \quad (4.4)$$

We will make use of other commonly used notations [78, 79, 26]. For the covariant derivative and for the external fields Γ_μ we have

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger R_\mu u - \frac{i}{2} u L_\mu u^\dagger \qquad (4.5)$$

where the fields R_μ and L_μ include the external fields (electromagnetic A_μ, weak Z_μ, W_μ):

$$R_\mu \;=\; e\,Q\,A_\mu \;+\cdots\,, \qquad L_\mu \;=\; e\,Q\,A_\mu \;+\cdots \qquad (4.6)$$

where $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ is the quark charge matrix. The meson fields are defined via:

$$u_\mu = i u^\dagger \nabla_\mu U u^\dagger \qquad (4.7)$$

and the mass breaking terms via:

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B\mathcal{M} + \cdots \qquad (4.8)$$

with the mass matrix of the current quark masses $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, \hat{m}_s\}$. We restrict to the isospin symmetry limit with $\hat{m}_u = \hat{m}_d = \hat{m} = 7$ MeV and the mass of the strange quark $\hat{m}_s$ is related to the nonstrange one as $\hat{m}_s = 25\,\hat{m}$. B is the quark condensate parameter denoted in the usual way by

$$B = -\frac{1}{F^2} < 0|\bar{u}u|0 >= -\frac{1}{F^2} < 0|\bar{d}d|0 > \ . \tag{4.9}$$

Also the tensor $F^+_{\mu\nu}$ in the Lagrangian is defined by

$$F^+_{\mu\nu} \ = \ u^\dagger F_{\mu\nu} Q u + u F_{\mu\nu} Q u^\dagger \ , \tag{4.10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the well known photon field strength tensor. We mention that the current quark masses are denoted by $\hat{m}_i$ while the constituent quark masses wear no additional symbol. The masses used for the pseudoscalar mesons are given in the leading order by

$$M_\pi^2 = 2\hat{m}B, \quad M_K^2 = (\hat{m} + \hat{m}_s)B, \quad M_\eta^2 = \frac{2}{3}(\hat{m} + 2\hat{m}_s)B \ . \tag{4.11}$$

In our numerical analysis we use the values: $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV $M_\eta = 574.75$ MeV. For the decay constants we use: $F_\pi = 92.4$ MeV, $F_K/F_\pi = 1.22$ and $F_\eta/F_\pi = 1.3$ [26].

Once armed with such a Lagrangian, we proceed to dress the bare quarks and construct out the dressed current operators. When considering electromagnetic properties we refer only to one-body operators, the γ couples to a constituent quark dressed by the pion. Two-body operators contributions are not considered in this work. Also we will consider the dressing only with pseudoscalar mesons, though the technique of dressing with vector mesons has already been studied in Ref. [12].

A bare one-body quark operator can be dressed using the above derived chiral Lagrangian. First, we start from the bare electromagnetic quark operator:

$$j^{\rm bare}_{\mu,{\rm em}}(x) = \bar{q}(x)\,\gamma_\mu\,Q\,q(x) \ , \tag{4.12}$$

which in momentum space reads as:

$$J^{\rm bare}_{\mu,\,{\rm em}}(q) = \int d^4x\, e^{-iqx}\, j^{\rm bare}_{\mu,\,{\rm em}}(x) \ . \tag{4.13}$$

The bare quark operator has a vector-like current form and has been used to compute current matrix elements also in the previous chapter. Dressing this operator by pions will generate a number of loop diagrams which contribute to the dressed electromagnetic quark operator. These diagrams (Fig. 4 a) are evaluated using the techniques of infrared dimensional regularization.

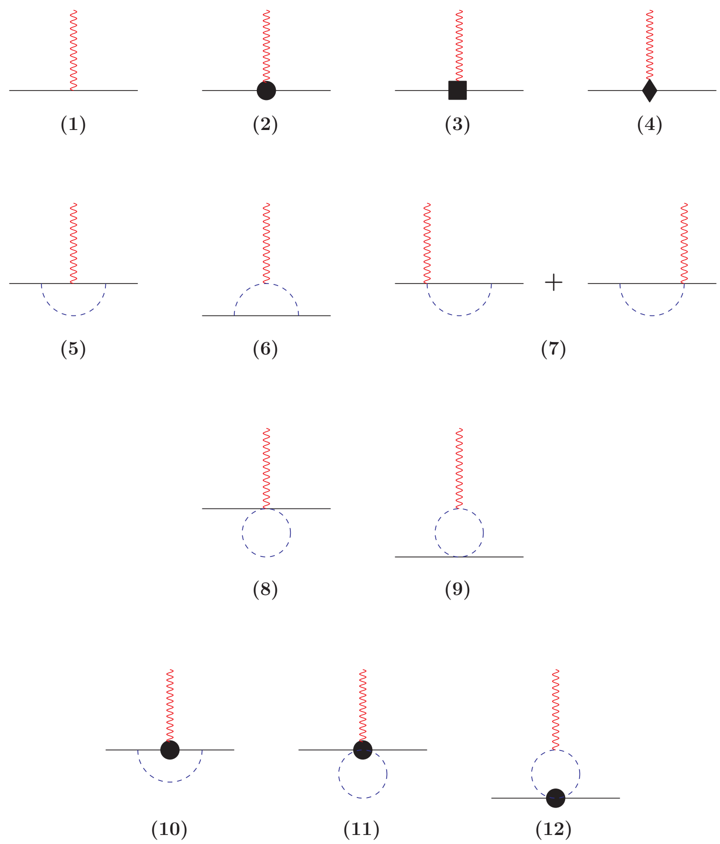

Fig. 4 a Diagrams including pseudoscalar meson contributions to the electromagnetic quark transition operator up to fourth order. Solid, dashed and wiggly lines refer to quarks, pseudoscalar mesons and the electromagnetic field, respectively. Vertices denoted by a black filled circle, box and diamond correspond to insertions from the second, third and fourth order chiral Lagrangian.

The formalism has been suggested and developed in Ref. [78]. Such technique provides the link between loop expansion and chiral expansion in terms of quark masses and external momenta. Here we will not illustrate the steps in performing the infrared dimensional regularization, but a good understanding of this matter can be found in Ref. [12] and references therein. The dressed quark operator in coordinate space is given by:

$$j^{\rm dress}_{\mu,\,\rm em}(x) = \sum_{q=u,d,s} \biggl\{ f_D^q(-\partial^2)\,[\bar q(x)\gamma_\mu q(x)] \,+\, \frac{f_P^q(-\partial^2)}{2m_q}\,\partial^\nu\,[\bar q(x)\sigma_{\mu\nu}q(x)]\biggr\} \tag{4.14}$$

and its Fourier transform reads as:

$$\begin{aligned} J^{\rm dress}_{\mu,\,\rm em}(q) &= \int d^4x\, e^{-iqx}\, j^{\rm dress}_{\mu,\,\rm em}(x) = \int d^4x\, e^{-iqx} \\ &\times \sum_{q=u,d,s} \bar q(x) \biggl[\gamma_\mu\, f_D^q(q^2) \,+\, \frac{i}{2m_q}\,\sigma_{\mu\nu}\, q^\nu\, f_P^q(q^2) \biggr] q(x)\ . \end{aligned} \tag{4.15}$$

The constituent quark mass m_q is generated by the chiral Lagrangian [12]. Since the photon does not couple point-like to a quark line, but to a quark dressed by mesonic clouds, we have the Dirac and Pauli form factors of the u, d and s quarks: $f_D^u(q^2), f_D^d(q^2), f_D^s(q^2)$ and $f_P^u(q^2), f_P^d(q^2), f_P^s(q^2)$. As the dressed quark operator satisfies current conservation $\partial^\mu j^{dressed}_{\mu,em}(x) = 0$, the normalization for the Dirac form factors follows from charge conservation, thus : $f_D^q(0) \equiv 0$.

Once the dressed current operator is given we can proceed further to project this operator between physical baryon states $B(p)$ and, moreover, obtain a parametrization of the cloud contributions.

For the diagonal $\frac{1}{2}^+ \to \frac{1}{2}^+$ transition we have:

$$\begin{aligned} &< B(p')|J^{\rm dress}_{\mu,\rm em}(q)|B(p) > \\ &= (2\pi)^4\delta^4(p'-p-q)\bar u_B(p')\Bigl\{\gamma_\mu F_1^B(q^2) + \frac{i}{2m_B}\sigma_{\mu\nu}q^\nu F_2^B(q^2)\Bigr\}u_B(p) \\ &= (2\pi)^4\delta^4(p'-p-q)\sum_{q=u,d}\Bigl\{ f_D^q(q^2) < B(p')|j^{\rm bare}_{\mu,q}(0)|B(p) > \\ &+ i\frac{q^\nu}{2m_q} f_P^q(q^2) < B(p')|j^{\rm bare}_{\mu\nu,q}(0)|B(p) > \Bigr\}\ . \end{aligned} \tag{4.16}$$

The baryon states are normalized as

$$< B(p')|B(p) >= 2E_B\,(2\pi)^3\,\delta^3(\vec{p}-\vec{p}\,') \tag{4.17}$$

and the spinor states $u_B(p)$ are normalized as

$$\bar{u}_B(p)u_B(p) = 2m_B\ . \tag{4.18}$$

m_B is the physical baryon mass and $E_B = \sqrt{m_B^2 + \vec{p}^{\,2}}$ is the baryon energy.

The first line of the master formula (4.16) is the well known generalization of the matrix element of a electromagnetic current coupling to a composite object with the momentum dependent baryonic form factors $F_1^B(q^2)$ (Dirac) and $F_2^B(q^2)$ (Pauli). However, adding in a chiral consistent fashion the meson cloud contributions (as given by the chiral quark Lagrangian), these form factors take a factorized form. Therefore, the second line connects the matrix element of a dressed quark operator to the matrix elements of the bare quark operators, while the Dirac and Pauli quark form factors parametrize the meson cloud contributions and they simply multiply the bare matrix elements. Finally, our approach allows to perform a model independent factorization between two effects: the chiral dynamic effects, contained in the relativistic Dirac and Pauli quark form factors and the hadronization/confinement effects contained in the bare quark operator matrix elements. It is thus possible to calculate these two contributions independently.

The bare matrix element multiplying the Dirac quark form factor has the vector current form,

$$j_{\mu,q}^{bare}(0) = \bar{q}(0)\gamma_\mu q(0)\ , \tag{4.19}$$

while the one multiplying the Pauli quark form factor has the tensor form

$$j_{\mu\nu,q}^{bare}(0) = \bar{q}(0)\sigma_{\mu\nu} q(0)\ . \tag{4.20}$$

To evaluate the matrix elements of the bare current operators we use the relativistic three-quark model which incorporates in a phenomenological manner hadronization and confinement. The technique is discussed in detail in the previous chapter. The calculation of the Dirac and Pauli quark form factors is based on the formalism of Ref. [12], with an effective chiral Lagrangian written in terms of constituent quarks and consistent with chiral symmetry. The explicit forms of $f_D^q(q^2)$ and $f_P^q(q^2)$ are given in Appendix C of Ref. [12].

4.1 Matrix elements of valence quark operators

Based on the detailed discussion in the previous chapters on the relativistic three-quark model, here we briefly discuss the calculation of the two baryonic matrix elements, relevant for computation of the magnetic moments of light baryons and, later on, of the $N \to \Delta$ transition:

$$< B(p')|\, j^{\rm bare}_{\mu,q}(0)\, |B(p) > \qquad \text{and} \qquad < B(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\, |B(p) > \tag{4.21}$$

induced by the vector-like and tensor-like bare quark operators, respectively:

$$j^{\rm bare}_{\mu,q}(0) \,=\, \bar q(0)\, \gamma_\mu\, q(0)\,, \quad \text{and} \quad j^{\rm bare}_{\mu\nu,q}(0) \,=\, \bar q(0)\, \sigma_{\mu\nu}\, q(0)\;. \tag{4.22}$$

The starting point is the effective interaction Lagrangian (2.1) with the general three-quark current of Eq. (2.2). The finite size of the baryon is characterized by the function F_B given in Eq. (2.4), with the correlation function of the three constituents Φ_B chosen to be a Gaussian function (2.9). The cutoff parameter Λ_B gives the distribution of the light quarks inside the baryon. We set one specific value for all the light octet baryons as based on a previous analysis of nucleon properties [7]. We keep the choice on the definition (though not unique) of the four-space quark coordinates in terms of the Jacobi coordinates and the center of mass coordinate as in Eq. (2.6). All calculations will be carried out in momentum space.

With respect to the choice of the three-quark currents, following the discussion in Chapter 2 and the SU(3) flavour symmetry of the light baryons, one can construct two independent currents: vector and tensor. In Appendix B we list the three-quark currents for the light baryon octet. The vector and the tensor currents of the octet are degenerate in the nonrelativistic limit. Moreover, we will show that they give similar predictions for the magnetic moments of the light baryons.

As before, we can make use of the compositeness condition to determine the couplings of the baryons to their constituents. This implies the calculation of the derivative of the baryon mass operator as shown in Eq. (2.54).

The electromagnetic interaction is incorporated in the effective Lagrangian by using the path-exponential formalism, discussed in Chapter 2 and developed in Ref. [44]. The full Lagrangian enables then to calculate matrix elements. These are described by one triangle diagram and two bubble diagrams as already pictured in Fig. 3 and in complete analogy to the heavy baryon case. The calculation of these graphs involves quark-loop integrals and trace algebra. We again use the free fermion propagator of Eq. (3.3), while the problem of confinement is handled by postulating $M_B < \sum_i m_{q_i}$.

The set of parameters for light baryons, determined in a previous work [7],[30] is given by:

$$\begin{aligned} m_u = m_d = 420 \;\; \text{MeV}\,, \\ m_s = 570 \;\; \text{MeV}\,, \quad \Lambda_B = 1.25 \;\; \text{GeV}\;. \end{aligned} \tag{4.23}$$

The strange quark mass has been set to the average value above as a consequence of the study made on $\Lambda_c^+ \to \Lambda^0 + e^+ + \nu_e$ decay [8]. The size parameter does not vary with the flavor content of the baryon, but we set one value for all light octet members.

4.2 Matching to ChPT

On the basis of some symmetry constraints it is possible to derive a set of relationships between the baryon form factors and quark form factors at zero momentum transfer. Ref. [12] exploits Lorentz and gauge invariance, chiral symmetry constraints, charge conservation and isospin invariance to derive a full set of such relations between nucleon and quark form factors, at their normalization point of zero momentum transfer.

We will briefly list these constraints for the nucleon, while for a full understanding of their consequences we recommend Ref. [12]. First of all, Lorentz covariance and gauge invariance allow us to decompose any bare matrix element as :

$$\begin{aligned}
&< B(p')|\, j^{\rm bare}_{\mu,q}(0)\, |B(p)> \\
&= \bar u_B(p') \Big\{ \gamma_\mu\, F_1^{Bq}(q^2) \,+\, \frac{i}{2\, m_B}\, \sigma_{\mu\nu}\, q^\nu\, F_2^{Bq}(q^2) \Big\} u_B(p)\,, \\
&i\, \frac{q^\nu}{2\, m_q} < B(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\, |B(p)> \\
&= \bar u_B(p') \Big\{ \gamma_\mu\, G_1^{Bq}(q^2) \,+\, \frac{i}{2\, m_B}\, \sigma_{\mu\nu}\, q^\nu\, G_2^{Bq}(q^2) \Big\} u_B(p)\;.
\end{aligned} \tag{4.24}$$

where $F_{1(2)}^{Bq}(q^2)$ and $G_{1(2)}^{Bq}(q^2)$ are the Dirac (index 1) and Pauli (index 2) form factors giving the u, d, s quark distributions in baryon B. Both the vector- and tensor-like bare current operators can be written in terms of Dirac and Pauli form factors. These form factors should not be confused with the quark form factors, nor with the baryon form factors. The baryon form factors $F^{B}_{1,2}(q^2)$ naturally express the extended structure of the baryons, as tested in the electromagnetic interaction. The constituent quark form factors $f^{q}_{D,P}(q^2)$ represent the "structure" of a constituent quark dressed by the meson cloud.

Due to charge conservation and isospin invariance we can obtain at $q^2 = 0$ the following relations:

$$\begin{aligned} F_1^{pu}(0) &= F_1^{nd}(0) = 2\,, \quad F_1^{pd}(0) = F_1^{nu}(0) = 1\,, \quad G_1^{Nq}(0) = 0\,, \\ &\quad F_2^{pu}(0) = F_2^{nd}(0)\,, \qquad F_2^{pd}(0) = F_2^{nu}(0)\,, \\ &\quad G_2^{pu}(0) = G_2^{nd}(0)\,, \qquad G_2^{pd}(0) = G_2^{nu}(0)\,. \end{aligned} \tag{4.25}$$

The $G^{Bq}_{1(2)}(q^2)$ is related to the bare nucleon tensor charge δ^{bare}_{Nq} (see Ref. [12]). The second set of constraints, the chiral symmetry constraints, enable the following identities:

$$\begin{aligned} 1 + F_2^{pu}(0) - F_2^{pd}(0) = G_2^{pu}(0) - G_2^{pd}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}\,, \\ 1 + F_2^{nd}(0) - F_2^{nu}(0) = G_2^{nd}(0) - G_2^{nu}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}\,, \end{aligned} \tag{4.26}$$

where $\bar{m} = m_u = m_d$ are the dressed nonstrange constituent masses in the isospin limit. These relations follow from the infrared-singular structure of QCD, resulting in the reproduction of the so-called leading non-analytic terms terms for the nucleons magnetic moments and charge radii [80]-[81]:

$$\begin{aligned} \mu_p &= -\frac{g_A^2}{8\pi}\,\frac{M_\pi}{F_\pi^2}\,\overset{\circ}{m}_N + \cdots\,, \\ < r^2 >^E_p &= -\frac{1+5g_A^2}{16\,\pi^2\,F_\pi^2}\ln\frac{M_\pi}{\overset{\circ}{m}_N} + \cdots\,, \\ < r^2 >^M_p &= \frac{g_A^2}{16\,\pi\,F_\pi^2\,\mu_p}\,\frac{\overset{\circ}{m}_N}{M_\pi} + \cdots \end{aligned} \tag{4.27}$$

where g_A and $\overset{\circ}{m}_N$ are the axial charge and mass of the nucleon in the chiral limit, respectively. Further conditions can be built up from the chiral symmetry constraints, like the matching between low-energy couplings at a certain order in the ChPT Lagrangian and the low-energy couplings in our Lagrangian (see Ref. [12]), which is beyond the scope of this work. Also, $SU(6)$ symmetry of the naive quark model involving ratios of the magnetic moments and tensor charges of the nucleon sets well-known relations between $F_2^{Ni}(0)$ and $G_2^{Ni}(0)$. Nevertheless, in this work we employ a relativistic model (beyond naive $SU(6)$ predictions)to derive the matrix elements and the contributions of the valence degrees of freedom.

