A. Eder-Hinterleitner
W. Neubauer
P. Melichar

Reconstruction of archaeological structures using magnetic prospection[1]

1 Introduction

Archaeological structures in the ground cause small anomalies in the earth's magnetic field due to different magnetic susceptibilities compared to the surrounding ground. These anomalies are measured by high precision magnetometers (Neubauer 1990, 1991). The measured data are preprocessed and displayed as images and manually interpreted by experts (Scollar 1990). The archaeological interpretation of magnetic anomalies is very difficult for several reasons:

1. it is a 2-dimensional projection of a 3-dimensional world;
2. the anomalies of nearby structures may be superimposed;
3. there is always a large amount of noise in the measurement caused by the susceptibility variance of the top soil, by geological structures and by other sources.

Although an expert can estimate whether there is an anomaly of an archaeological structure or not and the probable kind of structure, he or she can estimate only rough dimensions (depth, size, ...) of these structures.

We introduce a method to estimate the position, shape and size of buried archaeological structures by reconstructing a 3-dimensional magnetic model of the subsurface. Our method inverts the idea of simulating magnetic anomalies of archaeological structures of arbitrary shape by dipole sources. A magnetic model of the subsurface is built with homogeneous dipole sources of equal size in a regular grid with different magnetic susceptibilities for different materials (soil, stones, bricks, etc.). The distribution of the dipole sources is automatically arranged so that the differences between the magnetic anomalies of the model and the measured data are minimized.

While the computational costs for the calculation of the anomalies of a subsurface-model are negligible for today's computers, the inverse problem, the determination of the parameters of the subsurface-model is, also with known magnetic properties, a non-deterministic problem with great computational costs. We use the forward modelling method for calculating the anomalies of the modelled archaeological structure and determine the parameters of the model according to an optimization criterion. A special optimization algorithm which is fast enough to find good solutions with the computational power of conventional workstations within a few hours is used.

The reconstruction of filled ditches of the neolithic ring ditch system Puch 1 in Lower Austria is used to demonstrate this method (Trnka 1991). The preprocessed magnetic anomalies of Puch 1 are shown in figure 1. The differences between the total intensities of the earth's magnetic field in 0.5 m and 2.0 m are measured by a cesiumgradiometer in a 0.5 m regular grid. The measured area is 120 m × 120 m, the image therefore has 241 × 241 measuring values. This measurement was carried out by ARCHEO PROSPECTIONS® (Melichar/Neubauer 1993).

2 Method

Figure 2 gives a general view of our method and the data flow through it. After collecting the data in the field they are preprocessed to remove errors.

The reconstruction starts with a classification of the preprocessed data. The classification computes the probability for each data value that does not originate from the expected archaeological structure.

Then, by using the data and the classification the expected archaeological structures are reconstructed. No assumptions about the position and shape of the expected archaeological structure are made, except that the result has to be *smooth*. Therefore this first reconstruction is called *free*.

The free reconstruction is used to determine the nearly exact horizontal positions and a rough estimation of the depth of the expected structures. The detected structures and a modelling of the shape of the expected structures are used to reconstruct the exact position, depth and shape of the expected structures. As the shape of the expected structures and the positions are restricted, the second reconstruction is called *constrained*.

Both reconstruction steps use the same optimization algorithm but the optimization criteria are different. The constrained reconstruction uses a finer spatial resolution.

3 Subsurface model

The subsurface is magnetically modelled by homogeneous dipol sources of equal size in a 3-dimensional regular grid.

