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Rectification of aerial photographs by means of desk-top systems

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13.1 Introduction

The project described in this paper has occupied the author over a long period of time. Although no formal publication has been prepared after 1982 (Chamberlain & Haigh 1982, Haigh 1983), the work has progressed quite steadily since 1980. Several presentations have been made to the Aerial Archaeology Research Group, but the proceedings of that group do not normally appear in formal publication.

The project commenced in 1980 when the author was asked to interface a small digitising tablet and a small pen plotter to a microcomputer, in such a manner that outlines digitised from photographs were converted into map co-ordinates before being plotted. The original BASIC program ran on a Research Machines 380Z computer, using a HI-Pad digitiser and a HI-Plot plotter, both manufactured by Houston Instruments. The program was subsequently translated into FORTRAN, and has been adapted to run on a variety of different computers and using a range of peripherals. New features have continually been added, and existing features modified, during the period of development, so that the program now represents a reliable and versatile means of tackling a range of problems concerned with aerial photographs of archaeological sites. Two principles have been adhered to throughout the development: that the system should employ equipment which is within the means of any serious computer user and which is capable of being accommodated on a laboratory table; and that a minimum of technical knowledge should be required of the general user.

13.2 The use of the projective transformation

When the area of ground shown in the photograph is effectively a uniform plane (but not necessarily horizontal), it is possible to define the transformation from the photograph co-ordinates into map co-ordinates by means of projective geometry. This concept underlies many of the techniques which are used in hand-drawn transcription, such as Mobius networks and paper strips, and a good description has been given by Scollar 1975; it was adopted as a computer method by Palmer 1977, who employed the mainframe computing facilities available at Cambridge University. Palmer's program was used to transcribe features from many photographs in the Cambridge collection but, following changes in the computing facilities, seems to have been lost.

A projective transformation from photograph co-ordinates (x, y) into map co-ordinates (X, Y) may be expressed in the form:

$$X = \frac{ax + by + c}{gx + hy + 1}, Y = \frac{dx + ey + f}{gx + hy + 1}.$$

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In order to use these formulae to rectify digitised co-ordinates, it is necessary first to ascertain the values of the eight parameters denoted by the symbols a to h. These may be calculated, provided that there are at least four control points, whose co-ordinates are known both in the photograph and on the map. With exactly four control points, the parameters may be determined by the solution of simultaneous equations. With five or more control points, it is necessary to use a procedure for least-squares fitting; if the projective formulae are cross-multiplied by the denominator on the right-hand side, then they become linear in the eight parameters, making the fitting method quite straightforward (Haigh 1983).

With the author's AERIAL program, the user is invited first to fix his map to the digitising tablet, and to specify the South-West corner and size of a reference square. When the four corners of this square have been digitised, the computer is able to calculate the National Grid Reference of any other map control point digitised within the square. The user may then digitise all the control points on the map or, alternatively, their grid references may be typed from the keyboard. The use of the digitised map square obviates the need to specify the exact scale of the map.

Once the map control has been digitised, the user fixes the photograph to the digitiser and, having established the position of the photograph, digitises the photograph control in the same sequence that was used from the map. The computer calculates the optimal transformation, and displays a list of stated and fitted grid references, so that the user may check that no obvious errors have occurred during the digitisation of control points.

If the user is satisfied with the transformation, then the digitisation of the outlines of features of interest may commence. The user may choose to represent the outlines as a string of vector segments or as discrete points. With the normal operation of the program, a rectified plan is built up on screen as the digitisation proceeds and the digitised co-ordinates are stored in a file, so that they may be replotted at any future time. As each point is digitised, the corresponding grid reference is displayed on screen; this enables the user to note the location of any feature which may be required as a control for other photographs. The user is allowed to change the colour of the displayed line, or to insert a message into the output file, at any point; both the colour change and the message are recorded in the output file so that they may be used during replotting, whether on screen or on the pen plotter. Existing files and newly-digitised material may be displayed in any combination which is convenient to the user.

The author strongly recommends the use of the projective transformation to rectify any photograph of a genuinely flat landscape. It is often criticised for its lack of versatility, but it is very useful for archaeological photographs, which are frequently taken as the opportunity is offered, without time to ensure optimal conditions, and with comparatively simple equipment. Such circumstances are far from ideal for more advanced photogrammetric techniques.