4.3 Light baryon magnetic moments

Using the relativistic quark model to compute the bare matrix elements and the chiral Lagrangian to calculate the mesonic contributions we are able to display the light octet magnetic moments and to split the valence quark μ_B^{bare} and sea-quark μ_B^{cloud} contributions. First we give the definition of the magnetic moments in terms of Dirac and Pauli form factors, as in the previous chapter:

$$\mu_B \,=\, [\,F_1^B(0) + F_2^B(0)\,]\,\frac{e}{2m_B} \tag{4.28}$$

with $\hbar = 1$. In terms of the nuclear magneton $\mu_N = \frac{e\hbar}{2m_p}$, Eq. (4.28) is written as

$$\mu_B = \,[\,F_1^B(0) + F_2^B(0)\,]\,\frac{m_p}{m_B} \qquad \text{(in units of n.m.)} \tag{4.29}$$

The total contribution can be also written as:

$$\mu_B \,=\, \mu_B^{\rm bare} \,+\, \mu_B^{\rm cloud} \tag{4.30}$$

with

$$\begin{aligned} \mu_B^{\rm bare} &= \sum_{q=u,d,s} f_D^q(0)\,\Big(F_1^{Bq}(0) + F_2^{Bq}(0)\Big)\,, \\ \mu_B^{\rm cloud} &= \sum_{q=u,d,s} f_P^q(0) G_2^{Bq}(0)\,. \end{aligned} \tag{4.31}$$

Related to the factorization we observe that each contribution to the magnetic moment, meson-cloud part and bare part, is given by a product of a meson-cloud form factor with a valence-quark form factor at $q^2 = 0$.

In the expression of μ_B^{cloud} there is no contribution from the Dirac part $G_1^{Bq}(0)$, since this valence quark form factor is zero, due to charge conservation (4.25). Also, the meson-cloud Dirac form factor $f_D^q(0)$ is the quark charge with $f_D^q(0) \equiv e_q$ due to charge conservation. The mesonic cloud form factors f_D^q, f_P^q are explicitly calculated in Ref. [12], while the valence quark form factors F_i^{Bq}, G_i^{Bq} are calculated in a similar fashion as already demonstrated for the case of the heavy sector calculations of the previous chapter.

For the octet states two three-quark currents contribute to the observables of interest (see Chapter 2), the vector current and the tensor current. It was shown in Chapter 2 that besides being degenerate in the nonrelativistic limit, in a relativistic approach the two currents give similar contributions to the magnetic moments. This is valid for all the baryon states and does not depend on the flavor content of the barions. It is known that the magnetic moments of light baryons are dominated by nonrelativistic contributions, while the relativistic corrections are of higher order and small. This explains why the naive quark model has been so successful in describing static properties of the light octet. Therefore, a study of these properties possibly does not offer significant insight on the choice of the currents, but a study of other quantities, like the well-known ratios $E2/M1$ and $C2/M1$ in the $N - \Delta$ transition, which are sensitive to the choice of the currents, is more suitable. We will discuss this transition and its properties in the next chapter.

Now we can calculate the values of the light octet baryon, obtaining a very good agreement with experimental values. First of all, the set of parameters is fixed as follows: the constituent quark masses, working within the isospin limit, are $m_u = m_d = 420$ MeV, $m_s = 570$ MeV; the size parameter for all light baryons is set to have the value of $\Lambda_B = 1.25$ MeV. We test the sensitivity of the observables on the choice of the cutoff parameter, using another two values: $\Lambda_B = 0.8$ MeV and $\Lambda_B = 0.75$. In Table 7 we list the magnetic moments of the light octet baryons using the three-quark vector current. For completeness, we also show the results for the diagonal $\frac{1}{2}^+ \to \frac{1}{2}^+$ transition $\Sigma^0 \to \Lambda\gamma$ and for the non-diagonal $\frac{1}{2}^+ \to \frac{3}{2}^+$ transition $N \to \Delta\gamma$. The results and the characteristics of the $N \to \Delta\gamma$ transition will be discussed in the next chapter. We indicate the contribution of the valence quarks and the sea-quarks separately and also show the variation of all these values with respect to the size parameter Λ_B, as follows: Set I ($\Lambda_B = 1.25$ GeV), Set II ($\Lambda_B = 0.8$ GeV) and Set III ($\Lambda_B = 0.75$ GeV).

Table 7. Magnetic moments of the light baryon octet (in units of the nucleon magneton μ_N). Results are calculated for a purely vector current and for three sets of the size parameter Λ_B.

	Set I ($\Lambda_B = 1.25$ GeV)			Set II ($\Lambda_B = 0.8$ GeV)			Set III ($\Lambda_B = 0.75$ GeV)			Experiment [71, 82]
	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total	
μ_p	2.530	0.263	2.793	2.614	0.179	2.793	2.621	0.172	2.793	2.793
μ_n	-1.530	-0.383	-1.913	-1.634	-0.279	-1.913	-1.643	-0.270	-1.913	-1.913
μ_Λ	-0.575	-0.038	-0.613	-0.579	-0.034	-0.613	-0.578	-0.033	-0.613	-0.613 $\pm$ 0.004
μ_{Σ^+}	2.336	0.196	2.532	2.423	0.148	2.571	2.430	0.130	2.560	2.458 $\pm$ 0.010
μ_{Σ^-}	-0.942	-0.327	-1.269	-0.960	-0.223	-1.183	-0.962	-0.135	-1.197	-1.160 $\pm$ 0.025
μ_{Ξ^0}	-1.240	-0.096	-1.336	-1.303	-0.082	-1.385	-1.310	-0.076	-1.386	-1.250 $\pm$ 0.014
μ_{Ξ^-}	-0.599	0.033	-0.566	-0.567	0.012	-0.555	-0.562	0.014	-0.548	-0.6507 $\pm$ 0.003
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.273	0.293	1.566	1.372	0.335	1.617	1.385	0.202	1.607	1.61 $\pm$ 0.08
$\mu_{N\Delta}$	2.357	0.443	2.796	2.984	0.354	3.338	3.102	0.356	3.458	3.642 $\pm$ 0.019 $\pm$ 0.085

The mesonic cloud contributions to the magnetic moments have been already calculated in Ref. [12] using the chiral Lagrangian approach. The parameters used for such a calculation are the constituent quark mass m and the axial-charge g in the chiral limit. Another set of parameters is given by the low-energy coupling constants relevant at this order, c_2, c_6, e_7 and e_8. The first constant c_2 is fixed by the analysis of the nucleon mass, the meson-nucleon sigma term and the momentum transfer dependence of the nucleon form factors [12]. The rest of the couplings are fitted in such way to reproduce the experimental values of the magnetic moments μ_n, μ_p and μ_Λ. In our analysis we fit the last three parameters c_6, c_7 and c_8, which differ from the values of [12].

In the following we show the origin of these parameters and we briefly discuss the procedure of obtaining their values used in the specific calculation of the magnetic moments of light baryons and the $N \to \Delta\gamma$ properties. For a better understanding we show the low-energy dimension terms starting with dimension two in the chiral quark Lagrangian as derived in [12] up to order $O(p^4)$:

$$
\begin{aligned}
\mathcal{L}_q^{(2)} &= c_1\,\langle\chi_+\rangle\,\bar{q}q \,-\, \frac{c_2}{4m^2}\,\langle u_\mu u_\nu\rangle\,(\bar{q}\,D^\mu\,D^\nu\,q \,+\, \text{h.c.}) \,+\, \frac{c_3}{2}\,\langle u_\mu\,u^\mu\rangle\bar{q}q \\
&+ \frac{c_4}{4}\,\bar{q}\,i\,\sigma^{\mu\nu}\,[u_\mu,u_\nu]\,q \,+\, \frac{c_6}{8m}\,\bar{q}\,\sigma^{\mu\nu}\,F^+_{\mu\nu}\,q \,-\bar{q}\mathcal{M}q + c_5\bar{q}\hat{\chi}_+q \,+\, \ldots, \\
\mathcal{L}_q^{(3)} &= \frac{id_{10}}{2m}\,\bar{q}\,[D^\mu, F^+_{\mu\nu}]\,D^\nu\,q \,+\, \text{h.c.} \,+\ldots, \\
\mathcal{L}_q^{(4)} &= -\frac{e_1}{16}\langle\chi_+\rangle^2\,\bar{q}\,q \,+\, \frac{e_2}{4}\langle\chi_+\rangle(\bar{q}\,q) - \frac{e_3}{16}\langle\hat{\chi}_+^2\rangle\bar{q}q - \frac{e_4}{16}\langle\chi_+\rangle\bar{q}\hat{\chi}_+q - \frac{e_5}{16}\bar{q}\hat{\chi}_+^2q \\
&+ \frac{e_6}{2}\,\langle\chi_+\rangle\,\bar{q}\,\sigma^{\mu\nu}\,F^+_{\mu\nu}\,q \,+\, \frac{e_7}{4}\,\bar{q}\,\sigma^{\mu\nu}\,\{F^+_{\mu\nu}\hat{\chi}_+\}\,q \,+\, \frac{e_8}{2}\,\bar{q}\,\sigma^{\mu\nu}\,\langle F^+_{\mu\nu}\hat{\chi}_+\rangle\,q \\
&- \frac{e_{10}}{2}\,\bar{q}\,[D^\alpha,[D_\alpha,F^+_{\mu\nu}]]\sigma^{\mu\nu}\,q \,+\ldots,
\end{aligned}
\tag{4.32}
$$

The use of the physical masses and decay constants for the pseudoscalar mesons in Eq. (4.11) incorporates only partially the corrections due to $SU(3)$ flavor symmetry-breaking, while the quark mass term $\bar{q}Mq$ and the terms containing the low-energy coupling constants (LECs) c_5, e_4, e_5, e_7 and e_8 in Eq. (4.32) generate another part of the $SU(3)$ flavor symmetry-breaking corrections. In fact, the quark mas term and the terms containing the LECs c_5, e_4 and e_5 are responsible for the decoupling of the mass of the strange quark from the isospin-averaged value, m. The terms proportional to the fourth-order couplings e_7 and e_8 introduce explicit SU(3) symmetry breaking corrections to the magnetic moments of the constituents and baryons, which is sufficient to obtain agreement with the experimental values [80].

Although in [12] the meson cloud contributions have been calculated including heavier states, in the follwoing we restrict only to the pseudoscalar meson contributions. As already mentioned for the calculation of the static properties of the light octet and $N \to \Delta\gamma$ transition only the terms multiplied by the constants c_2, e_6, e_7 and e_8 are needed, while the additional constants c_4, d_{10} and e_{10} enter the calculation of the q^2-dependent quantities of the $N \to \Delta\gamma$ transition . We explain here how we extract the parameter values, although the full calculation of the $N \to \Delta\gamma$ transition properties is discussed in the next chapter. Together with the terms in (4.32), the full chiral quark Lagrangian serves to the calculation of the electromagnetic form factors of the nucleon and the meson-nucleon sigma-terms [12]. In fact, besides the c_6, e_7 and e_8 all the other constants required in the calculations performed in this work have been fixed in [12] and we will use their specific values. In [12] the constants c_6, e_7 and e_8 are fixed using the $SU(6)$ symmetry constraints of the naive non-relativistic quark model which draw relations among the form factors F_2^{Ni} and G_2^{Ni}.

The nucleon mass and the meson-nucleon sigma-term are important quantities of low energy nucleon physics. They are constrained by the Feynman-Hellmann theorem [83] which relates the derivative of the nucleon mass with respect to the current quark mass to the pion-nucleon sigma-term $\sigma_{\pi N}$ and to the strange quark condensate in the nucleon:

$$\begin{aligned}
\sigma_{\pi N}\, \bar{u}_N(p) u_N(p) &\doteq \hat{m}\langle N(p)|\bar{u}(0)u(0) + \bar{d}(0)d(0)|N(p)\rangle \\
&= \hat{m}\frac{\partial m_N}{\partial \hat{m}}\, \bar{u}_N(p)\, u_N(p)\,, \\
y_s\, \bar{u}_N(p) u_N(p) &\doteq \langle N(p)|\bar{s}(0)s(0)|N(p)\rangle = \frac{\partial m_N}{\partial \hat{m}_s}\, \bar{u}_N(p)\, u_N(p)\,.
\end{aligned}$$

In quantum field theory the nucleon mass m_N is defined as the matrix element of the trace of the energy-momentum tensor. Considering only the one-body interactions between the light quarks, this matrix element is given by:

$$m_N\, \bar{u}_N(p) u_N(p) \doteq \langle N(p)|\mathcal{H}_{\rm mass}(0)|N(p)\rangle\,. \tag{4.33}$$

The term $\mathcal{H}_{\rm mass}(x) = \bar{q}(x)\, m_q\, q(x)$ is the quark mass term in the Hamiltonian, where $m_q = {\rm diag}\{m_u, m_d, m_s\}$ is the matrix of constituent quark masses with

$m_u = m_d = \bar{m}$ (isospin invariance). We showed in (4.16) that the baryon matrix elements are calculated as expectation values of the dressed quark operators. In analogy, we define the nucleon mass and the sigma-terms as expectation values of the dresses quark operators. The bare quark mass term is given by:

$$\mathcal{H}_{\rm mass}^{\rm bare}(x) = m\,\bar{q}(x)\,q(x)\ , \tag{4.34}$$

which is in fact the quark mass term at leading order of the chiral expansion (in the chiral limit). We remind that m is the constituent quark mass in the chiral limit introduced in the Lagrangian (4.3). The nucleon mass in the chiral limit $\overset{\circ}{m}_N$ is defined by:

$$\overset{\circ}{m}_N\ \bar{u}_N(p)\,u_N(p) = \langle N(p)|\mathcal{H}_{\rm mass}^{\rm bare}(0)|N(p)\rangle = m\langle N(p)|\bar{q}(0)\,q(0)|N(p)\rangle\ . \tag{4.35}$$

On the other hand, the dressed quark mass term and the physical nucleon mass are given by:

$$\begin{aligned}\mathcal{H}_{\rm mass}^{\rm dress}(x) &= \bar{q}(x)\,m_q\,q(x)\,, \\ m_N\,\bar{u}_N(p)\,u_N(p) &= \langle N(p)|\mathcal{H}_{\rm mass}^{\rm dress}(0)|N(p)\rangle\ = \langle N(p)|\bar{q}(0)\,m_q\,q(0)|N(p)\rangle\ .\end{aligned} \tag{4.36}$$

In our case the constituent quark masses are the masses at one loop with inclusion of chiral corrections and with $m_u = m_d$ in the isospin limit. Clearly, using the Lagrangian (4.3) the nontrivial dependence of the constitutent quark masses $m_q \doteq m_q(\hat{m},\hat{m}_s)$ on the current quark massess, $\hat{m},\hat{m}_s$ can be calculated [12]. In [12] the constituent quark masses are calculated at one loop and at $O(p^4)$ with

$$m_q = m + \Sigma_q(m)\ , \tag{4.37}$$

where $\Sigma_q = {\rm diag}\{\Sigma_u,\Sigma_d,\Sigma_s\}$ is the quark mass operator, with $\Sigma_u = \Sigma_d = \bar{\Sigma}$ due to isospin invariance. The quark mass operator is evaluated on the mass-shell $\not{p} = m$, this quantity is ultraviolet-finite by construction. The renormalization of the fourth-order constants e_1, e_3, e_4 and e_5 which contribute to the mass operator Σ_q removes the UV divergencies. The mass operator Σ_q is described by the diagrams in Fig. 4 b.

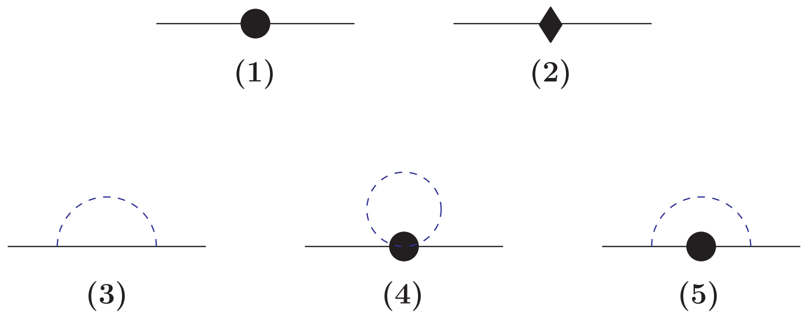

Fig. 4 b. Diagrams contributing to the mass operator of the quark at one loop. Vertices denoted by a black filled circle and diamond correspond to insertions from the second and fourth order chiral Lagrangian.

Further on, the quark mass operator is expanded in powers of pseudoscalar and vector meson masses, as it is done in [12]. Taking into consideration only the pseudoscalar mesons restricts the number of renormalized low-energy constants which enter this expansion. In the most general case, however, these constants are fixed in the follwoing way: using the chiral symmetry constraints and the matching of the nucleon mass calculated within the chiral quark Lagrangian approach to the model-independent derivation of [78] allows certain relations between the parameters used in [12] and the ones used in Baryon ChPT. An example is the term referd to in the literature as the leading nonanalytic term (LNA). Using this requirement the value of the axial charge of the constituent quark in the chiral limit and the axial charge of the nucleon in the chiral limit are related as:

$$g_A^2\, \bar{u}_N(p)\, u_N(p) = g^2 \langle N(p)|\bar{q}(0)\, q(0)|N(p)\rangle\,. \tag{4.38}$$

This matching condition gives a constraint for the scalar condensate, for which a resonable value is obtained using $g_A = 1.25$ and $g \sim 1$. Using (4.38) the final expression for the nucleon mass at one loop is:

$$m_N = \mathring{m}_N + \Sigma_N\,, \tag{4.39}$$

where

$$m_N = \left(\frac{g_A}{g}\right)^2 \bar{m}\,, \qquad \mathring{m}_N = \left(\frac{g_A}{g}\right)^2 m \tag{4.40}$$

and Σ_N is the nucleon mass operator written as an expansion in powers of meson masses, as well. Due to (4.39) the constituent quark mass is removed from the expressions of the nucleon (baryon) observables. The next step, as shown in [12], is to literally compare the expression of the nucleon mass as derived by [12] and the one derived by [78], which in fact leads to a set of conditions between the low-energy coupling constants (LEC) of the chiral quark Lagrangian (4.32) and the ones of the ChPT Lagrangian (at one loop and up to fourth-order).

