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Forward Trading and Collusion of Firms in Volatile Markets

by

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Forward trading and collusion of firms in volatile markets

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Abstract

Commodity markets are characterized by large volumes of forward contracts as well as high volatility. They are often accused of weak competitive pressure. This article extends the existing literature by analyzing tacit collusion of firms, forward trading and volatility simultaneously.

The expected collusive profit may depart from the monopoly outcome in a volatile market (Rotemberg and Saloner, 1986). Introducing forward trading enables firms to gain the expected monopoly profit for a broader range of parameters. In contrast to a deterministic market (Liski and Montero, 2006), trading forward in a volatile market may lead to an expected collusive profit below the monopoly one.

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1. Introduction

Commodity markets and especially the power market are often accused of oligopolistic market structures and weak competitive pressure. Among others, the following common market characteristics seam to be central: Few competitors due to high entry costs, a large market share that is sold either in long-term contracts or on future markets and a large volatility on the demand as well as on the supply side.

Stochastic influences play a crucial role in the power market and are one of the main reasons for trading forward. Thus, a volatile market context is added to the existing economic literature in order to gain a deeper insight into forward trading and collusion of firms. In figure 1 the

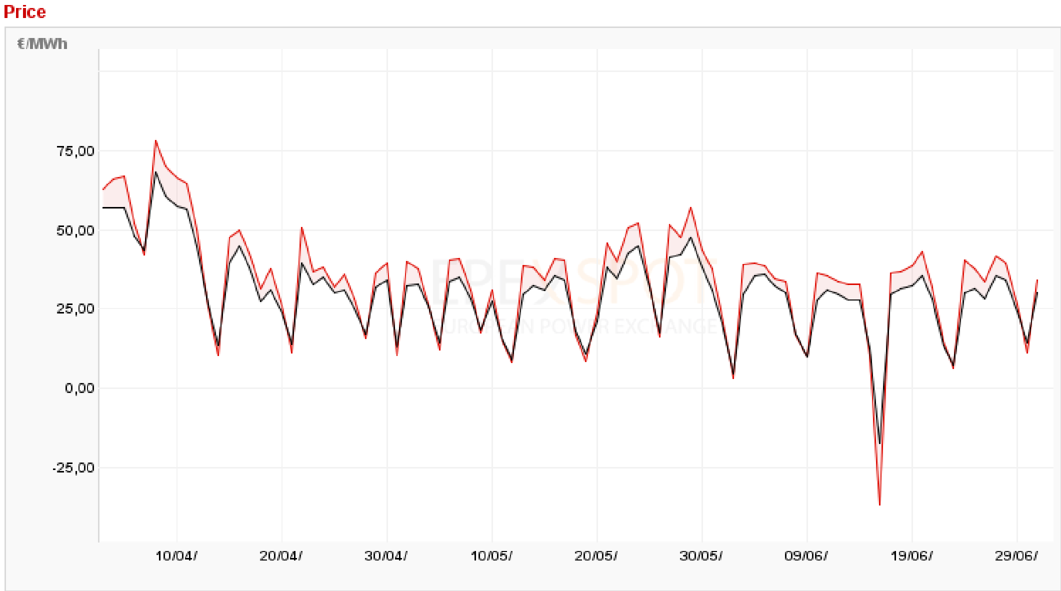

Figure 1: ELIX Day Base (black) and ELIX Day Peak(red line) for the second quarter 2013

European Electricity Index (ELIX) is illustrated for the second quarter of 2013. The ELIX is calculated by the Leipzig European-Energy-Exchange on the basis of the aggregated bid/offer curves of all EPEX Spot market areas. Thus, "the ELIX is a fundamental reference price for the common European market. It corresponds to the market price which would be determined in a market environment without bottlenecks" (European-Energy-Exchange, 2010). The red line plots the daily average value for peakload (ELIX Day Peak) and the black line the daily average for baseload (ELIX Day base). Obviously volatility plays a crucial role in the European power market, since e.g. in the second quarter of 2013 the price for one megawatt hour fluctuated regularly been between €10 and about €50. In the second quarter of 2013 the absolute bottom was reached on June 16 with a price of €-17,29 for baseload and a price of €-36,72 for peakload whereas the absolute peak was reached on April 8 with a price of €68,07 for baseload and price of €78,19 for peakload.

In table 1, volumes for different commodities traded at the Leipzig European-Energy-Exchange in 2009 and 2010 are presented, using data from the annual report of European-Energy-Exchange (2010). Spot market, forward market, total market volume as well as the ratio of forward traded volume and total market volume for power and natural gas are displayed in terrawatt-hours (TwH) and gigawatt-hours (GwH) respectively. The column Forwards m.share shows the ratio between forward contracted volume and total market volume (spot and forward market volume). Obviously for both commodities, most of the trading takes place on the forward market, since the market share of forwards exceeds 0.65 for all commodities and years. This illustrates the importance of trading forward on both markets.

Of course, there are important other reasons than collusive behavior for forward trading in

	Spot m.		Forward m.		Total m.		Forwards m.share	
Commodity	2009	2010	2009	2010	2009	2010	2009	2010
Power (TwH)	203	279	1025	1208	1228	1487	0,83	0,81
Gas (GwH)	3516	15026	11361	31863	14877	46889	0,76	0,68

Table 1: Commodity volumes traded at European-Energy-Exchange (2010)

these markets, e.g. risk sharing. However, the common effect of large forward traded amounts, volatility and (tacit) collusion of firms deserves a closer look.

Allaz (1992) and Allaz and Villa (1993) were the first, who introduced forward trading in industrial organization and analyzed its strategic aspects. Liski and Montero (2006) point out the effect of forward trading on (tacit) collusion of firms. They model an infinitely repeated oligopoly game where firms are allowed to act on the spot as well as on the forward market. They show under a deterministic demand and supply structure that forward trading has a stabilizing effect on a collusive agreement and does not alter the collusive profit. Thus, in a deterministic market structure forward contracts can be used to stabilize a collusive agreement without any disadvantage for the involved firms. Rotemberg and Saloner (1986) analyzed the effect of volatility on the collusive strategy when firms solely interact on the spot market and calculated that stochastic market conditions make collusive agreements harder to sustain. The contribution of this article is the connection of the findings of Liski and Montero (2006) and Rotemberg and Saloner (1986) by analyzing the effects of forward trading on collusive agreements in volatile markets.

The intuition behind the effect of forward trading on collusion is as follows: Firms fix a certain quantity at a certain price via forward trading. This induces two effects: On one hand it decreases the demand available for a deviating firm. Here, the consequence of forward trading is pro-collusive. On the other hand, forward trading decreases the demand available for collusive price-setting. Here, the consequence of forward trading is contra-collusive. Liski and Montero (2006) and Green and Coq (2010) show in a deterministic model that especially short-term forward contracts are suitable to stabilize collusive agreements. As will be shown in this paper trading short-term forward contracts strictly promotes collusion in volatile markets, as well. However, as will be pointed out in the upcoming analysis, trading forward more contracts than the respective monopoly quantity decreases the profits of colluding firms. This is a problem for colluding firms in a volatile market, especially when demand and cost parameters are continuously distributed, since firms cannot avoid having involuntarily contracted more than the corresponding monopoly quantity. This "over-contracting"leads to a decrease of the spot and forward market price and of the expected collusive profit.

The rest of the paper is organized as follows: In section 2.1 the main assumptions of the model and some general remarks are presented. Then in section 2.2 the effects of forward trading on a collusive agreement are modeled for a volatile market structure. In section 2.3 each firm's expected profit from forward trading is derived for any probability density function. Then an exponential distribution is used to show the profit decreasing effect of forward trading. Section 2.4 incorporates the possibility for firms to trade forward contracts, while setting a price below monopoly price. The properties of such a semi-collusive strategy are modeled for a two state distribution of cost and demand parameters. Section 3 concludes.

2. The model

2.1. Assumptions and general remarks

Collusive behavior of firms can occur if and only if there is no incentive for any firm to deviate from the collusive agreement unilaterally. If the net present value of profits gained by collusion is greater than or equal to the net present value of profits gained by ending collusion, no incentive for any firm to break the collusive agreement unilaterally exists.

The exact outcome of prices, quantities and profits is stochastic and depends on the difference between the reservation price (a) and marginal costs (c). I do not distinguish between demand and supply shocks. The difference between the reservation price and marginal costs, $\gamma = a - c$ will be denoted "spread" in the analysis. Whenever I use monopoly prices, quantities and profits for the argumentation, I refer to monopoly prices, quantities and profits for a given realization of the stochastic difference between reservation price and marginal costs. As shown by Liski and Montero (2006, p. 226) assuming a linear demand function is possible without loss of generality. I denote the price, quantity and profit associated with the one-period monopoly solution by $p^m = \frac{a+c}{2}$, $q^m = \frac{a-c}{2}$ and $\Pi^m = \frac{(a-c)^2}{4}$.