A recent and very successful output from the AERIAL program, using the projective transformation, is shown in Fig. 13.1. This shows a complicated set of cropmarks extending over an area of about 700 metres × 500 metres, digitised from two photographs taken in different seasons. The ground is so level and the control sufficiently good, that the two photographs agree with each other and with the map, to an accuracy of better than two metres, over the entire area. Under such circumstances, there is little to be gained by turning to photogrammetric techniques. For publication purposes, Fig. 13.1 has been plotted entirely in black, but the direct output from the plotter would normally show the results from the two photographs in contrasting colours.

Figure 13.1: A plan of a complicated group of crop-marks, digitised from two photographs taken in different seasons. Since the crop-marks were on level ground, rectification was performed on the basis of a projective transformation

13.3 Introduction of photogrammetric techniques

Clearly there are many circumstances where the projective technique cannot be expected to work. The most obvious cases are photographs taken in hilly terrain, but there are also more subtle cases where cropmarks in an apparently level field in fact lie on a slight rise in the middle of the field. In the latter case, a projective transformation might give an accurate rectification of the field boundary, and the problem would only be revealed by comparing plots of the cropmarks made from photographs taken from different directions, when the variation in ground height will cause a relative displacement of the plots.

Once it is clear that a particular problem cannot be solved on the basis of a plane-to-plane projection, it becomes necessary to turn to some three-dimensional analysis on the basis of photogrammetric technique. According to the standard works (e.g. Burnside 1979), two principal mathematical techniques are employed in photogrammetry: the collinearity equations and the coplanarity equations. The collinearity equations are used to establish the relationship between a single camera station and the scene of the image, on the basis of known control information. The coplanarity equations enable a three-dimensional model to be constructed from views of the same scene taken from two different camera stations; the model is initially described in terms of local co-ordinates, which can eventually be converted into global co-ordinates on the basis of control information; modern bundling techniques enable the local model to be extended over several different views, and the control information to be drawn from widely separated sources.

The collinearity equations do not in themselves provide for the construction of a three-dimensional model, since this cannot be done with any method which analyses just one photograph at a time. The coplanarity equations are employed in modern analytical plotting systems, which construct accurate three-dimensional models by relying on the brain's ability to fuse similar images in a binocular system. The author has succeeded in using the coplanarity equations to reconstruct archaeological cropmarks from two photographs of a hilly site, placed side by side on the digitising tablet. However, the need to locate precisely corresponding points in each of the two photographs proves to be an exacting and arduous requirement, and the method is unlikely to commend itself to the general user. Consequently the author has abandoned the coplanarity equations, at least for the time being, in favour of the method described in the following paragraphs, which allows good results to be produced from a single photograph.

As has been pointed out by Scollar 1975, the collinearity equations can be used to locate a point in space from a single photograph, provided that the position of the camera station has been determined, and that the ground height is known at the point concerned. Knowledge of the ground height effectively turns the problem into a surface-to-plane projection, making it an extension of the projective methods discussed in section 13.2. The question then arises as to how a digital terrain model (or DTM) may be constructed to describe the ground height over the area shown in the photograph. In Britain, the best general source of such height information is the 1:10000 series of maps published by the Ordnance Survey. The author has found that adequate DTMs can be constructed from such maps, preferably enlarged by a factor of about two before being placed on the digitising tablet.

The author's AERIAL program enables a DTM to be created by digitising a series of points from the contour lines shown on the map. It is important that the points should be distributed fairly uniformly over the area of the model; consequently it may be necessary to omit some of the contour lines in areas where the ground slopes steeply, and to sketch some additional contours in more level areas. The user is assisted during the digitisation of the DTM by a display of coloured symbols to indicate the positions of points already digitised. When the DTM is in use, it is necessary to interpolate between the points of specified height. Currently this is done using an extension of the methods discussed by Haigh & Kelly 1987. For each specified point of the DTM, a biquadratic surface is determined which passes through the point concerned and which is best fitted to the ten nearest neighbours. The height at any intermediate point is taken to be a weighted average of the heights given by the surfaces at each of the model points within a given radius. The weighting function is chosen to ensure that the correct height is taken at each model point, and that the height varies smoothly over the region between model points. There are many possible methods of interpolating between specified points on a surface, but the author has not had the opportunity to explore them. The method described above, using the weighted mixture of biquadratic surfaces, has proved to be quite satisfactory and reasonably efficient.