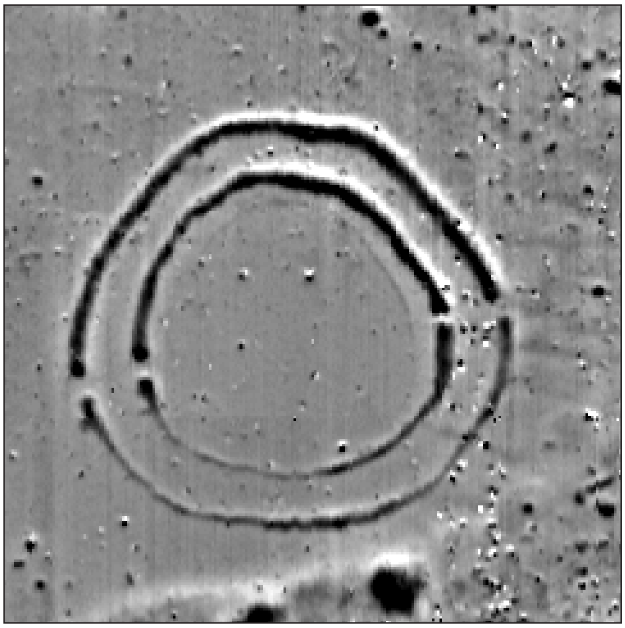

Figure 1. Preprocessed magnetic anomalies of Puch 1. [-4,8]nT → [white, black].

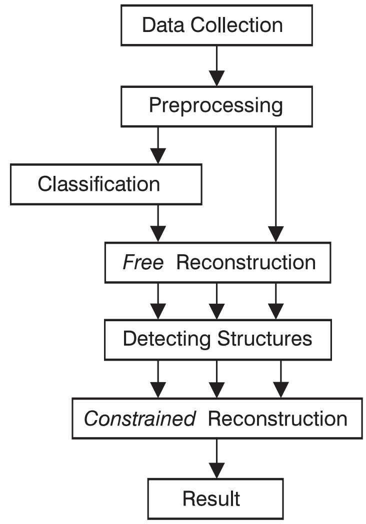

Figure 2. Method of reconstruction.

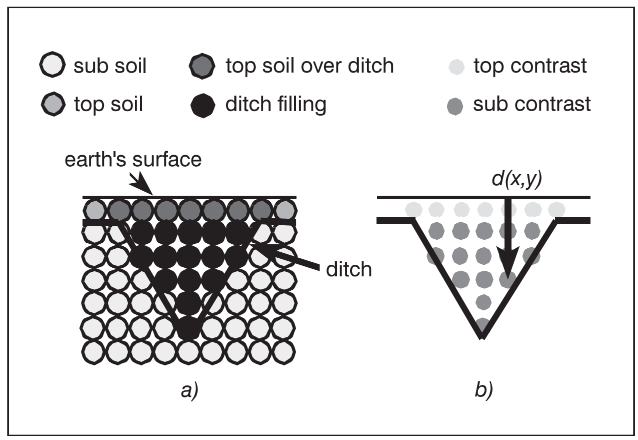

Figure 3. a. Profile of a ditch subsurface model with dipole sources; b. Susceptibility-contrast model of a ditch.

This method was proposed by I. Scollar to simulate anomalies of archaeological origin (Scollar 1969). The advantage of this method is that it is easy to calculate the anomaly of structures of any shape and any susceptibility distribution with any accuracy. The disadvantage, the computational costs for very accurate simulations, becomes less important due to the rapid progress of the power of computers.

Figure 3a, as an example, shows the profile of the modelling of a filled ditch. Each dipole source represents a cube whose sides are 0.5 m long according to the measuring grid of the prospection which was also 0.5 m. Susceptibility measurements of neolithic ditches in Austria lead to a model with four different layers and four different susceptibilities k:

1) top soil (k_t),
2) top soil above and near the ditch (k_d),
3) sub soil (k_s),
4) filling of the ditch (k_f).

The model can be simplified by subtracting horizontal layers which produce a constant magnetic anomaly. Therefore the top soil and the sub soil are removed. The result is a model of the filled ditch with the susceptibility-contrasts (top-contrast k_{tc}, sub-contrast k_{sc}) in an non-magnetic surrounding (fig. 3b).

$$k_{tc} = k_d - k_t \qquad k_{sc} = k_f - k_s$$

This simplification speeds up the computation of the anomalies because only the parts of the subsurface with a ditch are modelled.