On the other hand, the calculation of the meson-nucleon sigma-terms imposes also new restrictions on the parameters of the chiral quark model. In general the pion-nucleon sigma-term $\sigma_{\pi N}$ is calculated as the derivative of the term in the QCD Hamiltonian, which explicitly breaks the chiral symmetry, with respect to the current quark mass. In the context of the present model one uses the derivative of the dressed Hamiltonian $\mathcal{H}^{\rm dress}_{\rm mass}(x)$ with respect to the current quark mass of flavor $i = u, d, s$, $\hat{m}_i$. This defines the dressed scalar current operator $j_i^{\rm dress}(x)$ relevant for the calculation of meson-nucleon sigma-terms, as follows:

$$j_i^{\rm dress}(x) \doteq \frac{\partial \mathcal{H}^{\rm dress}_{\rm mass}(x)}{\partial \hat{m}_i} \; = \; \bar{q}(x) \, \frac{\partial m_q}{\partial \hat{m}_i} \, q(x) \; = \; \bar{q}(x) \, \frac{\partial \Sigma_q}{\partial \hat{m}_i} \, q(x) \, . \tag{4.41}$$

For the $\sigma_{\pi N}$ term we have:

$$\sigma_{\pi N} \, \bar{u}_N(p) \, u_N(p) \doteq \hat{m} \, \langle N(p) | \, j_u^{\rm dress}(0) \; + \; j_d^{\rm dress}(0) \, | N(p) \rangle \; , \tag{4.42}$$

which is consistent with the Feynmann-Hellmann theorem (see Ref.[12]). The calculation of the nucleon mass and the meson-nucleon sigma-terms involves the following set of parameters g, m, c_1, c_2, c_5, $\bar{e}_1$, $\bar{e}_3$, $\bar{e}_4$ and $\bar{e}_5$, where $\bar{e}_i$ are renormalized constants. However, for the study of the magnetic moments of the light baryons out of the above parameters only the constants g, m and c_2 are relevant. The parameters g and m are constrained by the second matching condition in (4.40). In literature the value of the axial charge of the constituent quark varies between 0.9 to 1 [84] to justify perturbation theory. The nucleon mass in the chiral limit was estimated in HBChPT: $\overset{\circ}{m}_N = 770 \pm 110$ MeV [85] and $\overset{\circ}{m}_N = 890 \pm 180$ MeV [86]. Using these values for the axial quark charge and the nucleon mass in the chiral limit and the experimental value $g_A = 1.267$ [71] in Eq. (4.39) one obtains the following limiting values for the constituent quark mass in the chiral limit:

$$m \simeq 500 \pm 167 \text{ MeV} \, . \tag{4.43}$$

The choice of g is constrained also by the bare magnetic moments of the nucleon and, as we will show later, the value of the axial quark charge is varied between 0.9 and 1, in the calculation of the magnetic moments of the light baryon octet. The LEC constants mentioned above, among which c_2 is of interest, are fixed by the following conditions:

$$m_N = 938.27 \text{ MeV}\,, \quad \sigma_{\pi N} = 45 \text{ MeV}\,, \quad y_N = 0.2\,, \quad m_s - \bar{m} \simeq 170 \text{ MeV}\,, \tag{4.44}$$

where the quantity y_N is the strangeness content of the nucleon. In [12] it is further discussed that the pion cloud contribution is dominant to the total value obtained for the $\sigma_{\pi N}$. In [12] the constraints imposed on the choice of g by the naive $SU(6)$ calculation of the bare contribution to the magnetic moments of the proton, give:

$$\mu_p^{\text{bare}} \equiv -\frac{3}{2}\mu_n^{\text{bare}} = \frac{3}{5}\left(\frac{g_A}{g}\right)^4 . \tag{4.45}$$

With a value of $g = 0.9$ one obtains a reasonable bare contribution of $\mu_{bare} = 2.357$. where the remaining value to match the experimental total value comes from the meson cloud.

Now we discuss briefly the parameters entering the meson cloud contributions to the magnetic moments of light baryons. These are the second-order low-energy constant c_2, c_6, and the fourth-order renormalized LECs $\bar{e}_6, \bar{e}_7$ and $\bar{e}_8$. The constant $\bar{e}_6$ is absorbed after a certain redefinition on c_6, therefore we remain with three constant only. To constrain these values we need also the experimental value of the magnetic moment of another light baryon, the Λ^0 baryon is chosen, besides the experimental values for the proton and neutron. In [12] the valence quark Sachs form factors associated with the expectation values of the vector and tensor currents are calculated using the naive picture of the $SU(6)$ symmetry group. Here, as already mentioned, the valence quark form factors are calculated using the relativistic quark model. However, in both cases at zero momentum transfer the parameters c_6, c_2 and the renormalized constants $\bar{e}_6, \bar{e}_7$ and $\bar{e}_8$ contribute to the Pauli quark form factor $f_P^q(0)$. The values obtained for the constants c_6 and e_7, e_8 differ from the ones obtained in [12] since in the present approach the bare contributions to the magnetic moments are calculated using the relativistic quark model, hence the total value to be fitted to the experimental value is slightly different from the one obtained within the naive picture.

As will be discussed in the next chapter, in the calculation of the q^2-dependence of the meson-cloud contribution to the $N \to \Delta\gamma$ transition, the following additional low-energy couplings of the chiral Lagrangian [12] and Eq. (4.3) enter: c_4 and $\bar{d}_{10}, \bar{e}_{10}$. In [12] the values for these LECs were obtained by a fit to the q^2-dependence of the electromagnetic form factors of the nucleon.

We now list the values of these parameters. As input parameters we use the physical nucleon mass $m_N = m_p = 938.27$ MeV and the axial charge of the nucleon $g_A = 1.267$. Using the chiral constraints of Eq. (4.26) we can easily determine the quark axial charge value for each set of baryon size parameters:

$$\begin{aligned} \text{SetI g} &= 0.94 \ , \\ \text{SetII g} &= 0.92 \ , \\ \text{SetIII g} &= 0.92 \ . \end{aligned}$$

For the low-energy coupling c_2 we use the value fixed in [12] of $c_2 = 2.502$ GeV^{-2}, while the other low-energy couplings have the following values:
Set I

$$\tilde{c}_6 = 0.163, \quad \bar{e}_7 = -0.426\ GeV^{-3}, \quad \bar{e}_8 = -0.097\ GeV^{-3}\ , \tag{4.46}$$

Set II

$$\tilde{c}_6 = 0.067, \quad \bar{e}_7 = -0.318\ GeV^{-3}, \quad \bar{e}_8 = -0.076\ GeV^{-3}\ , \tag{4.47}$$

Set III

$$\tilde{c}_6 = 0.067, \quad \bar{e}_7 = -0.314\ GeV^{-3}, \quad \bar{e}_8 = -0.082\ GeV^{-3}\ . \tag{4.48}$$

These are the relevant modified low-energy coupling constants for the meson cloud contributions at zero momentum transfer. The values of the relevant constants that enter the expressions of the q^2-dependent quantities are taken from [12].

$$c_4 = 01.693\ GeV^{-1}, \quad \bar{d}_{10} = 1.110\ GeV^{-2}, \quad \bar{e}_{10} = 0.039\ GeV^{-3}\ . \tag{4.49}$$

In all equations above the couplings $\bar{e}_7, \bar{e}_8, \bar{e}_{10}$ and $\bar{d}_{10}$ are in fact renormalized coupling constants [12] and the coupling $\tilde{c}_6$ is related to the renormalized $\bar{e}_6$ via $\tilde{c}_6 = c_6 - 16m(2\hat{m} + \hat{m}_s)B\bar{e}_6$.

Further details on the electromagnetic meson-cloud form factors and their calculation based on the infrared dimensional regularization procedure can be found in [12].

As already discussed, in Ref. [12] the valence quark contributions to the magnetic moments have been fixed by gauge, isospin and chiral symmetry constraints. Furthermore, previously naive $SU(6)$ symmetry constraints have been employed, whereas in this work these contributions are directly calulated employing the effective Lagrangian of the relativistic three-quark model. That does not mean that we will not make use of the symmetry constraints derived in Ref. [12] and listed in this work, but the contribution of the valence quarks does not emerge from these constraints, but from the systematic use of the relativistic three-quark model. The gauge invariance and the isospin invariance constraints are satisfied by the valence quark form factors, whereas the chiral symmetry constraints will be slightly violated in the direct calculation. With the set of parameters of Set I ($\Lambda_B = 1.25$ GeV) we get:

$$\begin{aligned} &1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.06 \,, \\ &G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.51 \,, \end{aligned} \tag{4.50}$$

with Set II ($\Lambda_B = 0.8$ GeV):

$$\begin{aligned} &1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.25 \,, \\ &G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.62 \,, \end{aligned} \tag{4.51}$$

and with Set III $\Lambda_B = 0.75$ GeV:

$$\begin{aligned} &1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.26 \,, \\ &G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.63 \,. \end{aligned} \tag{4.52}$$

The chiral identities of Eq. (4.26) are important model-independent relations, so it is desired to keep them valid in our approach. To do so, one has to modify the vector current $\bar{q}\gamma_\mu q$ or the tensor current $\bar{q}\sigma_{\mu\nu}q$ used in the evaluation of the F and G form factors. We show in the following that a modification of the G_2^{Nq} form factor is favoured to give the necessary constraints. This particular choice is motivated by physical considerations. If we choose to modify the F_1^{Nq} and F_2^{Nq} form factors, we modify the physical bare magnetic moments, as the charge conservation keeps the Dirac quark form factor fixed. This is not desired. A modification of the G_2^{Nq} form factor instead and an appropriate modification of the Pauli $f_P^q(0)$ form factor will lead to the invariance of the meson cloud contribution to the magnetic moment, but will render the fulfilment of the chiral symmetry constraints. To summarize, introducing the modifications

$$\begin{aligned} G_2^{Nq} &\rightarrow \tilde{G}_2^{Nq} \\ f_P^q(0) &\rightarrow \tilde{f}_P^q(0) \end{aligned} \tag{4.53}$$

gives

$$\mu_N^{cloud} \equiv \sum_{q=u,d} f_P^q(0) G_2^{Nq}(0) \equiv \sum_{q=u,d} \tilde{f}_P^q(0) \tilde{G}_2^{Nq}(0) \ . \tag{4.54}$$

Specifically, the modification $f_P^q(0) \rightarrow \tilde{f}_P^q(0)$ involves a redefinition of the low-energy coupling constans c_6, e_7 and e_8. This redefined form factor has no physical meaning. The modification of the low-energy coupling constants is just used to parametrize the modified form factor.

On the other hand, the modification for G_2^{Nq} is aquired by adding the so-called "chiral" counterterm. This term is constructed with nucleon fields. We analyze the case of the tensor current used in the evaluation of this form factor. The tensor current is built with quark fields, the corresponding matrix element is $< B(p^{'})|j_{\mu\nu,q}^{bare}(0)|B(p) >$ and can be modified by adding a term which consists of nucleon fields:

$$\bar{q}(x)\sigma_{\mu\nu}q(x) \rightarrow \bar{q}(x)\sigma_{\mu\nu}q(x) + \frac{\bar{m}}{m_N}\,\bar{N}(x)\sigma_{\mu\nu}\,\delta G^{Nq} N(x) \ , \tag{4.55}$$

where $\delta G^{Nu} = \mathrm{diag}\{\delta G^{pu}, \delta G^{nu}\}$ and $\delta G^{Nd} = \mathrm{diag}\{\delta G^{pd}, \delta G^{nd}\}$ are diagonal 2×2 flavor matrices and $q = u$ or d.

The matrix elements will now give an increase of $G_2^{Nq}(0)$ such that we can preserve the chiral constraints. Actually, from the direct calculation of the Pauli form factors $F_2^{Bq}(0)$ and $G_2^{Bq}(0)$ with the numerical values of Eqs. (4.50-4.52) we understand that we need to increase $G_2^{pu}(0) - G_2^{pd}(0)$ from 3.51 to 4.06, as for example in Set I, such that the chiral constraints (4.26) are fulfilled. But this modifications will affect the normalization of the G_2^{Nq} form factors. However, to fulfil the chiral constraints of Eq. (4.26) we have

to fix the constants δG^{Nq} in the expression of the modified tensor current Eq. (4.55) as follows:
Set I

$$\delta G^{pu} \equiv \delta G^{nd} \simeq -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.440 \ , \tag{4.56}$$

Set II

$$\delta G^{pu} \equiv \delta G^{nd} \simeq -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.504 \ , \tag{4.57}$$

and Set III

$$\delta G^{pu} \equiv \delta G^{nd} \simeq -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.504 \ . \tag{4.58}$$

The effect of introducing a counter term in the tensor current of the matrix elements translates into a modification of the G_2^{Nq} form factors as follows:

$$G_2^{Nq}(0) \to \tilde{G}_2^{Nq}(0) = G_2^{Nq}(0) + \delta G_2^{Nq}(0) \ . \tag{4.59}$$

With the two modifications operating on $f_P^q(0)$ and G_2^{Nq} the invariance of the meson-cloud contributions to the nucleon magnetic moment is not lost. We therefore extend this procedure to the other baryons. We will use the addition of the chiral counterterm in the expression of the quark tensor current and modify the Pauli quark form factors such that the chiral constraints are fulfiled and the meson-cloud contribution to the baryon magnetic moments is invariant:

$$\bar{q}(x)\sigma_{\mu\nu}q(x) \quad \to \quad \bar{q}(x)\sigma_{\mu\nu}q(x) + \frac{\bar{m}}{m_B}\sum_B \bar{B}(x)\sigma_{\mu\nu}\delta G^{Bq}B(x) + \cdots \tag{4.60}$$

where δG^{Bq} = are fixed due to the meson-cloud invariance

$$\mu_B^{\text{cloud}} \equiv \sum_{q=u,d,s} f_P^q(0)G_2^{Bq}(0) \equiv \sum_{q=u,d,s} \tilde{f}_P^q(0)\tilde{G}_2^{Bq}(0) \ . \tag{4.61}$$

The strange form factors $f_P^s \equiv \tilde{f}_P^s$ and $G_2^{Bs} \equiv \tilde{G}_B^{Bs}$ are not modified. It is not necessary to do so since there are no special constraints on the strange quark contribution. Also, the dots in the expression above stand for the modification operated in the non-diagonal terms, relevant for the transition $N \to \Delta$, which can be written in analogy.

Table 8. Sensitivity of the bare contributions to the light baryon magnetic moments on the choice of the octet baryon $3q$-current (in units of the nuclear magneton μ_N). The scale parameter is chosen to be $\Lambda_B = 0.8$ GeV.

	Vector current	Tensor current	Experiment [71, 82]
μ_p	2.614	2.804	2.793
μ_n	-1.634	-1.814	-1.913
μ_Λ	-0.579	-0.594	-0.613 $\pm$ 0.004
μ_{Σ^+}	2.423	2.509	2.458 $\pm$ 0.010
μ_{Σ^-}	-0.960	-0.973	-1.160 $\pm$ 0.025
μ_{Ξ^0}	-1.303	-1.385	-1.250 $\pm$ 0.014
μ_{Ξ^-}	-0.567	-0.560	-0.6507 $\pm$ 0.003
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.372	1.398	1.61 $\pm$ 0.08
$\mu_{N\Delta}$	2.984	2.740	3.642 $\pm$ 0.019 $\pm$ 0.085

In Table 8 we test the sensitivity of the bare contributions to the magnetic moments on the choice of the three-quark current. Again, for completeness, we show results for the diagonal transition $\Sigma^0 \to \Lambda\gamma$ and for the non-diagonal transition $N \to \Delta\gamma$. Here we chose the size parameter to be $\Lambda_B = 0.8$ GeV. Both forms, either the vector or the tensor current, yield numerical predictions which are very close to each other. Hence, the magnetic moments are not necessarily suitable to test the detailed three-quark current structure of the nucleon. Out of this similarity, in Table 7 we only showed the complete results for the vector current calculations.

As will be shown in the next chapter, observables of the $N \to \Delta$ transition are more suitable to elaborate on the relativistic current structure of the valence quarks. In fact, as will be demonstrated with a value of $0.75-0.8$ GeV for the size parameter Λ_B, the central values of the $N \to \Delta$ characteristics can be fixed in consistency with the reasonable results for the magnetic moments displayed in Table 7.

Chapter 5

$N - \Delta\gamma$ transition

We will employ the method outlined in the previous chapter for the calculation of important properties of the $N \to \Delta\gamma$ transition. This analysis is of particular importance as it allows to probe the structure of both the nucleon and the $\Delta(1232)-$isobar and can shed light on their possible deformation. Furthermore, this reaction represents itself a crucial test for the theoretical approaches. For example, the naive quark models based on the SU(6) symmetry in modeling the nucleon and its first resonance as the spherical symmetric 3q-configurations fail in generating non-vanishing values for the electric G_{E2} and Coulomb G_{C2} quadrupole form factors of the $N \to \Delta\gamma$ transition.

In Refs. [87, 88, 89] a model-independent analysis of the $N \to \Delta\gamma$ transition amplitude has been performed. Based on gauge and Lorentz covariance it was shown that the corresponding vertex function is expressed in terms of three linear independent form factors. All other characteristics like helicity or multipole amplitudes are expressed in terms of these form factors.

A comprehensive review of the role of nucleon resonances in nuclear structure has been done in [90]. A pedagogical introduction to the $N - \Delta$ transition underlying the main theoretical ideas and predictions of the constituent quark model (CQM) and its applications to the electromagnetic properties of nucleon and nuclei is given in [18]. The paper reviews the Isgur-Karl model and basic formulas for calculations of the baryon spectrum. Quarks are fundamental carriers of the electric charge of hadrons and the coupling of the photon is introduced at the quark level. The model is used in the calculation of the electromagnetic properties: nucleon form factors, the Δ electromagnetic form factors, excitations of the nucleon resonances. Further improvements are pointed out: inclusion of relativistic effects, introduction of pion degrees of freedom, etc.

An effective Lagrangian incorporating chiral symmetry has been used in [91]. This Lagrangian includes at the tree level the pseudo vector Born terms, leading $t-$channel vector-meson exchanges, $s-$ and $u-$channel Δ-isobar exchanges. The magnetic dipole M1 and electric quadrupole E2 amplitudes are expressed in terms of two independent gauge couplings at the $\gamma N\Delta$ vertex. The investigation of pion photoproduction from threshold through the $\Delta(1232)$ resonance region is done using various unitarization methods, therefore the errors obtained for both E2 and M1 reflect the theoretical uncertainties and model dependence.

Ref. [92] analyzed the vector and axial form factors of the NN and $N\Delta$ systems as well as the πNN and $\pi N\Delta$ coupling constants (calculated defining two effective Lagrangians for the πNN and $\pi N\Delta$ interactions) within a constituent quark model. The main observation is that while the Goldberger-Treiman relation remains still valid, the experimental couplings are found to be larger than the ones predicted in the model. The use of a constituent quark model provides significant mass-dependent corrections to the naive predictions of SU(6) symmetry.

Complex form factors were calculated to order $O(\epsilon^3)$ in the small scale expansion formalism (the inclusion of the Δ degrees of freedom consistent with the chiral symmetry) in the framework of a chiral effective theory [93]. The small scale ϵ denotes, collectively, small momenta, the pion mass and the delta-nucleon mass splitting. The small scale is used in [93] to perform a systematic power counting and to indicate which diagrams are included in the calculation up to a certain order in ϵ. It is shown that the low q^2 dependence of the three transition multipoles M1(q^2), E2(q^2) and C2(q^2) is governed by the πN and $\pi\Delta$ loop effects. The effective chiral lagrangian incorporates the spontaneous and explicit breaking of chiral symmetry. The way that the unknown low energy constants affects the ratios EMR(q^2)=E2(q^2)/M1(q^2) and CMR(q^2)=C2(q^2)/M1(q^2) is elucidated as well as estimated values for these couplings are obtained.

