Essays on Asset Pricing and Derivatives

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Chapter 1

Introduction

The theory of corporate finance builds on two ingenious ideas. The first idea was developed by Modigliani and Miller (1958). They prove that the firm's value is independent of the capital structure in a frictionless world. This result implies that firms cannot increase their value by the means of security design and risk management. The irrelevance theorem seems trivial and not to have much practical relevance. However, Modigliani and Miller devise an organizing principle for corporate finance research which still endures until today. To explain observed capital structure choices, one must provide evidence for a meaningful violation of Modigliani and Miller's assumptions. Such violations have been discovered, for example, in the form of taxes (Modigliani and Miller, 1963), bankruptcy costs (Robichek and Myers, 1966; Baxter, 1967; Warner, 1977), transaction costs (Williamson, 1981), information asymmetries (Stigler, 1961; Akerlof, 1970; Spence, 1973) and agency conflicts (Jensen and Meckling, 1976).

The second idea was introduced almost two decades later by Merton (1974). Merton was the first to interpret the firm's equity position as a call option on the firm's assets with the strike price being equal to the face value of the firm's debt. Or put more generally, all contracts of a firm can be thought of as claims contingent on the firm's asset value process. This structural model for the firm was enabled by the development of option pricing theory by Black and Scholes (1973) and Merton (1973b). Since then, a rich body of literature evolved from this idea implementing various of the aforementioned frictions (Kraus and Litzenberger, 1973; Scott Jr. 1976; Brennan and Schwartz, 1978). In addition, the static one-period model has been further developed in a dynamic setting, for example, by Leland (1994), Leland and Toft (1996), Fischer et al. (1989), and Goldstein et al. (2001).

The first part of this dissertation follows the tradition of these two ideas by applying the structural model in the context of the recent financial crisis. In the aftermath of the collapse of the prominent investment bank Lehman Brothers in 2008, the financing and risk-taking decisions of the financial sector have come under close scrutiny. This dissertation takes a closer look at two categories of debt contracts, which have been in the midst of the discussions among industry professionals, policy makers, regulators and academics. The first category, which is analyzed in chapter 2, comprises retail structured products. The second category consisting of contingent convertible debt contracts is evaluated in chapter 3.

Both types of debt contracts have in common that they improve the stability of the issuer and the financial system under certain conditions, for example, by reducing the probability of default or mitigating a credit crunch. Thus, they possess features which are desirable from the regulator's point of view. However, both types of debt contracts heavily distort the risk-taking incentives of the issuing financial institution. The objective of this thesis is to identify the conditions under which the specific contracts are beneficial and under which they foster agency conflicts. Furthermore, the optimal product design and regulatory recommendations are derived. Chapter 4 concludes the first part.

While this first part of the thesis applies derivative valuation techniques to consistently determine the value of risky corporate securities, the second part of the thesis is concerned with the origin of the risk premia which determine the prices of assets.

All asset pricing models build on the key idea that what matters for pricing is only the covariance of an asset with the pricing factor. In the capital asset pricing model (CAPM) developed by Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966), the risk premium of an asset is linear in the covariance of the asset's return with the market return. The risk arising from the portion of the asset's return, which is uncorrelated to the market, can be diversified in large portfolios. Hence, only the portion correlated to the market return is relevant for pricing. Due to its elegantly simple structure, the CAPM became the workhorse model in academia and disseminated into everyday business practice.

The pricing relevant beta factor can be determined as the coefficient of a linear time-series regression of the asset's excess return on the market's excess return. With the rising availability of data, the empirical literature added further variables to this regression and spotted a plethora of factors containing pricing relevant information. Harvey et al. (2015) identify as many as 315 distinct factors. The most well-known among these factors are firm size (Banz, 1981) and the market-to-book ratio (Basu, 1983), which led to the proposal of the three-factor model by Fama and French (1992, 1993, 1996).

As a consequence of the joint hypothesis problem (Fama, 1991), it is impossible to distinguish whether an empirical observation arises from a violation of market efficiency or whether the underlying asset pricing model is indeed wrong or incomplete. And albeit some of these factors can be interpreted as state variables in the intertemporal capital asset pricing model of Merton (1973a), there is no economic rationale for the inclusion of most other factors, for example, idiosyncratic volatility. As a consequence, one has to resort to frictions, for example, which hinder investors' ability to hold fully diversified portfolios (Levy, 1978; Merton, 1987), or behavioral biases, for example, that investors have a preference for lottery-like stocks with high idiosyncratic volatility (Barberis and Huang, 2008), as potential explanations for the empirical puzzle.

Another empirical puzzle, which is in the focus of the second part of this dissertation, results from the observation that stocks with low return volatility historically generated higher risk-adjusted returns than stocks with high return volatility. In other words, the return volatility contains pricing relevant information beyond the beta factor. Therefore, this puzzle is seemingly at odds with classical asset pricing theory. The objective of the analysis presented in chapter 5 is to reconcile this empirical puzzle with classical asset pricing theory without having to introduce a friction or behavioral bias.

The key insight of the analysis is that assets with low volatility react differently to increases in correlations than assets with high volatility. The beta factor of a low volatility asset, which is typically below one, increases in response to a correlation shock, while the beta factor of a high volatility asset, which is typically above one, decreases. When correlations increase, all assets move more in sync with each other and, thus, the assets' beta factors move closer to one. When this behavior of beta factors is taken into account by investors in a typical equilibrium setting, they demand a risk premium for holding low volatility assets and require a lower return than predicted by the standard CAPM from high volatility assets. Furthermore, the model is calibrated to standard market parameters. A structural model following the idea of Merton (1974) is used to derive the prices of different claims of the same firm. Finally, three testable hypotheses are developed. Chapter 6 concludes the second part.

Both parts of this dissertation have in common that they provide theoretical explanations for observed real-world behavior of financial institutions and investors. In all three analyses, the classical theory for the pricing of assets and derivatives is consistently applied. Furthermore, the structural model of the firm is used to evaluate the pricing consequences for different types of claims issued by the same firm. There is one notable difference between the two parts. In the first part, it is argued that a meaningful friction is required to explain the observed behavior. In the second part, the opposite view is taken. It is argued that the empirical puzzle can be already explained by classical theory in a setting without frictions.

Part I

Financial Institutions and Derivatives

Chapter 2

Retail structured products*

2.1 Introduction

When systemic risks are a matter of concern and banks are considered to be too big to fail, hedging between banks does little to help restore trust. Risks are passed on from one financial institution to another but can still spread within the financial sector. Hence, there is a need to transfer risks outside the financial system and for products capable of doing so. Retail structured products could be a suitable vehicle for this kind of risk transfer.

Retail structured products, which are often advertised under the generic term *certificate*, are part of the unsecured subordinated debt of a financial institution. Their repayment is tied to the performance of an arbitrary underlying asset (mostly equities, but these can also be commodities and interest rates). Thus, with the notable exception of the issuer's bankruptcy, the repayment is not linked to the issuer's own financial performance. In contrast to mutual funds, whose assets are separated from the assets of the managing firm, the issuer's use of the proceeds is not restricted or regulated, i.e., the funds can be used for purposes other than hedging.

These derivative products, which are tailored to the needs of retail investors, have themselves come under scrutiny in the aftermath of the financial crisis. Retail investors incurred significant losses from products issued by the defaulted investment bank Lehman Brothers. Subsequently, these products and their regulation have become subject to controversial

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debate among policymakers and industry professionals, which mainly focuses on transparency and risks from the retail investors' point of view. However, the debate does not include the more important point of the impact on the risk choice and stability of the issuing financial institution, which is our focus in this paper.

The literature on retail structured products so far has considered these products primarily as a source for profits for the issuing banks, since the products are sold at a price well above the value from stand-alone duplication (Wilkens et al., 2003; Stoimenov and Wilkens, 2005). We are adding two novel themes to this literature. Each retail structured product can be decomposed into a risk-free component and a derivative component. The first component is a valuable source of funds for the issuer's core business. We believe that the second component is an innovative tool for risk management.

For all standard product types, the first component is strictly positive, such that retail structured products generate a cash surplus[1]. We argue that the issuing financial institution uses the cash surplus to fund its ordinary business, for example, by granting loans, instead of purchasing risk-free government bonds. Thus, if the asset portfolio is illiquid or subject to price shocks, then the investors in retail structured products are exposed to the business risk of the issuer. The default by the prominent issuer Lehman Brothers provides anecdotal evidence for this risk exposure.

