THE APPLICATION OF CURVE FITTING TECHNIQUES TO THE STUDY OF MEGALITHIC STONE RINGS

by

IAN O. ANGELL

Department of Statistics and Computer Science,
Royal Holloway College, Egham, Surrey,

and

JOHN S. BARBER

Department of Archaeology, University College,
Cork.

In this talk we report on the two techniques we have developed to study the geometry of Stone Rings. Those of you who are familiar with the work of Professor Alexander Thom will be acquainted with his classification of Megalithic Sites into circles, ellipses, flattened circles, egg-shaped rings and a small number which fit into none of the previous classes. The first method we discuss is used to look at the circular and elliptical sites; the second is an alternative geometry for his flattened circles and egg-shaped rings.

To our knowledge the only method used at present to calculate 'best' circles (or ellipses) is the rather subjective one of taking an initial guess at the circle (or ellipse) and then making subtle adjustments depending on the deviation of the stone co-ordinates from the proposed shapes (i.e. successive approximation). The method we use is more objective in the sense that no initial assumptions are made, the algorithm is presented with the co-ordinate point data from the site and it furnishes us with a circle or ellipse (whichever is required).

STONEHENGE: AUBREY HOLES

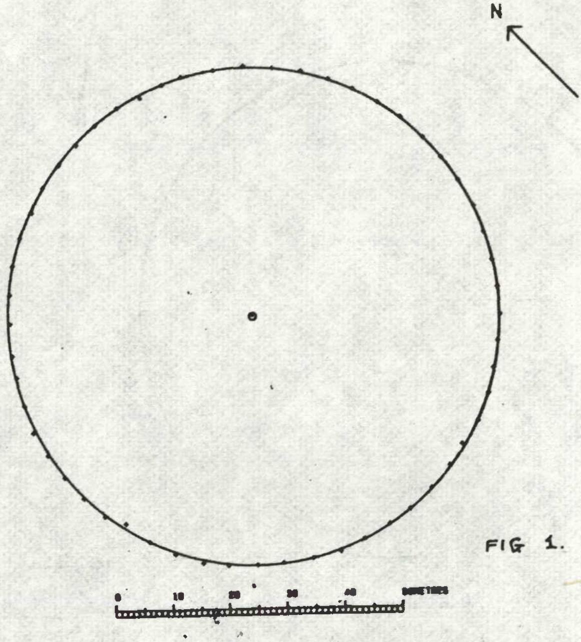

FIG 1.

BERRYBRAE

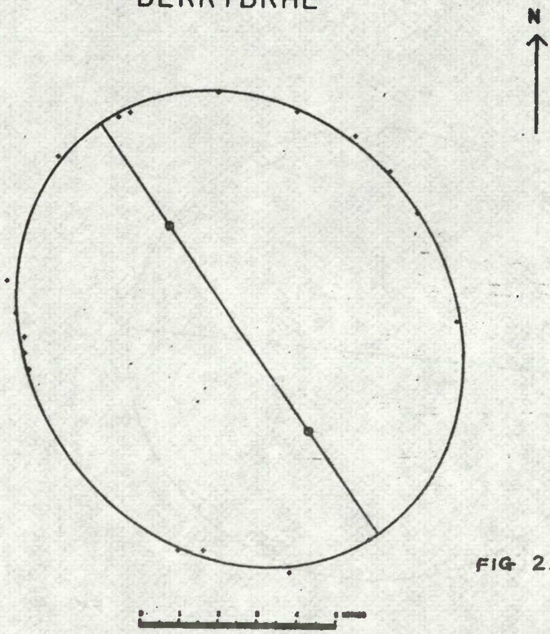

FIG 2.