In Ref. [94] it was shown that the $C2/M1$ ratio is related to the netron elastic form factor ratio G_C^n/G_M^n, not only at zero momentum transfer, but for the range of momenta transfer where data are available. In the context of this approach relations are given between the charge quadrupole transition form factor and the elastic nucleon charge form factor on one side, and the magnetic dipole transition form factor and the elastic neutron magnetic form factor, on the other side. For example, at zero momentum transfer, the transition quadrupole moment and the neutron charge radius are related, leading the authors to the conclusion that the phenomena of deviation from nucleon's spherical symmetry has the origin in a nonspherical cloud of quark-antiquark pairs in the nucleon. Performing an extrapolation of the C2/M1 result to $Q^2 \to \infty$ the ratio asymptotically approaches a small negative constant in qualitative agreement with perturbative QCD (pQCD).

In Ref. [95] it was studied the chiral behaviour (M_π dependence) of the ratios EMR and CMR of the multipole form factors of the $\gamma N\Delta$ transition, using a relativistic effective chiral Lagrangian of pion and nucleon fields supplemented with relativistic Δ-isobar fields. The calculation of observables is done corresponding to the pion electroproduction amplitude to next-to-leading order (NLO) in the δ-expansion. The δ-expansion scheme has been developed in a previous work of [95] to serve as power counting for the pion electroproduction amplitude. The small parameter δ is proportional to the excitation energy of the $\Delta-$resonance, which is treated as a light scale of the theory, and inversely proportional to $\Lambda \sim 1$ GeV, representing the heavy scale of the theory. The parameters entering the calculation of different cross sections are the couplings g_M, g_E and g_C characterizing the $M1, E2$ and $C2$ transitions.

In Ref. [96] a theoretical framework has been suggested for the calculation of the $\gamma^* N \to \Delta$ transition using the light-cone sum rule approach. All three possibilities for the virtual photon polarization are allowed, hence the transition is described by three independent form factors. This approach provides a rigorous separation of hard and soft dynamics and provides a tool for the study of the transition region between hard perturbative and soft non-perturbative QCD. The main conclusion on the result for the magnetic form factor is that the "soft" contribution is dominant at the experimentally accessible momentum transfers, while in the region above $Q^2 \sim 2$ GeV^2 calculations of the magnetic form factor are close to the experimental data.

There are a few interesting problems to address in this chapter. First we aim to investigate how important the contributions of both the valence quarks and the meson cloud to the $N \to \Delta$ transition observables are, second, in detail we elaborate on the necessary ingredients in our approach to explain the experimental data for the E2/M1 and C2/M1 ratios.

As before we will use the prescriptions of the relativistic three-quark model to compute the bare electromagnetic matrix elements and the chiral Lagrangian to calculate the meson cloud corrections. At the end, using the factorization of valence quarks and meson cloud contributions, one can calculate the dressed current matrix elements and extract the properties of the $N \to \Delta$ transition. We will underline the few additional differences which arise in the calculational technique of non-diagonal matrix elements.

Again we split the analysis into two parts: first, the evaluation of the bare vector and tensor quark operators between the N and the $\Delta(1232)$ states and the projection of the dressed quark operator between these two states. Replacing the final nucleon state in Eq. (4.16) with the Δ state, the dressed quark operator matrix element is:

$$\begin{aligned} &< \Delta(p')|J^{\rm dress}_{\mu,{\rm em}}(q)|N(p) >= (2\pi)^4\delta^4(p'-p-q)\bar{u}^{\nu}_{\Delta}(p')\Lambda_{\mu\nu}(p,p')u(p) \\ &= (2\pi)^4\delta^4(p'-p-q) \sum_{q=u,d}\Big\{ f^q_D(q^2) < \Delta(p')|j^{\rm bare}_{\mu,q}(0)|N(p) > \\ &+ i\frac{q^\nu}{2m_q} f^q_P(q^2) < \Delta(p')|j^{\rm bare}_{\mu\nu,q}(0)|N(p) > \Big\} \ . \end{aligned} \tag{5.1}$$

Here, $\Lambda_{\mu\nu}(p,p^{'})$ is the $N \to \Delta$ vertex function, $u_N(p)$ is the nucleon spin-1/2 Dirac spinor and $\bar{u}_\Delta(p^{'})$ is the spin-3/2 Rarita-Schwinger spinor. Spin-3/2 particles obey the Rarita-Schwinger equations. A Rarita-Schwinger spinor fulfils the conditions:

$$\bar{u}^{\nu}_{\Delta}(p')\,\gamma_\nu = 0 \quad \text{and} \quad \bar{u}^{\nu}_{\Delta}(p')\,p'^{\,\nu} = 0 \ . \tag{5.2}$$

Due to gauge invariance considerations one can decompose the vertex function $\Lambda_{\mu\nu}(p,p^{'})$ for the nucleon and the $\Delta-$isobar which are both on-shell, in terms of:

$$\Lambda_{\mu\nu}(p,p') = [g_{\mu\nu}b_1(q^2) + p_\mu q_\nu b_2(q^2) + \gamma_\mu q_\nu b_3(q^2) + q_\mu q_\nu b_4(q^2)]\gamma^5 \tag{5.3}$$

where the $b_i(q^2),\ i = 1,2,3,4$, are relativistic form factors. Moreover, gauge invariance dictates a relation for the fourth form factor as being a linear combination of the other three form factors with

$$b_1(q^2) + b_2(q^2)pq + b_3(q^2)m_+ = -q^2 b_4(q^2) \ , \tag{5.4}$$

where $m_\Delta = 1232$ MeV is the mass of the $\Delta-$isobar; the notations used in the previous equations are: $pq = (m_+m_- - q^2)/2$ and $m_\pm = m_\Delta \pm m_N$.

Therefore, the vertex function can be rewritten in a gauge invariant form, involving only the relativistic form factors b_1, b_2 and b_3 :

$$\Lambda_{\mu\nu}(p,p') \;=\; [g^{\perp}_{\mu\nu} b_1(q^2) + p^{\perp}_{\mu} q_\nu b_2(q^2) + \gamma^{\perp}_{\mu} q_\nu b_3(q^2)]\gamma^5 \;, \tag{5.5}$$

or, relating the value of b_1 in terms of b_2, b_3 and b_4 form factors from Eq. (5.4), the gauge invariant vertex function is:

$$\Lambda_{\mu\nu}(p,p') \;=\; \Big[L^{\perp}_{2\,\mu\nu}\, b_2(q^2) \;+\; L^{\perp}_{3\,\mu\nu}\, b_3(q^2) \;+\; L^{\perp}_{4\,\mu\nu}\, b_4(q^2)\Big]\gamma^5 \;. \tag{5.6}$$

The superscript $\perp$ refers to the following expressions:

$$\begin{aligned}
&g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,, \quad p^{\perp}_{\mu} = p_\mu - q_\mu \frac{pq}{q^2}\,, \quad \gamma^{\perp}_{\mu} = \gamma_\mu - q_\mu \frac{\not{q}}{q^2}\,, \\
&L^{\perp}_{2\,\mu\nu} \;=\; p^{\perp}_{\mu} q_\nu - g^{\perp}_{\mu\nu}\, pq\,, \quad L^{\perp}_{3\,\mu\nu} \;=\; \gamma^{\perp}_{\mu} q_\nu - g^{\perp}_{\mu\nu}\, m_-\,, \quad L^{\perp}_{4\,\mu\nu} \;=\; -g^{\perp}_{\mu\nu}\,,
\end{aligned} \tag{5.7}$$

and

$$g^{\perp}_{\mu\nu}\, q^\mu = 0\,, \quad p^{\perp}_{\mu}\, q^\mu = 0\,, \quad \gamma^{\perp}_{\mu}\, q^\mu = 0\;. \tag{5.8}$$

which are the transverse components relative to the photon momentum. All the last terms represent Lorentz structures perpendicular to the photon momentum, so that at the end the full vertex function we have written above is the perpendicular component. The longitudinal component also enters the expression of the vertex function. Though, we have shown for the diagonal transitions that such component is proportional to the mass operator, for non-diagonal transition this term is proportional to

$$\Sigma_{N\Delta}(p^{'}) - \Sigma_{N\Delta}(p)\,. \tag{5.9}$$

Although we use the same notation, the indices differ; these quantities are not mass operators of some baryon state, as the final and initial states are different.

Now it is easy to show that the matrix element of the nondiagonal $N \to \Delta$ transition fulfils the Ward-Takahashi identities and therefore gauge invariance

$$q^\mu\, \Lambda_{\mu\nu}(p,p') = 0\;. \tag{5.10}$$

The decomposition used in Eq. (5.3) is not unique. There are several ways of decomposing the matrix element in terms of relativistic form factors. A pedagogical introduction to this topic can be found in Ref. [18]. In Appendix F we indicate alternative sets of relativistic form factors defining the $N \to \Delta$ transition.

For the calculation of the $N \to \Delta$ matrix element we use the dressed electromagnetic quark operator of Eq. (4.15). The relativistic three-quark model is used to compute the bare matrix elements $< \Delta(p^{'})|j^{bare}_{\mu,q}(0)|N(p) >$ and $< \Delta(p^{'})|j^{bare}_{\mu\nu,q}(0)|N(p) >$, as we already proceeded for the case of the light octet baryons.

The diagrams which contribute to the vector-like bare matrix element are the triangle and the two bubble graphs, while for the tensor-like bare matrix element only the contribution from the triangle diagram is requested.The diagrams are summarized in Fig. 5. A particular additional diagram of Fig. 5 (d), the so-called "pole" diagram, gives a contribution to the non-diagonal $N \to \Delta$ transition. This diagram is included in the full "vector"-like matrix element in order to preserve gauge invariance. It describes the transition of the nucleon into a $\Delta-$isobar via a quark loop, followed by an interaction of the isobar with the electromagnetic field. An analogous diagram can be pictured for the nucleon itself: the nucleon interacts with the external field and then converts into a Δ. However, this contribution is zero because of the Rarita-Schwinger conditions.

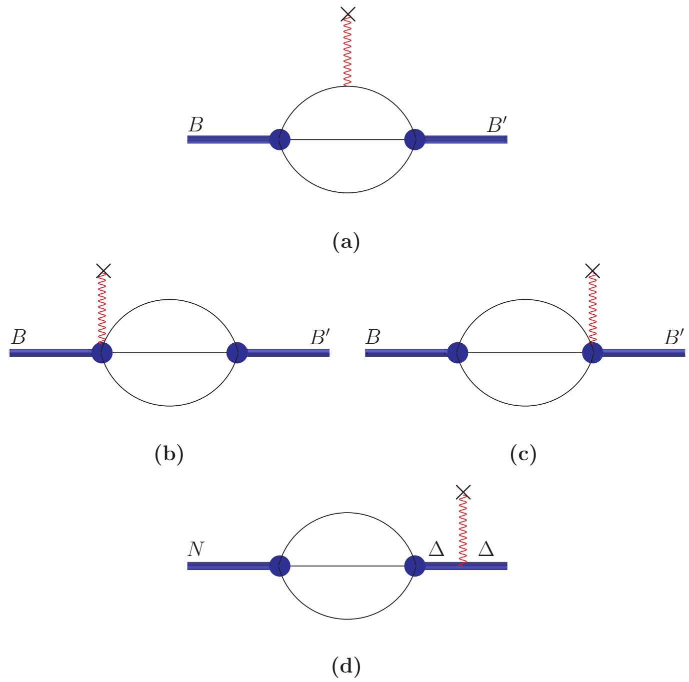

Fig. 5 Diagrams contributing to the matrix elements of the bare quark operators: triangle (a), bubble (b) and (c), pole (d) diagrams. Symbol $\times$ corresponds to the source of the external field.

In the discussion related to light baryon octet, we used a decomposition of the bare vector- and tensor-operators into Dirac and Pauli form factors,as based on Lorentz and gauge invariance, describing the distribution of the quarks inside the baryon Eq. (4.24). Here we will make use of a decomposition for the non-diagonal $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions as well, but now involving the three relativistic form factors

$$< \Delta(p')|\, j^{\rm bare}_{\mu,q}(0)\,|N(p) > \;=\; \bar{u}^{\nu}_{\Delta}(p') \sum_{i=2}^{4} L^{\perp}_{i\,\mu\nu}\, b^{V}_{i}(q^2)\, \gamma^5\, u_N(p) \tag{5.11}$$

and

$$i\,\frac{q^{\nu}}{2\,m_q}\; < \Delta(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\,|N(p) > \;=\; \bar{u}^{\nu}_{\Delta}(p')\; \sum_{i=2}^{4} L^{\perp}_{i\,\mu\nu}\, b^{T}_{i}(q^2)\, \gamma^5\, u_N(p) \tag{5.12}$$

where $b_i^V(q^2)$ are the relativistic form factors that arise from the general decomposition Eq. (5.3) corresponding to the vector-current operator. The form factors $b_i^T(q^2)$ are the ones corresponding to the tensor-current operator. The b_i form factors receive contributions from the bare matrix elements as well as from the meson cloud, where either of these contributions arise from vector-like or tensor-like contributions. To clearly express this combination, let us write the general form factor b_i of Eq. (5.3) as

$$b_i(q^2) \;=\; b_i^{\mathrm{bare}}(q^2) \;+\; b_i^{\mathrm{cloud}}(q^2)\;. \tag{5.13}$$

The bare part is

$$b_i^{\mathrm{bare}}(q^2) \;=\; \sum_{q=u,d} e_q b_i^V(q^2)\;, \tag{5.14}$$

while the meson cloud part is given by

$$b_i^{\mathrm{cloud}}(q^2) \;=\; \sum_{q=u,d} [(f_D^q(q^2)-e_q)b_i^V(q^2) + \; f_P^q(q^2)b_i^T(q^2)]\;. \tag{5.15}$$

From the above relations, the relativistic form factors b_i can be also written as a sum of vector and tensor contributions.

We next define important properties exhibited in the $N\to\Delta$ transition. First of all, the observables we study are easily expressed in terms of the relativistic form factors (expressions in terms of form factors can be found using the relations given in Appendix F and in Refs. [18],[87]-[96]) as follows:
1) Magnetic form factor $G_{M1}(Q^2)$:

$$\begin{aligned} G_{M1}(Q^2) \;&=\; \frac{1}{4}\Big\{b_3(Q^2)\frac{m_+(3m_\Delta+m_N)+Q^2}{m_\Delta} \\ &+\; b_2(Q^2)(m_+m_- + Q^2) \;-\; 2b_4(Q^2)Q^2\Big\}\;. \end{aligned} \tag{5.16}$$

2) Electric form factor $G_{E2}(Q^2)$:

$$\begin{aligned} G_{E2}(Q^2) \;&=\; \frac{1}{4}\Big\{b_3(Q^2)\frac{m_+m_- - Q^2}{m_\Delta} \\ &+\; b_2(Q^2)(m_+m_- + Q^2) \;-\; 2b_4(Q^2)Q^2\Big\}\;. \end{aligned} \tag{5.17}$$

3) Coulombic form factor $G_{C2}(Q^2)$:

$$G_{C2}(Q^2) \;=\; \frac{|\vec{q}\,|}{2}\Big\{b_3(Q^2) \;+\; b_2(Q^2)E_N \;+\; b_4(Q^2)\omega\Big\}\;. \tag{5.18}$$

4) Helicity amplitudes $A_{3/2}$ and $A_{1/2}$:

$$\begin{aligned} A_{3/2}(Q^2) &\;=\; -\sqrt{\frac{\pi\,\alpha\,\omega}{2m_N^2}}\;\big[G_{M1}(Q^2) \;+\; G_{E2}(Q^2)\big]\,, \\ A_{1/2}(Q^2) &\;=\; -\sqrt{\frac{\pi\,\alpha\,\omega}{6m_N^2}}\;\big[G_{M1}(Q^2) \;-\; 3\,G_{E2}(Q^2)\big]\;. \end{aligned} \tag{5.19}$$

5) Ratios EMR $= E2/M1 = -G_{E2}(Q^2)/G_{M1}(Q^2)$ and
CMR $= C2/M1 = -G_{C2}(Q^2)/G_{M1}(Q^2)$:

$$\mathrm{EMR}(Q^2) \;=\; -\,\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}\,, \quad \text{and} \quad \mathrm{CMR}(Q^2) \;=\; -\,\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}\;. \tag{5.20}$$

6) Transition dipole moment $\mu_{N\Delta}$:

$$\mu_{N\Delta} \;=\; \frac{2}{\sqrt{6}}\,G_{M1}(0)\;. \tag{5.21}$$

7) Transition quadrupole moment $Q_{N\Delta}$:

$$Q_{N\Delta} \;=-\,\sqrt{6}\,\frac{m_+\,m_-}{m_\Delta\,m_N}\,G_{E2}(0)\;. \tag{5.22}$$

8) $\Delta^+ \to N + \gamma$ decay width:

$$\Gamma(\Delta^+ \to p\gamma) = \frac{m_\Delta\,m_N}{8\,\pi}\left[1 - \frac{m_N^2}{m_\Delta^2}\right]^2 \Big\{\,|A_{1/2}(0)|^2 + |A_{3/2}(0)|^2\Big\} \tag{5.23}$$

where $Q^2 = -q^2$ is an Euclidean momentum squared, $\alpha = 1/137$ is the fine structure coupling and we made use of the follwoing notations for the nucleon energy

$$E_N \;=\; m_\Delta - \omega = \frac{m_\Delta^2 + m_N^2 + Q^2}{2m_\Delta} \tag{5.24}$$

and for the photon energy

$$\omega = \frac{m_\Delta^2 - m_N^2 - Q^2}{2m_\Delta}\;. \tag{5.25}$$

The 3-momentum of the virtual photon in the Δ-isobar rest frame is given formally by

$$|\vec{q}| \; = \; \frac{\lambda^{1/2}(m_\Delta^2, m_N^2, -Q^2)}{2m_\Delta} \; , \tag{5.26}$$

where

$$\lambda(x,y,z) \; = \; x^2+y^2+z^2-2xy-2xz-2yz \tag{5.27}$$

is the Källen triangle function. Further ways of equivalent definitions of all the observables above in terms of relativistic form factors are listed in Appendix F and also in Refs. [18],[87]-[96]. It is possible to simplify the definitions for the magnetic form factor and the quardupole electric form factor of the transition by making use of the relation (5.4). This allows us to re-express these two form factors in terms of $b_1(Q^2)$ and $b_3(Q^2)$ only:

$$\begin{aligned} G_{M1}(Q^2) &= \frac{1}{2}\biggl\{-b_1(Q^2)+b_3(Q^2)\frac{m_+^2+Q^2}{2m_\Delta}\biggr\} \, , \\ G_{E2}(Q^2) &= \frac{1}{2}\biggl\{-b_1(Q^2)-b_3(Q^2)\frac{m_+^2+Q^2}{2m_\Delta}\biggr\} \, . \end{aligned} \tag{5.28}$$

Thus, the sum of the magnetic and electric form factors equals the relativistic form factor $b_1(Q^2)$, while the difference of these two form factors equals the $b_3(Q^2)$ form factor

$$\begin{aligned} G_{M1}(Q^2)+G_{E2}(Q^2) &= -b_1(Q^2) \, , \\ G_{M1}(Q^2)-G_{E2}(Q^2) &= b_3(Q^2)\frac{m_+^2+Q^2}{2m_\Delta} \, . \end{aligned} \tag{5.29}$$

In the following we will show the results obtained for above properties and will discuss in detail the value obtained for the experimental ratio $E2/M1$. As shown in the previous chapter, the results for the magnetic moments of the light baryons are only slightly sensitive to the choice of either the vector or the tensor current. We concluded that these quantities do not offer sensitive test on the linear combination of these currents. However, the situation changes in the case of the nondiagonal transition $N \to \Delta$. The important quantities $E2/M1$ and $C2/M1$ are sensitive to relativistic effects, thus one can learn about a suitable choice of currents for the nucleon. We have underlined that for the $\Delta-$isobar only the vector current is possible .