Although there is remnant magnetization in the soil, only induced magnetism is considered for the model. It is assumed that the field vector of the ditch anomaly has the same direction as the field vector of the earth magnetic field (Oehler 1987). The remnant magnetization of the ditch is modelled by a higher susceptibility for the induced magnetization.

The magnetic anomaly (A_M) of a ditch is calculated by

$$A_M(x_s,y_s) = \sum_{x_d} \sum_{x_d} \sum_{z_d=0}^{d(x_s,y_s)} Fk(x_d,y_d,z_d)\ VM(x_s,y_s,x_d,y_d,z_d)$$

The subscript s stands for the positions of the sensor(s) and the subscript d for the positions of dipole sources. F is the total intensity of the earth's magnetic field. V is the volume and k the susceptibility-contrast of the dipole source. d is the depth of the ditch at the place (x_s, y_s). The influence of each dipole source on the measuring device is described by M.

For a gradient measuring device with one sensor in 0.5 m and one in 2.0 m M is calculated by (Linnington 1972):

$$M(x_s,y_s,x_d,y_d,z_d) = D(x_s-x_d,y_s-y_d,0.5-z_d) - D(x_s-x_d,y_s-y_d,2.0-z_d)$$

$$D(x,y,z) = \frac{x^2(3\cos^2 I-1) + z^2(3\sin^2 I-1) - y^2 - 6xz\sin I\cos I}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

D is the anomaly produced by a single dipole source and I is the inclination of the earth's magnetic field. The declination of the earth's magnetic field is neglected.

This model is used for the *free* reconstruction where a first rough estimation of the ditches is calculated by a 0.5 m resolution in the depth. For the *constrained* reconstruction the dipole sources are divided into 5 slices to enhance the resolution to 0.1 m.

4 Reconstruction problem

The reconstruction problem is to find the distribution of the dipole sources of the subsurface model to minimize the difference between the model-anomalies and the measured data. All other parameters, the susceptibilies of the dipole sources, the inclination and the total intensity of the earth's magnetic field, are assumed to be known and constant.

To reconstruct ditches according to our susceptibility-contrast model, the depth d of the filling of the ditch at each measuring point (x_s, y_s) determines the position and shape of the ditch (fig. 3b). It is thus possible to estimate the shape of the ditch by estimating the depth-points d.

Our reconstruction problem is to estimate $d(x_s, y_s)$ for all measuring values by minimizing the square of the difference (E_D) between the model-anomalies (A_M) and the measuring data (A_D):

$$E_D = \sum_x \sum_y (A_D(x,y) - A_C - A_M(x,y))^2$$

A_C is the constant anomaly of the measuring device produced by the removed horizontal layers and all other influences on the sondes. A_C is equal to the mean value of all measuring values.

Two problems appear when using this minimization criterion:

1. The least-square-criterion is not a robust criterion. Big anomalies not caused by a ditch or noise lead to unrealistically deep ditches.
2. The intensity of the anomaly of a dipole source decreases with the third power of the distance of the dipole sources to the measuring device. Thus, deep structures like deep parts of a ditch have very little influence.

Two extensions of the minimization term E_D to solve these two problems are described in the following.

4.1 Robustness

A weighting of the least-squares term is used to make the criterion robust. The weights $w(x,y)$ are a preclassification of the anomalies and represent the correctness of each data value. The weights have values between 1 and 0. 1 stands for a correct and 0 for an incorrect data value. By multiplying the data fitting (E_D) by these weights, anomalies which definitely do not originate from the expected source are neglected. E_D is extended to

$$E_D = \sum_x \sum_y w(x,y)\,(A_D(x,y) - A_C - A_M(x,y))^2$$

For anomalies of ditches, the possible maximum and minimum value (A_{min}, A_{max}) of an anomaly caused by a ditch and the difference between each data value and its four neighbours b are considered. The limits A_{min}, A_{max}, b_{min}, b_{max} are determined interactively for each prospected site.

$$b(x,y) = \log(\mathrm{abs}(4A_D(x,y) - A_D(x-1,y) - A_D(x+1,y) - A_D(x,y-1) - A_D(x,y+1)))$$

$$w(x,y) = \begin{cases} 0 & \textit{if } b(x,y) > b_{\max} \vee A_D(x,y) < A_{\min} \vee A_D(x,y) > A_{\max} \\ 1 & \textit{if } b(x,y) < b_{\min} \wedge A_D(x,y) > A_{\min} \wedge A_D(x,y) < A_{\max} \\ (b(x,y) - b_{\min})/(b_{\max} - b_{min}) & \textit{otherwise} \end{cases}$$

Figure 4 shows the weights used to reconstruct the ditches of Puch. Black areas prevent a fitting of the data.