KENMARE K6

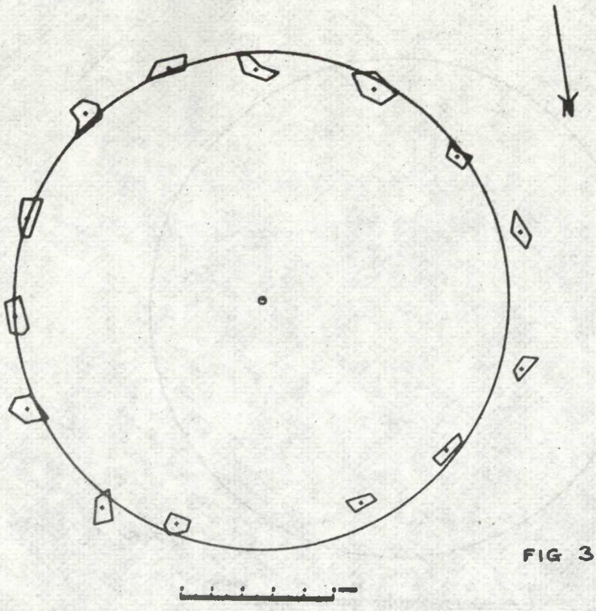

FIG 3.

KENMARE K6

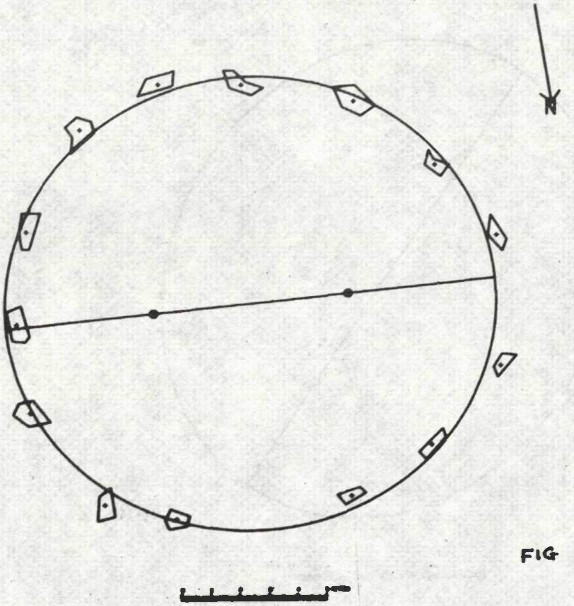

FIG 4.

We define M, a measure of goodness of fit of a function $F(x,y)$ to a set of n points $\{(x_i,y_i)|i=1,..,n\}$ as follows:-

$$M = \sum_{i=1}^{n} F(x_i,y_i)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} F(x_i,y_i)\right)^2$$

This measure was chosen for a number of reasons.

(i) M is always non-negative (the Cauchy inequality);

(ii) if the pairs (x_i,y_i) are all on the curve $F(x,y) = 0$ (a perfect fit) then $M = 0$;

(iii) but most important, it furnishes an elementary minimization technique - simply differentiate M with respect to the various coefficients of $F(x,y)$ and equate each to zero.

We consider the two special cases of circles and ellipses

$F(x,y) = x^2+y^2+2fx+2gy+c$ ... (A)

$F(x,y) = x^2+2hxy+by^2+2fx+2gy+c$ ... (B).

Differentiating M with respect to each coefficient and setting to zero case A yields two equations in two unknowns f and g and case B yields four equations in four unknowns b,f,g and h. In each case c is calculated by making $\sum_{i=1}^{n} F(x_i,y_i) = 0$. As examples of this technique we look at the Aubrey Holes (circle) and Berrybrae (ellipse).

Fig. 1 shows a circle of radius 43.2 metres; the root mean square (R.M.S.) of the errors is $\simeq 0.2$ metres.

Fig. 2 shows an ellipse with major axis 6.4 metres and eccentricity 0.5006 ($\simeq \frac{1}{2}$); R.M.S. of the errors is $\simeq 0.12$ metres.

As was mentioned earlier, Thom has demonstrated that there are many non-elliptical sites; Kenmare is a good example. The unsatisfactory circle and ellipse fits for this site show this quite clearly (Figures 3 and 4).

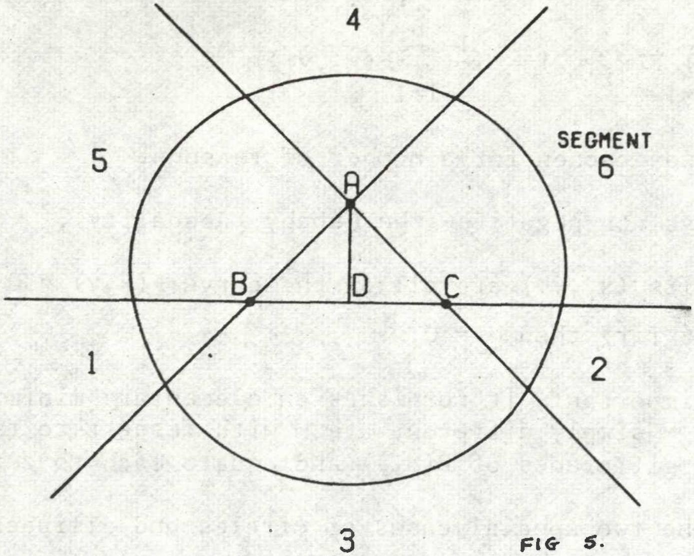

FIG 5.