For example, using a pure vector current for the nucleon, one recovers the correct negative sign for the ratios $E2/M1$ and $C2/M1$, while the use of the tensor current delivers a wrong negative sign for both ratios. Based on this observation we will further proceed to employ only the vector-vector combination for the $N \to \Delta$ observables. In fact, the best choice would be the use of a linear combination of vector and tensor currents for the nucleon. We show explicitly the sensitivity of the ratios $E2/M1$ and $C2/M1$ on the relative weight of these two current contributions.

To support the idea of a pure vector form in the description of the leading nucleon current we also refer to a QCD sum-rule analysis [97], where the vector current is shown to be preferred. However, the currents used in QCD sum-rules are written down in terms of current quarks, while in the present approach we deal with constitutent quarks.

Table 9. Results for the N $\to \Delta\gamma$ transition (Set I: $\Lambda_B = 1.25$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [71, 82, 98]
EMR (%) at $Q^2 = 0$	-3.22	0.29	-2.93	-2.5 $\pm$ 0.5; -3.07 $\pm$ 0.26 $\pm$ 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.14	0.42	-2.72	-2.28 $\pm$ 0.29 $\pm$ 0.20
CMR (%) at $Q^2 = 0$	-3.69	0.34	-3.35	
CMR (%) at $Q^2 = 0.06$ GeV^2	-4.75	0.44	-4.31	-4.81 $\pm$ 0.27 $\pm$ 0.26
$A_{1/2}(0)$ in 10^{-3} GeV$^{-1/2}$	-87.4	-11.8	-99.2	-135 $\pm$ 6
$A_{3/2}(0)$ in 10^{-3} GeV$^{-1/2}$	-173.0	-20.9	-193.9	-250 $\pm$ 8
$G_{E2}(0)$	0.093	0.002	0.095	0.137 $\pm$ 0.012 $\pm$ 0.043
$G_{M1}(0)$	2.887	0.359	3.246	4.460 $\pm$ 0.023 $\pm$ 0.104
$G_{C2}(0)$	0.107	0.008	0.115	
$Q_{N\Delta}$ (fm^2)	-0.073	-0.001	-0.074	-0.108 $\pm$ 0.009 $\pm$ 0.034
$\mu_{N\Delta}$	2.357	0.439	2.796	3.642 $\pm$ 0.019 $\pm$ 0.085
$\Gamma_{\Delta\to\gamma}$ (MeV)	0.30	0.09	0.39	0.58 - 0.67

Table 10. Results for the N $\to \Delta\gamma$ transition (Set II: $\Lambda_B = 0.8$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [71, 82, 98]
EMR (%) at $Q^2 = 0$	-3.41	0.31	-3.10	-2.5 ± 0.5; -3.07 ± 0.26 ± 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.34	0.33	-3.01	-2.28 ± 0.29 ± 0.20
CMR (%) at $Q^2 = 0$	-3.95	0.26	-3.69	
CMR (%) at $Q^2 = 0.06$ GeV^2	-5.13	0.35	-4.78	-4.81 ± 0.27 ± 0.26
$A_{1/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-110.0	-14.3	-124.3	-135 ± 6
$A_{3/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-219.4	-25.3	-244.7	-250 ± 8
$G_{E2}(0)$	0.125	0.002	0.127	0.137 ± 0.012 ± 0.043
$G_{M1}(0)$	3.655	0.434	4.089	4.460 ± 0.023 ± 0.104
$G_{C2}(0)$	0.144	0.007	0.151	
$Q_{N\Delta}$ (fm^2)	-0.098	-0.001	-0.099	-0.108 ± 0.009 ± 0.034
$\mu_{N\Delta}$	2.984	0.354	3.338	3.642 ± 0.019 ± 0.085
$\Gamma_{\Delta\to N\gamma}$ (MeV)	0.49	0.12	0.61	0.58 - 0.67

Table 11. Results for the N $\to \Delta\gamma$ transition (Set III: $\Lambda_B = 0.75$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [71, 82, 98]
EMR (%) at $Q^2 = 0$	-3.43	0.30	-3.13	-2.5 ± 0.5; -3.07 ± 0.26 ± 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.35	0.30	-3.05	-2.28 ± 0.29 ± 0.20
CMR (%) at $Q^2 = 0$	-3.98	0.25	-3.73	
CMR (%) at $Q^2 = 0.06$ GeV^2	-5.17	0.33	-4.84	-4.81 ± 0.27 ± 0.26
$A_{1/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-114.3	-14.3	-128.6	-135 ± 6
$A_{3/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-228.1	-25.4	-253.5	-250 ± 8
$G_{E2}(0)$	0.130	0.002	0.132	0.137 ± 0.012 ± 0.043
$G_{M1}(0)$	3.800	0.435	4.235	4.460 ± 0.023 ± 0.104
$G_{C2}(0)$	0.151	0.007	0.158	
$Q_{N\Delta}$ (fm^2)	-0.102	-0.002	-0.104	-0.108 ± 0.009 ± 0.034
$\mu_{N\Delta}$	3.102	0.356	3.458	3.642 ± 0.019 ± 0.085
$\Gamma_{\Delta\to N\gamma}$ (MeV)	0.53	0.13	0.66	0.58 - 0.67

In Tables 9, 10 and 11 we list the complete results for the $N \rightarrow \Delta$ observables for three different values of the size parameter $\Lambda_B = 1.25,\ 0.8,$ and $0.75\ GeV$, respectively : the helicity amplitudes, the G_{E2}, G_{M1} and G_{C2} form factors at zero momentum transfer, the dipole $\mu_{N\Delta}$ and quadrupole $Q_{N\Delta}$ moments, the decay width and the ratios $EMR = E2/M1$ and $CMR = C2/M1$. We indicate our results for these ratios both at zero recoil and for finite $Q^2 = 0.06\ GeV^2$ (recently the A1 Collaboration [98] measured these quantities at this kinematic point).

Our predictions are in good agreement with further experimental data by the LEGS Collaboration at Brookhaven [82] and by the GDH, A1, A2 Collaborations at Mainz [98, 99]. The electric, magnetic and Coulomb form factor, the helicity amplitudes, the dipole and quadrupole moments and the decay width are sensitive to the choice of the size parameter. An easy explanation of such a dependence can be given just by looking at the behaviour of the dipole moment $\mu_{N\Delta}$ (and implicitly to the G_{M1} form factor) which seems to be the most sensitive among the observables. The definition of G_{M1} in terms of the relativistic form factors of Eq. (5.16) shows a strong dependence on the variation of the size parameter Λ_B, since at zero momentum transfer it has the following dependence on b_2 and b_3:

$$\mu_{N\Delta} \sim b_3(0) \frac{m_+(3m_\Delta + m_N)}{m_\Delta} + b_2(0) m_+ m_- \ . \tag{5.30}$$

The main contribution actually comes from the $b_3(0)$ form factor which has dimension $1/M$. The contribution of the valence quarks to $\mu_{N\Delta}$ shows that b_3 scales like $1/\Lambda_B$. Hence, decreasing the value of Λ_B leads to an increase of the value of $b_3(0)$ and therefore of the dipole moment, closer to the experimental value. The value $\Lambda_B \simeq 0.75$ GeV gives the best fit to data. The same arguments hold for G_{M1} and other quantities in the tables. We conclude that a good description of data can be achieved with values for the size parameter between $0.75 - 0.8$ GeV.

In Figs. 6-12 we display the Q^2 dependence of the most important experimental quantities: the G_{E2}, G_{M1}, G_{C2} form factors, the helicity amplitudes $A_{1/2}, A_{3/2}$ and the ratios EMR and CMR. The values are indicated for $Q^2 = 0 \div 0.2$ GeV. The solid line corresponds to the full contribution (valence quarks and meson-cloud corrections), while the dashed line corresponds to the bare contributions only. For the Figs. 6-12 we fix the value of the size parameter at $\Lambda_B = 0.8$ GeV.

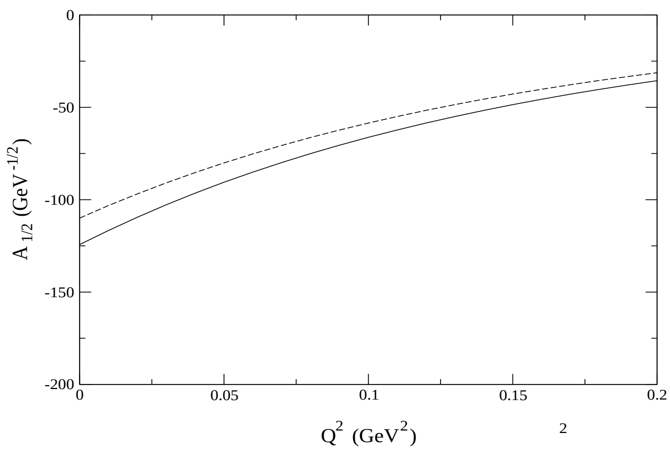

Fig. 6 Helicity amplitude $A_{1/2}(Q^2)$. The solid line is the total result, whereas the dashed line corresponds to the valence quark contribution.

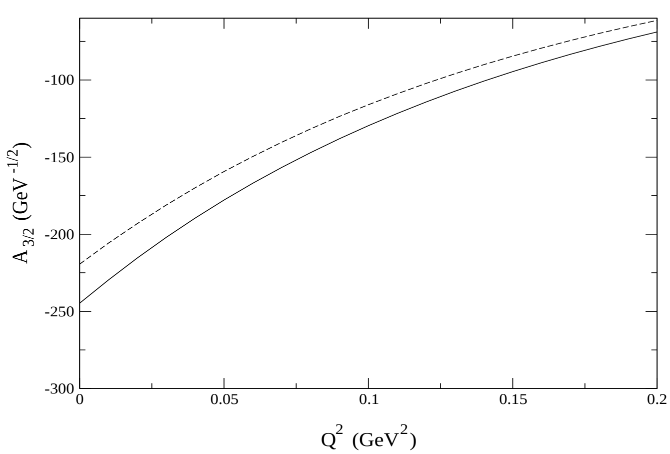

Fig. 7 Helicity amplitude $A_{3/2}(Q^2)$. Otherwise as in Fig. 6.

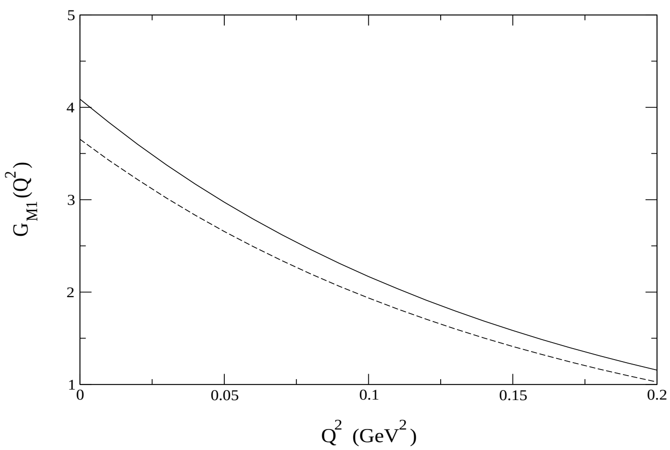

Fig. 8 Form factor $G_{M1}(Q^2)$. Otherwise as in Fig. 6.

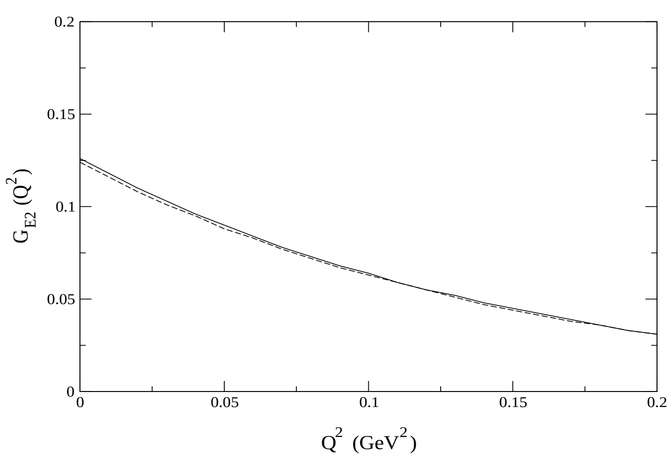

Fig. 9 Form factor $G_{E2}(Q^2)$. Otherwise as in Fig. 6.

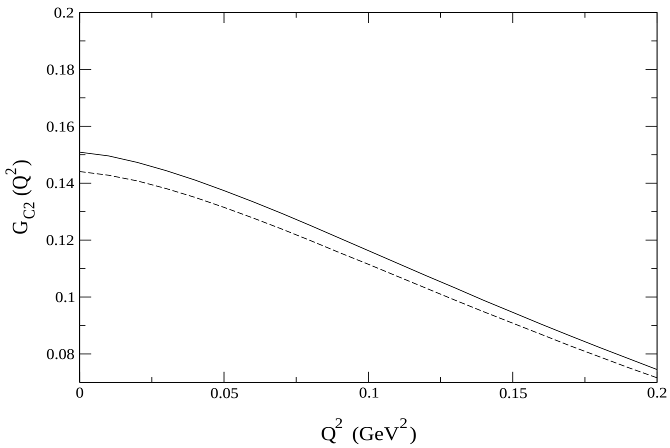

Fig. 10 Form factor $G_{C2}(Q^2)$. Otherwise as in Fig. 6.

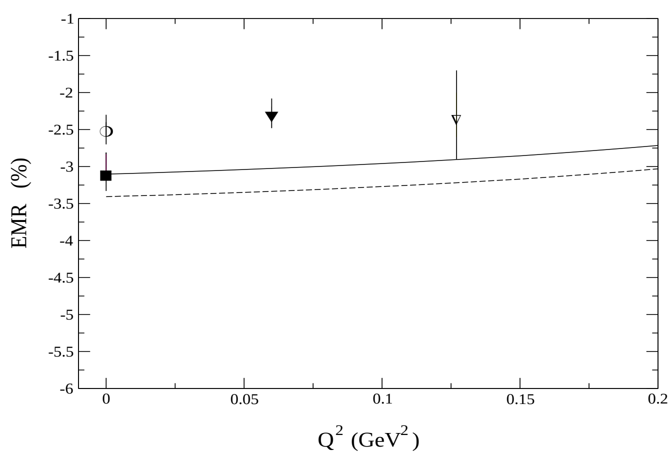

Fig. 11 Ratio EMR$(Q^2) = -G_{E2}(Q^2)/G_{M1}(Q^2)$. Data are taken from Refs. [98] (filled triangle),[82] (filled box), [100] (opened circle) and [101] (opened triangle). Othewise as in Fig. 6.

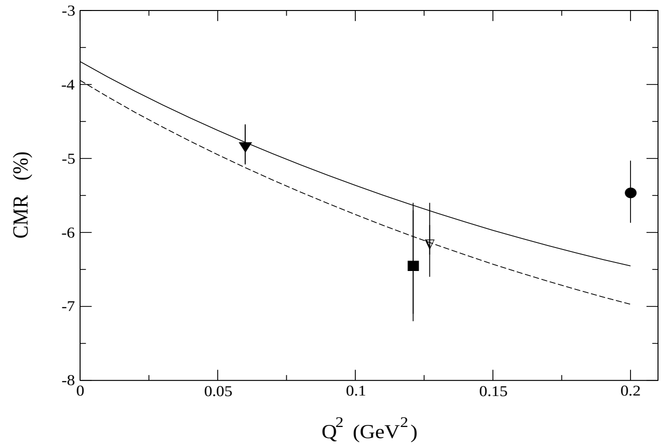

Fig. 12 Ratio $\mathrm{CMR}(Q^2) = G_{C2}(Q^2)/G_{M1}(Q^2)$. Data are taken from Refs. [98] (filled triangle), [101] (opened triangle), [102] (filled box) and [103] (filled circle). Otherwise as in Fig. 6.

The curves obtained within our approach are in good agreement with the present data. We remind that the meson-cloud contributions to the static quantities and to the q^2-dependent quantities depend on a set of low-energy parameters: the constituent quark mass m, the axial quark charge in the chiral limit and a set of low-energy coupling constants, as shown in the chiral quark Lagrangian (4.3). The determination of these parameters has been widely discussed in the Section 4.3.

We shall also discuss the importance of choosing the vector current for nucleon. As mentioned above, a correct negative sign for the two ratios, the EMR and CMR, is achieved with the choice of a pure vector current. The occurence of the desired sign is based on the dependence of these two ratios on the relativistic form factors. We will neglect the b_4 contribution as it appears in the ratios as $b_4(0)/b_3(0) \simeq -1/10$ and $b_4(0)/b_2(0) \simeq 1/5$. Neglecting temporarily also the $b_2(0)$ form factor we recover a result well-known in the literature [87]-[96], when the two rations become degenerate and equal to

$$\text{EMR} \ = \ \text{CMR} \ = -\frac{m_-}{3m_\Delta + m_N} \simeq -6\% \ . \tag{5.31}$$

Therefore to achieve a good agreement with data the $b_2(0)$ dependence is rather important. First of all the sign of this form factor is negative either for a pure vector or a tensor current. However, in the case of the tensor current, the b_2 form factor has a value twice as large than required phenomenologically. This results in a change of sign for G_{E2} and G_{C2} from positive to negative. As G_{M1} carries a negative sign, the overall result of the EMR and CMR ratios will be positive. It is not the same for the vector current, since b_4 contributes with a reasonable value, such that G_{E2} and G_{C2} keep a positive sign and bring the desired negative sign to the ratios. The vector current for the nucleon is therefore preferred by the phenomenology of the $N \to \Delta$ transition. To further elaborate quantitatively on this point, in Table 12 we test the sensitivity of EMR and CMR on the choice of the three-quark current at $Q^2 = 0$ for all three values of the cutoff parameter $\Lambda_B = 0.75, \ 0.8, \ 1.25$ GeV. We use a linear combination of vector and tensor currents:

$$J_p \ = \ (1-\beta)J_p^V \ + \ \beta J_p^T \tag{5.32}$$

with β as mixing parameter. $\beta = 0$ corresponds to a pure vector current, whereas $\beta = 1$ indicates the limit of a to pure tensor current. We observe that an increased contribution by the tensor current leads to a stronger deviation from data. Future and more precise experiments on the ratios EMR and CMR could give a narrow range for the mixing parameter β. Of course, there is room for improvement in our approach as well: tests of the functional form of the vertex function as well as including confinement in the quark propagator.