The spot and the forward market are connected similar to the deterministic model of Liski and Montero (2006): In the first period, both firms simultaneously choose the amount of forward contracts they want to trade (forward market period). In the second period, contracts are settled and firms choose the amount they want additionally to sell on the spot market (spot market period). This structure of a forward market, that is directly followed by a spot market is indefinitely repeated.

In order to ensure comparability with pure spot market super games (e.g. Rotemberg and Saloner (1986), Friedman (1971) and Tirole (1988)), there is no discounting between a consecutive forward and spot market. Discounting only takes place between two spot markets or two forward markets. One can think of firms deciding around Christmas each year about forward contracts to be delivered in the following year. See Liski and Montero (2006, p.217) for a more detailed discussion about discounting.

Firms compete in prices and sell a homogenous product, which seems a valid assumption especially for the power market. Whenever firm i sets a price lower than its competitor j firm i meets the whole spot market demand. When prices are equal, firms split the market equally. The trigger strategy played by each firm can be characterized as follows: As long as both firms have set the (semi-)collusive price p_{sc} and have contracted forward the (semi-)collusive amount of F_{sc}, each firm sets the (semi-)collusive price p_{sc} on the spot market and on the forward market each firm sells the (semi-)collusive quantity forward F_{sc}. When at least one firm has deviated from the (semi-)collusive price and forward quantity, the competitor sets a price equal to marginal cost on the spot market and sells any arbitrarily amount forward. This can be seen as the grim trigger strategy for games, where firms are allowed to trade on a spot as well as on a forward market, analogous to the spot market grim trigger strategy analyzed by Friedman (1971). See Liski and Montero (2006, p.218) for more details.

In general, two possibilities of deviation exist. Firstly, setting a price lower than the collusive price in the spot market. Secondly, increasing the forward sales in the forward market. The latter is never profitable as speculators, which take the counterpart, immediately realize any deviation from collusion in the forward market and are not willing to pay any price higher than the next period's stock market price, which is given by marginal costs. This restricts profitable deviation to the spot market and a deviating firms knows the actual state of the economy.

The demand that can be achieved on the spot market for a deviating firm is restricted by already sold future contracts. Each firm has a secured supply of f_i. The secured supply of both firms is given by $F = f_i + f_j$. Total traded amount decreases accessible demand $(a - F$

instead of a). This gives the (residual) demand function on the spot market:

$$D_i^R = \begin{cases} (a - F - p_i) & \text{if} \quad p_i < p_j, \\ \frac{1}{2}(a - F - p_i) & \text{if} \quad p_i = p_j, \\ 0 & \text{if} \quad p_i > p_j \end{cases} \tag{1}$$

2.2. Effects of forward trading on the stability of a collusive agreement

A firm deviating from collusion maximizes its profit over its (deviation) price. This leads to the following optimal deviation price and quantity:

$$\begin{aligned} &\max_{p} \quad \Pi_i = (p_i - c)(a - F - p_i) \\ &p^d = \frac{1}{2}[a + c - F], q^d = \frac{1}{2}(a - F - c), \Pi^d = \frac{1}{4}[a - c - F]^2 \end{aligned} \tag{2}$$

Deviation price, quantity and profit are quite similar to price, quantity and profit in a deviation from collusion without forward trading. However, the already contracted amount decreases the demand that is reachable on the spot market and quantity and profit become smaller.

When the total contracted amount exceeds or equals the Bertrand quantity (q^B), which is given by twice monopoly quantity ($F \geq q^B = 2q^m = a - c$), no positive deviation profit can be earned since any deviation would require a price that is lower than the Bertrand price on the spot market, which is given by marginal costs. As described in section 2.1 deviation yields zero profits in all following forward and spot market periods. Therefore, the net present value of deviation is given solely by the deterministic deviation profit of this single period:

$$E_{NPV}[Deviation] = \begin{cases} \frac{1}{4}[a - c - F]^2 & \text{if} \quad F < 2q^m \\ 0 & \text{if} \quad F \geq 2q^m, \end{cases} \tag{3}$$

The demand that can be reached by collusive behavior in this period is restricted by already sold forward contracts,too. As long as firms are able to fully-collude they set monopoly prices behaving as if no forward trading had occurred ($p^m = \frac{a-c}{2}$ instead of $p^m = \frac{a-F-c}{2}$). If they would not do so, they would not be able to sell collusive forward contracts at expected (monopoly) prices as speculators would anticipate the (expected) price discount on the spot market (see section 2.4 for a collusive price below the monopoly price). When firms set this collusive price, they split residual demand given by $D^R = a - F - p^m$ and earn a per-unit-profit of $\pi^C = p^m - c$ and each firms' collusive profit on the spot market can be stated as:

$$\begin{aligned}\Pi^C &= \frac{1}{2} D^R \pi^C = \frac{1}{2}\left(a - F - p^m\right)\left(p^m - c\right) \\ &= \frac{1}{8}\gamma^2 - \frac{1}{4}\gamma F = \frac{1}{2}\left[\frac{1}{4}\left(\gamma^2 - 2\gamma F + F^2\right) - \frac{1}{4}F^2\right] \\ &= \frac{1}{2}\left[\frac{1}{4}\left(a - c - F\right)^2 - \frac{1}{4}F^2\right] = \frac{1}{2}\Pi^d - \frac{1}{8}F^2\end{aligned} \tag{4}$$

Whenever the total forward traded amount does not exceed or equal monopoly quantity ($F < q^m$), collusive behavior leads to collusive profits in this period. Additionally collusive profits given by half of the expected monopoly profit are expected in all upcoming periods.

Whenever the total forward traded amount exceeds or equals monopoly quantity ($F \geq q^m$) no collusive profits can be earned in this period, since the total demand for the monopoly price is already satisfied. However, not deviating from collusion promises half of the expected monopoly profit in all upcoming periods. This defines the net present value of collusion as:

$$E_{NPV}[Collusion] = \begin{cases} \frac{1}{2}\Pi^d - \frac{1}{8}F^2 + \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] & \text{if} \quad F < q^m \\ \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] & \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{5}$$

The different collusive profits in the period of (possible) deviation lead to two scenarios. In the first scenario (I), the total forward traded amount is less than the monopoly quantity ($F < q^m$). In the second scenario (II), the total forward traded amount exceeds monopoly quantity ($q^m < F$). A firm that is involved in an (explicit or tacit) collusive agreement with its competitor has two alternative strategies. Firstly, it can collude and gain a profit in the corresponding period and in future periods. Secondly, it can deviate and gain an additional profit in the corresponding period but forgo all collusive profits in future periods. A firm chooses the strategy yielding the highest expected net present value of profits. Comparing the net present values leads to an inequality, which represents the trade-off between collusion and deviation. This inequality is used to find the critical discount factor, that is applied in supergames to measure the stability of non-cooperative collusive behavior.

*Scenario **I**: The monopoly quantity exceeds the total forward traded amount ($F < q^C$)*

For a stable collusive agreement, the net present value of collusion must be larger than the net present value of deviation. Hence, the forward traded amount is below collusive quantity and the following no deviation constraint has to be fulfilled for a stable collusive agreement:

$$\begin{aligned} &E_{NPV}[Deviation] \leq E_{NPV}[Collusion] \\ &\frac{1}{4}(a-F-c)^2 \leq \frac{1}{2}(a-F-p)(p-c) + \frac{\delta}{1-\delta}E\left[\Pi_i^C\right] \end{aligned} \tag{6}$$

Inserting the monopoly price and profit gives the critical discount factor for full-collusion and a forward traded amount below monopoly quantity, that is given in Proposition 2.1.

*Scenario **II**: The total forward traded amount exceeds the collusive quantity ($q^C < F$)*

In scenario **II** no collusive profits are earned on the spot market, since the total forward traded amount exceeds monopoly quantity ($q^m < F$). Hence, the net present value of collusion is restricted to half of the future expected monopoly profits. For the forward traded amount exceeding monopoly quantity this gives following no deviation constraint for a stable collusion:

$$\begin{aligned} &E_{NPV}[Deviation] \leq E_{NPV}[Collusion] \\ &\frac{1}{4}(a-F-c)^2 \leq \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] \end{aligned} \tag{7}$$

Rearranging again yields the critical discount factor for fully-collusive behavior and an forward traded amount above the corresponding monopoly quantity, that is given in proposition 2.1.

Proposition 2.1. *The critical discount factor for any forward traded amount under full-collusion is given by:*

$$\delta^* = \begin{cases} 1 - \frac{E[\gamma]^2 + Var[\gamma]}{E[\gamma]^2 + Var[\gamma] + \gamma^2 - 2F\gamma + 2F^2} & \text{if} \quad F < q^m \\ \\ 1 - \frac{E[\gamma]^2 + Var[\gamma]}{E[\gamma]^2 + Var[\gamma] + 2\gamma^2 - 4F\gamma + 2F^2} & \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{8}$$

See equation A.5 and equation A.6 in the Appendix for a detailed derivation.