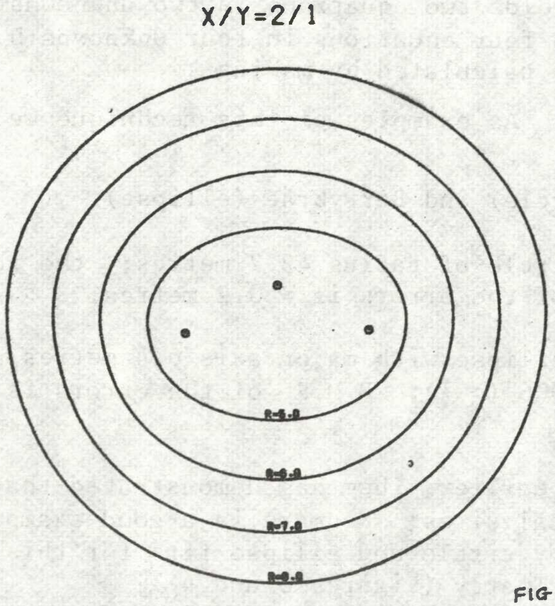

FIG 6.

X/Y=1/1

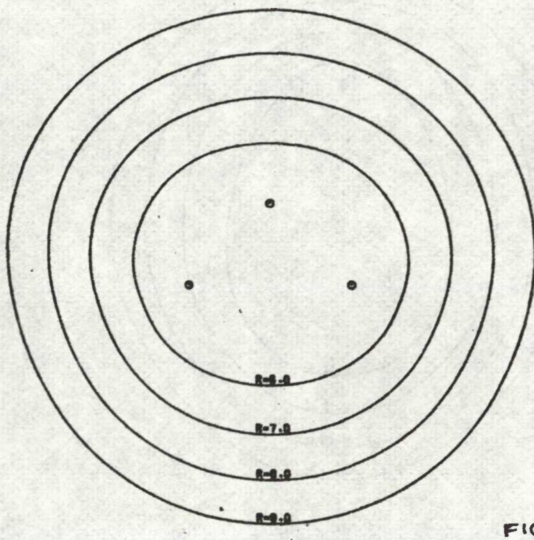

FIG 7.

X/Y=1/2

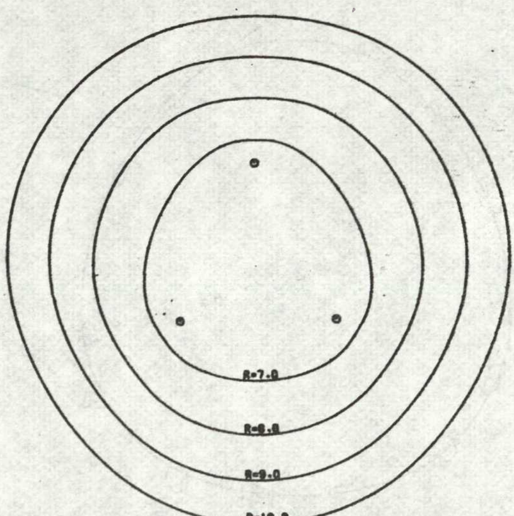

FIG 8.

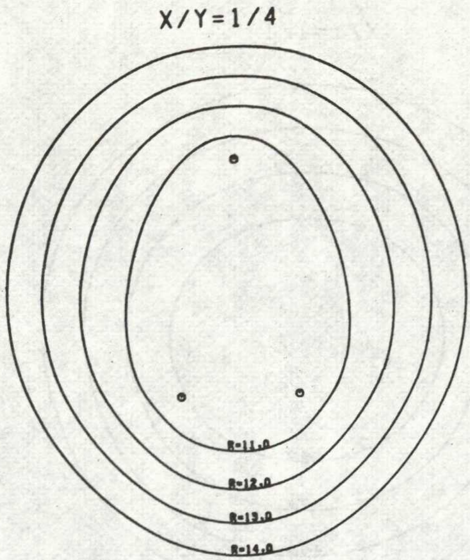

FIG 9.