Having established the DTM, it is necessary to determine the location and orientation of the camera station from which the photograph was taken. This involves the estimation of six parameters, three for the co-ordinates of the camera in the ground-based system, and three for the photogrammetric angles, using control information in a similar manner to that described in section 13.2. Given the map co-ordinates (X, Y) of any control point, its height Z may be estimated from the DTM. The corresponding co-ordinates in the photograph are:

$$x = -\frac{M_{11}(X - X_c) + M_{12}(Y - Y_c) + M_{13}(Z - Z_c)}{M_{31}(X - X_c) + M_{32}(Y - Y_c) + M_{33}(Z - Z_c)} f,$$

$$y = -\frac{M_{21}(X - X_c) + M_{22}(Y - Y_c) + M_{23}(Z - Z_c)}{M_{31}(X - X_c) + M_{32}(Y - Y_c) + M_{33}(Z - Z_c)} f.$$

where f is focal length of the camera, (X_c, Y_c, Z_c) are the map co-ordinates of the camera, and M is the rotation matrix given in Scollar 1975. The parameters are adjusted to minimise the sum of squares of the differences between the calculated photograph co-ordinates and those observed. Since the relations above are non-linear in the parameters, there is no simple method of minimisation. Instead, it is necessary to resort to a quasi-linearisation technique, whereby the above relations are approximated by ones linear in the parameters. From the initial starting values, a best fit is achieved from the linear approximation; the fitted values are then used as the starting values for a new linear approximation and so on, iteratively, until no further improvement can be achieved.

Relations which are non-linear in the parameters do not in general provide a unique minimum for the best fit. In fact, the relations quoted above have many separate minima, and it is very difficult to ensure that the search will finish at the minimum giving the best fit. As with the projective model, the situation can be improved by cross-multiplying by the denominators on the right-hand side. The relations are still non-linear, since the elements of the rotation matrix M contain trigonometric functions of the photogrammetric angles, but there are now far fewer spurious minima. The cross-multiplication is carried out in Scollar 1975, although no specific reference is made to the fact. Scollar refers to one spurious minimum, when the photograph is apparently taken from a considerable distance underground below the site. A careful examination of the geometry shows that such a minimum is bound to occur whenever the control points are coplanar, or almost coplanar. Not only is the camera underground, but the photograph is apparently taken through the back of the camera. Both anomalies are mathematical possibilities, but hardly physical ones.

Scollar suggests that the spurious minimum can be avoided by restricting the amount by which the parameters can change at each step of the iterative procedure. Although this may avoid the difficulty on some occasions, it is hard to see how it can be entirely reliable; if the search is initiated in the attractor of the spurious minimum, then it will ultimately achieve that minimum, no matter what adjustments are made to the step length. The present author finds that it is better to allow the search to find the spurious minimum, and then to reflect the camera position in the plane of the control points; a new search initiated from the reflected camera position should then find the correct minimum.

A problem with many archaeological photographs is that the exact focal length f is not known. Even when the focal length of the camera is known, the photograph is often in the form of a print, whose enlargement factor has not been recorded. Under these circumstances the search procedure must determine the effective focal length f, as well as the six external parameters of the camera station. The author has found that an unconstrained search on all seven parameters is liable to yield a spurious minimum in which the focal length is reduced almost to zero. Experience has shown that this problem can be avoided by making an initial estimate of the focal length based on the dimensions of the photograph, carrying out a preliminary search with this as a fixed focal length (i.e. on only six parameters), and using the new values as the starting point for a final search on all seven parameters.

With these two modifications, the reflection in the control plane to avoid the spurious minimum and the double search on seven parameters, it has been found that the quasi-linearisation technique provides a reliable means of searching from a standard starting set to the correct camera parameters. There is no need for the user to understand anything of the technicalities of the process, but merely to ensure that the computer is given the correct control information. The technique's reliability has been proved both by the experience of users and in extensive laboratory tests, where a video camera interfaced to a computer framestore was suspended above a test rig. In many trials, the search routine failed in only one laboratory test, when it got into a loop, apparently through repeatedly returning to the spurious underground minimum. It should be noted that the author's search routine differs in several details from that set out in Scollar 1975.