Effects of forward trading on the critical discount factor

In the following I will analyze how the critical discount factor is influenced by the realization of the random difference between reservation price and marginal costs (γ), the amount of forward contracts (F), the expected difference between reservation price and marginal cost ($E[\gamma]$) and the variance of the difference between reservation price and marginal cost ($Var[\gamma]$).

The partial derivative of the critical discount factor with respect to the difference between reservation price and marginal costs is given by:

$$\frac{\partial\delta^*}{\partial\gamma} = \begin{cases} 2\frac{[\gamma-F][E[\gamma]+Var[\gamma]]}{\left[E[\gamma]^2+Var[\gamma]+\gamma^2-2F\gamma+2F^2\right]^2} \quad \geq 0 \quad \text{if} \quad F < q^m \\ \\ 4\frac{[\gamma-F]\left[E[\gamma]^2+Var[\gamma]\right]}{\left[E[\gamma]^2+Var[\gamma]+2\gamma^2-4F\gamma+2F^2\right]^2} \quad \geq 0 \quad \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{9}$$

A higher difference between reservation price and marginal costs leads to a higher profit leading to a higher critical discount factor, because deviation becomes more attractive.

The partial derivative of the critical discount factor due to forward contracts is given by:

$$\frac{\partial\delta^*}{\partial F} = \begin{cases} -2\ \frac{[\gamma-2F]\left[E[\gamma]^2+Var[\gamma]\right]}{\left[E[\gamma]^2+Var[\gamma]+\gamma^2-2F\gamma+2F^2\right]^2} \quad \leq 0 \quad \text{if} \quad F < q^m \\ \\ -\ 4\frac{[\gamma-F]\left[E[\gamma]^2+Var[\gamma]\right]}{\left[E[\gamma]^2+Var[\gamma]+2\gamma^2-4F\gamma+2F^2\right]^2} \quad \leq 0 \quad \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{10}$$

A higher forward contracted amount strictly reduces the critical discount factor, since for forward traded amounts less than the monopoly quantity ($0 \leq F < q^m$) the deviation profit is cut more sharply than the collusive profit in the corresponding period. This is derived analytically in the Appendix (equations A.1 - A.4). If the forward traded amount is larger than the monopoly quantity ($q^m \leq F$), no collusive profit can be earned in the actual period. Thus, only the deviation profit is reduced and forward contracts strictly promote collusion.

The partial derivative of the critical discount factor with respect to the expected difference between reservation price and marginal costs is given by:

$$\frac{\partial \delta^*}{\partial E\left[\gamma\right]} = \begin{cases} -2\ \frac{\left[\gamma^2-2F\gamma+2F^2\right]E[\gamma]}{\left[E[\gamma]^2+Var[\gamma]+\gamma^2-2F\gamma+2F^2\right]^2} & \leq 0 \quad \text{if} \quad F < q^m \\ \\ -2\ \frac{\left[2\gamma^2-4F\gamma+2F^2\right]E[\gamma]}{\left[E[\gamma]^2+Var[\gamma]+2\gamma^2-4F\gamma+2F^2\right]^2} & \leq 0 \quad \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{11}$$

A higher expected difference of reservation price and marginal costs decreases the critical discount factor. Deviation from collusion becomes less attractive. A higher expected difference increases future collusive profits which cannot be earned after a deviation. Hence, the additional profits earned by deviating become smaller in relative terms.

The partial derivative of the critical discount factor with respect to the variance of the difference between reservation price and marginal costs is given by:

$$\frac{\partial \delta^*}{\partial Var\left[\gamma\right]} = \begin{cases} -\ \frac{\gamma^2-2F\gamma+2F^2}{\left[E[\gamma]^2+Var[\gamma]+\gamma^2-2F\gamma+2F^2\right]^2} & \leq 0 \quad \text{if} \quad F < q^m \\ \\ -\ \frac{2\gamma^2-4F\gamma+2F^2}{\left[E[\gamma]^2+Var[\gamma]+2\gamma^2-4F\gamma+2F^2\right]^2} & \leq 0 \quad \text{if} \quad q^m \leq F < 2q^m \end{cases} \tag{12}$$

A higher variance of the difference of reservation price and marginal costs decreases the critical discount factor. At a first glance this seems to be counter-intuitive since fluctuations are said to threaten collusions. One should keep in mind the relationship between variance squared, expectation and expectation squared used above ($E\left[\gamma^2\right] = E\left[\gamma\right]^2 + Var\left[\gamma\right]$). As can be seen, expected profit given by $\frac{1}{4}E\left[\gamma^2\right]$ *ceteris paribus* increases by an increasing variance. As presented above, a higher expected profit increases the stability of collusion. Thus, it is not the variance itself that decreases the stability of an collusive agreement, but more precisely the appearance of a high realization of the random difference between reservation price and marginal costs. For a higher variance, this high realization of the random variable is more likely to be drawn. However, for a given realization of the random variable, a higher variance decreases the critical discount factor. Table 2 summarizes these partial effects on the critical discount factor.

Figure 2 shows the evolution of the critical discount factor due to forward contracts and due

Variable	Partial Effect	I Monopoly quantity exceeding contracts	II Contracts exceeding monopoly quantity
"Spread"	$\frac{\partial\delta^*}{\partial\gamma}$	$\Downarrow$	$\Downarrow$
Forwards	$\frac{\partial\delta^*}{\partial F}$	$\Uparrow$	$\Uparrow$
Expected "spread"	$\frac{\partial\delta^*}{\partial E[\gamma]}$	$\Uparrow$	$\Uparrow$
Variance of "spread"	$\frac{\partial\delta^*}{\partial Var[\gamma]}$	$\Uparrow$	$\Uparrow$

Table 2: Summary of partial effects on the stability of a collusive agreement. Note: A higher critical discount factor implies a lower stability

to the ratio of boom and expected profits. The discount factor is plotted for positive ratios of contracted amount and monopoly quantity. Neither collusive nor deviation profits can be earned for a higher amount of contracts than the Bertrand quantity and the critical discount factors becomes zero. Hence, the graph starts at a ratio of the forward traded amount and monopoly quantity of zero and stops at a ratio of two. It is known from Rotemberg and Saloner (1986) that deviation from collusion is more profitable in booms. The graph in figure 2 starts at a ratio of profit over the expected profit of 1, since in booms per definition profits are higher than the expected ones. It ends in this dimension at a profit that is ten times the expected one.

The horizontal front-line of figure 2 shows the evolution of the discount factor for expected profit equal to actual profit $\left(\frac{\gamma^2}{E[\gamma^2]+Var[\gamma]} = 1\right)$. This represents the case of certainty described by Liski and Montero (2006), since without any forward contracts and without any volatility the critical discount factor is one half and when total monopoly quantity is traded forward the discount factor is one-third. For forward contracts between these two extreme cases ($0 \leq \frac{F}{q^m} < 1$), the critical discount factor strictly decreases in forward contracts. When firms have contracted more than the monopoly quantity of the corresponding state (scenario II), the critical discount factor still decreases in forward contracts. In scenario II the critical

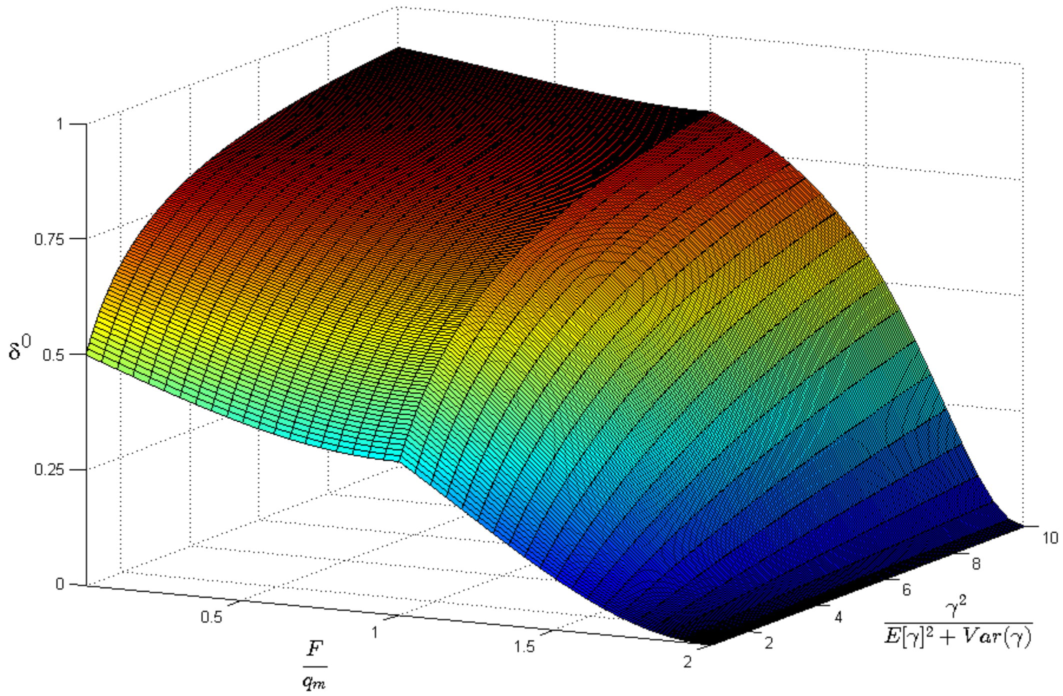

Figure 2: Effects of forward trading and ratio of boom and expected profit on discount factor

discount factor decreases more rapidly than in scenario I, since in scenario II forward trading solely cuts the deviation profit, whereas in scenario I it cuts the deviation profit as well as the collusive profit.