Table 12. Sensitivity of the EMR and CMR ratios to the choice of the proton $3q$-current

	Mixing parameter β													
	0	0.025	0.05	0.075	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.5	0.75	1
Set I ($\Lambda_B = 1.25$ GeV)														
EMR (%)	-2.93	-2.54	-2.28	-2.04	-1.80	-1.35	-1.08	-0.55	-0.19	0.15	0.47	1.05	2.29	3.19
CMR (%)	-3.35	-3.03	-2.72	-2.42	-2.13	-1.59	-1.08	-0.61	-0.17	0.25	0.63	1.35	2.81	3.95
Set II ($\Lambda_B = 0.8$ GeV)														
EMR (%)	-3.10	-2.83	-2.56	-2.30	-2.06	-1.60	-1.17	-0.77	-0.40	-0.05	0.28	0.87	2.09	3.03
CMR (%)	-3.69	-3.35	-3.03	-2.64	-2.35	-1.80	-1.29	-0.81	-0.37	0.04	0.43	1.14	2.58	3.69
Set III ($\Lambda_B = 0.75$ GeV)														
EMR (%)	-3.13	-2.84	-2.58	-2.33	-2.07	-1.62	-1.19	-0.79	-0.41	-0.07	0.26	0.85	2.10	3.00
CMR (%)	-3.73	-3.39	-3.06	-2.75	-2.44	-1.87	-1.34	-0.85	-0.40	0.03	0.43	1.16	2.65	3.80

Chapter 6

Summary

In this work we pursue a fully covariant quark model (RQM) to study the electromagnetic properties of baryons, both heavy and light. In our framework the baryon is in leading order described as a composite object of three constituent quarks.

The first part of this work was dedicated to the underlying principles of the relativistic quark model. The relativistic quark model can be viewed as a quantum field theory approach based on a phenomenological Lagrangian of light and heavy baryons interacting with their constituent quarks. Such an interaction Lagrangian is written in terms of baryon and quark fields. The quark fields are hidden in the relativistic baryon three-quark currents. We have used an universal Gaussian shape for the baryon-quark vertex and we argue about the choice of the relativistic vertex function. Since a solution of the Faddeev-Popov equations for baryons as bound states is not available, we have free choice on the shape of the relativistic vertex function, as long as the particular form chosen falls rapidly enough in the ultraviolet region. The wave function of the three constituent quarks as given by an analysis in the Bethe-Salpeter approach should indicate a shape for our phenomenological distribution of the constituent quarks inside the baryon, possibly from first principles. As in the previous applications of the relativistic quark model, the choice of the Gaussian shape is universal for all baryon states, while we allow for the free cut-off parameter to vary with the flavour content. However, the cut-off parameters fall into three categories for the heavy sector: single-, double- and triple-type parameters for single-, double-, and triple-heavy baryons, respectively. The specific values of these parameters have been extracted from the best fit to the data. One can elaborate more on the choice of the vertex function and test the sensitivity of the basic hadronic observables, as has been done in [35].

The constituent quark fields enter the Lagrangin via the three-quark currents. One considers two important aspects when deriving such currents: the description of ground state baryons is restricted to the use of non-derivative currents and the structures involved should meet the right quantum numbers of the baryons. We explicitly discussed the formalism involved in the derivation of the three-quark currents and exemplified the structures with the similar mathematical procedure used in the QCD sum rule approaches. Of course, in our case the relevant degrees of freedom are the constituent quarks and a linear combination of all the possible currents is taken into consideration. However, one can choose specific forms for the three-quark currents as restricted by the symmetry or antisymmetry properties of the baryon wave function under the permutation of relevant flavor indices. Moreover, as expected, some classes of relativistic currents give similar predictions to those observables which have low sensitivity to relativistic effects. On the other hand, there are observables highly dominated by relativistic effects and, therefore, highly sensitive to the combination of the currents.

One important ingredient of the relativistic quark model is the compositeness condition. Originally, Weinberg [41] proposed this condition for the description of the deuteron as a bound state of proton and neutron. Generally speaking, the compositeness condition applies to the description of any bound state and sets an upper limit for the coupling of the physical particle to its own constituents. This limit is achieved when the matrix element between the physical particle and its corresponding bare state is zero. By this means, the baryon is a composite object and exists dressed by the interaction only through its virtual constituent quarks.

After we specified all the ingredients of the Lagrangian and after deriving the Feynman rules one can proceed with the calculation of the S-matrix elements describing the hadron-hadron interaction. The observables will be expressed by a set of quark diagrams. We have shown that for the calculation of the magnetic moments several processes contribute, the triangle and the two bubble diagrams. These diagrams are generated by the coupling of the photon to the individual quark lines in the baryon-quark vertex. We have shown that the incorporation of the electromagnetic interaction into the effective Lagrangian is consistenly done within the path-exponential formalism. The relativistic quark model provides a self consistent powerful calculational technique, a reduced number of free parameters (the cut-off parameter and the constituent quark masses entering the free fermion propagator).

In the second part of this work we concentrated on the calculation of the magnetic moments of single-, double- and triple-heavy baryons using the relativistic quark model. An interesting aspect is the ability of the RQM to recover the model-independent HHChPT predictions, as any model should aim for. Therefore, the analysis of the magnetic moments of heavy baryons has been carried out taking into consideration the heavy-quark limit, both in the baryonic correlation functions (BCF) and in the BCF and the heavy quark propagators (HQP). We also performed the calculation of the magnetic moments within a non-relativisic framework, where the non-relativistic spin-flavor wave functions of the baryon are chosen to match the non-relativistic limit of our relativistic currents. In the numerical discussion, we showed that the magnetic moments of $\Lambda-$type baryons are practically the same in all three models (full approach, heavy-quark limit in BCF and heavy-quark limit in both BCF and HQP), because the main contribution comes from the coupling of the photon to the heavy degree of freedom. However, for the $\Sigma-$type baryons the main contribution comes from the coupling of the photon to the light diquark system (as also predicted by the non-relativistic quark model) in the full approach, while in the heavy quark limit this contribution is suppressed. The last affirmations are in full agreement with the HHChPT prescriptions. We showed explicitly that we exactly recover the magnetic moments of the $\Lambda-$type and $\Sigma-$type baryons in the heavy quark limit in correspondence with the model-independent predictions of HHChPT. We also noticed that the contribution of the bubble diagrams is suppressed with respect to the contribution of the triangle diagram. Further on, a comparison of our results to the ones obtained in various theoretical approaches, for example in the relativistic quark models of [58] and [60], in QCD sum rules [64], shows a good agreement. However, we do not agree with the Skyrme model predictions [63].

In the last part of this work we extended the relativistic quark model to the analysis of the electromagnetic properties of light baryons. In particular we calculated the magnetic moments of the light baryon octet and the properties of the $N \to \Delta\gamma$ transition including the pseudoscalar meson cloud corrections. We proved the consistency when introducing the chiral corrections into the relativistic quark model by deriving a master formula which highlights a model-independent factorization: cloud effects are encoded in a set of relativistic quark form factors, whereas the effects of confinement and hadronization are contained in the bare electromagnetic quark operators.

Thus, the dressed electromagnetic quark operators sandwiched between the baryon states can be expressed in terms of bare matrix elements multiplied by meson cloud contributions. The dressing of the bare quark operator is done using the baryon chiral perturbation theory prescriptions by writing a Lorentz covariant chiral quark Lagrangian in terms of constituent quarks and chiral fields as effective degrees of freedom. We considered only one body operators for the dressed quark operators. Such an operator has been calculated [12] up to the order $O(p^4)$ in external momenta using the infrared dimensional regularization technique. The calculation of bare matrix elements has been done using the relativistic quark model. In addition to the triangle and bubble diagrams, for the non-diagonal transition there is an extra "pole"-diagram contribution, due to gauge invariance requirements. In the practical calculations we used the vector and tensor currents for the nucleon and we tested the sensitivity of the magnetic moments of light baryons on the choice of these currents. Since the relativistic contributions to the magnetic moments are small and only of higher order, we argued that the vector and tensor currents are expected to give similar predictions for these quantities. That explains the success of the naive quark model results. For the $N \to \Delta\gamma$ transition we calculated helicity amplitudes, electromagentic form factors and the long disputed multipole ratios $E2/M1$ and $C2/M1$, indicating also results for the recent kinematical points. We performed a q^2-dependent analysis of the helicity amplitudes and of the multipole ratios which shows a good agreement with the latest data. In the analysis of the properties of the light baryons we indicated results separately for the contributions of the valence quarks and for the meson-cloud contributions and therefore we offered a good understanding of the necessity of taking such corrections into consideration.We conclude that the contribution of the meson cloud to the static properties of light baryons counts up to 20%, a value consistent with the perturbative nature of such contributions. Together with the relativistic corrections we can improve the naive predictions offered in the $SU(6)$ picture. Also, the two experimental values for the multipole ratios are succesfully recovered with our approach. We have analyzed how relativistic effects count to recover the correct negative sign and reasonable values for these quantities. The ratios $E2/M1$ and $C2/M1$ are significantly dominated by relativistic effects, thus it is worth testing the sensitivity of these quantities on the choice of the currents. The correct minus sign is achieved with high or maximum contribution of the nucleon vector current.

Further improvement is needed with respect to the weight of the nucleon tensor current. In future, chiral corrections due to heavier states may be included in a similar fashion. In the heavy baryon sector one could elaborate and include meson cloud contributions, as well. On the other hand, the relativistic quark model predictions can be improved by considering an appropriate vertex function as derived for example from Bethe-Salpeter approaches. Another important aspect to be considered in the future is related to the choice of the baryon three-quark current.

Appendix A

Representations of the Lorentz group

The Lorentz group is defined as a set of 4×4 matrices which leave the space-time interval

$$s^2 = c^2t^2 - x^i x^i = x^\mu g_{\mu\nu} x^\nu \tag{A.1}$$

invariant and $g_{\mu\nu}$ is the metric of the group. If all signs within the metric were positive then the group would be $O(4)$. As the signs alternate, the Lorentz group can be labeled as $O(3,1)$, that means three positive signs and one negative sign within the metric. Following that, the difference between the Lorentz group and the $O(4)$ group translates into that the invariant distance can be both positive or negative within the Lorentz group and only positive within the $O(4)$ group. Despite this crucial difference, the representations of the two groups share common properties.
The action of the Lorentz group on some fields is written:

$$L^{\mu\nu} = i(x^\mu \partial^\nu - x^\nu \partial^\mu) \ . \tag{A.2}$$

with $p^\mu = i\partial_\mu$ and which generates the algebra of the Lorentz group

$$[l^{\mu\nu}, L^{\rho\sigma}] = ig^{\nu\rho}L^{\mu\sigma} - ig^{\mu\rho}L^{\nu\sigma} - ig^{\nu\sigma}L^{\mu\rho} + ig^{\mu\sigma}L^{\nu\rho} \ . \tag{A.3}$$

Defining $U(\Lambda) = exp(i\epsilon^{\mu\nu}L_{\mu\nu})$, the action of the Lorentz group on a vector field is given by

$$U(\Lambda)\Phi^\mu(x)U^{-1}(\Lambda) = (\Lambda^{-1})^\mu_\nu \Phi^\nu(x^{'}) \tag{A.4}$$

The Lorentz boosts of coordinates $x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\nu}$ can be written in terms of $M^{\mu\nu}$, generators of $O(N)$, with the definitions

$$\begin{aligned} J^i &= \tfrac{1}{2}\epsilon^{ijk} M^{jk} \ , \\ K^i &= M^{0i} \ . \end{aligned} \tag{A.5}$$

The Lorentz boost in the x direction is $e^{iK^x v^x} = cosh\,\Phi + i\,sinh\,\Phi K^x$ with v^x the $x-$component of the velocity (velocity components are the parameters of the Lorentz boosts). The K matrices do not form a closed algebra (a boost in the y direction followed by a boost in the x direction does not generate another Lorentz boost), therefore the rotation group $O(3)$ is introduced, with the three generators J^x, J^y, J^z. Then the K and J generate a closed algebra, the algebra of the Lorentz boosts and rotations. They fulfil the commutation relations

$$[K^i, K^j] = -i\epsilon^{ijk} J^k \ , \qquad [J^i, J^j] = i\epsilon^{ijk} J^k \ , \qquad [J^i, K^j] = i\epsilon^{ijk} K^k \ . \tag{A.6}$$

Taking a linear combination of these generators

$$\begin{aligned} A^i &= \frac{1}{2}(J^i + iK^i) \ , \\ B^i &= \frac{1}{2}(J^i - iK^i) \ , \end{aligned} \tag{A.7}$$

the algebra of the Lorentz group splits into two pieces

$$[A^i, B^j] = 0 \ . \tag{A.8}$$

Additionally, if one exploits the similarities between the Lorentz group algebra and the $SO(4)$ algebra and the fact that $SO(4) = SU(2) \times SU(2)$, we notice that each piece A^i and B^j generates a separate $SU(2)$. Changing the sign in the metric, the Lorentz group is written as $SU(2) \times SU(2)$, so that irreducible representations (j) of SU(2), $j = 0, 1/2, 3/2, ...$ etc, can be used to construct representations of the Lorentz group, labeled $(j, j^{'})$, by pairing two representations of SU(2). A spinorial representation of the Lorentz group is obtained, if $j + j^{'}$ is half-integral.

A four-spinor can be decomposed as

$$\Psi = \begin{pmatrix} \Psi_R \\ \Psi L \end{pmatrix} \tag{A.9}$$

The two spinors, right- and left-handed can be chosen as

$$\begin{aligned} (1/2,0) &= \frac{1+\gamma^5}{2}\Psi \ , \\ (0,1/2) &= \frac{1-\gamma^5}{2}\Psi \ . \end{aligned} \tag{A.10}$$

To construct higher spin fields one can take tensor products between the spinors. Vectors can be constructed as the product of two spinors

$$(1/2,0)\otimes(0,1/2) = (1/2,1/2) \ . \tag{A.11}$$

A spin-3/2 field can be written in the most common way as the product of a spinor and a vector

$$(1/2,1/2)\otimes(1/2,0) = (1,1/2)\oplus(0,1/2) \ , \tag{A.12}$$

which corresponds to the construction of a four-spinor with a vector index attached

$$\Psi_\mu = \begin{pmatrix} \Psi_{\mu R} \\ \Psi_{\mu L} \end{pmatrix} \ . \tag{A.13}$$

The $(0,1/2)$ spinor corresponds to the contraction of spin-3/2 spinor with a gamma matrix

$$(0,1/2) = \gamma^\mu \Psi_\mu \ . \tag{A.14}$$

The $(1,1/2)$ representation corresponds to a spin-3/2 field and has zero contraction on a gamma matrix.

$$(1,1/2) = \Psi_\mu - \frac{1}{4}\gamma_\mu\gamma^\nu\Psi_\nu \ . \tag{A.15}$$

This discussion is useful for a complete understanding of the decomposition of a three-quark current operator into the irreducible representations of the Lorentz group.

Appendix B

Three-quark currents

B.1 Light baryons

Here we present a list of the three-quark currents of the baryon octet.
I. Vector currents:

$$\begin{aligned}
J_p^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,, \\
J_n^V &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,, \\
J_{\Sigma^+}^V &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,, \\
J_{\Sigma^0}^V &= \sqrt{2}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu d^{a_3}\,, \qquad (B.1) \\
J_{\Sigma^-}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,, \\
J_{\Xi^-}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,, \\
J_{\Xi^0}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,, \\
J_{\Lambda^0}^V &= \sqrt{\frac{2}{3}}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 (u^{a_1}d^{a_2}C\gamma_\mu s^{a_3} - d^{a_1}u^{a_2}C\gamma_\mu s^{a_3})\,. \qquad (B.2)
\end{aligned}$$

II. Tensor currents:

$$\begin{aligned}
J_p^T &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 d^{a_1}u^{a_2}C\sigma_{\mu\nu}u^{a_3}\,, \\
J_n^T &= -\varepsilon^{a_1a_2a_3}\sigma_{\mu\nu}\gamma^5 u^{a_1}d^{a_2}C\sigma_{\mu\nu}d^{a_3}\,,
\end{aligned}$$

$$
\begin{aligned}
J^T_{\Sigma^+} &= -\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 s^{a_1}u^{a_2}C\sigma_{\mu\nu}u^{a_3}\,, \\
J^T_{\Sigma^0} &= \sqrt{2}\;\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 s^{a_1}u^{a_2}C\sigma_{\mu\nu}d^{a_3}\,, \qquad \text{(B.3)} \\
J^T_{\Sigma^-} &= \varepsilon^{a_1a_2a_3}\gamma^{\mu}\gamma^5 s^{a_1}d^{a_2}C\gamma_{\mu}d^{a_3}\,, \\
J^T_{\Xi^-} &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 d^{a_1}s^{a_2}C\sigma_{\mu\nu}s^{a_3}\,, \\
J^T_{\Xi^0} &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 u^{a_1}s^{a_2}C\sigma_{\mu\nu}s^{a_3}\,, \\
J^T_{\Lambda^0} &= \sqrt{\frac{2}{3}}\;\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5(u^{a_1}d^{a_2}C\sigma_{\mu\nu}s^{a_3} - d^{a_1}u^{a_2}C\sigma_{\mu\nu}s^{a_3})\,. \qquad \text{(B.4)}
\end{aligned}
$$

The three-quark (vector) currents of the Δ-isobar are:

$$
\begin{aligned}
J^{\mu}_{\Delta^{++}} &= \varepsilon^{a_1a_2a_3}u^{a_1}u^{a_2}C\gamma^{\mu}u^{a_3}\,, \\
J^{\mu}_{\Delta^{+}} &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(d^{a_1}u^{a_2}C\gamma^{\mu}u^{a_3} + 2u^{a_1}u^{a_2}C\gamma^{\mu}d^{a_3})\,, \\
J^{\mu}_{\Delta^{0}} &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(u^{a_1}d^{a_2}C\gamma^{\mu}d^{a_3} + 2d^{a_1}d^{a_2}C\gamma^{\mu}u^{a_3})\,, \\
J^{\mu}_{\Delta^{-}} &= \varepsilon^{a_1a_2a_3}d^{a_1}d^{a_2}C\gamma^{\mu}d^{a_3}\,. \qquad \text{(B.5)}
\end{aligned}
$$

We also display the currents for a diquark subsystem of two identical quarks (two “up” or two “down” quarks):

$$
\begin{aligned}
J^{\mu}_{\Delta^{+}} &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(2d^{a_1}u^{a_2}C\gamma^{\mu}u^{a_3} - i\gamma_{\nu}d^{a_1}u^{a_2}C\sigma^{\mu\nu}u^{a_3})\,, \\
J^{\mu}_{\Delta^{0}} &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(2u^{a_1}d^{a_2}C\gamma^{\mu}d^{a_3} - i\gamma_{\nu}u^{a_1}d^{a_2}C\sigma^{\mu\nu}d^{a_3})\,. \qquad \text{(B.6)}
\end{aligned}
$$

B.2 Heavy baryons

We display the complete list of three-quark curents used in the calculation of magnetic moments of single-, double-, triple-heavy baryons.