4.2 Regularization

To get plausible results the *smoothest* result is selected by regularizing the parameters which are optimized. A regularization term E_R is defined describing the relation of each parameter to its neighbours. E_R is multiplied by α to regulate the influence of the regularization. The new minimizing term E_G is calculated by:

$$E_G = E_D + \alpha E_R$$

The depth d of ditches cover a surface representing the border between the ditch filling and the sub soil. Due to the decreasing influence of a dipole source with the third power of the distance between the dipole source and the measuring sensor(s), ditches with too deep positions near too flat ones may occur. To avoid such unplausible ditches the depth d is regularized by smoothing the free reconstruction.

$$E_R = \sum_x \sum_y (2d(x,y) - d(x-1,y) - d(x+1,y))^2 + (2d(x,y) - d(x,y-1) - d(x,y+1))^2$$

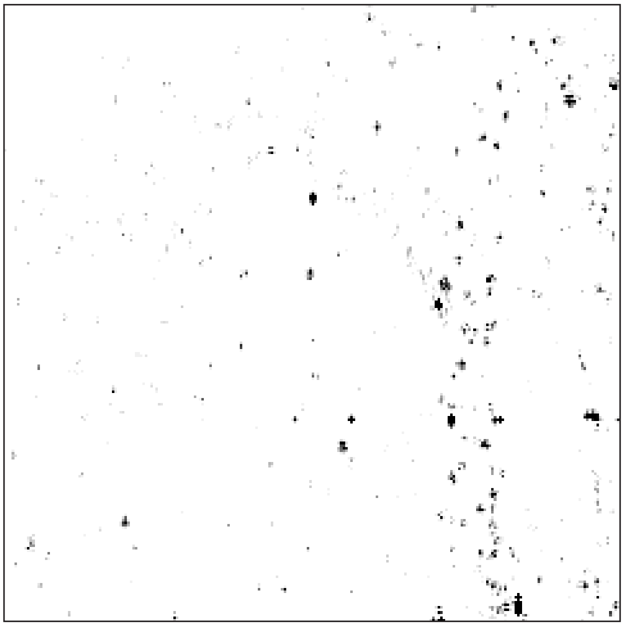

Figure 4. Classification w of Puch 1; [0, 1] → [black, white].

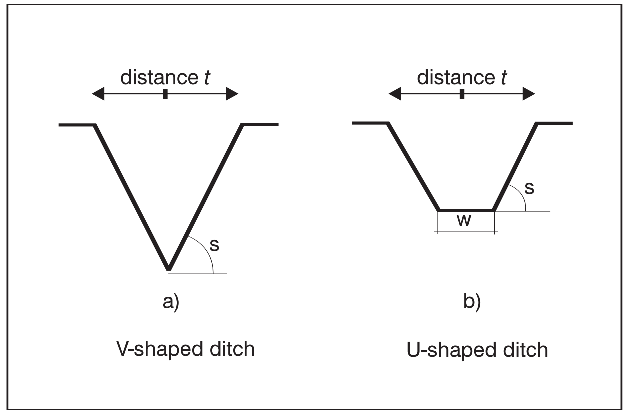

Figure 5. Ditch profile model.

4.3 Modelling

For the constrained reconstruction the expected structure is modelled. A new regularization term E_R describes how close the reconstructed and the modelled structures are. A rough estimation of the position and size of the expected structures is necessary to have good starting solutions for the annealing process.