Introducing a volatile market creates an incentive to deviate from collusion during booms. Without forward contracts ($F = 0$) the critical discount factor strictly increases and converges to one for boom profits increasing to infinity. The functional form of the critical discount factor depends on the ratio of boom and expected profit and is given by $\frac{\gamma^2}{E[\gamma^2]+Var[\gamma]+\gamma^2} = \delta^0 \leq \delta$, which is equivalent to the findings of Rotemberg and Saloner (1986). When contracts are traded forward and at the same time boom profits are larger than expected profits, the evolution of the critical discount factor described above does not change fundamentally. Other things being equal, a higher amount of contracts decreases the critical discount factor, whereas boom profits exceeding expected profit increase the critical discount factor. This is shown graphically in figure 2 by the evolution of the plane between the above described front-lines. When firms contract a sufficiently high quantity, stable collusion becomes possible for any discount factor.

2.3. Effects of forward trading on the profitability of a collusive agreement

Proposition 2.2. *When firms set a collusive price, for which spot market quantity exceeds the forward traded amount, firms profit is not altered by the forward traded amount:*

$$\Pi_i^{SC} = \frac{1}{2}(a-p)(p-c) \quad \forall \quad F < (a-p) \tag{13}$$

The profit of colluding firms, that trade a certain amount forward has two sources: Firstly, the profit coming selling production on the spot market. Secondly, the profit coming from selling production on the spot market. As long as the forward traded amount does not exceed the collusive quantity, the spot market profit for colluding firms is given by equation 4. Inserting an an arbitrarily collusive price leads to collusive spot market profit of:

$$\Pi_i^{SM} = \frac{1}{2}\left(a-p^{SM}\right)\left(p^{SM}-c\right) - \frac{1}{2}F\left(p^{SM}-c\right) \quad \forall\ F < (a-p) \tag{14}$$

The profit on the forward market is given by each firms forward traded amount multiplied by the difference of the forward price and the marginal costs. As mentioned before, the forward market price is given by the anticipated spot market price, since speculators build rational expectations. Thus, the expected profit on the forward market is given by the expected difference of the spot market price and marginal costs times each firms forward traded amount

$$\Pi_i^{FM} = \frac{1}{2}F\left(p^{FM}-c\right) = \frac{1}{2}F\left(p^{SM}-c\right) \qquad \forall\ F \leq (a-p) \quad \square \tag{15}$$

The total (semi-)collusive profit for a firm is given by the spot and the forward market profit:

$$\begin{aligned}\Pi_i^{SC} &= \frac{1}{2}\left(a-p^{SM}\right)\left(p^{SM}-c\right) - \frac{1}{2}F\left(p^{SM}-c\right) + \frac{1}{2}FE\left[p^{SM}-c\right] \\ &= \frac{1}{2}\left(a-p^{SM}\right)\left(p^{SM}-c\right) \qquad \forall\ F < (a-p) \quad \square\end{aligned} \tag{16}$$

Thus, the increase of the expected forward market profit from forward trading is totally offset by a decrease of the expected spot market profit. Therefore, as long as forward traded amount does not exceed the spot market quantity, firms profit is not changed by forward trading

Proposition 2.3. *When firms set a collusive price, for which the forward traded amount exceeds spot market quantity, forward traded amount decreases firms profit:*

$$\Pi_i^{SC} = \frac{1}{2}\left(2q^m F - F^2\right) \quad \forall \quad F > (a-p) \tag{17}$$

When firms set a price, for which the already forward traded amount exceeds the spot market quantity, that is associated with this price, firms cannot sell any unit on the spot market. Speculators always supply the total forward traded amount to the market, since by assumption they cannot store the commodity. Hence, the price on the spot market is given by $p^{sm} = a-F$, which is below the monopoly price ($p^{SM} = a - F < p^m = \frac{1}{2}(a-c)$) and colluding firms do not earn any profit on the spot market. However, both firms earn a profit from the amount that they have traded forward. Thus, when firms have traded forward an amount above the amount, that is associated with their price on the spot market, the profit is solely given by the profit from forward trading:

$$\Pi_i^{SC} = \frac{1}{2}F(p^{SM} - F) = \frac{1}{2}F\left(a - F - c\right) = \frac{1}{2}\left(2q^m F - F^2\right) \quad \forall \quad F \leq (a-p) \tag{18}$$

Proposition 2.4. *For any distribution function each firms expected total collusive profit can be stated as:*

$$E[\Pi_i^{sc}] = \frac{1}{2}\left[E\left[2q^m F - F^2 \mid F > (a-p)\right] + E\left[(a-p)\left(p-c\right) \mid F \leq (a-p)\right]\right] \tag{19}$$

The total collusive profit for each firm is given by the profit, when the total forward traded amount does not exceed the quantity sold by firms on the spot market as well as the profit, when firms set a price, for which the already forward traded amount exceeds the spot market quantity. Combining profits of Proposition 2.2 and Proposition 2.3 leads to Proposition 2.4.

Proposition 2.5. *The profit function for an exponential distributed spread and firms that always set the monopoly price is*

$$E[\Pi_i] = \frac{1}{2}\frac{F}{\lambda} - \frac{1}{2}F^2 + \frac{1}{4}\frac{1}{\lambda^2}e^{-2\lambda F} \tag{20}$$

This profit is found by calculating the profit in Proposition 2.4 for the exponential distribution. See equation A.9 in the Appendix for the detailed derivation. The effect of forward trading on the expected collusive profit can be analyzed by taking the first and second order derivatives with respect to the forward traded amount:

$$\begin{aligned} \frac{\partial E[\Pi_i]}{\partial F} &= \frac{1}{2}\frac{1}{\lambda}\left[1 - e^{-2\lambda F}\right] - F \quad < 0 \quad \forall\, F > 0 \\ \frac{\partial^2 E[\Pi_i]}{\partial F^2} &= -1 + e^{-2\lambda F} \qquad\qquad\quad < 0 \quad \forall\, F > 0 \end{aligned} \tag{21}$$

Thus, the total expected profit for colluding firms is concavely decreasing in the contracted amount. When for example colluding firms trade the total expected monopoly quantity forward ($F = \frac{1}{2}\frac{1}{\lambda}$), they earn only about 87% of the profit compared to a situation where firms do not trade any forward contracts, since :

$$\frac{E\left[\Pi_i | F = \frac{1}{2\lambda}\right]}{E[\Pi_i | F = 0]} = \frac{1}{2} + e^{-1} \;\approx\; 0.8679 \tag{22}$$

Figure 3 shows the collusive profit for firms depending on the forward traded amount, when they could sustain a full collusion at any price ($\delta \to 1$). Figure 3 shows the expected collusive per period profit for an expected monopoly quantity of $E[q^m] = \frac{1}{2}$, $E[q^m] = \frac{2}{3}$ and $E[q^m] = 1$, since for an exponentially distributed spread the expected monopoly quantity is $E[q^m] = \frac{1}{2}\frac{1}{\lambda}$. For moderate amounts traded forward the profit decreasing effect of forward trading is rather small mainly due to two reasons. Firstly, when firms only trade a moderate amount forward, the probability, that the forward traded amount exceeds the collusive monopoly quantity is rather

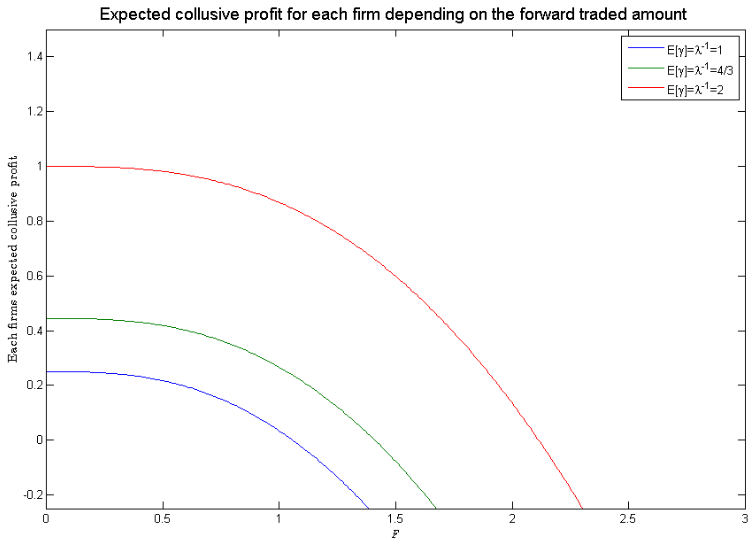

Figure 3: Effect of the forward traded amount on the collusive profit for $\delta \to 1$

small. Secondly, even if the forward traded amount exceeds the collusive monopoly quantity, only rather small monopoly profits on the spot market are crowded out by forward trading. Higher realizations of the random difference between the reservation price and marginal costs, which contribute much more to the expected profit, are not affected. The opposite is true for excessive amounts traded forward. Then, it becomes rather likely that the forward traded amount exceeds the monopoly quantity and even relatively large realizations of the spread are affected. This illustrates the fundamental finding that is in contrast to the deterministic market conditions modeled by Liski and Montero (2006): Stabilizing a collusive agreement using forward contracts is costly in volatile markets.

