

THE ORTHOGRAPHIC APPROXIMATION - A SIMPLE GEOMETRICAL MODEL FOR AVOIDING PERSPECTIVE ERROR IN CONSTRUCTING PHOTOMOSAICS

ABSTRACT

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With appropriate apparatus, ortho-photomosaics created on the desktop computer from near-range vertical photographs can be used to plan excavations more quickly than by conventional drawing techniques. However, a good fit of adjacent photographs is impossible when there is excessive perspective error, and the software "stretches" we might make to correct gross problems of alignment of features will often create error elsewhere in the image. A simple mathematical model, the Orthographic Approximation, provides a simple means of calculating the metric parameters which will give a result which is tolerably close to being orthographic, making the creation of an ortho-photomosaic a simple task.

INTRODUCTION

Since the advent of photography and the ready availability of film cameras, there have been many archaeologists who have sought to speed up the recording process by replacing plan drawing with ortho-photomosaics made from near-range vertical photographs taken from atop tripods or similar apparatus (see Sterud and Pratt 1975, for a useful history of these apparatus and their use, also Poulter and Kerlake 1997, Gruel, Buchsenschutz, Alliot and Murgalé 1993). Yet despite this, the technique remains infrequently used. The author originally presumed that this was due, in part at least, to the time and labour spent in developing and printing the photographs, and piecing together the mosaic, and so set out to test the notion that the process could be made much faster and simpler using today's affordable graphics software and digital cameras. It soon became apparent that there was another important issue.

The primary focus of the fore mentioned papers is usually a detailed description of the construction and use of some apparatus for holding the camera on high, the purpose of which is obviously to ameliorate the problem of perspective error, yet these authors pay little or no attention to ascertaining whether excessive perspective distortion is still present in their photographic results or not. Certainly, it was this author's experience that, despite photographing from elevations of 5m or 7m, considerable perspective error was still present in the photographs, making the task of creating a photomosaic a far from straight-forward task. This turned our attention from camera and software to an investigation of perspective error through the geometry of near-range photography. The results of this investigation provide a model for quantifying, or rather, approximating perspective error. This allows us to then define a region within the photograph where perspective error is acceptably small or, to put it another way, a region which is tolerably close to being orthographic. This model we call the Orthographic Approximation.

DEFINING THE QUESTION

In attempting to construct an ortho-photomosaic from near-range vertical photographs we are attempting to place a series

of perspective views into a formal grid or loose framework to produce a mosaic image which is close to being an orthographic projection. In doing so the problem of perspective distortion within individual photographs must be addressed; firstly, from a purely functional stance, because of the probable difficulty of aligning features which overlap photographs or mosaic pieces, and secondly, because some or more of the remaining information in the photograph or mosaic piece may be similarly, if less conspicuously, affected by perspective distortion. Typically, attempts to minimise perspective distortion entail both photographing from a height and using only the central portion of the resulting photographs.

Thus far we have talked of the "problem", i.e. perspective distortion, and have perhaps implied moving from a bad situation towards something better. Let us turn this around and start from a "point of perfection", that is the only truly orthographic point in a photograph which lies at the point where the photographic axis intersects the terrain. A second point, very close to the first, might not technically be orthographic but it is almost certainly close enough for our requirements. The further we move away from the photographic axis, the more perspective distortion is introduced and the further we can consider to be from an "orthographic situation". The question of where to draw the line between useable and non-useable parts of a photograph now becomes "How far from the photographic axis can we move before the perspective error becomes too large?" To answer this we will seek a means of quantifying, or at least approximating, perspective error and go on to investigate its relationship to topographic relief, the height of the camera and the size of that part of the photographic field which yields a good orthographic approximation.

THE BASIC GEOMETRY

Figure 1 shows the situation of a vertical photograph of a horizontal surface containing two substantial topographical irregularities. Note three values; I which is half the photographic angle, d which is the distance between the camera lens and the centre of the field, and f which is half the photographic field width.

Photogrammetry

We can assign some further values in the regions of the topographical irregularities. We let the vertical heights of the irregularities be h and note that the angle I is formed between the photographic projection and a vertical line through point x_0 . In the resulting photograph, the point x_0 (the corner of the irregularity) will appear to lie at x_1 , introducing an apparent lateral translocation of the true x and y coordinates, an error due to perspective distortion which we label e .

In figure 1 we can identify three similar right-angled triangles, sharing the angle I . From these triangles we can form the equation;

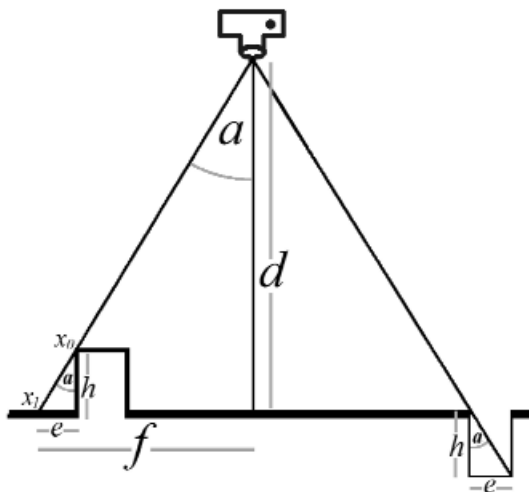


Figure 1 Basic geometry of vertical photography

Equation 1: $f/d = e/h$

This very simple equation relates the vertical topography of the terrain, the photographic geometry and the lateral perspective error, which is very useful for us. For a given (measured) amount of vertical relief in the terrain we want to record, and knowing the maximum height of the photographic apparatus, we can choose a value for e which represents the maximum perspective error that we are prepared to accept and the equation will give us f , which is the radius of a circle centred on the photographic axis, within which perspective error will be less than or equal to our chosen limit.¹