The ditch profile model assumes that the direction and the middle of the ditch are known and that the ditch is symmetric. The normal distance t of each position (x,y) to the middle of the ditch is computed. By using t, the relative difference between each depth d and its four neighbours can be calculated locally. This local information is necessary for optimizing in subimages (see below).

For a V-shaped ditch (fig. 5a) only the slope s, for a U-shaped ditch (fig. 5b) also the width w of the bottom of the ditch has to be defined. No assumptions about the true depth d are made. The ditches are modelled from the bottom to the top. This model takes into account that filled ditches are eroded from the top to the bottom. The new regularization term E_R for a V-shaped ditch (for areas above a ditch) is:

$$E_R = \sum_x \sum_y \sum_{i=1}^{4} diff_i^2(x,y)$$

$$diff_1(x,y) = d(x-1,y) - s(t(x,y) - t(x-1,y)) - d(x,y)$$
$$diff_2(x,y) = d(x+1,y) - s(t(x,y) - t(x+1,y)) - d(x,y)$$
$$diff_3(x,y) = d(x,y-1) - s(t(x,y) - t(x,y\text{-}1)) - d(x,y)$$
$$diff_4(x,y) = d(x,y+1) - s(t(x,y) - t(x,y+1)) - d(x,y)$$

The middle of the profile and the direction of the ditch are calculated in the detecting structures step (fig. 1). The detection of ditches is described below.

5 Optimization algorithm

For reconstructing ditches by using the susceptibility-contrast model, the depths d have only discrete values. Therefore the minimization problem is a combinatorial optimization problem. But this optimization problem has some further special conditions:

1. Many parameters have to be determined. The number of parameters is equal to the number of measuring values ($p = n \times m$).
2. The parameters have only a few discrete values. The number of different values (v) is ~10 for the free reconstruction and ~50 for the constraint reconstruction.
3. There are v^p different solutions. For the site Puch 1 with an area of 14,400 m^2 there are $10^{58,081}$ different solutions for the free reconstruction. (It is not possible to evaluate all of them!)
4. The parameters have a limited spatial relation due to the decreases of the magnitude of the magnetic field of a dipole source with the third power of the distance.

A partially iterative random search algorithm called *leaped annealing* is used to find a solution to this optimization problem (Eder-Hinterleitner 1994). Leaped annealing is similar to *simulated annealing* (Kirkpatrick *et al*. 1983; Romeo/Santigiovanni-Vincentelli 1991). Both have a term T, called temperature, which decreases with the progress of the algorithm and which determines the ability to leave a local minimum. The higher the temperature the easier it is to leave a local minimum. Whereas this is done in simulated annealing by accepting worse solutions temporarily, in leaped annealing it is done by changing the

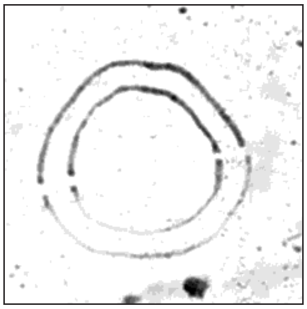

Figure 6. Free reconstruction of Puch 1; *d:* [0, 2] m → [white, black].

Figure 7. Ditches at depth d = -0.5 m.

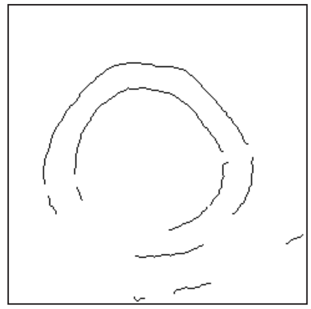

Figure 8. Middle line of the detected ditches.

Figure 9. Distance t to the middle of the ditch; [0, 5] m → [black, white].

possible distance of the new solution to the old one. While the solution has to move up and down along the optimization-function in simulated annealing, it jumps from one random place to another and it never has to accept a worse solution in leaped annealing. At the beginning of the leaped annealing algorithm every possible state in the search space can be reached from every other state in one step.