Proposition 2.6. *If firms have (involuntarily) traded forward an amount above half prohibitive price ($F < \frac{1}{2}a$), it is profitable to buy back own production. However, as long as firms face marginal costs this profit is below half monopoly profit, since:*

$$E\left[\Pi_i\right] = \frac{1}{2}F(a - F - c) < \frac{1}{2}\left[\frac{1}{4}a^2 - Fc\right] \leq \frac{1}{2}\left[\frac{1}{4}(a-c)^2\right] \quad \forall\, F > \frac{1}{2}a \tag{23}$$

When firms buy back their own production they do not gain any profit on the spot market. Quite the opposite, they bear the cost of buying back their production. This cost is given by the amount firms buy back $(F-\tilde{x})$ times the price associated with the amount, that is left for consumers $(p(\tilde{x})=a-\tilde{x})$.

On the forward market firms benefit from buying back production, since this increases the forward price to $p^{FM}=a-\tilde{x}$. Therefore, the profit of buying back own production is:

$$\begin{aligned}\Pi_i &= \frac{1}{2}\left[F\left(a-\tilde{x}-c\right)-\left(F-\tilde{x}\right)\left(a-\tilde{x}\right)\right] \\ &= -\tilde{x}^2+\tilde{x}a-Fc\end{aligned} \tag{24}$$

As easily can be seen, the optimal amount left for consumers is given by $\tilde{x}=\frac{1}{2}a$, since the marginal can be seen as sunk costs. The profit associated with this amount is given by ${\Pi_i}^*=\frac{1}{8}a^2-\frac{1}{2}Fc$. See equation A.11 and A.12 in the Appendix for the comparison of profits. One might think, that it could be profitable to increase production to $\tilde{x}=\frac{1}{2}a$, when the forward traded amount is below $(F<\tilde{x})$. This is not profitable, since marginal cost cannot be seen as sunk costs any more. and restricting the amount available for consumers to $\tilde{x}=\frac{1}{2}a$ is profitable if and only if forward traded amount exceeds this amount $(F>\frac{1}{2}a)$.

However, especially on the electricity market there is a huge direct cost of buying back own production, since storage or disposal are not that easy. The missing possibility of (profitable) storage or disposal is a severe problem on the european energy market, which even leads sometimes to negative prices. Therefore, this possibility is not analyzed more detailed.

2.4. Forward trading and the optimal semi-collusive strategy

Proposition 2.7. *Each firms expected collusive profit is given exactly by half of the expected monopoly profit ($E[\Pi_i]=\frac{1}{2}\left[\mu\Pi_R^M+(1-\mu)\Pi_B^M\right]$) as long as their discount factor is above the threshold discount factor of:*

$$\delta>\delta^*=1-\frac{\Pi_B^M\left(1-\mu\right)+\mu\Pi_R^M}{\Pi_B^M\left(2-\mu\right)+\mu\Pi_R^M-q_R^Mq_B^M+\frac{1}{2}{q_R^M}^2} \tag{25}$$

See equation A.7 Appendix for a detailed derivation.

The critical discount factor for full-collusion without forward trading (δ^0) ("Rotemberg and Saloner (1986) or Tirole (1988) style"') is above the critical discount factor with forward trading, since

$$\delta^0 = 1 - \frac{\Pi_B^M (1-\mu) + \mu \Pi_R^M}{\mu \Pi_R^M + (2-\mu)\Pi_B^M} > 1 - \frac{\Pi_B^M (1-\mu) + \mu \Pi_R^M}{\Pi_B^M (2-\mu) + \mu \Pi_R^M - q_R^M q_B^M + \frac{1}{2} {q_R^M}^2} = \delta^* \tag{26}$$

For a two state distribution the recessive amount is exactly known. A forward traded amount less or equal the recession monopoly quantity stabilizes collusion, but is not altering the profit. Thus, for a discrete distribution colluding firms can trade up to this recessive monopoly quantity forward, without altering the expected profit. This is in contrast to the findings for an exponential distribution in section 2.3, where the recessive monopoly amount can be any positive real number and firms are always in danger of "over-contracting".

Proposition 2.8. *When colluding firms trade forward an amount that is above the monopoly quantity in recession, the expected collusive profit for a two-state distribution is given by:*

$$E\left[\Pi_i^{SC}\right] = \frac{1}{2}\left[\mu\left(2q_R^M F - F^2\right) + (1-\mu)\left(a^B - p\right)\left(p - c^B\right)\right] \quad < \quad \frac{1}{2} E\left[\Pi^M\right] \qquad \forall\, F > \; q_R^M \tag{27}$$

Proposition 2.8 follows straightforward from Proposition 2.4, since for a two state distribution with probability μ a recession and with probability $1-\mu$ a boom occurs. Thus, the expected recession profit is given by $\mu\left(2q_R^M F - F^2\right)$, since firms have traded forward an higher amount than the corresponding monopoly quantity. However, the expected boom profit remains unaffected and is given by $(1-\mu)\left(a^B - p\right)\left(p - c^B\right)$:

$$\begin{aligned} E[\Pi_i^{sc}] &= \frac{1}{2}\left[E\left[2q^m F - F^2 \mid F > (a-p)\right] + E\left[(a-p)(p-c) \mid F \leq (a-p)\right]\right] \\ &= \frac{1}{2}\left[\mu\left(2q^m F - F^2\right) + (1-\mu)(a-p)(p-c)\right] \end{aligned} \tag{28}$$

Proposition 2.9. *When firms cannot collude by contracting the total recessive quantity forward, firms adopt their price in boom as well as sell more than the recessive monopoly quantity* *forward. The the optimal boom price (p_{sc}) and forward traded amount (F_{sc}) is:*

$$\begin{aligned} F_{sc} &= q_R^M + \frac{1}{2}\frac{1-\mu}{\mu}\left(a-2p+c\right)\frac{\partial p}{\partial F} > q_R^M && \forall\ p\ < p_B^M \\ p_{sc} &= p_B^M - \frac{\mu}{1-\mu}\left(F-q_R^M\right)\frac{\partial F}{\partial p} < p_B^M && \forall\ F\ > q_R^M \end{aligned}$$

Firms will choose the forward traded amount F and the boom price p, such that they maximize the expected collusive profit. Unfortunately, optimization of the expected collusive profit such that the no deviation constraint holds, cannot be solved analytically. Therefore, the total differential is used to show the structure of optimal collusive design.

When firms cannot fully-collude, firms choose price and forward traded quantity exactly to match the no deviation constraint ($C \stackrel{!}{=} 0$). The partial effect of the semi-collusive price on the forward traded amount is: (For derivation see equation A.13 to A.20 in the Appendix.)

$$\begin{aligned} \frac{\partial p}{\partial F} &= \frac{(a-F-p)(1-\delta)+2\delta\mu(q_R^M-F)}{(1-\delta)F-(1-\delta\mu)(a-2p+c)} > 0 \\ \frac{\partial F}{\partial p} &= \frac{(1-\delta)F-(1-\delta\mu)(a-2p+c)}{(a-F-p)(1-\delta)+2\delta\mu(q_R^M-F)} > 0 \end{aligned} \tag{29}$$

For the upcoming analysis, the most important factor for this partial effect are:

$$\frac{\partial p}{\partial F} > 0, \quad \frac{\partial F}{\partial p} > 0, \quad \frac{\partial \frac{\partial F}{\partial p}}{\partial \mu} > 0, \quad \frac{\partial \frac{\partial p}{\partial F}}{\partial \mu} < 0 \tag{30}$$

Maximizing the expected collusive profit due to the forward traded amount leads to:

$$\begin{aligned} \frac{\partial E[\Pi]}{\partial F} &= \mu\left(2q_R^M - 2F\right) + (1-\mu)\left((a+c)\frac{\partial p}{\partial F} - 2p\frac{\partial p}{\partial F}\right) \stackrel{!}{=} 0 \\ F_{sc} &= q_R^M + \frac{1}{2}\frac{1-\mu}{\mu}\left(a-2p+c\right)\frac{\partial p}{\partial F} \ > \ q_R^M && \forall\ p\ < p_B^M \end{aligned} \tag{31}$$

Maximizing the expected collusive profit due to the boom price leads to:

$$\begin{aligned} \frac{\partial E[\Pi]}{\partial p} &= \mu\left(2q_R^M\frac{\partial F}{\partial p} - 2F\frac{\partial F}{\partial p}\right) + (1-\mu)\left(a-2p+c\right) \stackrel{!}{=} 0 \\ p_{sc} &= p_B^M - \frac{\mu}{1-\mu}\left(F-q_R^M\right)\frac{\partial F}{\partial p} \ < \ p_B^M && \forall\ F\ > q_R^M \end{aligned} \tag{32}$$

As long as semi-colluding firms set a price below the monopoly boom price, they choose an forward traded amount above recessive monopoly quantity and vice versa ($p_{sc} < p_B^M \Leftrightarrow F_{sc} > q_R^M$). Therefore, in recession as well as in booms the optimal strategy departs from the monopoly outcome.