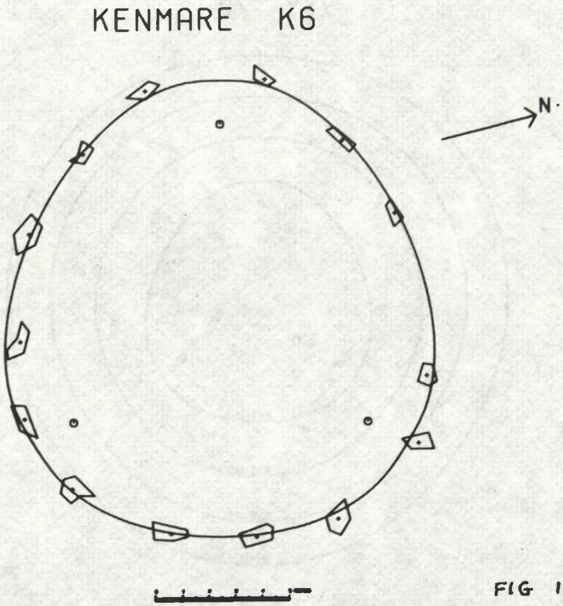

FIG 10.

The second technique we consider is for fitting a special set of closed convex curves to the co-ordinate data of such sites. Before describing the fitting procedure we look at the set of curves. This set is produced by a practical 'rope and pegs' method, and which is proposed as an alternative to the more complex designs of Professor Thom. Pegs are placed in the ground (Figure 5) at vertices A,B,C of an isosceles triangle; D is the mid-point of BC (these points may be produced by pacing). A loop of rope, whose length is a multiple R of BD, is thrown over the triangle, and holding it taut complete a circuit of the site. This is the natural evolution of circle and ellipse construction - a circle uses one central peg, an ellipse two pegs. By changing the ratio of BD (X paces?) to AD (Y paces?) and also the value of R, we get a wide range of shapes (some examples are shown in Figures 6,7,8 and 9). A catalogue of these shapes was produced together with a vital statistic in each case, viz. the ratio of the vertical diameter (along the line of symmetry) to the maximum horizontal diameter. For any given megalithic site the ratio of the apparent diameters may be calculated, and with reference to the catalogue it is possible to draw up a 'short list' of suitable outlines. The technique for fitting each of these outlines to the co-ordinate data from the site uses an IMLAC interactive graphics terminal. The screen contains two superimposed frames, the first showing the co-ordinate data (a cross marks the approximate centre of a stone) and the second holds the chosen outline. The relative origins, scale and angular difference may be altered. As well as a visual impression of the fit, the machine also returns a numerical value, the R.M.S. of the distances of the co-ordinate points to the curve.

Using this feedback information we may iteratively approach a solution. Some examples of this technique are Kenmare (Fig. 10) and Cappanaboul (Fig. 11).

Kenmare:- $x/y = \frac{1}{2}$ $R = 7$ R.M.S. value $\simeq$ 0.2 metres.

Cappanaboul:- $x/y = \frac{1}{2}$ $R = 7$ R.M.S. value $\simeq$ 0.1 metres.

This technique is capable of producing all the shapes of Thom's more elaborate geometry. Both methods give excellent fits for the non-elliptical sites, yet they are mutually incompatible. So which do we accept?

CAPPANABOUL C4

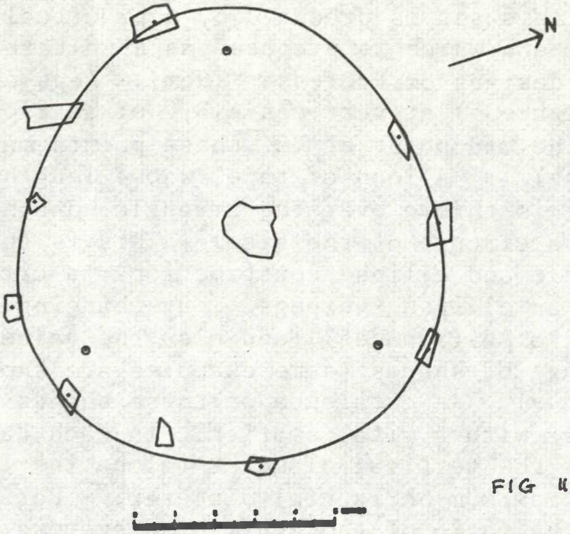

FIG 11.