The annealing process is not applied to the whole image at once but to subimages of 2 by 2 pixels in size due to the limited spatial relations of the dipole sources to each other. These subimages are optimized separately but in parallel to consider the mutual influence. The splitting into subimages reduces the solution space and is necessary to reach every possible state from every other state in one step. The algorithm converges as fast as possible when only about 10 percent of the subimages are changed during each iteration. With leaped annealing only 10^4 of $10^{58,081}$ possible solutions have to be evaluated to get a *good* result.

The algorithm is used for both the free and the constrained reconstruction.

6 Reconstructing ditches

The method is demonstrated by the reconstruction of the neolithic ring ditch system Puch 1. The result of the magnetic prospection survey is visualized in figure 1, the classification in figure 4. The magnetic parameters for the reconstruction are:

$F = 48000$ nT $I = 65°$ $V = 0.125$ m^2
$k_{tc} = 70\ 10^{-5}$ $k_{sc} = 100\ 10^{-5}$

The result of the free reconstruction is visualized in figure 6. It can be clearly seen that the upper half of the ditch is well preserved while the lower half is mostly destroyed. The regularization leads to a smooth ditch, yet, the ditch is too wide at the top and not deep enough in the middle. The varying shape of the ditch is caused by the inhomogeneous susceptibilies of the ditch filling. Although the ditch is too wide, it is well located. Many pits are also reconstructed.

6.1 Detecting ditches

To localize the ditches the result of the free reconstruction is first convolved with a 5 × 5 mean filter for smoothing. Then a threshold (fig. 7) at d = -0.5m is taken. The black areas are an estimation of the shape of the ditch after removing the A-horizon.

The middle of the ditch (fig. 8) is calculated by thinning the threshold image and removing short lines. Figure 9 visualizes the normal distance t of pixels which are above the ditch using the middle line (fig. 8) and the thresholded image (fig. 7). To overcome the disadvantage of the discretization in a 0.5 m grid the normal distances t are calculated with subpixel precision. The normal distances to a regression line calculated by using the next five pixels on the middle line are computed.

6.2 Constrained reconstruction

The constrained reconstruction uses the modelling of the profile with a discretization of the depth d of 0.1 m. A V-shaped ditch with a slope s=45° is modelled. The 3-dimensional visualization (fig. 10) gives a realistic impression of the remains of the ditches. In the best preserved areas the ditches are 4.5 m wide and 2 m deep. The two entrances are between 3 m and 5 m wide. The extensive destruction of both ditches towards the front was caused by soil removal when the site was graded. The soils removed fill the large pits at the very front of the reconstruction.

The many small pits look like flat basins due to the smoothing of the depth d. A modelling of the expected shape of the pits would lead to more realistic results.

The remains of the palisade, which can be seen partly in the anomalies, are not reconstructed due to the large horizontal grid of 0.5 m.

The whole reconstruction procedure, the determination of 58,081 parameters, of Puch 1 requires 2 hours of processing time on a Sun SPARCstation 20.

7 Conclusion

We present a method for the reconstruction of a 3-dimensional magnetic subsurface model with dipole sources. The reconstruction problem is formulated as a minimization problem. The difference between the model anomalies and the measured data as well as a regularization or modelling term are minimized by determining the distribution of the dipole sources using an iterative random search annealing algorithm. Although the optimization problem has a very large solution space, a practicable method by dividing the problem into many small subproblems is achieved. Dividing into subproblems offers the possibility of using massive parallel computers to speed up the annealing process by the number of available processors.

The method has two reconstruction steps to combine the following characteristics:

1. no assumptions about the location of archaeological structures are necessary,
2. pre-information about the expected archaeological structure can be integrated into the reconstruction process.

The first step determines rough positions and depths of the expected structures by using a rough subsurface model. The second one uses a finer resolution and a modelling of the expected structures to estimate the exact positions, depths and shapes of the archaeological structures.

The ring ditch system Puch 1 is modelled, reconstructed and visualized to demonstrate the method.