The effect of the recession probability μ on the semi-collusive outcome is given by the derivatives of the optimal semi-collusive price and forward traded amount with respect to the recession probability μ

$$\begin{aligned}\frac{\partial F_{sc}}{\partial \mu} &= (a-2p+c)\left[-\frac{1}{(1-\mu)^2}\frac{\partial p}{\partial F}+\frac{1-\mu}{\mu}\frac{\partial \frac{\partial p}{\partial F}}{\partial \mu}\right] < 0 \\ \frac{\partial p_{sc}}{\partial \mu} &= -\left[F-q_R^M\right]\left[\frac{1}{(1-\mu)^2}\frac{\partial F}{\partial p}+\frac{\mu}{1-\mu}\frac{\partial \frac{\partial F}{\partial p}}{\partial \mu}\right] \quad < 0\end{aligned} \tag{33}$$

For a given discount factor, that forces firms to semi-collude, firms can either trade forward more than the corresponding recession monopoly quantity or set a boom price below the monopoly one. Ceteris paribus a higher recession probability μ leads to an lower forward traded amount as well as to a lower collusive boom price. This means firms stabilize their collusive agreement rather by adopting boom price than by trading forward. Quite the opposite is true, when the probability for a boom $1-\mu$ is increased. Then firms trade a rather large amount forward but are reluctant to adopt boom price.

The economic intuition of this result is straight forward: Semi-colluding firms have to choose whether they sacrifice an larger amount of boom or of recession profit. When the expected recession profit increases, they prefer sacrificing more of the boom profit. When in contrast the expected boom profit increases, firms prefer sacrificing more of the recession profit.

3. Conclusion

Uncertainty, volatility and fluctuations are the most frequent reasons given for forward trading. The contribution of this paper is the simultaneous analysis of fluctuations and forward contracts on collusive agreements. The incorporation of stochastic market conditions leads to a more precise understanding of the effects of forward trading and collusion. In terms of the economic literature, the gap between Rotemberg and Saloner (1986) and Liski and Montero (2006) has been closed.

The first part answers the question, whether forward trading can be used in volatile markets to stabilize a collusive agreement. Therefore, the critical discount factor has been determined and the partial derivatives of the critical discount factor were analyzed. Main findings are: High realizations of the random difference between reservation price and marginal costs ("spread") have a destabilizing effect, whereas a higher expectation of the "spread" has a stabilizing effect on collusive agreements. The results are totally in line with the analysis of Rotemberg and Saloner (1986). However, decomposition of the expectation of the squared "spread" into its squared expectation and variance led to an interesting insight: For a given positive fluctuation (boom), a higher variance increases the stability of collusion, since a higher variance makes a boom more common. Hence, it is not the variance itself that decreases the stability of a collusive agreement in volatile markets, but rather the appearance of high realizations of the "spread" that destabilizes collusive agreements. However, extraordinary booms only occur if the distribution of the spread is characterized by a sufficient degree of dispersion. As a further insight we found that short term forward contracts can be used by firms to strictly stabilize collusion. This is in line with the analysis of Liski and Montero (2006) and Green and Coq (2010).

The second part answers the question, how the expected collusive profit is influenced by forward trading. For deterministic market conditions the profit that is earned by colluding firms, is not at all influenced by the forward traded amount (Liski and Montero, 2006). As shown in this article for continuous distributed cost and demand parameters the expected profit earned by colluding firms strictly decreases in the forward traded amount. When firms trade forward on a volatile market, they do not know in advance the demand and cost structure they will face at the date of delivery. For colluding firms this always leads to the problem of involuntarily having contracted more or less than the optimal collusive amount. When firms have contracted less than the optimal collusive amount, colluding firms can sell an additional amount on the spot market, which gives them the possibility to share the monopoly profit. However, for rather small contract volumes (in relation to the total accessible demand) a deviation could become profitable for "impatient firms". When firms have contracted more than the optimal collusive amount, solely the speculators decide about the price on the spot market, which leads to a lower price. This lowers forward price, since the forward price is determined on the basis of rational expectations. As a consequence, the expected profit from trading forward a certain amount is beneath the expected profit from selling the same amount on the spot market. Therefore, the total expected value of the profit for each colluding firm is decreased by forward trading. The more forward contracts are sold, the more severe is the reduction of collusive profit by (additional) forward contracts.

The third part describes for a two-state distribution of cost and demand parameters the optimal semi-collusive strategy. Semi-colluding firms choose a forward traded amount above recession monopoly quantity and a boom price below the monopoly price. Therefore, neither in recession nor in boom the monopoly outcome is generated.

The three main result of this article can be stated as follows: Firstly, forward contracts can be used in deterministic as well as in volatile markets to stabilize a collusive agreement. Secondly, in volatile markets forward trading decreases the expected total profit of colluding firms, when they "involuntarily" trade forward an amount above the recession quantity. For a discrete distribution, the lowest recession quantity is known. Therefore, this is not a severe problem for colluding firms. When in contrast to this for a continuous distribution the lowest recession monopoly quantity is not known, firms expected profit is strictly decreasing in forward contracts. Thirdly, semi-colluding firms will generate neither in boom nor in recession the monopoly outcome.

4. Appendix

4.1. Properties of the profit for a deviating and a collusive firm

Deviation profit (equation 2) can be rearranged to

$$\begin{aligned}\Pi^d &= \frac{1}{4}\left[a-c-F\right]^2 = \frac{1}{4}\left[(a-c)^2 - 2F(a-c) + F^2\right] \\ &= \Pi^m\left[1 - \frac{F}{\frac{1}{2}(a-c)} + \frac{1}{4}\frac{F^2}{\frac{1}{4}(a-c)^2}\right] = \Pi^m\left[1 - \frac{1}{2}\frac{F}{q^m}\right]^2\end{aligned} \tag{A.1}$$

Collusive profit in a spot market period (equation 4) can be brought to:

Remember: Collusive profit in a spot market period can be earned if and only if $F < q_m$

$$\begin{aligned}\Pi^C &= \frac{1}{2}\left[\frac{1}{4}\left(a-c\right)^2 - \frac{1}{2}F(a-c)\right] = \frac{1}{2}\left[\Pi^m - \frac{2}{4}(a-c)^2\frac{F}{a-c}\right] \\ &= \frac{1}{2}\Pi^m\left[1 - \frac{F}{q^m}\right]\end{aligned} \tag{A.2}$$

As can easily be seen , deviation profit as well as collusive profit in a spot market period is decreased by forward contracts. However, as long as the total amount of forward contracts is less then the monopoly quantity, the decreasing effect is stronger on deviation profit. This is due to the fact that forward trading influences deviation profit squared ($\Pi^D = \Pi^m\left[1 - \frac{1}{2}\frac{F}{q^m}\right]^2$) whereas collusive profit is influenced linearly ($\Pi^C = \frac{1}{2}\Pi^m\left[1 - \frac{F}{q^m}\right]$).

Partial derivatives of collusion and deviation profit in a spot market period are given by:

$$\frac{\partial \Pi^C}{\partial F} = -\frac{1}{2}\frac{\Pi^m}{q^m}, \qquad \frac{\partial \Pi^D}{\partial F} = -\frac{\Pi^m}{q^m}\left[1 - \frac{1}{2}\frac{F}{q^m}\right] \tag{A.3}$$

Comparing both partial derivatives leads to

$$-\frac{1}{2}\frac{\Pi^m}{q^m} \quad \geq \quad -\frac{\Pi^m}{q^m}\left[1 - \frac{1}{2}\frac{F}{q^m}\right] \quad \Longrightarrow q^m \quad \geq \quad F \tag{A.4}$$

If the forward traded amount is less than the respecting monopoly quantity ($F < q^m$), additional forward contracts decrease deviation profit more sharply than collusive profit.