The first component links the payoff of retail structured products to the financial performance of the issuer; the derivative component creates an exposure to the underlying security. On the one hand, the issuer can effectively transfer a risk exposure to the retail investor, i.e., outside the banking system. On the other hand, the retail investor explicitly wants to have this exposure to the underlying asset, which is usually in the focus of the advertisements of these products. The bundle of the derivative component with a risk-free component ensures that there is no future cash flow from the retail investor to the issuer, i.e., from the issuer's perspective there are no settlement costs and no counterparty risk. Our main objective in this paper is to evaluate the conditions under which the issuers can benefit from retail structured products as a risk management tool.

To meet our main objective, we incorporate retail structured products in a simple Merton-type model. We focus on the two most prominent types of claims, principal-protected notes and discount notes. The payoff of principal-protected notes is convex in the value of the underlying asset, while the payoff of discount notes is concave. We assert that these two claims represent the class of claims with convex or concave payoffs, respectively. We

[1]The German Derivative association, which represents the issuing financial institutions in Germany, estimates a market size of EUR 90.2 bn (as end of 2013). This corresponds to 1.2% of total bank liabilities and 24.4% of aggregated bank equity in Germany. For some banks, the market value of issued retail structured products already exceeds the volume of equity financing.

use the option pricing theory developed by Black and Scholes (1973) and Merton (1973b) for the consistent valuation of the retail structured products as well as all other claims in the Merton model.

According to the seminal work of Modigliani and Miller (1958), the value of the issuer is invariant to its capital structure. There is no optimal capital structure in a world without frictions. Similarly, there is no additional value to be created by risk management. Hence, there is no rationale for the existence of retail structured products in a frictionless world. As a consequence of this central result of Modigliani and Miller, we have to consider market frictions to explain the issuer's capital structure choice. Hence, we incorporate the classical trade-off between tax benefits of debt and bankruptcy cost.

We find that when the issuer's assets are highly correlated with the underlying security, retail structured products increase the value of the issuer. We show that compared to the case of straight debt financing, a high-risk issuer can always improve its value and simultaneously lower the default probability for any given target leverage ratio. The opposite is true for a low-risk issuer, whose assets are uncorrelated to the underlying security.

Nevertheless, the issuer is subject to risk-shifting and has an incentive to optimally adjust its leverage and asset risk weight. Even when accounting for these optimal decisions, risky issuers prefer to optimally add retail structured products to the financing mix. Thereby, issuers with high asset risk increase the probability of default when issuing principal-protected notes, but reduce it by issuing discount notes. The results also hold when the issuer can optimally design the retail structured products.

This chapter is organized as follows. In section 2.2 we survey the relevant literature. In section 2.3 we introduce the model and describe the valuation of all relevant claims. In section 2.4 we analyze the issuer value for a given asset composition and leverage, and evaluate the issuer's optimal financing choice in section 2.5. In section 2.6, we analyze the risk-taking incentives of the issuer. In section 2.7 we derive the optimal design of retail structured products. We also discuss further product types and product complexity. Section 2.8 concludes the chapter.

2.2 Literature review

Our work reconciles two strands of studies. First, there is a predominantly empirical literature on retail structured products. Second, our analysis is also related to the literature dealing with the capital structure and risk management of firms and especially financial institutions.

The focus of the empirical literature on retail structured products is on the pricing from the investors' perspective. In one of the most comprehensive empirical studies of the German market, Stoimenov and Wilkens (2005) document that retail structured products are traded at a markup compared to their stand-alone duplication values. They attribute this observation to information asymmetries and retail investors' limited market access. Their results are confirmed by many further studies, e.g., Wilkens et al. (2003), Baule et al. (2008), Entrop et al. (2009), and Baule (2011). In addition, Baule et al. (2008) show that the default risk of the issuer is not appropriately reflected in the pricing of retail structured products. Henderson and Pearson (2011) provide similar evidence for equity linked products in the U.S., which are also mainly traded by retail investors.

Carlin's (2009) model supplements this empirical evidence on the pricing of retail structured products. His key result is that producers of financial products can increase the profits they make from selling these products to uninformed retail investors by making the products more complex. Breuer and Perst (2007) make another interesting theoretical contribution. These authors explore why utility-maximizing retail investors want to add retail structured products to their portfolios in the first place. According to their results, the purchase of retail structured products is particularly beneficial for investors with low levels of competence in investing.

Our work also follows the tradition of structural models in corporate finance. Considering typical frictions such as the tax benefits of debt and bankruptcy costs, these models are capable of deriving an optimal capital structure. One of the first models to implement the trade-off between tax benefits and bankruptcy costs is that of Brennan and Schwartz (1978), which builds on the option theoretic approach of Merton (1974). This approach has been further developed in continuous time by Fischer et al. (1989), Leland (1994) and Goldstein et al. (2001). Decamps et al. (2004) apply the framework to financial institutions and derive implications for the risk-taking incentives and stability of banks.

Following Modigliani and Miller (1958), there is no optimal capital structure in a frictionless world. Similarly, firms cannot add value with risk management. Hence, the need for risk management arises when firms try to avoid the costs related to frictions; for example, the costs of financial distress, which is also the motive for hedging in our model.

So far, the literature has not considered the linkages between these two strands. Since issuers, however, have access to highly sophisticated financing claims such as retail structured products, it is essential to analyze the impact of issuing such products on the issuers risk-taking incentives and stability.

2.3 Model

2.3.1 Investment and financing choices

We consider an initially unlevered financial institution (issuer) in a one-period setting with initial time $t = 0$ and maturity time $t = T$. The issuer holds an asset portfolio with value $\widetilde{A}_t$ at time t. The asset structure remains static until maturity. We consider different compositions of the issuer's asset portfolio.

The financial institution may choose to issue zero coupon bonds and retail structured products (RSPs). The raised capital is immediately paid out as a cash dividend to equity holders. The demand is sufficiently large such that the issuer can raise any desired amount of debt. We focus our analysis on the two most prominent claims, principal-protected notes (PPNs) and discount notes (DCNs). The issuer can issue only one type of product at a time. We do not require a specific seniority structure; we model debt as one claim. Thus, the split among the debtors in the case of default is arbitrary and does not impact the results.

The issuer promises holders of the bond a fixed payment of $\overline{B}$ at maturity T. The RSP payoff is linked to the performance of an underlying security $\widetilde{R}$, for example, a stock market index such as the Euro STOXX 50 or the Dow Jones Industrial Average. The promised payoff of the principal-protected note CP_T at maturity T is given by

$$CP_T = \left(1 + \pi \cdot \max\left\{\tfrac{\widetilde{R}_T - X_P}{X_P}, 0\right\}\right) \cdot \overline{P}, \tag{2.1}$$

where $\overline{P}$ denotes the minimum payment to investors (see left-hand side of figure 2.1).

Investors participate at the rate of π in the performance of the underlying asset above the threshold X_P, which usually matches the initial value of the underlying asset $X_P = R_0$. Hence, the investor is protected against decreases in the underlying value as long as the issuer remains solvent. The promised payoff is equivalent to that of a portfolio comprising a risk-free zero bond with face value $\overline{P}$ and $\pi \cdot \overline{P}$ times a call option with strike price X_P.

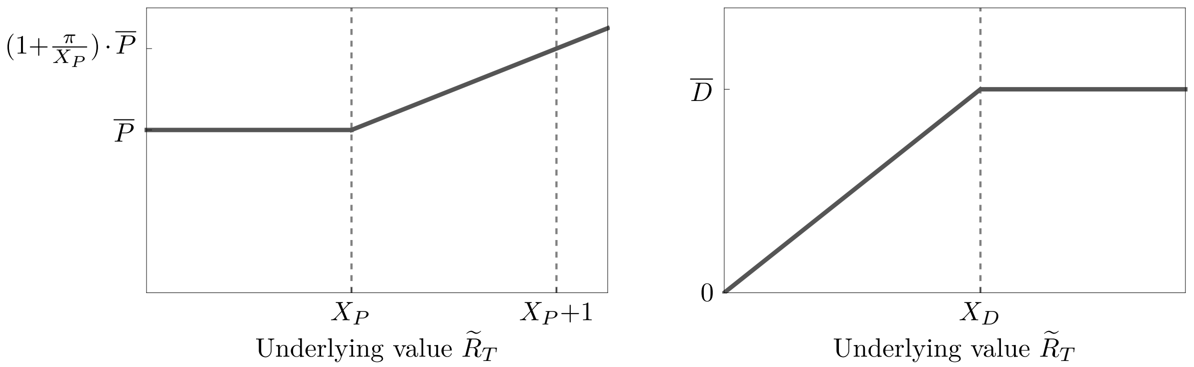
The promised payoff of the discount note CD_T at maturity T is given by

$$CD_T = \min\left\{1, \gamma \cdot \widetilde{R}_T\right\} \cdot \overline{D}, \tag{2.2}$$

where $\overline{D}$ denotes the maximum payment to investors (see right-hand side of figure 2.1). We define $\gamma \equiv \frac{1}{X_D}$. If the price of the underlying $\widetilde{R}_T$ falls below the threshold X_D, then the investors are paid the value of the underlying asset. This promised payoff can be

Figure 2.1: Promised payoff of standard products

The graph on the left shows the promised payoff CP_T of a principal-protected note with strike price $X_P = R_0$ and participation rate π. The graph on the right shows the promised payoff CD_T of a discount note with strike price X_D.



duplicated with a portfolio consisting of a risk-free zero bond with face value $\overline{D}$ and $\gamma \cdot \overline{D}$ times a short put with strike price X_D.

