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Methods for finding calendar date bands from from multiple-valued radiocarbon calibration curves

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14.1 Introduction

Multiple-valued radiocarbon calibration curves, including the latest agreed high-precision curves (Pearson & Stuiver 1986, Stuiver & Pearson 1986), transform the normal distribution implied by a single radiocarbon date and error into a multimodal distribution of calendar dates. This 'bunching' of the calendar scale means that the form of the distribution of calendar dates, given a radiocarbon measurement, is highly unusual and standard statistical techniques are thus of little help in finding bands of calendar dates. As will be shown, the various empirical methods which are available can give rise to different sets of confidence bands, depending on the criteria used to construct them. In order to develop these points, an example is taken from the article of Hassan & Robinson 1987 which illustrates the multimodal nature of the distributions and provides a starting point for the discussion. This is mainly concerned with single dates, but the problems involved in moving on to multiple-date series are also briefly discussed.

14.2 Methods for finding calendar date bands

The conventional method for finding 68% or 'one-sigma' confidence bands using a multiple-valued curve is well known and is described, for example, by Pearson 1987. The method, illustrated for a particular example in Fig. 14.1a, gives one or more bands which together contain 68% of the calendar dates consistent with the radiocarbon date and error. In this example there is one intercept where the horizontal line through 4122 bp just touches the curve (at 4680 calBP); this might be reasonably be ignored or quoted as a single possible date; the two bands separated by less than 10 years would, if we follow the recommended procedure, be amalgamated. Thus we obtain in this instance bands 4810–4766, 4680, 4643–4632 and 4612–4552 calBP.

A second method is given by Hassan & Robinson 1987, who introduce the idea of transforming the normal distribution of radiocarbon dates into the corresponding multimodal distribution of calendar dates using a computer algorithm. This can be done by approximating the normal radiocarbon distribution as a histogram, defined within ± three standard deviations of the mean. The mid-point of each bin is calibrated and the probability attached to each bin in the normal radiocarbon histogram is divided equally among the one or more bins in the corresponding

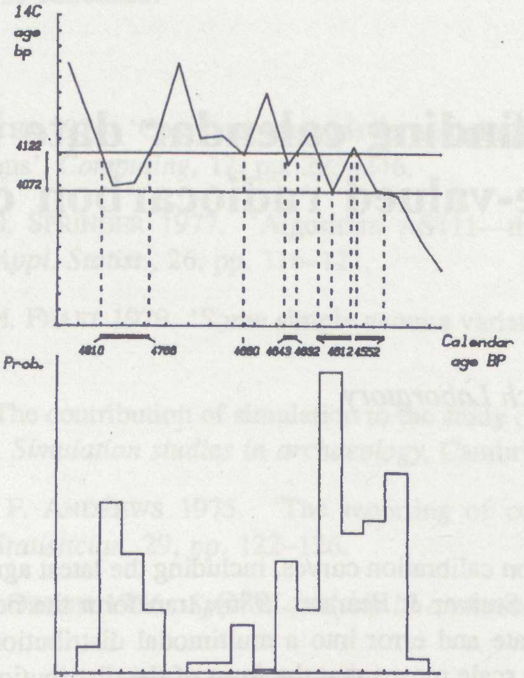

Fig. 14.1: Three methods for finding calendar confidence bands from a radiocarbon date

(a) The conventional method

(b) A method based on determining the central part of the transformed distribution of calendar dates

(c) A method based on searching for dates having maximum probability in the transformed distribution

The bands in each case are indicated thus ▬▬. The example (radiocarbon date 4097 ± 25bp) is taken from Hassan & Robinson 1987, Fig. 1.

transformed calendar histogram. The confidence band is then determined by finding the central 68% of the whole transformed distribution. The band in this case is 4795–4564 calBP, and is illustrated in Fig. 14.1b.

Comparison of Figs. 14.1a and 14.1b shows that different results are obtained by the two methods, and highlights the need for a way of choosing between the various possible 'confidence bands' that can be derived from a single radiocarbon measurement. This requires a consideration of criteria for their construction and it is suggested here that it would be reasonable to insist that any date *inside* any of the bands (at a given confidence level) should be more probable than any date *outside* them. Fig. 14.1c illustrates the application of this rule, which gives rise to what can be termed the *maximum probability* method. In this method, peaks are identified in the histogram representing the transformed distribution, starting at the bin with the highest frequency. Further bins are added by moving the line downwards until the relative frequency contained within them totals 68%. The intervals on the calendar scale which correspond to these peaks are then the confidence bands. The method guarantees to contain the most probable dates, within a certain accuracy determined by the bin sizes used to represent the distributions. Here the process results in the two bands 4810–4790 and 4610–4530 calBP.