$$
\begin{aligned}
J^{\Sigma_c^{++}} &= \gamma^{\mu}\gamma^5 c^a(u^bC\gamma_{\mu}u^c)\epsilon^{abc} \\
J^{\Sigma_c^{0}} &= \gamma^{\mu}\gamma^5 c^a(d^bC\gamma_{\mu}d^c)\epsilon^{abc} \\
J^{\Sigma_c^{+}} &= \sqrt{2}\gamma^{\mu}\gamma^5 c^a(u^bC\gamma_{\mu}d^c)\epsilon^{abc}
\end{aligned}
$$

$$
\begin{aligned}
J^{\Omega_c^0} &= \gamma^\mu\gamma^5 c^a(s^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Lambda_c^+} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[d^a(u^bC\gamma_\mu c^c) - u^a(d^bC\gamma_\mu c^c)]\epsilon^{abc} \\
J^{\Xi_c^{S+}} &= \sqrt{2}\gamma^\mu\gamma^5 c^a(u^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Xi_c^{A+}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[s^a(u^bC\gamma_{mu} c^c) - u^a(s^bC\gamma_\mu c^c)]\epsilon^{abc} \\
J^{\Xi_c^{S0}} &= \sqrt{2}\gamma^\mu\gamma^5 c^a(d^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Xi_c^{A0}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[s^a(d^bC\gamma_{mu} c^c) - d^a(s^bC\gamma_\mu c^c)]\epsilon^{abc} \\
J^{\Sigma_b^+} &= \gamma^\mu\gamma^5 b^a(u^bC\gamma_\mu u^c)\epsilon^{abc} \\
J^{\Sigma_b^-} &= \gamma^\mu\gamma^5 b^a(d^bC\gamma_\mu d^c)\epsilon^{abc} \\
J^{\Sigma_b^0} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(u^bC\gamma_\mu d^c)\epsilon^{abc} \\
J^{\Omega_b^-} &= \gamma^\mu\gamma^5 b^a(s^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Lambda_b^0} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[d^a(u^bC\gamma_\mu b^c) - u^a(d^bC\gamma_\mu b^c)]\epsilon^{abc} \\
J^{\Xi_b^{S0}} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(u^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Xi_b^{A0}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[s^a(u^bC\gamma_{mu} b^c) - u^a(s^bC\gamma_\mu b^c)]\epsilon^{abc} \\
J^{\Xi_b^{S-}} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(d^bC\gamma_\mu s^c)\epsilon^{abc} \\
J^{\Xi_c^{A-}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[s^a(d^bC\gamma_{mu} b^c) - d^a(s^bC\gamma_\mu b^c)]\epsilon^{abc} \\
J^{\Xi_{cc}^{++}} &= -\gamma^\mu\gamma^5 u^a(c^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Xi_{cc}^{+}} &= -\gamma^\mu\gamma^5 d^a(c^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Omega_{cc}^{+}} &= -\gamma^\mu\gamma^5 s^a(c^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Xi_{bb}^{0}} &= -\gamma^\mu\gamma^5 u^a(b^bC\gamma_\mu b^c)\epsilon^{abc} \\
J^{\Xi_{bb}^{-}} &= -\gamma^\mu\gamma^5 d^a(b^bC\gamma_\mu b^c)\epsilon^{abc} \\
J^{\Omega_{bb}^{-}} &= -\gamma^\mu\gamma^5 s^a(b^bC\gamma_\mu b^c)\epsilon^{abc} \\
J^{\Xi_{bc}^{S+}} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(u^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Xi_{bc}^{A+}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[c^a(u^bC\gamma_{mu} b^c) - u^a(c^bC\gamma_\mu b^c)]\epsilon^{abc}
\end{aligned}
$$

$$
\begin{aligned}
J^{\Xi^{S0}_{bc}} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(d^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Xi^{A0}_{bc}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[c^a(d^bC\gamma_{mu}b^c) - d^a(c^bC\gamma_\mu b^c)]\epsilon^{abc} \\
J^{\Omega^{S0}_{bc}} &= \sqrt{2}\gamma^\mu\gamma^5 b^a(s^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Omega^{A0}_{bc}} &= \sqrt{\frac{2}{3}}\gamma^\mu\gamma^5[c^a(s^bC\gamma_{mu}b^c) - s^a(c^bC\gamma_\mu b^c)]\epsilon^{abc} \\
J^{\Omega^+_{bcc}} &= \gamma^\mu\gamma^5 b^a(c^bC\gamma_\mu c^c)\epsilon^{abc} \\
J^{\Omega^0_{bbc}} &= -\gamma^\mu\gamma^5 c^a(b^bC\gamma_\mu b^c)\epsilon^{abc}
\end{aligned}
\tag{B.7}
$$

B.3 Baryon wave functions

Below we give the explicit construction of baryon wave functions based on symmetry principles.

Light Baryons (19=8[octet]+1[singlet]+10[decuplet] states) where "[...]" is the the spin and flavour antisymmetric combination and {...} is the symmetric one:

$$
\begin{array}{ccc}
B^{1[23]} = \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}}, & B^{2[23]} = \Sigma^-, & B^{3[23]} = -\Xi^-, \\
B^{1[31]} = \Sigma^+, & B^{2[31]} = -\frac{\Sigma[0]}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}}, & B^{3[31]} = \Xi^0, \\
B^{1[12]} = p, & B^{2[12]} = n, & B^{3[12]} = -\frac{2\Lambda^0}{\sqrt{6}}
\end{array}
$$

$$
F^{[123]} = \Lambda^{*0}
$$

$$
\begin{array}{cccc}
D^{\{111\}} = \Delta^{++}, & D^{\{112\}} = \frac{\Delta^+}{\sqrt{3}}, & D^{\{122\}} = \frac{\Delta^0}{\sqrt{3}}, & D^{\{222\}} = \Delta^-, \\
D^{\{113\}} = \frac{\Sigma^{*+}}{\sqrt{3}}, & D^{\{123\}} = \frac{\Sigma^{*0}}{\sqrt{6}}, & D^{223} = \frac{\Sigma^{*-}}{\sqrt{3}}, & \\
D^{\{133\}} = \frac{\Xi^{*0}}{\sqrt{3}}, & D^{\{233\}} = \frac{\Xi^{*-}}{\sqrt{3}}, & D^{\{333\}} = \Omega^- &
\end{array}
$$

Single Charm Baryons (18=9+3+6 states):

$$
\begin{array}{ccc}
B^{1[24]} = \frac{\Sigma_c^+}{\sqrt{2}} + \frac{\Lambda_c^+}{\sqrt{6}}, & B^{2[41]} - \frac{\Sigma_c^+}{\sqrt{2}} + \frac{\Lambda_c^+}{\sqrt{6}}, & B^{4[12]} = -\frac{2\Lambda_c^+}{\sqrt{6}} \\
B^{3[14]} = \frac{\Xi_c^{+S}}{\sqrt{2}} + \frac{\Xi_c^{+A}}{\sqrt{6}}, & B^{1[43]} = -\frac{\Xi_c^{+S}}{\sqrt{2}} + \frac{\Xi_c^{+A}}{\sqrt{6}}, & B^{4[31]} = -\frac{2\Xi_c^{+A}}{\sqrt{6}}, \\
B^{3[24]} = \frac{\Xi_c^{0S}}{\sqrt{2}} + \frac{\Xi_c^{0A}}{\sqrt{6}}, & B^{2[43]} = -\frac{\Xi_c^{0S}}{\sqrt{2}} + \frac{\Xi_c^{0A}}{\sqrt{6}}, & B^{4[32]} = -\frac{2\Xi_c^{0A}}{\sqrt{6}}, \\
B^{1[14]} = \Sigma_c^{++} & B^{2[24]} = \Sigma_c^0 & B^{3[34]} = \Omega_c^0.
\end{array}
$$

$$
F^{[124]} = \Lambda_c^{*+}, \qquad F^{[134]} = \Lambda_{cs}^{*+}, \qquad F^{[234]} = \Lambda_{cs}^{*0},
$$

$$
\begin{array}{ccc}
D^{\{114\}} = \frac{\Sigma_c^{*++}}{\sqrt{3}}, & D^{\{124\}} = \frac{\Sigma_c^{*+}}{\sqrt{6}}, & D^{\{224\}} = \frac{\Sigma_c^{*0}}{\sqrt{3}}, \\
D^{\{134\}} = \frac{\Xi_c^{*+}}{\sqrt{6}}, & D^{\{234\}} = \frac{\Xi_c^{*0}}{\sqrt{6}}, & D^{\{334\}} = \frac{\Omega_c^{*0}}{\sqrt{3}}.
\end{array}
$$

Single Bottom Baryons (18=9+3+6 states):

$$
\begin{array}{ccc}
B^{1[25]} = \frac{\Sigma_b^0}{\sqrt{2}} + \frac{\Lambda_b^0}{\sqrt{6}}, & B^{2[51]} = -\frac{\Sigma_b^0}{\sqrt{2}} + \frac{\Lambda_b^0}{\sqrt{6}}, & B^{5[12]} = -\frac{2\Lambda_b^0}{\sqrt{6}} \\
B^{3[15]} = \frac{\Xi_b^{0S}}{\sqrt{2}} + \frac{\Xi_b^{0A}}{\sqrt{6}}, & B^{1[53]} = -\frac{\Xi_b^{0S}}{\sqrt{2}} + \frac{\Xi_b^{0A}}{\sqrt{6}}, & B^{5[31]} = -\frac{2\Xi_b^{0A}}{\sqrt{6}}, \\
B^{3[25]} = \frac{\Xi_b^{-S}}{\sqrt{2}} + \frac{\Xi_b^{-A}}{\sqrt{6}}, & B^{2[53]} = -\frac{\Xi_b^{-S}}{\sqrt{2}} + \frac{\Xi_b^{-A}}{\sqrt{6}}, & B^{5[32]} = -\frac{2\Xi_b^{-A}}{\sqrt{6}}, \\
B^{1[15]} = \Sigma_b^+ & B^{2[25]} = \Sigma_b^- & B^{3[35]} = \Omega_b^-.
\end{array}
$$

$$
F^{[125]} = \Lambda_b^{*0}, \qquad F^{[135]} = \Lambda_{bs}^{*0}, \qquad F^{[235]} = \Lambda_{bs}^{*-},
$$

$$
\begin{array}{ccc}
D^{\{115\}} = \frac{\Sigma_b^{*+}}{\sqrt{3}}, & D^{\{125\}} = \frac{\Sigma_b^{*0}}{\sqrt{6}}, & D^{\{225\}} = \frac{\Sigma_b^{*-}}{\sqrt{3}}, \\
D^{\{135\}} = \frac{\Xi_b^{*0}}{\sqrt{6}}, & D^{\{235\}} = \frac{\Xi_b^{*-}}{\sqrt{6}}, & D^{\{335\}} = \frac{\Omega_b^{*-}}{\sqrt{3}}.
\end{array}
$$

Double Charm Baryons (6=3+3 states):

$$
\begin{array}{ccc}
B^{4[41]} = \Xi_{cc}^{++}, & B^{4[42]} = \Xi_{cc}^{+}, & B^{4[43]} = \Omega_{cc}^{+}, \\
D^{\{144\}} = \frac{\Xi_{cc}^{*++}}{\sqrt{3}}, & D^{\{244\}} = \frac{\Xi_{cc}^{*+}}{\sqrt{3}}, & D^{\{344\}} = \frac{\Omega_{cc}^{*+}}{\sqrt{3}}.
\end{array}
$$

Double Bottom Baryons (6=3+3 states):

$$
\begin{array}{ccc}
B^{5[51]} = \Xi_{bb}^{0}, & B^{5[52]} = \Xi_{bb}^{-}, & B^{5[53]} = \Omega_{bb}^{-}, \\
D^{\{155\}} = \frac{\Xi_{bb}^{*0}}{\sqrt{3}}, & D^{\{255\}} = \frac{\Xi_{bb}^{*-}}{\sqrt{3}}, & D^{\{355\}} = \frac{\Omega_{bb}^{*-}}{\sqrt{3}}.
\end{array}
$$

$$B^{1[45]} = \frac{\Xi_{bc}^{+S}}{\sqrt{2}} + \frac{\Xi_{bc}^{+A}}{\sqrt{6}}, \qquad B^{4[51]} = -\frac{\Xi_{bc}^{+S}}{\sqrt{2}} + \frac{\Xi_{bc}^{+A}}{\sqrt{6}}, \qquad B^{5[14]} = -\frac{2\Xi_{bc}^{+A}}{\sqrt{6}},$$
$$B^{2[45]} = \frac{\Xi_{bc}^{0S}}{\sqrt{2}} + \frac{\Xi_{bc}^{0A}}{\sqrt{6}}, \qquad B^{4[52]} = -\frac{\Xi_{bc}^{0S}}{\sqrt{2}} + \frac{\Xi_{bc}^{0A}}{\sqrt{6}}, \qquad B^{5[24]} = -\frac{2\Xi_{bc}^{0A}}{\sqrt{6}},$$
$$B^{3[45]} = \frac{\Omega_{bc}^{0S}}{\sqrt{2}} + \frac{\Omega_{bc}^{0A}}{\sqrt{6}}, \qquad B^{4[53]} = -\frac{\Omega_{bc}^{0S}}{\sqrt{2}} + \frac{\Omega_{bc}^{0A}}{\sqrt{6}}, \qquad B^{5[34]} = -\frac{2\Omega_{bc}^{0A}}{\sqrt{6}}.$$

$$F^{[145]} = \Lambda_{bc}^{*+}, \qquad F^{[245]} = \Lambda_{bc}^{*0}, \qquad F^{[345]} = \Lambda_{bcs}^{*0}.$$

$$D^{\{145\}} = \frac{\Xi_{bc}^{*+}}{\sqrt{6}}, \qquad D^{\{245\}} = \frac{\Xi_{bc}^{*0}}{\sqrt{6}}, \qquad D^{\{345\}} = \frac{\Omega_{bcs}^{*0}}{\sqrt{6}}.$$

Triple Heavy Baryons (6=2+4 states):

$$B^{4[45]} = \Omega_{bcc}^{+}, \qquad B^{5[45]} = \Omega_{bbc}^{0}.$$
$$D^{\{444\}} + \Omega_{ccc}^{*++}, \qquad D^{\{445\}} = \frac{\Omega_{bcc}^{*+}}{\sqrt{3}} \qquad D^{\{455\}} = \frac{\Omega_{bbc}^{*0}}{\sqrt{3}} \qquad D^{\{555\}} = \Omega_{bbb}^{*-}.$$

Appendix C

Feynman rules for the non-local electromagnetic vertex

In this appendix we derive the Feynman rules for the non-local vertex of Fig. 1 describing the coupling of a baryon, three quarks and the photon field. The path integral over the electromagnetic field

$$I(x,y,P) \equiv \int_x^y dz_\mu A^\mu(z) \tag{C.1}$$

enters the vertex

$$\Gamma_{B3q\gamma} = \int d^4x_1 \int d^4x_2 \, \Phi(x_1^2+x_2^2) \, e^{ip_1x_1+ip_2x_2} I(x_+,x,P) \ , \tag{C.2}$$

where $x_+ = x \pm a_1 x_1 \pm a_2 x_2$ and where p_1 and p_2 are linear combination of the loop momenta. The parameters $a_{1,2}$ are related to the kinematical quantities $\omega_i = \frac{m_i}{\sum\limits_{i=1}^{3} m_i}$ and the loop momenta, their specific form is not necessary for the further derivation.
The following operator identity will be used

$$\begin{aligned}
\Phi(x_1^2+x_2^2) &= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \, \tilde{\Phi}(-k_1^2-k_2^2) \, e^{ik_1x_1+ik_2x_2} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \, \tilde{\Phi}(\partial_{x_1}^2+\partial_{x_2}^2) \, e^{ik_1x_1+ik_2x_2} \\
&= \delta^4(x_1) \, \delta^4(x_2) \, \tilde{\Phi}(\partial_{x_1}^2+\partial_{x_2}^2) \ ,
\end{aligned} \tag{C.3}$$

where $\tilde{\Phi}$ is the Fourier transform of the vertex function Φ resulting in

$$\Gamma_{B3q\gamma} = \int d^4x_1 \int d^4x_2 \; \delta^4(x_1) \; \delta^4(x_2) \; \tilde{\Phi}(\partial^2_{x_1} + \partial^2_{x_2}) \, e^{ip_1x_1+ip_2x_2} I(x_+, x, P) \, . \tag{C.4}$$

Applying the partial derivative operators on the exponentials we obtain

$$\tilde{\Phi}(\partial^2_{x_1} + \partial^2_{x_2}) e^{ip_1x_1+ip_2x_2} I(x_+, x, P) = e^{ip_1x_1+ip_2x_2} \tilde{\Phi}(\mathcal{D}^2_{x_1} + \mathcal{D}^2_{x_2}) I(x_+, x, P) \, . \tag{C.5}$$

with $D_{x_i \equiv \partial_{x_i}} + ip$.
The expression needed to be evaluated is

$$\tilde{\Phi}(\mathcal{D}^2_{x_1} + \mathcal{D}^2_{x_2}) I(x_+, x, P) \; = \; \sum_{n=0}^{\infty} \frac{\tilde{\Phi}^{(n)}(0)}{n!} \, [\mathcal{D}^2_{x_1} + \mathcal{D}^2_{x_2}]^n \, I(x_+, x, P) \;\; , \tag{C.6}$$

and we will make use of the definition of the derivative of a line integral as shown in Section 2.4. The derivative of a line integral

$$I(x, y, P) \equiv \int_x^y A_\mu dz^\mu \tag{C.7}$$

can be defined as follows:

$$\begin{aligned} & \lim_{dy_\mu \to 0} dy_\mu \frac{\partial}{\partial y^\mu} I(x, y, P) \;, \\ = \;\; & \lim_{dy_\mu \to 0} I(x, y + dy, P^{'}) - I(x, y, P) \;. \end{aligned} \tag{C.8}$$

One then obtains:

$$\partial^\mu_x I(x, y, P) = A^\mu(x) \, , \tag{C.9}$$

resulting in

$$[\mathcal{D}^2_{x_1} + \mathcal{D}^2_{x_2}] I(x_+, x, P) \; = \; L(A) - (p_1^2 + p_2^2) I(x_+, x, P) \;, \tag{C.10}$$

where

$$L(A) \equiv (\partial_{x_1} + \partial_{x_2}) A(x) + 2i(p_1 + p_2) A(x) \, . \tag{C.11}$$

Applying this as many times as requested by (C.6) we obtain the sequence

$$\begin{aligned}(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)^2 I(x_+,x,P) &= [\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2-(p_1^2+p_2^2)]L(A) \\ &+ (-p_1^2-p_2^2)^2 I(x_+,x,P)\end{aligned} \tag{C.12}$$

$$\begin{aligned}(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)^3 I(x_+,x,P) = [(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)^2-(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)(p_1^2+p_2^2) \\ +(-p_1^2-p_2^2)^2]L(A)+(-p_1^2-p_2^2)^3 I(x_+,x,P)\,,\end{aligned} \tag{C.13}$$

$$\cdots$$

$$\begin{aligned}(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)^n I(x_+,x,P) = \sum_{k=0}^{n-1}(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)^{n-1-k}\,(-p_1^2-p_2^2)^k\,L(A) \\ +\,(-p_1^2-p_2^2)^n I(x_+,x,P) = n\int_0^1 dt\;[(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)t\,-\,(p_1^2+p_2^2)(1-t)]^{n-1}\,L(A) \\ +\,(-p_1^2-p_2^2)^n\,I(x_+,x,P)\,.\end{aligned} \tag{C.14}$$

$$\begin{aligned}\tilde{\Phi}(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)I(x_+,x,P) &= \int_0^1 dt\,\tilde{\Phi}'[(\mathcal{D}_{x_1}^2+\mathcal{D}_{x_2}^2)t\,-\,(p_1^2+p_2^2)(1-t)]\,L(A) \\ &+ \tilde{\Phi}(-p_1^2-p_2^2)\,I(x_+,x,P) \\ &= \int\frac{d^4q}{(2\pi)^4}\,\tilde{A}_\mu(q)\left\{iK^\mu e^{-iqx_+}\int_0^1 dt\tilde{\Phi}'[w(t)]\right. \\ &+ \left.\tilde{\Phi}[w(0)]\int_x^{x_+} dz^\mu e^{-iqz}\right\},\end{aligned} \tag{C.15}$$

where

$$K^\mu = a_1[2p_1-q]^\mu + a_2[2p_2-q]^\mu\,,$$

$$w(t) = -(p_1-a_1q)^2t-(p_2-a_2q)^2t-(p_1^2+p_2^2)(1-t)\,.$$

$\tilde{A}_\mu(q)$ is the Fourier-transform of the electromagnetic field and $\tilde{\Phi}'(z) = d\tilde{\Phi}(z)/dz$. The last term of (B.10) vanishes due to the delta functions $\delta^4(x_1)$ and $\delta^4(x_2)$ in the expression (B.4).