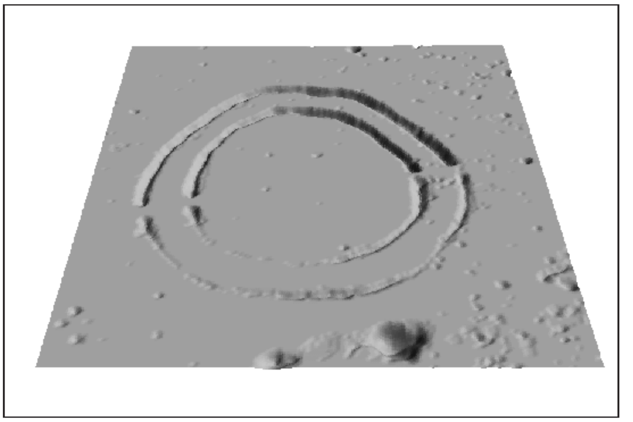

Figure 10. 3-dimensional visualization of reconstructed ditches of Puch 1.

This procedure can be easily applied to other archaeological structures, like pits, walls, etc. New regularization and modelling terms have to be developed, but the modelling with dipole sources and leaped annealing for solving the resulting optimization problem can also be used.

Acknowledgement

The authors would like to thank Amy Krois-Lindner, Axel Pinz and Christian Cenker for reading earlier drafts of this paper and Brigitta Galian for many useful hints.

note

1 This work was supported by the Austrian Science Foundation under grant P9242-HIS.

references

Eder-Hinterleitner, A. 1994 Ein Robustes Rekonstruktionsverfahren zur Bestimmung der Form von Gräben für die archäologische magnetische Prospektion. In: W.G. Kropatsch/H. Bischof (eds), *Tagungsband Mustererkennung 1994*, Informatik Xpress 5, 532-539.

Kirkpatrick, S., C.D. Gelatt, P. Vecchi 1983 Optimization by simulated annealing, *Science* 220 (4598), 671-680.

Linnington, R.E. 1972 A summary of simple theory applicable to magnetic prospection in archaeology, *Prospezioni archeologiche* 7, 9-27.

Melichar, P.
W. Neubauer — 1993 — Magnetische Prospektion von Kreisgrabenanlagen in Niederösterreich, *Mitteilungen der Österreichischen Gesellschaft für Ur- und Frühgeschichte* 43, 61-68.

Neubauer, W. — 1990 — Geophysikalische Prospektion in der Archäologie, *Mitteilungen der Anthropologischen Gesellschaft in Wien* 120, 1-60.

1991 — Magnetische Prospektion von Kreisgrabenanlagen. In: G. Trnka, Studien zu mittelneolithischen Kreisgrabenanlagen, *Verlag der Österreichischen Akademie der Wissenschaften*, 331-338.

Oehler, A. — 1987 — *Zweidimensionale Modellrechnung zu magnetischen Prospektionsmessungen in der Archäologie*. Master thesis, Inst. f. Allgemeine und Angewandte Geophysik, Ludwigs-Maximilians-University, Munich, Germany.

Romeo, F.
A. Santigiovanni-Vincentelli — 1991 — A theoretical framework for simulated annealing, *Algorithmica* 6, 302-345.

Scollar, I. — 1969 — A program for the simulation of magnetic anomalies of archeological origin in a computer, *Prospezioni archeologiche* 4, 59-83.

1990 — *Archeological prospecting and remote sensing*. Cambridge: Cambridge University Press, Topics In Remote Sensing.

Trnka, G. — 1991 — *Studien zu mittelneolithischen Kreisgrabenanlagen.* Verlag der Österreichischen Akademie der Wissenschaften.

A. Eder-Hinterleitner
Dept. f. Pattern Recognition a. Image Processing
Technical University of Vienna
Treitlstr. 3/1832
1040 Vienna
Austria
e-mail: ahi@prip.tuwien.ac.at

W. Neubauer
Institute for Prehistory
University Vienna
F. Kleing. 1,
1190 Vienna
Austria
e-mail: Wolfgang.Neubauer@univie.ac.at

P. Melichar
Central Inst. f. Meteorology and Geodynamics
Hohe Warte 38
1190 Vienna
Austria