If the forward traded amount is greater than the respective monopoly quantity ($F > q^m$), no collusive profits in the corresponding period can be earned. Additional forward contracts decrease deviation profit. Hence, the effect of additional forward contracts on the critical discount factor increases.

4.2. No deviation constraint and critical discount factor

Derivation of the critical discount factor (Proposition2.1). To find the critical discount factor, the no deviation constraint (equation 6), which represents the trade-off between collusion and deviation, is solved for the discount factor δ. As long as firms trade less than the monopoly quantity forward, the critical discount factor is given by:

$$\begin{aligned} NPV\left(Collusion\right) &\geq NPV\left(Deviation\right) \\ \Pi^{d} &\leq \frac{1}{2}\Pi^{d} - \frac{1}{8}F^{2} + \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^{m}] \\ 4\Pi^{d} + F^{2} &\leq \frac{\delta}{1-\delta}E[\gamma^{2}] \\ \gamma^{2} - 2\gamma F + 2F^{2} &\leq \frac{\delta}{1-\delta}\left[E[\gamma]^{2} + Var\left[\gamma\right]\right] \\ \delta \geq \frac{\gamma^{2} - 2\gamma F + 2F^{2}}{E[\gamma]^{2} + Var\left[\gamma\right] + \gamma^{2} - 2\gamma F + 2F^{2}} &= 1 - \frac{E[\gamma]^{2} + Var\left[\gamma\right]}{E[\gamma]^{2} + Var\left[\gamma\right] + \gamma^{2} - 2\gamma F + 2F^{2}} \end{aligned} \tag{A.5}$$

When firms trade more than the monopoly quantity forward, the no deviation constraint in equation 7 has to hold and the critical discount factor is given by:

$$\begin{aligned} NPV\left(Collusion\right) &\geq NPV\left(Deviation\right) \\ \frac{1}{4}\left(a - F - c\right)^{2} &\leq \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^{m}] \\ 2\gamma^{2} - 4F\gamma + 2F^{2} &\leq \frac{\delta}{1-\delta}\left[E\left[\gamma\right]^{2} + Var\left[\gamma\right]\right] \\ \delta \geq \frac{2\gamma^{2} - 4F\gamma + 2F^{2}}{E\left[\gamma\right]^{2} + Var\left[\gamma\right] + 2\gamma^{2} - 4F\gamma + 2F^{2}} &= 1 - \frac{E\left[\gamma\right]^{2} + Var\left[\gamma\right]}{E\left[\gamma\right]^{2} + Var\left[\gamma\right] + 2\gamma^{2} - 4F\gamma + 2F^{2}} \end{aligned} \tag{A.6}$$

Inserting the two state distribution function into the no deviation constraint (equation A.5):

$$
\begin{aligned}
&\frac{1}{4}\gamma^2 - \frac{1}{2}\gamma F + \frac{1}{2}F^2 \leq \frac{\delta}{1-\delta}E\left[\Pi^M\right] \\
&\Pi_B^M - q_B^M F + \frac{1}{2}F^2 \leq \frac{\delta}{1-\delta}\left[\mu\Pi_R^M + (1-\mu)\Pi_B^M\right] \\
&\delta \geq \frac{\Pi_B^M - Fq_B^M + \frac{1}{2}F^2}{\mu\Pi_R^M + (1-\mu)\Pi_B^M + \Pi_B^M - Fq_B^M + \frac{1}{2}F^2} \\
&\delta \geq 1 - \frac{\mu\Pi_R^M + (1-\mu)\Pi_B^M}{\mu\Pi_R^M + (2-\mu)\Pi_B^M - q_R^M q_B^M + \frac{1}{2}{q_R^M}^2} \qquad \square
\end{aligned}
\tag{A.7}
$$

Where the last line comes from the fact, that the highest forward traded amount without a loss in (recession) profit is given by recession monopoly quantity ($F = q_R^M$).

4.3. Using the exponential distribution to specify the total expected profit

An exponential distribution for the spread ($\gamma = a - c$) is introduced into the expected collusive profit (Proposition 2.4), to derive the total expected collusive profit in Proposition 2.5. Note: As long as the forward traded amount does not exceed the monopoly quantity, each firm earns half monopoly boom profit ($\frac{1}{2}\Pi^M = \frac{1}{8}(a-c)^2 = \frac{1}{8}\gamma^2$), since they set the monopoly price. When the forward traded amount exceeds monopoly quantity, they solely earn a profit from forward trading of $\frac{1}{2}F(2q^m F - F^2) = \frac{1}{2}F((a-c)F - F^2) = \frac{1}{2}F(\gamma F - F^2)$

$$
\begin{aligned}
E[\Pi_i^{sc}] &= \frac{1}{2}\left[E\left[2q^m F - F^2 \mid F > \frac{1}{2}(a-c)\right] + E\left[\frac{1}{8}(a-c)^2 \mid F \leq \frac{1}{2}(a-c)\right]\right] \\
&= \frac{1}{2}\left[\int_0^{2F}\left(\gamma F - F^2\right)\widehat{f}(\gamma)d\gamma + \int_{2F}^{\infty}\frac{1}{4}\gamma^2\widehat{f}(\gamma)d\gamma\right] \\
&= \underbrace{\frac{1}{2}F\int_0^{2F}\gamma\widehat{f}(\gamma)d\gamma}_{A} - \underbrace{\frac{1}{2}F^2\widehat{F}\left(2F\right)}_{B} + \underbrace{\frac{1}{8}\int_{2F}^{\infty}\gamma^2\widehat{f}(\gamma)d\gamma}_{C}
\end{aligned}
\tag{A.8}
$$

A, B and C can be brought to:

$$\begin{aligned}
A &= \frac{1}{2}F\lambda\int_0^{2F}\gamma e^{-\lambda\gamma}d\gamma = \frac{1}{2}F\lambda\left[-2F\frac{1}{\lambda}e^{-2F\lambda}+0+\frac{1}{\lambda}\int_0^{2F}e^{-\lambda\gamma}d\gamma\right]\\
&= -F^2e^{-2F\lambda}+\frac{1}{2}\frac{F}{\lambda}\left[1-e^{-2F\lambda}\right]\\
B &= -\frac{1}{2}F^2\left[1-e^{-2F\lambda}\right]\\
C &= \frac{1}{8}\lambda\int_{2F}^{\infty}\gamma^2e^{-2F\lambda}d\gamma = \frac{1}{8}\lambda\left[\frac{1}{\lambda}4F^2e^{-2F\lambda}+2\frac{1}{\lambda}\int_{2F}^{\infty}e^{-\lambda\gamma}d\gamma\right]\\
&= \frac{1}{2}F^2e^{-2F\lambda}+\frac{1}{4}\left[\frac{1}{\lambda}e^{-2F\lambda}2F+\frac{1}{\lambda}\int_{2F}^{\infty}e^{-\lambda\gamma}d\gamma\right]\\
&= \frac{1}{2}F^2e^{-2F\lambda}+\frac{1}{2}\frac{F}{\lambda}e^{-2F\lambda}+\frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda}
\end{aligned} \tag{A.9}$$

Summing up the first (A), the second (B) and the third part (C) yields:

$$\begin{aligned}
E\left[\Pi_i\right] = &-F^2e^{-2F\lambda}+\frac{1}{2}\frac{F}{\lambda}\left[1-e^{-2F\lambda}\right]-\frac{1}{2}F^2\left[1-e^{-2F\lambda}\right]+\frac{1}{2}F^2e^{-2F\lambda}+\frac{1}{2}\frac{F}{\lambda}e^{-2F\lambda}\\
&+\frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda} = \frac{1}{2}\frac{F}{\lambda}-\frac{1}{2}F^2+\frac{1}{4}\frac{1}{\lambda^2}e^{-2F\lambda}
\end{aligned} \tag{A.10}$$

4.4. Profits of buying back own production

As long as firms bear marginal costs, firms profit, when buying back their own production is below the monopoly profit, since:

$$\begin{aligned}
\Pi_i^{BuyBack} &< \frac{1}{2}\Pi_i^M\\
\frac{1}{2}\left(\frac{1}{4}a^2-Fc\right) &< \frac{1}{8}\left(a-c\right)^2\\
\frac{1}{2}a-F &< \frac{1}{4} \quad \forall F > \frac{1}{2}a \quad \square
\end{aligned} \tag{A.11}$$

When firms do not buy back any production, their profit is given by $\Pi_i = \frac{1}{2}F(a-F-c)$, this profit is below the profit when buying back own production, since:

$$\begin{aligned}
&\frac{1}{2}\Pi_i{}^* = \frac{1}{8}a^2-\frac{1}{2}Fc > \frac{1}{2}F(a-F-c)\\
&\frac{1}{4}a^2-Fa+F^2 > 0 \qquad F_{1,2} = \frac{a\pm\sqrt{a^2-a^2}}{2} = \frac{1}{2}a
\end{aligned} \tag{A.12}$$

4.5. Derivation of the optimal semi-collusive strategy

When firms trade more than the recessive monopoly quantity forward, the no deviation constraint looks as follows:

$$\begin{aligned}
&\frac{1}{4}(a-F-c)^2 \leq \frac{1}{2}(a-F-p)(p-c)+\frac{1}{2}\frac{\delta}{1-\delta}\left[\mu(2q_R^M F-F^2)+(1-\mu)(a-p)(p-c)\right] \\
&0 \leq -\frac{1}{2}(a-F-c)^2+(a-F-p)(p-c)+\frac{\delta}{1-\delta}\left[\mu\left(2q_R^M F-F^2\right)+(1-\mu)(a-p)(p-c)\right] \\
&C := -\frac{1}{2}(a-F-c)^2+(a-p)(p-c)\frac{1-\delta\mu}{1-\delta}-F(p-c)+\frac{\delta\mu}{1-\delta}\left(2q_R^M F-F^2\right) \stackrel{!}{=} 0
\end{aligned} \tag{A.13}$$

Firms that collude and need to adopt forward traded amount above the recessive monopoly quantity and/or set a price in booms below monopoly price, choose the contracted amount and the price exactly to match no deviation constraint.