14.3 Computer implementation of the maximum probability method

The method is simple to incorporate into a computer program, once the basic algorithm for producing the transformed distribution, and the database containing the curve(s), have been stored. Authors who have developed calibration programs include Robinson (referred to above) in WBASIC; Avery (see chapter in this volume) in BBC BASIC; the present author in FORTRAN; Stuiver & Reimer 1986 in FORTRAN. The latter does not include any probabilistic computations, but refers to work in progress on a program which does. In computational terms the method for finding confidence bands described above is as follows:

1. Store the transformed multimodal distribution as a histogram;
2. Sort the histogram bins into order;
3. Accumulate the frequencies in the sorted bins until the desired confidence level is attained, marking those individual bins accumulated;
4. Re-sort the bins into their original order;
5. Work through the bins, picking out marked bins and amalgamating any of these that are contiguous, thus forming the bands.

Bands obtained in this way will be imprecise because the two distributions involved have been digitised as histograms; however, this imprecision can be reduced to an acceptable level by reducing the bin sizes, down to a minimum determined by the precision of the curve itself. Bin sizes of 5–10 years for the normal (radiocarbon) distribution and 10–20 years for the multimodal (calendar) distribution are usually appropriate.

14.4 The limitations of confidence bands

One point that has not yet been discussed is the choice of confidence level and, more generally, the interpretation of the confidence bands. Pearson 1987 suggests using the 95% (or 'two-sigma') level for the radiocarbon error band, though 68% also seems to be widely used for both radiocarbon and calendar bands. Whatever level is chosen, examination of the histograms shows that, in this multimodal situation, the probabilities of dates excluded from the band(s) do not show any predictable pattern. This makes the interpretation of bands at, say, the 68% level, very difficult since we do not know where the remaining 32% is concentrated. Indeed, there is an argument for saying that the only adequate calendar interpretation of a radiocarbon date is the whole distribution, approximated for practical purposes as a histogram.

Clearly there are problems of interpretation in this multimodal situation, which stem from borrowing the confidence band idea from the more usual unimodal situation. It might be preferable in these circumstances to take a different approach, avoiding altogether the need for an arbitrary choice of confidence level. At its simplest, this would be to try, wherever possible, to pose the following question: what is the probability that a given band contains the true calendar date, given a radiocarbon date and error? The transformed distribution approach has the potential to give an unequivocal answer to this question, since the relative frequency within any band can be found by summing the relative frequencies in the appropriate bins. This feature has been incorporated in the program used here.

The question posed above would only be relevant in very specific circumstances, and it will also be necessary to consider the treatment of the more realistic archaeological problems involving series of related dates. Such series are presently few and far between, as noted for example by Needham 1986 in the context of Bronze Age metalwork. However, trends seem to be towards the formulation of definite hypotheses based on stratigraphic relationships and/or associations and eventually sets of related dates will become more commonplace. The new situation will demand more sophisticated statistical methods similar to those developed by Orton 1983 for uncalibrated dates, but with modifications to take account of calibration bunching. This will entail further methodological work, as also noted by Ottaway 1987, which it is hoped will lead to an extension of the program used here to cover the analysis of multiple-date series.

14.5 Summary and conclusions

The discussion above argues for a move away from confidence bands for calendar dates since they are difficult to interpret in a multimodal situation. Nevertheless, in the current absence of suitable alternatives, archaeologists will naturally continue to require confidence bands as a convenient summary of their dates. In view of this, it seems worthwhile considering a logical approach to the construction of such bands for multimodal data and this has been the main purpose of this note. Of the multiplicity of calendar confidence bands that can be derived from a single radiocarbon date, one particular method, that of maximum probability, has been shown to have certain advantages. Firstly, it is based on the transformed distribution method described by Hassan and Robinson. It thus provides a diagram of the relative probabilities of the possible calendar dates in the form of a histogram, and ensure that no important feature of the distribution of calendar dates has been overlooked. Secondly, it finds the most probable dates. This is not necessarily true of the other two methods illustrated. For example, the conventional method (Fig. 14.1a) includes the band 4643–4632 calBP which from the histogram (Fig. 14.1b) can be seen to be relatively unlikely. The method illustrated in Fig. 14.1b gives rise to a band which is centred on the 'valley' between the two main peaks, and thus also contains relatively unlikely dates. The method of Fig. 14.1c automatically gives the bands containing the most probable dates, for a given total probability, and is thus recommended if confidence bands are required.

Acknowledgements

The author is grateful to colleagues Dr S. Bowman and Ms J. Ambers for their advice in formulating the computer program used for this article, and to Dr S. Robinson and Dr M. Avery for advance information regarding their computer programs.

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