When the search is completed, the user is given the opportunity to check the results, by comparing the stated and calculated locations of the control points on the ground. The DTM is then projected from the ground through the calculated position of the camera lens on to the plane of the photograph; this is necessary in order to associate a height with each point digitised from the photograph. Once the projected DTM has been calculated, the user can digitise the outlines of features in exactly the same manner as for the projective transformation. One extra piece of information appears on screen, namely, the ground height at the last point digitised; this provides a check that the DTM is correctly set up.

An example of output from the AERIAL program, using a DTM and the collinearity equations, is shown in Fig. 13.2. This site is on the edge of the Lincolnshire Wolds (Jones 1988); there is a steep valley to the west and north of the cropmarks, and there are appreciable undulations in the southern field. The plan is constructed from the evidence of five photographs, one taken in a different season from the others, but it is impossible to differentiate between them in a black-and- white diagram. The technique has been entirely successful in pulling together the different pieces of evidence, except for some divergence in the north-east corner, where there is some suspicion that the map contours may not portray an entirely accurate impression of the ground surface.

13.4 Timings and portability

The current version of the AERIAL program was written for a PC/AT compatible computer with an 80386 20MHz processor, an 80387 mathematics coprocessor, and a

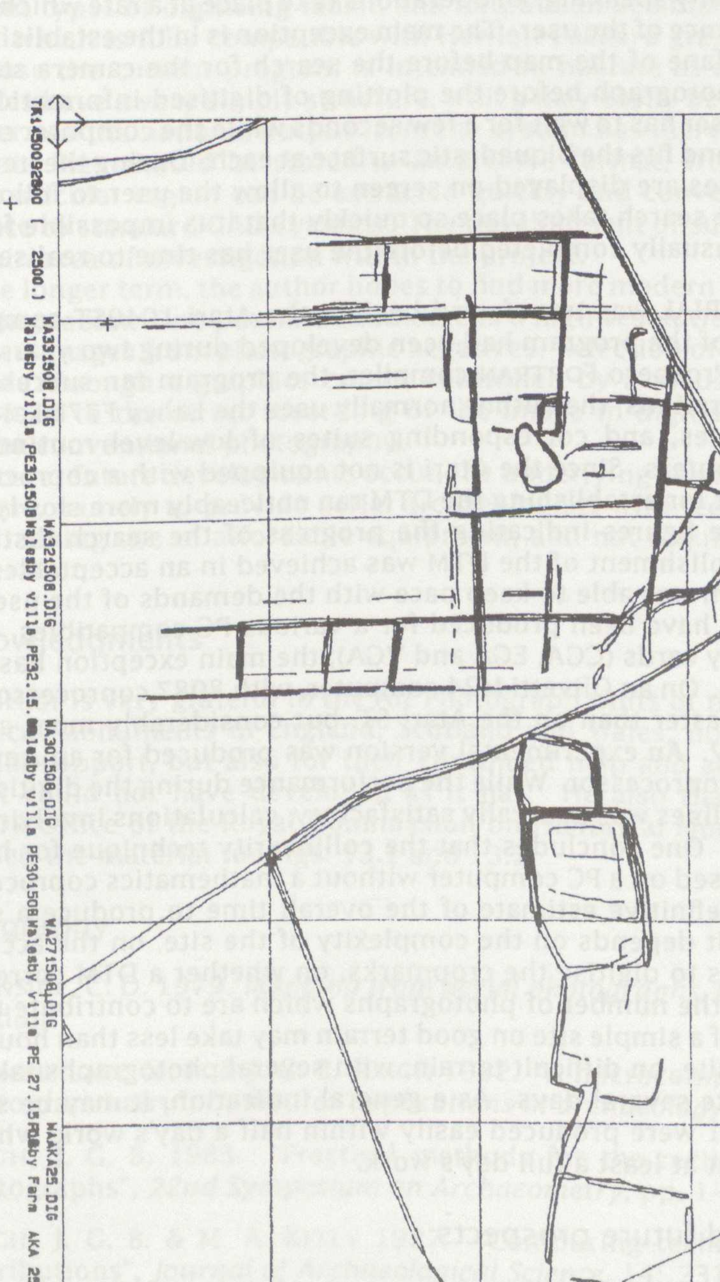

Figure 13.2: A plan of a group of crop-marks on a Romano-British site, digitised from five photographs taken in two seasons. As the site is on hilly ground, the rectification was performed by the collinearity equations with the aid of a digital terrain model