All market participants have perfect information. Investors observe market prices as well as the structure of the issuer's asset portfolio. They are able to anticipate the issuer's decision and appropriately incorporate the information in the pricing of the claims.

The issuer is operated by managers on behalf of the equity holders. The managers choose the face value $\overline{B}$ of the discount bond and the product parameters $\overline{P}$ and $\overline{D}$ to maximize the value of the equity holders' position at time $t = 0$. According to the well known result of Modigliani and Miller (1958), the manager's choice is arbitrary in a world of complete and efficient markets. Hence, we incorporate the classical trade-off between tax benefits of debt and bankruptcy cost.

At maturity T, the issuer repays its debt and pays taxes at rate $\tau > 0$. The tax deductibility of interest payments allows the issuer to derive value from debt financing.[2] Similarly, the issuer can derive tax benefits from retail structured products, for which the tax deductible cost of financing is equal to the difference between the repayment and the issuance price. Since the repayment is linked to the underlying asset $\widetilde{R}$, the size of the tax shield also depends on the realization of the underlying asset and can possibly turn negative in some states of the world.

The issuer defaults if the value of its debt exceeds the value of its assets. In this case, the debt holders receive a share $1-\alpha$ of the issuer's assets, where $\alpha \in (0,1]$ denotes the proportional cost of bankruptcy. A potentially positive tax shield is lost.

Alternatively to the tax benefits, we could assume that the issuer has a franchise value, i.e., the capability to generate additional revenues from business related to issuing retail structured products. Such revenues include fees for sales, trading, and depository of the securities. We analyze such a setup in section 2.7.4. Hence, our model framework can accommodate a wide spectrum of market frictions.

2.3.2 Valuation of claims

We build on the approach of Merton (1974), who interprets the equity holders' payoff at maturity T as a call option on the issuer's assets with the issuer's liabilities corresponding to the strike price. Hence, the established valuation framework for contingent claims can

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[2] We do not consider the personal income tax of equity holders and debt holders. Their effect is negligible if all investors pay the same tax rate on dividends, interest income, and gains in the value of traded securities, which has been the case in Germany since 2009.

be applied. Our model differs in one dimension: the issuer's liability at maturity T, i.e., the strike price of the option, is itself contingent on the price of a risky asset.

We follow the set of assumptions provided by Black and Scholes (1973) and Merton (1973b).[3] The price of the underlying asset $\widetilde{R}$ follows a diffusion process of the form

$$\mathrm{d}R = \mu_R R_t \ \mathrm{d}t + \sigma_R R_t \ \mathrm{d}z_R, \tag{2.3}$$

where μ_R denotes the underlying asset's expected rate of return, σ_R denotes the standard deviation of returns, and z_R is a Wiener process. The underlying asset $\widetilde{R}$ is not paying a dividend. The term structure of interest rates is constant and flat. The value of the risk-free asset F_t at any point in time t is determined by the risk-free interest rate r with

$$F_t = F_0 \cdot e^{rt}. \tag{2.4}$$

We consider two settings for the asset value $\widetilde{A}$. In the most general case (see section 2.5), the asset value also follows a diffusion process of the form

$$\mathrm{d}A = \mu_A A_t \ \mathrm{d}t + \sigma_A A_t \ \mathrm{d}z_A, \tag{2.5}$$

where μ_A denotes the asset's expected rate of return, σ_A denotes the standard deviation of returns, and z_A is a Wiener process, which is correlated to the Wiener process z_R determining the value of the underlying, i.e., $\mathrm{d}z_R \ \mathrm{d}z_A = \rho \ dt$ with $\rho \in (-1,1)$. Using risk-neutral valuation, the value of the issuer V_0 at time $t = 0$ equals

$$V_0 = D_0 + e^{-rT} \int_0^{\infty} \int_0^{\infty} (A_T - D_T + \tau (D_T - D_0)) \cdot \mathbb{1}_{\text{solvency}} \cdot f_{RA}(R_T, A_T) \ \mathrm{d}R_T \ \mathrm{d}A_T, \tag{2.6}$$

where D_t denotes the value of total debt including retail structured products at time t, and $f_{RA}(R_T, A_T)$ is the joint risk-neutral probability density function of the underlying asset $\widetilde{R}_T$ and the issuer's asset value $\widetilde{A}_T$ at time T. The indicator function $\mathbb{1}_{\text{solvency}}$ for the survival of the issuer takes the value of one for $A_T - D_T \geq 0$ and zero otherwise.

When the issuer is able to fully repay the debt, it generates a tax benefit with present value $\tau(D_T - D_0)e^{-rT}$. The tax benefit is lost if the issuer defaults. The bankruptcy cost are included in the pricing of the debt claim D_0. The value V_0 is given by the value of the assets A_0 of the unlevered issuer plus the present value of the tax-shield minus the present value of the bankruptcy cost.

[3] With the exception of taxes and bankruptcy costs, the market is free of frictions. There are no transaction costs or bid-ask-spreads. Trading in the underlying asset is continuous and all securities are perfectly divisible. All market prices are observable and short selling is not restricted. Investors are assumed to be non-satiable and agree on σ, but not necessarily on μ.

In a simplified setting (see section 2.6), we consider only one single source of uncertainty. In this case, the asset portfolio $\widetilde{A}$ of the issuer is linked to the development of the underlying asset $\widetilde{R}$. The expression of the issuer value V_0 simplifies to

$$V_0 = D_0 + e^{-rT} \int_0^\infty \left(A_T - D_T + \tau \left(D_T - D_0\right)\right) \mathbb{1}_{\text{solvency}} \cdot f_R(R_T) \; \mathrm{d}R_T. \tag{2.7}$$

We can derive a closed-form solution for the equity holders' claim V_0 (issuer value). The functions are piecewise defined depending on the managers' choice of $\overline{B}$, $\overline{P}$, and $\overline{D}$. To improve readability, we present the exhaustive derivation of the formulae in appendix 2.A.

We introduce a measure for the stability of the issuer. For this purpose, we use the risk-neutral probability of default *pd*, which we calculate as

$$pd = \int_0^\infty \Big(1 - \mathbb{1}_{\text{solvency}}\Big) \cdot f_{RA}(R_T, A_T) \; \mathrm{d}R_T \; \mathrm{d}A_T. \tag{2.8}$$

Since the quotes of credit default swaps written on the issuer monotonically increase with the risk-neutral default probability, *pd* is a reasonable market-oriented measure for stability.

2.4 Constant leverage issuer

Before we take a look at the optimal financing and risk choices, we inspect the issuer value and the probability of default depending on the leverage ratio $\lambda = \frac{D_0}{V_0}$. By doing so we can draw important conclusions on the value generated by RSPs and on the stability of the issuer. We focus on two polar cases. First, we consider a high-risk issuer whose assets are the same as the underlying asset of the RSP, i.e.,

$$\widetilde{A}_t = \widetilde{R}_t. \tag{2.9}$$

Second, we analyze a low-risk issuer investing only in risk-free government bonds, i.e.,

$$\widetilde{A}_t = F_t. \tag{2.10}$$

In addition, we restrict the issuer to issuing one single debt claim. This approach has the advantage that the valuation formulae simplify and general results can be derived analytically.

2.4.1 High-risk issuer

We first consider the case of $\widetilde{A}_t = \widetilde{R}_t$. On the one hand, this case represents an issuer taking the maximum amount of risk. On the other hand, this issuer also has the greatest capability to produce RSPs, which depend on the same risky asset that is part of the issuer's balance sheet.

Before analyzing the issuer value, we examine the risk-neutral default probability of the issuer, which is depicted in figure 2.2. The graph on the left shows the default probability of an issuer financed with PPNs (solid line) and the graph on the right shows the default probability of an issuer financed with DCNs (solid line). Both plots also show the default probability under straight debt financing as a reference case (dashed line).