Finally we have

$$\Gamma_{B3q\gamma} = i \int \frac{d^4q}{(2\pi)^4}\, e^{-iqx}\, \tilde{A}_\mu(q)\, K^\mu \int_0^1 dt\, \tilde{\Phi}'[w(t)]\,. \tag{C.16}$$

Appendix D

Calculation of matrix elements

To illustrate the technique of calculating matrix elements we display as an example the evaluation of the baryon mass operator. The generic integral is

$$I_B(p) \;=\; \int dk_{123}\; \tilde{\Phi}^2(z_0)\, R_\Sigma(k_1^+, k_2^+, k_3^+)\,. \tag{D.1}$$

We set $w_1 = 1$ and $w_2 = w_3 = 0$, then the integral becomes :

$$\begin{aligned} I_B(p) \;&=\; \int \frac{d^4k_1}{\pi^2 i} \int \frac{d^4k_2}{\pi^2 i}\; \tilde{\Phi}^2(-12[k_1^2 + k_1k_2 + k_2^2])\, \Gamma_{1f}\, S_{q_1}(k_1 + p)\, \Gamma_{1i} \\ &\times\; \mathrm{tr}\left[\Gamma_{2f}\, S_{q_2}(k_2)\, \Gamma_{2i}\, S_{q_3}(k_1 + k_2)\right]\;. \end{aligned} \tag{D.2}$$

The technique used is based on the following main ingredients:

- use of the Laplace transform of the vertex function, its derivative and integral:

$$\tilde{\Phi}(z_0) = \int\limits_0^\infty ds\, \Phi_L(s)\, e^{-sz_0}\,,$$

$$\tilde{\Phi}'(z_0) = -\int\limits_0^\infty ds\, s\, \Phi_L(s)\, e^{-sz_0}\,,$$

$$\int\limits_0^\infty d\alpha\, \alpha^n\, \tilde{\Phi}(z_0 + \alpha) = \Gamma(n+1)\int\limits_0^\infty \frac{ds}{s^n}\, \Phi_L(s)\, e^{-sz_0}\,,$$

- α-transform of the propagator functions S_{q_1}, S_{q_2} and S_{q_3}

$$\frac{1}{m_q^2-(k+p)^2} = \int\limits_0^\infty d\alpha \, e^{-\alpha(m^2-(k+p)^2)} \,,$$

- differential representation of the numerator

$$(\, m+ \not{k}+ \not{p}) \; e^{kq} = \left(m + \gamma^\mu \frac{\partial}{\partial q^\mu} + \not{p} \right) \; e^{kq} \,.$$

- Gaussian integral over virtual momenta k_1 and k_2

$$\prod_{j=1}^{n} \int \frac{d^4k_j}{\pi^2 i} \; \exp[\, kAk + 2Bk \,] = \frac{1}{[\mathrm{det}A]^n} \exp[\, -B \, A^{-1} \, B \,]$$

  where, in a general approach, A is a $n \times n$ matrix and B a n-component vector. In the present application we have $n = 2$.

The integral becomes:

$$\begin{aligned} I_B(p) \;&=\; \int\limits_0^\infty d\alpha_1 \int\limits_0^\infty d\alpha_2 \int\limits_0^\infty d\alpha_3 \int\limits_0^\infty d\beta \; \tilde{\Phi}^2[-12(z+\beta)] \Big\{ \Gamma_{1f} \, D_1 \, \Gamma_{1i} \mathrm{tr} \, [\Gamma_{2f} \, D_2 \, \Gamma_{2i} \, D_3] \\ &\times \; -\beta \, \Gamma_{1f} \, D_1 \, \Gamma_{1i} \mathrm{tr} \, [\Gamma_{2f} \, \gamma^\mu \, \Gamma_{2i} \, \gamma_\mu] \, (A_{12}^{-1} + A_{22}^{-1}) \\ &- \; \beta \, \Gamma_{1f} \, \gamma^\mu \, \Gamma_{1i} \mathrm{tr} \left[\Gamma_{2f} \, \gamma_\mu \, \Gamma_{2i} \, D_3 A_{12}^{-1} + \Gamma_{2f} \, D_2 \, \Gamma_{2i} \, \gamma_\mu (A_{11}^{-1} + A_{12}^{-1}) \right] \Big\} \,, \end{aligned} \tag{D.3}$$

where

$$D_i \;=\; m_{q_i} + \not{P}_i \,, \quad P_1 \;=\; p - B_1 A_{11}^{-1} \,, \quad P_2 \;=\; -B_1 A_{12}^{-1} \,, \tag{D.4}$$

$$P_3 \;=\; -B_1 (A_{11}^{-1} + A_{12}^{-1}) \,, \quad z \;=\; -\sum_{i=1}^{3} \alpha_i m_{q_i}^2 + p^2 \alpha_1 - B_1^2 A_{11}^{-1} \,. \tag{D.5}$$

Here $B_1 = p\alpha_1$ and A_{ij}^{-1} are the elements of the inverse matrix A_{ij}:

$$\begin{aligned} A \;&=\; \begin{pmatrix} 1+\alpha_1+\alpha_3 & \frac{1}{2}+\alpha_3 \\ \frac{1}{2}+\alpha_3 & 1+\alpha_2+\alpha_3 \end{pmatrix} \\ A^{-1} \;&=\; \frac{1}{\mathrm{det}A} \begin{pmatrix} 1+\alpha_2+\alpha_3 & -(\frac{1}{2}+\alpha_3) \\ -(\frac{1}{2}+\alpha_3) & 1+\alpha_1+\alpha_3 \end{pmatrix} \,. \end{aligned} \tag{D.6}$$

The baryonic correlation function $\tilde{\Phi}$ is specified in the last step of calculation after the integration over the virtual momenta. The traces which appear in the evaluation of matrix elements, as well as the Dirac algebra, are performed analytically using FORM programs. The three- and four-dimensional integrals are performed using FORTRAN.

Appendix E

Non-relativistic spin-flavor wave functions and magnetic moments

We present analytical results for the magnetic moments of heavy baryons within the non-relativistic quark model (NRQM). It is well known that the NRQM is able to describe successfully static properties, therefore it is desired to check our relativistic model predictions with those of the NRQM. The spin-flavor wave functions in the NRQM arise in our case by taking the non-relativistic limit of the covariant three-quark currents carrying the quantum numbers of the ground-state ($J^P = 1/2^+$) baryons. With the expressions established in Section 2.2 (Eq. (2.46) and (2.47)) for the baryonic wave functions we derive the magnetic moments using the formula

$$\mu_{B_{q_1 q_2 q_3}} = \langle B_{q_1 q_2 q_3} | \sum_{i=1}^{3} \frac{e_{q_i}}{2m_{q_i}} \, \sigma_3^i | B_{q_1 q_2 q_3} \rangle \, , \tag{E.1}$$

where σ_3^i is the third component of the spin operator acting on the quark i and e_{q_i} is the electric charge of the quark i. The following notations are used for the symmetric χ_S and antisymmetric χ_A spin wave functions:

$$\chi_A = \sqrt{\frac{1}{2}} \left\{ \uparrow (\uparrow\downarrow - \downarrow\uparrow) \right\}, \qquad \chi_S = \sqrt{\frac{1}{6}} \left\{ \uparrow (\uparrow\downarrow + \downarrow\uparrow) - 2 \downarrow\uparrow\uparrow \right\}. \tag{E.2}$$

We offer as an example the calculation of the non-relativistic magnetic moment of the $\Lambda^+_{c[ud]}$ baryon which corresponds to the calculation of the projection of the relevant operator on the baryon spin-flavor states. First, we remind that the relativistic current for this state is antisymmetric under the permutation of the flavor indices u and d, as indicated by [...]. The non-relativistic spin-flavor wave function is written as:

$$|\Lambda^+_{c[u]} >= \chi^A_{\Lambda^+_c} f^A_{\Lambda^+_c} \ , \tag{E.3}$$

where $\chi^A_{\Lambda^+_c}$ stands for the antisymmetric spin wave function and $f^A_{\Lambda^+_c}$ stands for the antisymmetric flavor wave function. For the antisymmetric flavor wave function $f^A_{\Lambda^+_c}$ we use (see Section 3.3, Table 5):

$$f^A_{\Lambda^+_c} = \frac{1}{\sqrt{2}}(c[ud - du]) \ . \tag{E.4}$$

Now we calculate the magnetic moment using (E.1) as follows:

$$\begin{aligned} \mu_{\Lambda^+_c} &= \langle \Lambda^+_c | \sum_{i=1}^{3} \frac{e_{q_i}}{2m_{q_i}} \sigma^i_3 | \Lambda^+_c \rangle \\ &= \langle \sqrt{\frac{1}{2}}[cud - cdu] \sqrt{\frac{1}{2}} \left\{ \uparrow (\uparrow\downarrow - \downarrow\uparrow) \right\} | \frac{e_{q_1}}{2m_1} \sigma_3(1) + \frac{e_{q_2}}{2m_2} \sigma_3(2) \\ &+ \frac{e_{q_3}}{2m_3} \sigma_3(3) | \sqrt{\frac{1}{2}} \left\{ \uparrow (\uparrow\downarrow - \downarrow\uparrow) \right\} \sqrt{\frac{1}{2}}[cud - cdu] \rangle \ . \end{aligned} \tag{E.5}$$

Using the following properties [52]:

$$e_{q_i} c = e_c c \quad , e_{q_i} u = e_u u \quad , e_{q_i} d = e_d d \tag{E.6}$$

and

$$\sigma_3 \ \uparrow = \uparrow \quad \sigma_3 \ \downarrow = - \downarrow \ , \tag{E.7}$$

we calculate the matrix element of $e_{q_i} \sigma^i_3$ between the baryon spin-flavor states and obtain for the magnetic moment:

$$\mu_{\Lambda^+_c} = \frac{1}{4} \, (4 \frac{e_c}{2m_c}) \ = \mu_c . \tag{E.8}$$

The contributions of the second and third quark vanish. Keeping in mind that the correct non-relativistic spin-flavor wave functions are the ones corresponding to the non-relativistic limit of the three-quark baryon currents and using the formula (E.1) one can calculate the non-relativistic magnetic moments for all the baryon states (see Section 3.3, Table 5).

Appendix F

Relativistic form factors for the $N \to \Delta\gamma$ transition

In the study of the $N \to \Delta$ transition there are several ways for decomposing the vertex function $\Lambda_{\mu\nu}(p, p^{'})$ [87]-[96] :

$$
\begin{aligned}
\Lambda^{(1)}_{\mu\nu}(p,p') &= [(g_{\mu\nu} \not{q} - \gamma_\mu q_\nu)G_1(q^2) + (g_{\mu\nu}p'q - p'_\mu q_\nu)G_2(q^2) \\
&+ \quad (g_{\mu\nu}q^2 - q_\mu q_\nu)G_3(q^2)]\gamma^5 \\
\Lambda^{(2)}_{\mu\nu}(p,p') &= -[(g_{\mu\nu}q^2 + q_\mu p_\nu)a_1(q^2) + a_2(g_{\mu\nu}m_+m_- + P_\mu p_\nu)a_2(q^2) \\
&+ \quad (g_{\mu\nu}m_+ + \gamma_\mu p_\nu)a_3(q^2)]\gamma^5 \\
\Lambda^{(3)}_{\mu\nu}(p,p') &= \frac{1}{2m_N}[(g_{\mu\nu} \not{q} - \gamma_\mu q_\nu)c_1(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\frac{c_2(q^2)}{2m_N} \\
&+ \quad (g_{\mu\nu}pq - p_\mu q_\nu)\frac{c_3(q^2)}{2m_N}]\gamma^5
\end{aligned}
\tag{F.1}
$$

where $P = p + p'$ and $m_\pm = m_\Delta \pm m_N$.
The sets of the relativistic form factors G_i, a_i, b_i and c_i are related to each other as:

$$
\begin{aligned}
&G_1 = -a_3 = b_3 = \frac{c_1}{2m_N}\,, \\
&G_2 = -2a_2 = b_2 = \frac{c_3}{4m_N^2}\,, \\
&G_3 = -a_1 + a_2 = -b_2 + b_4 = \frac{c_2 - c_3}{4m_N^2}\,, \qquad \text{(F.2)} \\
&c_4 \equiv 0\,. \qquad \text{(F.3)}
\end{aligned}
$$

Appendix G

Uniqueness of the $\Delta^+(1232)$ three-quark current

In Ref. [39], applying Fierz transformations and using the identities (2.26), one concludes that for the Δ^+ current two forms are available. Here we will show explicitly the calculation which makes these two forms equivalent to the more general current, the vector current:

$$J^\mu_{\Delta^+} = \epsilon^{abc}\Gamma_1 d^{Ta}(y_1)u^b(y_2)C\Gamma_2 u^c(y_3)\ . \tag{G.1}$$

Starting with the expression of Ref. [39] we already derived an equivalent form of the three-quark current in Section 2.2, Eq. (2.49). Considering that for the Δ^+-state one can write a totally symmetric in spin-flavor wave function (see Appendix B, Section B.3) , the three-quark current thus includes the following structures:

$$\begin{aligned} J^\mu_{\Delta^+} &= \frac{\epsilon^{abc}}{\sqrt{3}}\{u^{Ta}u^bC\gamma^\mu d^c - \frac{i}{2}\gamma_\nu u^{Ta}u^bC\sigma^{\mu\nu}d^c \\ &+ u^{Ta}d^bC\gamma^\mu u^c - \frac{i}{2}\gamma_\nu u^{Ta}d^bC\sigma^{\mu\nu}u^c \\ &+ d^{Ta}u^bC\gamma^\mu u^c - \frac{i}{2}\gamma_\nu d^{Ta}u^bC\sigma^{\mu\nu}u^c\}\ . \end{aligned} \tag{G.2}$$

Now we use the following Fierz transformation: for the first and second tensor-like terms, $T = -\tilde{V}+\tilde{A}$; for the third tensor-like term, $T = V+A-2\tilde{V}$; it follows that

$$J^\mu_{\Delta^+} = \frac{\epsilon^{abc}}{\sqrt{3}}\{u^{Ta}u^bC\gamma^\mu d^c - \frac{1}{2}(-d^{Ta}u^bC\gamma^\mu u^c + \gamma^5 d^{Ta}u^bC\gamma^\mu\gamma^5 u^c)$$

$$
\begin{aligned}
&+ \quad u^{Ta}d^bC\gamma^\mu u^c - \frac{1}{2}(-d^{Ta}u^bC\gamma^\mu u^c + \gamma^5 d^{Ta}u^bC\gamma^\mu\gamma^5 u^c) \\
&+ \quad d^{Ta}u^bC\gamma^\mu u^c - \frac{1}{2}(d^{Ta}u^bC\gamma^\mu u^c + \gamma^5 d^{Ta}u^bC\gamma^\mu\gamma^5 u^c - 2u^{Ta}u^bC\gamma^\mu d^c)\} \; .
\end{aligned}
\tag{G.3}
$$

The third, sixth and ninth terms vanish; the fifth and the eight terms cancel each other and the fourth term equals $u^{Ta}u^bC\gamma^\mu d^c$. Hence, the current can be expressed as

$$
J^\mu_{\Delta^+} = \frac{3}{2}\frac{\epsilon^{abc}}{\sqrt{3}}(d^{Ta}u^bC\gamma^\mu u^c + 2u^{Ta}u^bC\gamma^\mu d^c) \; . \tag{G.4}
$$

which proves the choice of a vector-type currrent for Δ^+. Note the factor 3 in front which underlines the spin-flavor symmetric behaviour of the Δ^+ wave function.

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Curriculum Vitae

Personal Information

Name:	Diana Nicmorus
Email:	diana.nicmorus@uni-tuebingen.de
Nationality:	Romanian
Date of Birth :	20 March 1979
Gender:	Female

Academic preparation

Dates:	June 2004 - March 2007
Title of qualification:	PhD.
Thesis title:	Electromagnetic Properties of Light and Heavy Baryons in the Relativisitc Quark Model
Advisors :	Prof. Dr. Thomas Gutsche
Institution:	Institute of Theoretical Physics, University of Tuebingen, Germany

Dates:	October 2002 - February 2004
Title of qualification:	MSc.
Thesis title:	New Symmetries describing Medium and Heavy Nuclei
Advisors :	Prof. Dr. A.A. Raduta, Prof. Dr. Amand Faessler
Institution:	Faculty of Physics, University of Bucharest, Romania

Dates:	October 1998 - June 2002
Title of qualification:	BSc.
Thesis title:	Serial Communications in Transmitting Informations with Microcontrollers
Advisors :	Prof. Dr. Sever Spanulescu
Institution:	Faculty of Physics, “Hyperion” University of Bucharest, Romania

Professional experience

Dates:	March 2007 - present
Ocupation/Position held:	Wissenschaftliche Ausgestellte
Institution:	Institute of Theoretical Physics, University of Tuebingen, Germany

Dates:	Fall 2005 - Summer 2006
Ocupation/Position held:	Teaching Assistant/ Exercise classes, Particle and Nuclear Physics
Institution:	Institute of Theoretical Physics, University of Tuebingen, Germany

Dates:	Fall 2003 - Spring 2004
Ocupation/Position held:	Research Assistant / Software development
Institution:	Institute of Space Sciences, Bucharest, Romania

Fellowships awarded

Dates:	March 2003 - October 2003
Fellowship:	European Eramus - Socrate Program
Host university:	University of Tuebingen, Germany

Dates:	March 2004 - March 2007
Fellowship:	DFG -Doctoral Fellowship
Host university:	University of Tuebingen, Germany

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