Lowering the boom price stabilizes a collusive agreement, if and only if the partial derivative according to the price is negative ($\frac{\partial C}{\partial p} < 0$):

$$\begin{aligned}
\frac{\partial C}{\partial p} &= (a-2p+c)\frac{1-\delta\mu}{1-\delta}-F<0 \\
p &> \frac{1}{2}(a+c)-F\frac{1-\delta}{1-\delta\mu}
\end{aligned} \tag{A.14}$$

The partial derivative of the constraint according to the price is negative for all prices above the residual monopoly price ($\frac{\partial C}{\partial p} \leq 0 \quad \forall\, p \geq \frac{1}{2}(a-F-c)$), as long as:

$$\frac{1-\delta}{1-\delta\mu} \quad > \frac{1}{2} \quad \Leftrightarrow \quad \delta < \frac{1}{2-\mu} \tag{A.15}$$

This condition is fulfilled for any discount factor, that forces firms to semi-collude (see equation 26 for the condition of semi-collusion without forward trading), since:

$$\begin{aligned}
\frac{1}{2-\mu} &> \delta^0 = \frac{\Pi_B^M}{\mu\Pi_R^M+(2-\mu)\Pi_B^M} \\
\frac{\mu}{2-\mu}\Pi_R^M+\Pi_B^M &> \Pi_B^M \;\Rightarrow \frac{\mu}{2-\mu}\Pi_R^M > 0 \quad \square
\end{aligned} \tag{A.16}$$

Selling an higher amount than the recessive monopoly quantity forward, stabilizes a collusive agreement, if and only if the partial derivative according to forward contracts is positive ($\frac{\partial C}{\partial F} > 0$):

$$\begin{aligned} \frac{\partial C}{\partial F} &= (a - F - c) - (p - c) + \frac{\delta\mu}{1-\delta}\left(2q_R^M - 2F\right) > 0 \\ &\frac{(1-\delta)\,(a-p)}{1-\delta(1-2\mu)} + \frac{2\delta\mu}{1-\delta(1-2\mu)} q_R^M > F \end{aligned} \tag{A.17}$$

The first part of the condition is given by a factor depending on the discount factor and the recession probability multiplied with the boom quantity ($\frac{(1-\delta)(a-p)}{1-\delta(1-2\mu)}$). The second part is given by a factor depending on the discount factor and the recession probability multiplied with the recession quantity ($\frac{2\delta\mu}{1-\delta(1-2\mu)} q_R^M$). This condition is fulfilled for forward traded quantities that do not "exceed too much" the recessive collusive quantity. If the condition had been negative, firms exactly choose $F = \frac{(1-\delta)(a-p)}{1-\delta(1-2\mu)} + \frac{2\delta\mu}{1-\delta(1-2\mu)} q_R^M$, since a higher amount would decrease the stability of a collusive agreement and simultaneously decrease the profit.

To identify the partial effect of the forward traded amount and the boom price the total differential of the no deviation constraint is used.

$$\begin{aligned} \frac{\partial F}{\partial p} &= -\frac{\frac{\partial C}{\partial p}}{\frac{\partial C}{\partial F}} = \frac{(1-\delta)F - (1-\delta\mu)(a-2p+c)}{(a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)} > 0 \\ \frac{\partial p}{\partial F} &= -\frac{\frac{\partial C}{\partial F}}{\frac{\partial C}{\partial p}} = \frac{(a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)}{(1-\delta)F - (1-\delta\mu)(a-2p+c)} > 0 \end{aligned} \tag{A.18}$$

This leads to following optimal forward traded amount and boom price:

$$\begin{aligned} F_{sc} &= q_R^M + \frac{1}{2}\frac{1-\mu}{\mu}(a-2p+c)\frac{(a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)}{(1-\delta)F - (1-\delta\mu)(a-2p+c)} > \quad q_R^M \\ p_{sc} &= p_B^M - \frac{\mu}{1-\mu}(F - q_R^M)\frac{(1-\delta)F - (1-\delta\mu)(a-2p+c)}{(a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)} \qquad < \quad p_B^M \end{aligned} \tag{A.19}$$

The partial derivatives of the relationship between forward traded amount and semi-collusive boom price with respect to recession probability μ are:

$$\frac{\partial \frac{\partial p}{\partial F}}{\partial \mu} = \frac{2\delta\left(q_R^M - F\right)\left((1-\delta)F - (1-\delta\mu)(a-2p+c)\right) - \delta\left(a-2p+c\right)\left((a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)\right)}{\left((1-\delta)F - (1-\delta\mu)(a-2p+c)\right)^2} < 0$$
$$\frac{\partial \frac{\partial F}{\partial p}}{\partial \mu} = \frac{\delta\left(a-2p+c\right)\left((a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)\right) - 2\delta\left[q_R^M - F\right]\left((1-\delta)F - (1-\delta\mu)(a-2p+c)\right)}{\left((a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F)\right)^2} > 0 \tag{A.20}$$

Where the signs can easily be deducted from the fact that, the forward traded amount exceeds recessive monopoly quantity $F > q_R^M$ and the fact that $(1-\delta)F - (1-\delta\mu)(a-2p+c) > 0$ (see equation A.14) and $(a-F-p)(1-\delta) + 2\delta\mu(q_R^M - F) > 0$ (see equation A.17)

4.6. Negligible uncertainty as a special case

Under certainty, firms never trade more than the monopoly quantity in a full collusive agreement, since trading forward more than (a priori known) monopoly quantity would decrease profits. Total traded amount is given by summing up the single (symmetrically) traded amount where x gives the proportion of monopoly quantity that is traded forward ($F = f_i + f_j = 2f = xq^m = \frac{1}{2}\gamma x$). Under certainty, the "spread" equals its expectation and the variance of the "spread" is equal to zero. Then the critical discount factor (equation A.5) can be brought to:

$$\begin{aligned} \delta \geq \delta^* &= 1 - \frac{E\left[\gamma\right]^2 + V\left[\gamma\right]}{E\left[\gamma\right]^2 + V\left[\gamma\right] + \gamma^2 - 2F\gamma + 2F^2} = 1 - \frac{\gamma^2}{2\gamma^2 - x\gamma^2 + \frac{1}{2}x^2\gamma^2} \\ &= 1 - \frac{2}{(2-x)^2 + 2x}, \qquad \frac{\partial \delta^*}{\partial x} = \frac{-4\left[1-x\right]}{(2-x)^2 + 2x} \leq 0 \end{aligned} \tag{A.21}$$

The partial derivative of the critical discount factor due to proportion of monopoly quantity traded forward is strictly negative. Hence, in a deterministic market structure, trading forward is able to stabilize collusive agreements as well. The critical discount factor neglecting uncertainty (equation A.21) is equivalent to the factor found by Liski and Montero (2006, p.219).

Representation of the critical discount factor used for plotting in figure 2:

$$\begin{aligned}\delta^* &= 1 - \frac{E\left[\gamma\right]^2 + Var\left[\gamma\right]}{E\left[\gamma\right]^2 + Var\left[\gamma\right] + \gamma^2 - 2F\gamma + 2F^2} \\ &= 1 - \frac{2}{2 + \frac{\gamma^2}{E[\gamma]^2 + Var[\gamma]}\left[2 - 2\frac{F}{\frac{1}{2}\gamma} + \frac{F^2}{\frac{1}{4}\gamma^2}\right]} = 1 - \frac{2}{2 + \frac{\gamma^2}{E[\gamma]^2 + Var[\gamma]}\left[2 - 2\frac{F}{q^m} + \frac{F^2}{q^{m2}}\right]}\end{aligned} \tag{A.22}$$

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