In line with our expectations, the curves monotonically increase with the leverage ratio λ. For PPN financing, we have to distinguish two cases. For low issuance volumes $\overline{P} < X_P$, the issuer defaults only when the value of the underlying asset drops below the issued principal amount, i.e., $R_T < \overline{P}$. But when the issued amount $\overline{P}$ exceeds X_P, the issuer is also not able to repay the promised participation in the underlying asset even though the value of the underlying asset appreciates. Figure 2.2 shows that the graph has a kink at the transition point between these two cases at $\overline{P} = X_P$.

For an issuer with DCN financing, we also observe two cases. The issuer can repay its liabilities in all states of the world as long as the issued amount $\overline{D}$ is less than the maximum repayment X_D, i.e., we have $pd_{DCN} = 0$. However, the default probability jumps up when $\overline{D}$ exceeds X_D, since the issuer is defaulting for all values of the underlying asset, $R_T < \overline{D}$. In this case, the default probability corresponds to that of an issuer with straight debt financing with an issued amount $\overline{B} = \overline{D}$.

The main finding from figure 2.2 is that the probability of default with RSP financing is either equal to or strictly lower than the default probability of an issuer with straight debt financing. This observation can be generalized due to the closed form solutions for all claim values. We provide proofs in appendix 2.B[4].

We summarize this important result as:

Proposition 2.1 (Risk reduction of high-risk issuer)
For any attainable leverage ratio $\widehat{\lambda} < 1$, the risk-neutral default probability of a high-risk issuer financed with RSPs never exceeds the probability of default of a high-risk issuer financed with straight debt, i.e., $pd_{RSP}\left(\widehat{\lambda}\right) \leq pd_B\left(\widehat{\lambda}\right)$.

[4] The proof for DCN requires the technical condition $N(d_2(y)) - N(d_1(y)) \leq \bar{\varepsilon}$ for all y. The proof for PPN requires $\overline{P} < \frac{X_P}{\pi}$ for $\pi > 1$.

The next logical step in our analysis is to consider the issuer value, which is depicted in figure 2.3. The graph on the left shows the value of an issuer financed with PPNs (solid line) and the graph on the right shows the value of an issuer financed with DCNs (solid line). Both plots also show the issuer value for straight debt financing as a reference (dashed line).

The issuer value increases with the leverage ratio λ and then decreases to $(1-\alpha)A_0$ when λ approaches one. This behavior is consistent with the results of Leland (1994). Analogous to the corresponding graph of the default probability, the issuer value under PPN financing has a kink at $\overline{P} = X_D$. Due to the zero default probability, the issuer value under DCN financing increases linearly until $\overline{D} = X_D$ and then drops down to the issuer value under straight debt financing.

We observe that the issuer value under RSP financing is always equal to or higher than the value under straight debt financing. Again, we can generalize this important result. (See appendix 2.B for proof.)

Proposition 2.2 (Value creation of high-risk issuer)
For any attainable leverage ratio $\widehat{\lambda} < 1$, the value of a high-risk issuer financed with RSPs is always greater than or equal to the value of a high-risk issuer financed with straight debt, i.e., $V_{0,RSP}\left(\widehat{\lambda}\right) \geq V_{0,B}\left(\widehat{\lambda}\right)$.

In summary, the high-risk issuer always benefits from the issuance of RSPs. Propositions 2.1 and 2.2 show that the issuer can increase its value and at the same time reduce the probability of default for fixed leverage ratios as compared to the case of straight debt financing.

Surprisingly, this result holds for both types of products, i.e., concave payoffs as well as convex payoffs. The benefit of PPNs compared to straight debt financing is that given the same probability of default, PPNs can create higher tax benefits. This increase in tax benefits is achieved by selling a fraction of the assets only in good states $\widetilde{R}_T > X_P$ at maturity time T. In contrast, the benefit of DCNs financing originates from a lower repayment to debt holders in bad states $\widetilde{R}_T < X_D$ at maturity, which allows the issuer to reduce its expected bankruptcy cost compared to straight debt financing.

This result is certainly only valid for a fixed leverage ratio. It is apparent from figure 2.3 that the optimal leverage for RSP financing is higher than that for straight debt financing. We analyze this optimal choice in more detail in section 2.6. Nevertheless, we can still derive an important implication here for the regulator. Due to the one-to-one correspondence between the leverage ratio and probability of default, the regulator can easily impose restrictions on the leverage to fit the maximum amount of risk that the issuer should take from the social planner's perspective.

Figure 2.2: Probability of default depending on leverage (high-risk)

The graph on the left shows the probability of default pd_{PPN} for an issuer with PPN financing (solid line). The graph on the right shows the default probability pd_{DCN} for an issuer with DCN financing (solid line). Both graphs also show the default probability pd_B with straight debt financing (dashed line). We compute the values using the model parameters $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

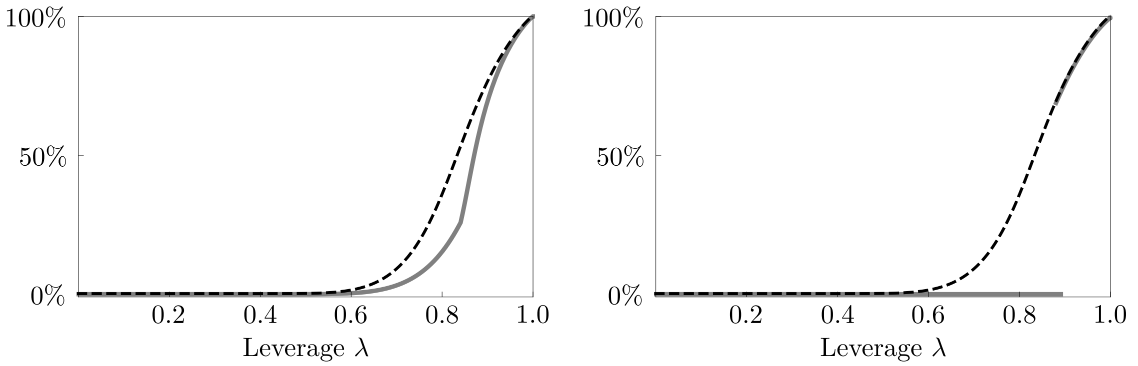
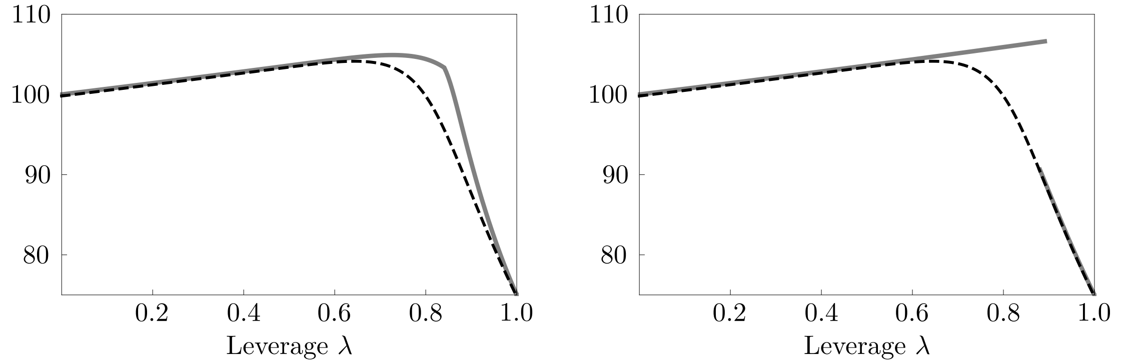


Figure 2.3: Issuer value depending on leverage (high-risk)

The graph on the left shows the issuer value $V_{0,PPN}$ with PPN financing (solid line). The graph on the right shows the issuer value $V_{0,DCN}$ with DCN financing (solid line). Both graphs also show the issuer value $V_{0,B}$ with straight debt financing (dashed line). We compute the values using the model parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.



2.4.2 Low-risk issuer

The issuer with $\widetilde{A}_t = \widetilde{R}_t$ considered so far is well capable to issue RSPs, due to the high exposure to the risky underlying on the balance sheet. In this section, we evaluate the opposite case of an issuer with no exposure to the risky underlying asset. The assets of the issuer characterized by $\widetilde{A}_t = F_t$ are completely free of risk. This asset structure implies that the issuer could borrow a face value up to $F_T = A_0 \cdot e^{rT}$ at the risk-free rate.

Again, we first examine the risk-neutral probability of default. The case of straight debt financing is apparently simple. As long as the face value of the bond $\overline{B}$ is lower than the asset payoff F_T, the default probability is zero. If more debt is issued, then both the leverage ratio and the default probability increase to one.