VGA colour display. Generally speaking, all operations take place at a rate which more than matches the performance of the user. The main exception is in the establishment of the DTM, both in the plane of the map before the search for the camera station, and in the plane of the photograph before the plotting of digitised information; on both these occasions the user has to wait for a few seconds while the computer counts through the model points and fits the biquadratic surface at each. During the iterative search routine, some figures are displayed on screen to allow the user to follow the progress of the search; the search takes place so quickly that it is impossible for the figures to be read, and is usually completed before the user has time to realise what is happening.

The latest version of AERIAL was transferred back to the Atari 1040ST computer, on which earlier versions of the program had been developed during two years from 1986 to 1988. Using the Prospero FORTRAN compiler, the program ran successfully after only a few minor alterations; the author normally uses the Lahey F77L FORTRAN compiler on PC compatibles, and corresponding suites of low-level routines are available for the two computers. Since the Atari is not equipped with a coprocessor, the search routine and that for establishing the DTM ran noticeably more slowly than on the PC/AT, to make the figures indicating the progress of the search distinctly visible. However, the establishment of the DTM was achieved in an acceptable time, and the rest of the program was able to keep pace with the demands of the user.

Versions of the program have been produced for a various PC compatibles, using a range of different display cards (CGA, EGA and VGA); the main exception has been the Hercules graphics card. On an Olivetti M24 computer, with 8087 coprocessor, the program runs somewhat faster than on the Atari ST, but considerably more slowly than on the the 80386/387. An experimental version was produced for an Amstrad 1512 computer without a coprocessor. While the performance during the digitisation and plotting of feature outlines was generally satisfactory, calculations involving the DTM were extremely slow. One concludes that the collinearity technique for height correction should not be used on a PC computer without a mathematics coprocessor.

It is difficult to give a definitive estimate of the overall time to produce a set of results from the system. It depends on the complexity of the site, on the accuracy with which the user wishes to digitise the cropmarks, on whether a DTM correction is to be employed, and on the number of photographs which are to contribute to the results. The rectification of a simple site on good terrain may take less than hour; the preparation of a complex site, on difficult terrain, with several photographs taken in different seasons, may take several days. As a general indication, it may be stated that the results of Fig. 13.1 were produced easily within half a day's work, whereas those of Fig. 13.2 represent at least a full day's work.

13.5 Current state and future prospects

The AERIAL program offers a choice of techniques for the rectification of aerial photographs, using either a projective transformation or the collinearity equations of photogrammetry. Although both methods are available to the user in a manner which makes minimal technical demands, the photogrammetric technique does require some special care in the setting up of the DTM and in the establishment of control.

The program has run successfully on a variety of PC compatible computers, as well as on Atari ST machines. To obtain the best results from the screen graphics, it is recommended that a PC computer should have a high-resolution colour display (EGA or VGA). Users who wish to employ DTMs to correct for height variation will find that the heavy requirement for floating-point arithmetic makes a mathematics coprocessor almost essential on PC compatibles. Input has been taken from several different types of digitising tablet, and output can be obtained from almost any pen plotter which is fully compatible with Hewlett-Packard graphics language.

Because the current program is intended to operate as an interactive system, its data files have a very simple structure, which may easily be corrected or amended by the user. As the national output from the system has increased, it has become clear that data files should be stored in an archive format, from which all information for a particular region can be extracted quickly and conveniently, possibly with an interface to standard CAD systems. The development of such an archive is currently the main area of investigation within the project.

In the longer term, the author hopes to find more modern technology to replace the digitising tablet. One possible candidate is a high-resolution video camera to obtain digitised images from photographic negatives. Selection of features of interest could be made through a graphics cursor controlled by a mouse. Further development might lead to the direct recording of the image in digitised form, eliminating the need for conventional photography.

Whatever future developments occur, the underlying aims of the project will remain the same, namely to provide aerial archaeologists with technically reliable methodology, making use of affordable equipment, and not requiring special expertise for its operation.

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