MODIFYING THE EQUATION FOR PRACTICAL USE

With a little modification, Equation 1 becomes somewhat more practical. When planning the photography of the site or working with photographs on computer, most people will find it easier to work with square portions of a photograph rather than circles. To this end, we calculate the size of the largest square which is encompassed by this circle (Fig.2).

From this diagram, the right-angled isosceles triangle allows us to define

$F = \sqrt[4]{2f}$, or in another form,

$f = \sqrt[4]{F^2/2}$, which can be substituted into Equation 1.²

Topo. Relief (H) in cm.	5	10	20	30	40
Useable Field (F) in m.	2.83	1.41	0.71	0.47	0.35

Equation 2: $F = \sqrt{2\left(\frac{2e.d}{H}\right)^2}$

The term F gives us a square that we use both on the terrain and in the photograph. On the terrain it is both the area covered by any individual photograph, and the distance between photographic centres, allowing a grid to be set out (or at least a plan for the photography) and the calculation of the number of photographs required for the job. In the photograph, F (scaled down to that of the photograph) defines the region of the photograph to be used, it allows us to create a software mask of that size for easy cutting, and it allows us to create a

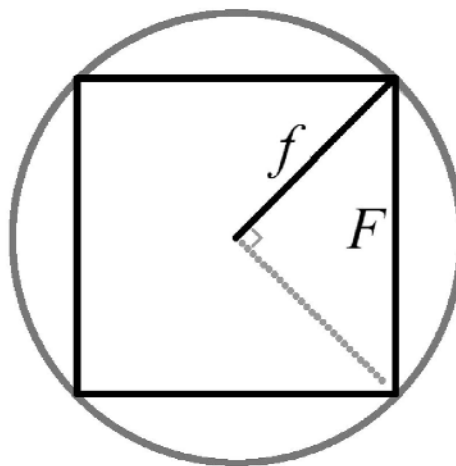


Figure 2 Calculation of F, the length of the sides of the largest square which will fit in a circle of radius f

gridded blank sheet (grid size= F) for easy pasting of pieces to create the mosaic.

CALCULATIONS IN THE FIELD

Any three of the four variables in Equation 2 can be assigned values which will yield a value for the fourth, allowing us to answer the following questions;

- "Is there too much topographical relief in this terrain for the technique?"

- "How high must I position the camera?"
- "How many photographs will I need to take?"

And, if any of the above questions do not yield satisfactory answers, it is possible to plug in practical or achievable values for each to learn just how large an error could result and whether you may judge it still small enough to warrant proceeding with the exercise.

EASY REFERENCE TABLES

Clearly, knowing the maximum height of the available apparatus on which we mount our camera, and having a consistent notion of what is an acceptable error, it is possible to use Equation 2 to draw up simple reference tables for use in the field.

The following is a table for use of an apparatus which holds the camera at height 5m, and chosen acceptable error of +/-1 cm, to calculate the useable field within our photographs.

A similarly useful table might be for using an apparatus giving maximum $d = 5\text{m}$ and a photographic field width $F = 0.5\text{m}$ to determine how large our error will be.

Topo. Relief (H) in cm.	5	10	20	30	40
Error (+/-e) in cm.	0.18	0.35	0.71	1.06	1.41

CASE STUDY: LE CHESLÉ, BELGIUM.

Le Cheslé is a Vth-Vth century BCE (late Hallstatt/early La Tène) fortification of the éperon-barre type, on the Ourthe River, near the village of Bérisménil in the Provence of Luxembourg, southern Belgium (Fig.3). Professor P. Bonenfant (Service des Fouilles, Université Libre de Bruxelles) has been excavating the main barrage wall of the fort (Fig.3) since 1994.

Tranchée A was photographed in June 2000 using an Olympus Camedia C-2000Z digital camera (one of the few digital cameras to have an infra-red remote shutter operator) mounted at a height of 5 m on the Statif (Fig.4) is

an apparatus conceived by Yves Glotz of Mons (Belgium), designed by engineers at the Université Libre de Bruxelles and fabricated at the Polytechnic at Mons in 1984 to facilitate near-range vertical photography in linear sequences. The base of the Statif is a large, horizontal aluminium beam (5 m long, 12 cm square section) standing on 2 pairs of adjustable-length legs. A carriage, which slides along the length of the beam, has an inverted "L"-shaped aluminium arm (3.5 m high) clamped to it, on the end of which is a camera mount. The camera typically sits at a height of 5m to 7m and linear series of photographs are made by sliding the carriage across the beam. Parallel series of photographs may be acquired without necessarily needing to move the entire apparatus by altering the length of the horizontal section of the inverted "L"-shaped arm, and/or by rotating the arm 180° to photograph on the other side of the beam.