Figure 2.4 illustrates the default probability of RSP issuers. The graph on the left shows the default probability of an issuer financed with PPNs and the graph on the right shows the default probability of an issuer financed with DCNs.

The default probability of the PPN issuer increases monotonically as long as $\overline{P} \leq F_T$. The issuer defaults for high values of the underlying asset. When more debt is issued, i.e., for $\overline{P} > F_T$, the default probability rises to one. The DCN issuer does not default as long as $\overline{D} \leq F_T$. For higher debt volumes of $\overline{D} > F_T$, the default probability jumps up and tends to one, as the issuer is now defaulting for high realizations of the underlying asset $\widetilde{R}_T > \frac{F_T}{\gamma \overline{D}}$.

Since the issuer of straight debt never defaults for $\lambda < 1$, the issuer of RSP is always worse off. The low-risk issuer introduces a dependency to the risky asset by issuing RSPs. This dependency increases the probability of default for some leverage ratios, but can never decrease it. This result again can be generalized. (See appendix 2.B for proof.)

Proposition 2.3 (Risk increase of low-risk issuer)
For any attainable leverage ratio $\widehat{\lambda} < 1$, the risk-neutral probability of default of a low-risk issuer financed with RSP is always greater than or equal to the default probability of a low-risk issuer financed with straight debt, i.e., $pd_{RSP}\left(\widehat{\lambda}\right) \geq pd_B\left(\widehat{\lambda}\right)$.

This result at first seems problematic from the regulator's point of view, since he is naturally concerned about increasing default probabilities. But to evaluate if the issuer actually prefers to issue RSPs over straight debt, we again need to inspect the issuer value. The corresponding issuer values are depicted in figure 2.5. The graph on the left shows the value of an issuer financed with PPNs (solid line) and the graph on the right shows the value of an issuer financed with DCNs (solid line). Both plots also show the issuer value under straight debt financing as a reference case (dashed line).

Figure 2.4: Probability of default depending on leverage (low-risk)

The graph on the left shows the probability of default pd_{PPN} for an issuer with PPN financing. The graph on the right shows the default probability pd_{DCN} for an issuer with DCN financing. We compute the values using the model parameters $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

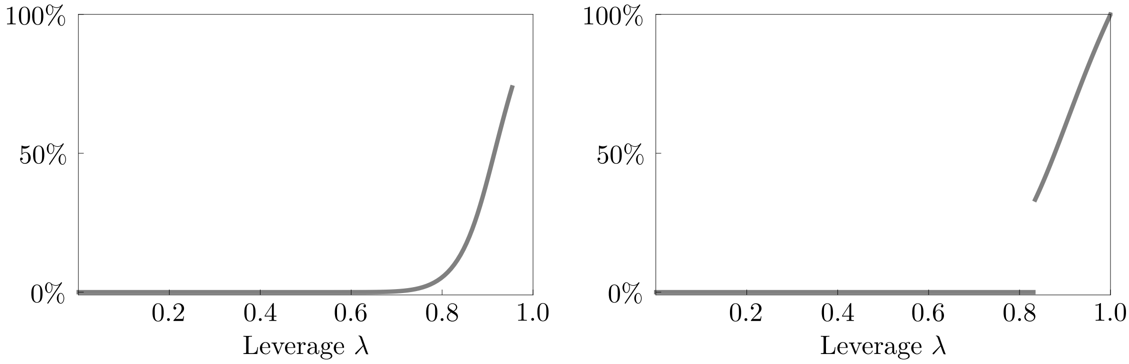
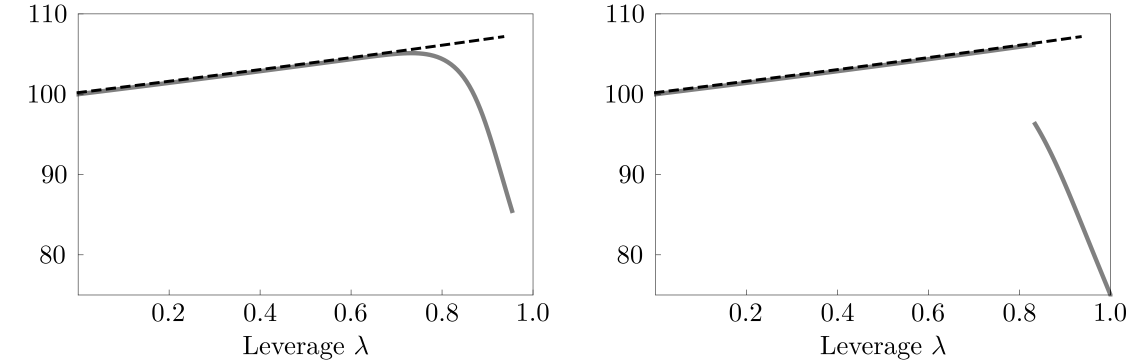


Figure 2.5: Issuer value depending on leverage (low-risk)

The graph on the left shows the issuer value $V_{0,PPN}$ with PPN financing (solid line). The graph on the right shows the issuer value $V_{0,DCN}$ with DCN financing (solid line). Both graphs also show the issuer value $V_{0,B}$ with straight debt financing (dashed line). We compute the values using the model parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.



The issuer value under straight debt financing increases linearly with leverage λ, since tax benefits can be generated at no additional cost. In contrast, the RSP issuer incurs an additional bankruptcy cost when the probability of default rises. Consequently, the issuer value with RSP financing lies below the value under straight debt financing whenever there is a positive default probability. The issuer value with PPN financing decreases for high leverage ratios up to $\overline{P} = X_P$. The issuer value with DCN financing agrees with the value under straight debt financing up to $\overline{D} = F_T$. It drops down and decreases towards $(1-\alpha)A_0$ when the face value $\overline{D}$ is further increased. We summarize this important result in the following proposition. (See appendix 2.B for proof.)

Proposition 2.4 (Value destruction of low-risk issuer)
For any attainable leverage ratio $\widehat{\lambda} < 1$, the value of a low-risk issuer financed with RSPs never exceeds the value of a low-risk issuer financed with straight debt, i.e., $V_{0,RSP}\left(\widehat{\lambda}\right) \leq V_{0,B}\left(\widehat{\lambda}\right)$.

We conclude from propositions 2.3 and 2.4 that low-risk issuers never benefit from the issuance of RSPs. The highest tax benefits are generated by issuing risk-free debt. In contrast, the issuance of RSPs may increase the default probability. In these cases, the bankruptcy costs eat up the tax benefits. Again, this result holds for both types of contracts, i.e., for concave as well as for convex payoff structures. The regulator does not have to consider the danger of RSP financing for low-risk issuers, since they do not voluntarily issue them.

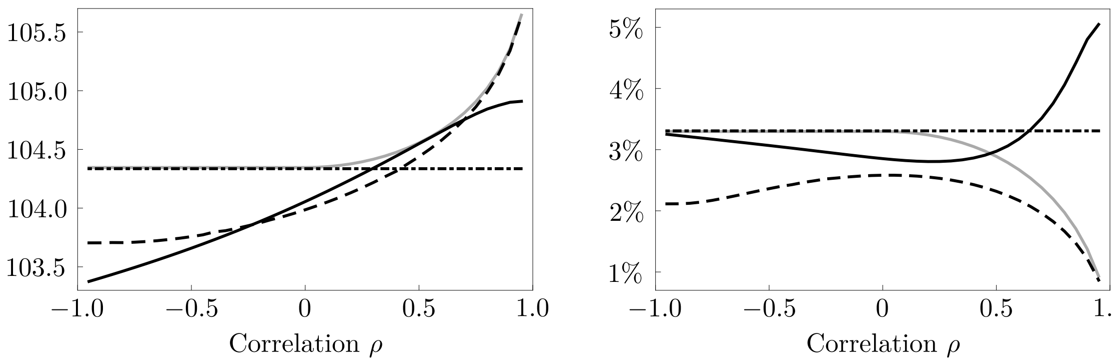
We conclude that the benefits of issuing RSPs depend critically on the risk of the issuer's asset portfolio. The high-risk issuer can use RSPs to reduce the probability of default, i.e., as a form of insurance. The low-risk issuer has no need for insurance. Thus, RSPs have the opposite effect in this case. They increase the riskiness of the issuer, since they introduce a dependency on the risky underlying asset.

2.5 Optimal financing choice

In the next step, we evaluate the impact of RSPs on the optimal financing choice of the bank. To accommodate a more general and realistic set of scenarios, we assume that the bank's assets and the underlying asset of the RSPs are not the same. Hence, the asset portfolio is exogenous and has a constant volatility σ_A. However, the returns of the assets and the underlying asset are correlated with coefficient $\rho \in (-1, 1)$. A perfect correlation of $\rho = 1$ corresponds to the high-risk issuer described in section 2.4.1. We resort to numerical solutions for the claim values in this section.