DEREENATAGART C1

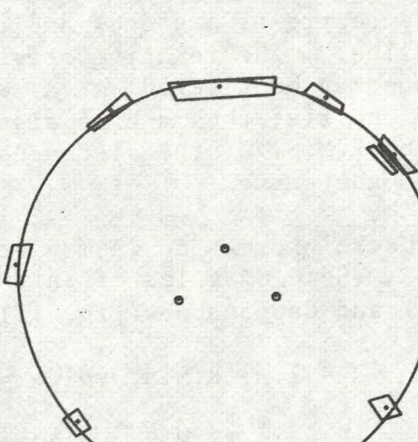

FIG 12.

Of course they may both be wrong. It is always dangerous to accept diagrams on face value alone - 'seeing is believing' is a fallacious dictum. The human eye is a biased instrument which may be easily misled - a fit may appear good but if there is no comparison with other fits we will not know how good! Similarly numbers (such as measures of goodness of fit) should not on their own be a justification of validity. Alternative interpretations and also the physical features of a site must be taken into account.

For example we may have the problem of two fits - one sophisticated with a good measure of fit, and a second less sophisticated still with a reasonable fit - which do we accept? This is especially true of near circular and near elliptical sites; consider Dereenatagart. Figure 12 shows a 'triangle fit' with x/y = 1, R = 11 and the R.M.S. of the errors is 0.08 metres. Note that the largest stone, which is recumbent, is on an axis of symmetry. Next, Figure 13, we have an ellipse fit eccentricity = 0.21 and R.M.S. of errors = 0.12 metres. Not quite as good as the triangle method, but since an ellipse is a simpler construction we must prefer it. Or do we? Note the recumbent stone is not on an axis of symmetry, and we have an irrational eccentricity value. Of course we have not yet tried to fit a circle (Figure 14). This is a reasonable fit (Radius = 4.2 metres, R.M.S. of errors = 0.13 metres) and realistically we must accept this interpretation in place of the previous two. Can we now use the calculated diameter to search statistically for a megalithic yard? This is highly questionable since once we accept the existence of non-elliptical sites, we cannot totally dismiss this possibility from near-circular sites. We must be far more objective about circle diameter values before subjecting them to statistical treatment.

We deliberately leave you in this confused state as a warning to all archaeologists - do not believe your eyes or your local mathematician without first questioning the basic assumptions and alternative interpretations. Statistics is not all black and white - it is full of grey areas - as many statisticians, who have put numbers before commonsense, have found to their cost.

DEREENTAGART C1

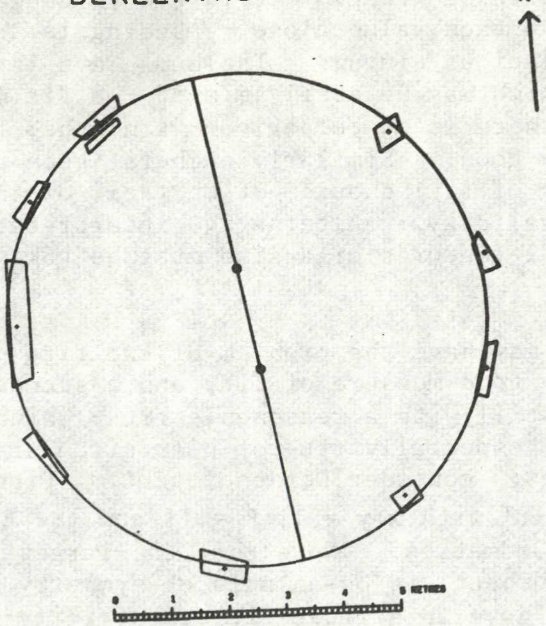

FIG 13

DEREENATAGART C1

FIG 14.