RESULTS

Photographs of Tranchée A taken from the Statif were cropped to a size calculated from Equation 2 and compiled into an ortho-photomosaic (Fig.5).

Having drawn Tranchée A by hand with a fellow student in the preceding year, I could compare the time taken to produce the final drawings against that of producing the photomosaic;

	Duration	Labour (person days)
1999 - Drawing by hand	3.5 weeks	5 person-days
2000 - Ortho-photo with Statif	1 week	8 person-days

SUMMARY

By examining the geometry of vertical photography a simple equation was derived which allows us to make an approximate calculation of perspective error. The same equation can be expressed in a form which allows us to delimit a region around the photographic axis which is tolerably close to being orthographic. This "Orthographic Approximation" allows us to be predictive of "goodness of fit" at the initial photographic stage rather than reactive at the stage of constructing the photomosaic, with efficiencies accruing at both stages as demonstrated in the case study at Le Cheslé, Belgium.

Closing Note: While the geometry described above is obviously elementary in the fields of optics and photography, the author has not been able to find any reference in the archaeological literature to any similar considerations in the application of photography to field archaeology. Any claim of novel and original work that we might make in this respect we must, therefore, limit to its application in archaeology and to its dissemination amongst the archaeological community.

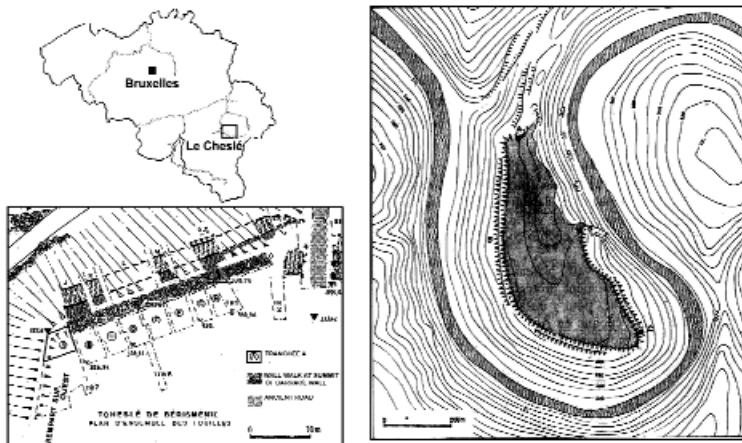


Figure 3 Locality of Le Cheslé. Lower left: Plan of excavations of the barrage wall (modified: after Bonenfant, 1995). Right: Topographic plan of Le Cheslé showing barrage wall, A (from Papeleux, 1988)



Figure 4 The Statif

Photogrammetry

¹ In reality, the geometry of the situation described above is more complex, however, the influence of these factors is considered to be sufficiently small to be ignored in the derivation of the approximation of the situation.

² We also introduce the term H . In Equation 1, topographical relief, h , can be in either the positive or negative direction from the "average field height". Here, we let $H = 2h$ where H is the total topographical relief, being the difference in measured height between the highest and lowest points of the terrain, not necessarily within just a single photograph but possibly across the entire area to be photographed.

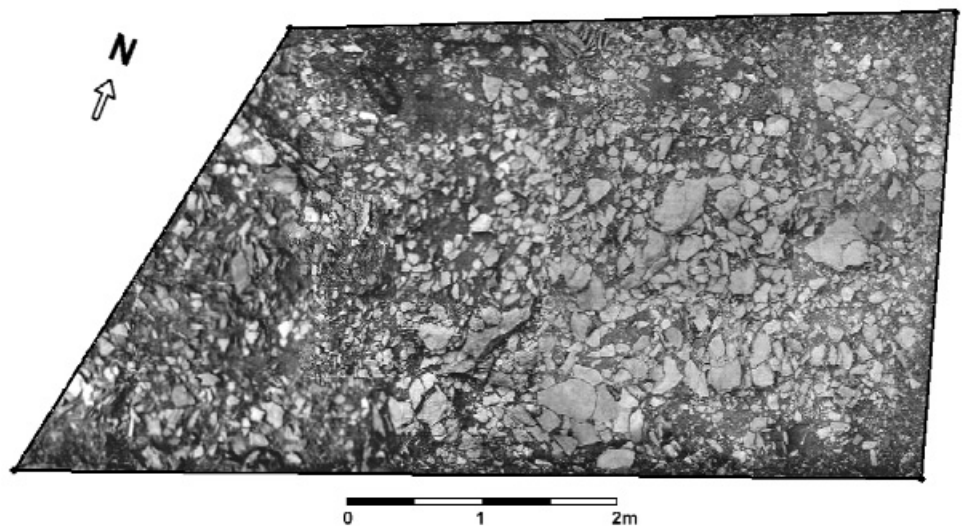


Figure 5 Ortho-photomosaic of Tranchée A, Le Cheslé

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