Figure 2.6: Issuer value and default probability depending on correlation

The plot on the left shows the optimal issuer value V_0 with PPN financing (solid black line), with DCN financing (dashed line) and with straight debt financing (dot-dashed line) depending on the correlation ρ. The plot on the left also shows the optimal value when the issuer can finance itself with any mix of straight debt and DCNs (solid gray line). The plot on the right shows the corresponding risk-neutral default probabilities. We compute the values using the model parameters $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.



We first inspect the issuer value V_0 which depends on the correlation ρ between the returns of the assets and the underlying asset (see left-hand plot of figure 2.6). The value of an issuer financed with only straight debt (the dot-dashed line) is obviously independent of the risky underlying asset. The issuer values under PPN financing (solid black line) and under DCN financing (dashed line) both increase with the correlation. For negative and low positive correlations, financing with RSPs reduces the issuer value compared to straight debt financing. Since the payoff of both products increases in the value of the underlying asset, financing with RSPs is only beneficial when the correlation is high, i.e., when the values of the issuer's assets and the underlying security behave similarly.

This finding is confirmed when we examine a mix of different debt contracts. The issuer value for a financing mix consisting of straight debt and DCNs is also depicted in the left-hand plot of figure 2.6 (solid gray line). For negative correlations, the issuer uses only straight debt. However, the issuer always adds a strictly positive fraction of RSPs to the financing mix when the correlation turns positive. The weight of RSPs in the financing mix increases monotonically with the correlation up to a share of 100%. The results for a financing mix which includes PPNs (not shown) are qualitatively the same.

Next, we evaluate the impact of RSP financing on the default risk of the issuer. We plot the risk-neutral probability of default *pd* depending on the correlation ρ on the right-hand side of figure 2.6. Again, the default risk of an issuer financed with only straight debt is independent of the correlation. PPN financing (solid black line) turns out to reduce the default probability of the issuer for low positive and for negative correlations. However, it is not optimal to finance with PPNs for those correlations. But PPN financing increases default risk for high correlations, when PPN have an advantage over straight debt in terms of value maximization.

In contrast, for all possible correlations DCN financing reduces the default risk of the issuer compared to straight debt financing. The effect is also large in magnitude. For example, the default risk is reduced from 3.3% to 0.9% for a correlation of $\rho = 1$. The hedging benefit is still present when we examine an optimal mix of DCNs and straight debt (solid gray line). DCNs are not added to the financing mix for negative correlations. The default risk of the issuer declines with an increasing share of DCNs in the financing mix and thus with an increasing correlation. The issuer could further decrease the default probability by issuing only DCNs, but doing so is not optimal in terms of value maximization.

Clearly, adding RSP to the financing mix is always beneficial for positive correlations between the assets and the underlying. When issuing DCNs, the issuer can thereby reduce its default probability. In contrast, PPN financing causes the default risk of the issuer to increase when the correlation is high. A comparative static analysis of these results is contained in appendix 2.C.

2.6 Optimal risk-taking

We have shown in section 2.4 that high-risk issuers prefer RSP over straight debt. Low-risk issuers prefer the opposite. Furthermore, we have shown in section 2.5 that issuers optimally add RSP to their financing mix whenever the correlation between the assets and the underlying is positive. In this section, we tackle the question of how the issuer's choice of asset risk is influenced when RSPs are available as an instrument for financing and risk management.

For this purpose, we consider an asset portfolio that is a linear combination of the underlying $\widetilde{R}$ with weight $\delta \in [0,1]$ and the risk-free asset F with weight $1-\delta$. Thus, the asset value $\widetilde{A}_t$ of the unlevered issuer at time t is given by

$$\widetilde{A}_t = \delta \cdot \widetilde{R}_t + (1-\delta) \cdot F_t. \tag{2.11}$$

The financial institution trades in securities, lends money to consumers and enterprises, purchases government bonds, and holds central bank deposits. We assume that all such investments are separable into a component impacted by the source of uncertainty $\widetilde{R}$ and a residual component F, which is free of risk. The high-risk and low-risk issuers discussed in section 2.4 are represented by $\delta = 1$ and $\delta = 0$, respectively. The asset structure described here corresponds to the case $\rho = 1$ discussed in the previous section 2.5, i.e., the case in which RSPs add most value. However, the volatility of the assets is no longer constant. The parameter δ scales the volatility of the assets such that $\sigma_A = \delta \cdot \frac{R_t}{A_t} \cdot \sigma_R$.

In the following, we analyze the issuer's optimal financing choice for a given asset risk weight δ as well as the optimal risk weight choice. In section 2.6.3, we consider the risk-shifting incentives of equity holders. We use numerical optimization techniques, since solutions for the optimal decisions cannot be obtained in closed form. We control the optimization results for many different scenarios. The comparative static analysis can be found in appendix 2.C.

2.6.1 Principal-protected notes

The issuer can finance with straight debt, PPNs, or a mix of both. We determine the optimal leverage ratio λ^* for each risk weight δ. Figure 2.7 shows the resulting optimal issuer values on the left-hand side and the corresponding probability of default given the optimal leverage on the right. The graphs show the values for the issuer financed with straight debt (thin black line), for an issuer financed with PPNs only (thick gray line), and for an issuer financed with a mix of bonds and PPNs (dashed line).

We look first at an issuer with one single debt claim outstanding. The optimal value with straight debt financing strictly decreases with the asset risk weight δ. The maximum is at $\delta = 0$. The probability of default pd_B strictly increases with δ from zero for $\delta = 0$ up to 3.3% for $\delta = 1$. These findings reproduce the well known results of Merton (1974) and (Leland, 1994). We use this case as a reference to evaluate the impact of RSP financing.

We can reconcile the results shown in figure 2.7 with the findings from section 2.4. The optimal value of a low-risk issuer with $\delta = 0$ financed with RSPs is below the value of the issuer financed with straight debt. The opposite is true for a high-risk issuer with $\delta = 1$. Hence, there must be an asset risk weight for which the issuer is indifferent between financing with bonds and RSPs. For the chosen parameter values, this risk weight is approximately at $\delta = 0.29$. For lower risk weights, the issuer prefers to finance with straight debt. For higher risk weights, the issuer prefers to finance with PPNs.

The optimal issuer value under PPN financing is a hump-shaped curve with its maximum at $\delta = 0.39$. For all tested scenarios, the maximum issuer value with PPN financing never exceeds the maximum value when the issuer uses straight debt. Given the optimal choice, the corresponding default probability decreases for low risk weights and increases sharply around the maximum issuer value at $\delta = 0.39$, thereby surpassing the default probability under straight debt financing. It continues to increase up to the maximum of 5.1% for $\delta = 1$. At first, this finding seems to contradict proposition 2.1, which states that the default probability with PPN financing should be reduced compared to straight debt financing. However, the issuer has an incentive to optimally increase the leverage ratio λ. In the case of PPN financing, this increase in leverage eats up the beneficial effect of RSPs on the default probability.

We next consider an issuer who can choose any arbitrary mix of zero bonds and PPNs to finance itself. Since this financing mix adds an extra degree of freedom to the optimization, the issuer can never be worse off compared to the case of a single debt claim.

The most important finding is that the issuer always chooses to finance itself with a positive amount of PPNs for all positive risk weights $\delta > 0$. The low-risk issuer with $\delta = 0$ finances itself with straight debt only as shown in propositions 2.3 and 2.4. The issuer combines bonds and principal-protected notes for $0 < \delta < 0.39$. For higher risk weights, the issuer relies only on PPNs for financing. The resulting curve for the issuer value is a monotonically decreasing function in the risk weight δ. The maximum is at $\delta = 0$, i.e., the case of straight debt financing. We also observe that given optimal leverage, the probability of default is always equal to or higher than the default probability of the bond financed issuer.

Finally, we consider the choice of the optimal risk weight δ^*. An issuer always has the incentive to reduce the risk weight as much as is feasible, i.e., an issuer with full flexibility chooses a risk weight of $\delta = 0$. This is good news for the regulator, since the default probability at the optimum is zero. However, should the issuer be constrained from further reducing the risk weight, the regulator might be concerned in two cases. In the first case, when the minimum attainable risk weight is below 0.39, the issuer optimally chooses a mix between straight debt and PPNs. However, a financing with only PPNs would result in a lower probability of default. In the second case, when the minimum attainable risk weight is above 0.39, the issuer relies only on PPNs for financing. Again, a lower probability of default can be achieved by financing with straight debt only.

As noted earlier, the regulator can exploit the one-to-one relation between the default probability and the leverage ratio to limit the risk taking incentive of the issuer. Unfortunately, this relation changes fundamentally with the risk weight δ. For example, if we look at the issuer value under PPN financing, the value maximizing leverage ratio at $\delta = 0.39$ is higher than the leverage ratio at $\delta = 1$. However, the resulting probability of default at $\delta = 1$ is more than five times as high. Hence, the maximum leverage ratio prescribed by the regulator should either incorporate the asset risk of the issuer or it should be geared towards the worst-case scenario, i.e., $\delta = 1$.

We conclude that adding PPN to the financing mix of the issuer can increase the issuer value. However, there is also the danger that the default risk of the issuer increases. This increase is especially severe for an issuer who inherits a high exposure to the risky asset and is either not capable of adjusting this exposure in the short run or incurs a high cost when doing so.

2.6.2 Discount notes

Next, we analyze an issuer who is financed with straight debt, DCNs, or a mix of both. Again, we determine the optimal leverage ratio λ^* for each risk weight δ. Figure 2.8 depicts the resulting optimal issuer values on the left-hand side and the corresponding probability of default at the optimum on the right. The graphs show the values for the issuer financed with straight debt (thin black line), for an issuer financed with DCNs only (thick gray line), and for an issuer financed with a mix of bonds and DCNs (dashed line).

We first analyze an issuer with one single debt claim outstanding. The optimal value with straight debt financing is still our reference scenario. Moreover, figure 2.8 mirrors the results from section 2.4. The optimal value of a low-risk issuer with $\delta = 0$ financed with RSPs is below the value of the issuer financed with straight debt. The opposite is true for a high-risk issuer with $\delta = 1$. The issuer is indifferent at a risk weight of approximately $\delta = 0.22$. For lower risk weights, the issuer prefers to finance with straight debt. For higher risk weights, the issuer prefers to finance with DCNs.

Figure 2.7: Optimal issuer value and probability of default with PPN financing

The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage. The plots show the values for an issuer financed with straight debt (thin black line), for an issuer financed with PPN only (thick gray line) and for an issuer financed with a mix of bonds and PPN (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, $X_P = 100$, *and* $\pi = 0.5$.

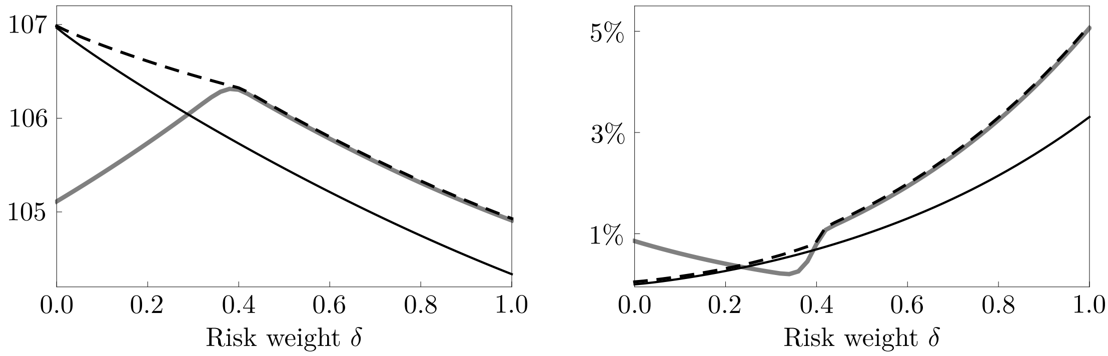
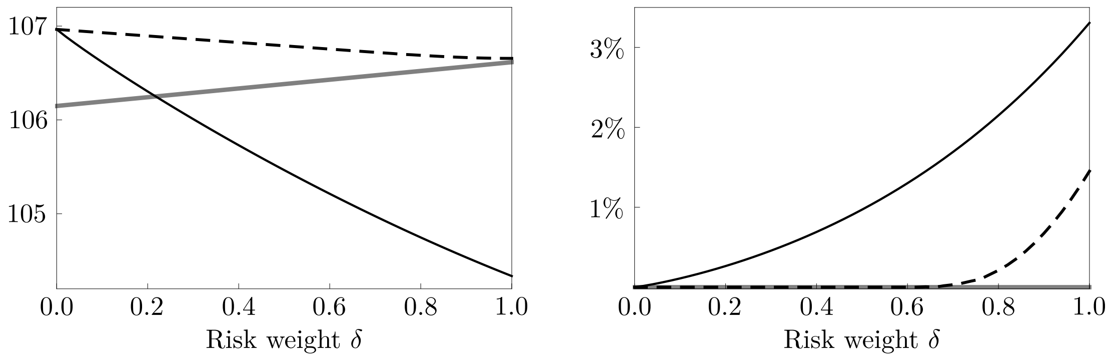


Figure 2.8: Optimal issuer value and probability of default with DCN financing

The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage choice. The plots show the values for an issuer financed with straight debt (thin black line), for an issuer financed with DCN only (thick gray line) and for an issuer financing with a mix of bonds and DCN (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, *and* $X_D = 125$.



The issuer value under DCN financing increases linearly with the risk weight δ. The maximum is at $\delta = 1$. Remarkably, the default probability drops to zero for all risk weights δ. This drop is due to the DCNs' insurance property discussed earlier. The issuer reduces the repayment in bad states of the world and consequently lowers both the default risk and the expected bankruptcy costs.

When the issuer can mix the two debt claims, it chooses to issue a positive amount of DCNs for all positive risk weights $\delta > 0$. The curve slightly decreases, i.e., the optimal risk weight δ^* is again zero. The corresponding default probability is zero for risk weights lower than 0.7 and then increases monotonically up to 1.5% for $\delta = 1$. The default probability at $\delta = 1$ is positive, since the issuer optimally includes a small but positive fraction of straight debt in the financing mix. Most importantly, the probability of default, given the optimal leverage, is always lower than in the case of straight debt financing. Hence, the issuance of DCNs is desirable from the regulatory point of view and should be actively encouraged.

In short, low-risk issuers with $\delta = 0$ optimally issue bonds, and risky issuers, i.e., $\delta > 0$, prefer to add RSPs to the financing mix. Issuers with high asset risk thereby increase the probability of default when issuing PPNs and reduce it by issuing DCNs, compared to the benchmark case of straight debt financing.

2.6.3 Risk-shifting incentives

We have shown that unconstrained issuers prefer to reduce their asset risk weight to $\delta = 0$. This result implies that only straight debt is used and RSPs are not issued. Only an issuer constrained in the choice of the asset risk weight adds RSP to the financing mix. On the one hand, the issuer might voluntarily keep an exposure to the underlying security, for example, as inventory for trading or due to related businesses. On the other hand, the issuer might not be able to adjust the asset risk weight — at least, not in the short run — due to liquidity constraints or transactions costs.

In addition, the equity holders might not behave optimally in terms of firm value maximization when it is possible to adjust the asset risk weight after debt is issued. Our model considers an initially unlevered issuer. The issuer pays out the value of issued debt as a special dividend to equity holders. This setup ensures that if asset risk is contractible, then equity holders maximize the total value of the firm, i.e., the sum of debt and equity value. In the Merton model, the equity value of a levered firm can be thought of as a call option on the firm's assets with the face value of debt corresponding to the strike price. The value of the call option increases with the volatility of the underlying asset. Hence, once debt is issued, equity holders have an incentive to increase the asset risk.

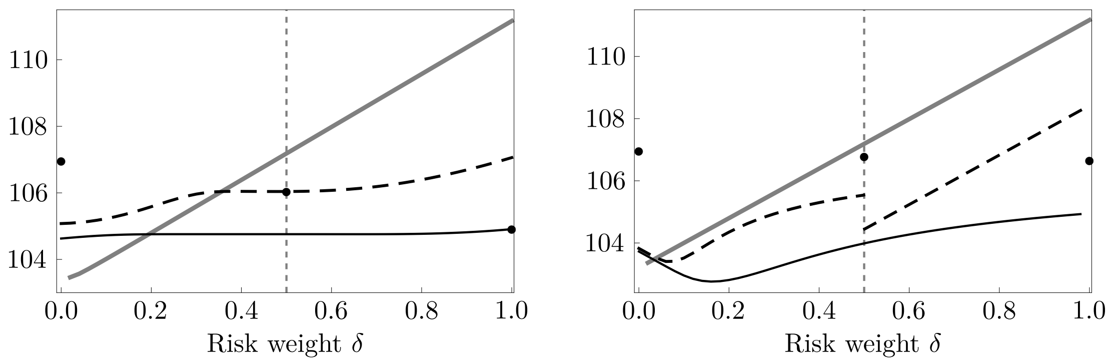
Figure 2.9 shows the total shareholder wealth when the equity holders engage in risk-shifting behavior. The issuer determines the face value of debt and the debt value, which is paid as a special dividend to equity holders, based on an initial risk weight $\widehat{\delta}$. After debt is issued, equity holders are able to adjust the asset risk weight from $\widehat{\delta}$ to δ. Debt holders do not anticipate this behavior. The plot shows the resulting optimal shareholder wealth, i.e., the sum of the special dividend given $\widehat{\delta}$, and the equity value at the final risk weight δ, for financing with a mix of straight debt and PPNs on the left and for a mix of straight debt and DCNs on the right. We consider three different initial risk weights: $\widehat{\delta}=0$ (thick gray line), $\widehat{\delta}=0.5$ (dashed line) and $\widehat{\delta}=1$ (thin black line). The graphs show the respective values for $\delta=\widehat{\delta}$ as black dots, since the functions are not continuous.

The low-risk issuer with $\widehat{\delta}=0$ is financed only with straight debt. The issuer has an initial default probability of zero. Increasing the asset risk weight causes the default probability to increase, which leads to a drop in value. Shareholder wealth increases linearly with the final risk weight δ and surpasses the initial value for risk weights of $\delta > 0.47$. The maximum at $\delta = 1$ results in a shareholder wealth of 111.16 compared to an initial value of 106.94. Similarly, an issuer with initial risk weight $\widehat{\delta} = 0.5$ is willing to increase the asset risk weight up to $\delta = 1$. However, the magnitude of the effect is not as large as it is for the low-risk issuer. In contrast, an issuer with initial risk weight $\widehat{\delta} = 1$ is not willing to reduce the risk weight. Hence, risk-shifting is beneficial for the issuer, who in all three cases chooses a final risk weight of $\delta = 1$.

Because the risk-shifting phenomenon is well known, we assert that debt holders anticipate the behavior of the issuer. Risk-shifting is to the disadvantage of debt holders, since the subsequent increase in bankruptcy costs reduces the value of debt. Hence, debt holders value their claims as if the issuer chooses an initial risk weight of $\widehat{\delta} = 1$. A lower choice of asset risk weight by the issuer is not credible. Consequently, equity holders determine their optimal response for an initial risk weight $\widehat{\delta}=1$ and adjust the debt mix accordingly. This debt mix includes RSPs, since RSPs are added to the financing mix for all positive risk weights $\delta > 0$.

Figure 2.9: Issuer value and risk-shifting incentives

The plot on the left shows the total shareholder wealth when financing with a mix of straight debt and PPN allowing for a change of the risk weight δ after issuance. Debt is issued assuming at an initial risk weight of $\widehat{\delta} = 0$ (thick gray line), $\widehat{\delta} = 0.5$ (dashed line) or $\widehat{\delta} = 1$ (thin black line). The plot on the right shows the total shareholder wealth when financing with a mix of straight debt and DCN for the same initial risk weight scenarios. The graphs show the respective values for $\delta = \widehat{\delta}$ as black dots since the functions are not continuous. We compute the values using the parameters $A_0 = 100,\ \sigma_R = 0.2,\ r = 0.15,\ T = 1,\ \tau = 0.5,\ \alpha = 0.25,\ X_P = 100,\ \pi = 0.5,$ and $X_D = 125$.



2.7 Optimal product design

Although our focus has been on RSPs with exogenous product properties, we note that these parameters are mainly determined by the preferences of the retail investors. However, the properties of the offered product are, at least to some extent, at the discretion of the issuer. Hence, we analyze what set of parameters the issuer optimally chooses and how this choice affects issuer value and default probability.

2.7.1 Principal-protected notes

We first focus on an issuer financed only with PPNs. As before, the guaranteed amount is fixed to the level $X_P = R_0$. The issuer optimizes over two parameters, the face value $\overline{P} \geq 0$ and the participation rate $\pi \geq 0$.

The optimal participation rate π^* nearly linearly increases with the risk weight δ, whereby we always observe $\pi^* \cdot \overline{P}^* > \delta$. The issuer defaults for high realizations of the underlying $\widetilde{R}_T$. The optimal parameter values range from $\pi^* = 0$ for a low-risk issuer with $\delta = 0$ up to $\pi^* = 1.33$ for a high-risk issuer with $\delta = 1$.

Figure 2.10 shows the optimal issuer value on the left and the corresponding probability of default on the right. Both plots present the values for an issuer financed with the standard PPN contract with $\pi = 0.5$ (thick gray line) and the values for the optimally designed PPN contract (dashed line). For comparison, we also include the issuer value with straight debt (thin black line), which coincides with $\pi = 0$.

We examined the hump-shaped curve for PPN financing with $\pi = 0.5$ earlier in section 2.6.1. The issuer can always increase the value by adjusting the participation rate. Both curves agree for $\delta = 0.4$, where the optimal participation rate is approximately $\pi^* = 0.5$. For values below $\delta = 0.4$, the issuer can increase the value by lowering the participation rate. For values above $\delta = 0.4$, the issuer is better off by increasing the participation rate. Since the optimal participation rate is $\pi^* = 0$ for the low-risk issuer with $\delta = 0$, the corresponding issuer value agrees with the case of straight debt financing. The issuer value under the optimal participation rate declines with the risk weight δ up to values of $\delta = 0.9$ and then increases slightly.

The optimization over the participation rate also has important consequences on the risk profile of the issuer. For low risk weights, the default probability is close to that of an issuer financed with straight debt. The probability of default monotonically increases with the risk weight δ, whereby it is always larger compared to an issuer financed purely with straight debt. For values of $\delta > 0.74$, the default probability of the optimally designed

Figure 2.10: Issuer value and default probability with optimally designed PPN

The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage. The plots show the values for an issuer financed with straight debt (thin solid line), for an issuer financed with the standard PPN contract (thick gray line) and for an issuer financed with the optimally designed PPN contract(dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, *and* $X_P = 100$.

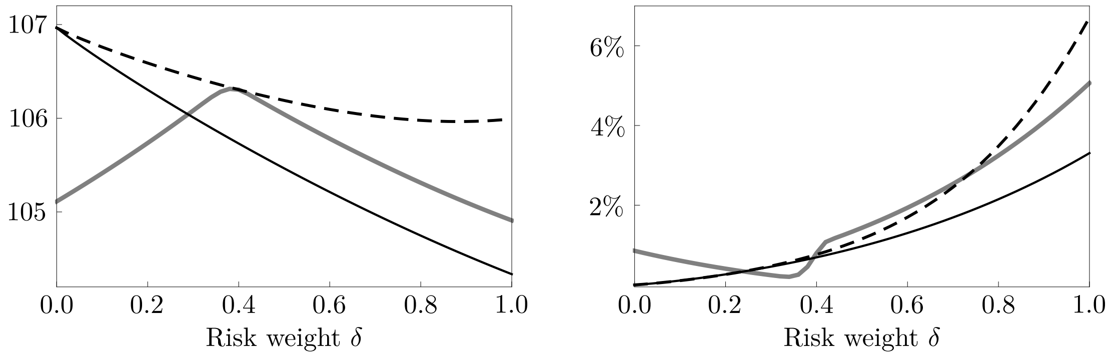
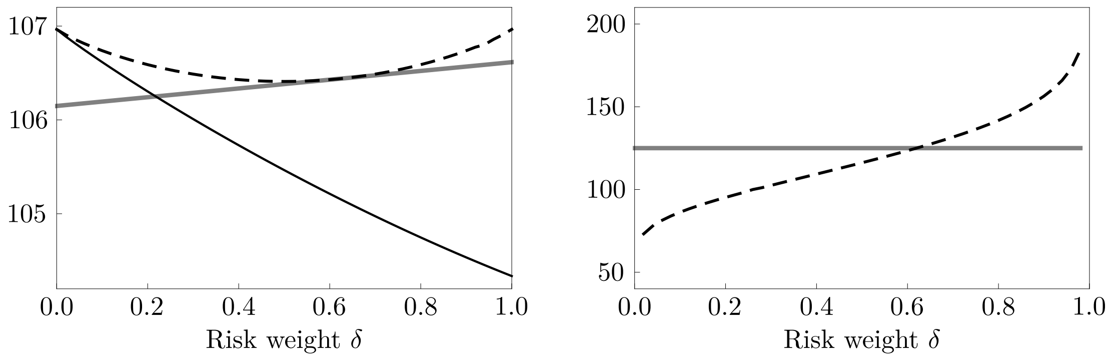


Figure 2.11: Issuer value and default probability with optimally designed DCN

The graph on the left shows the optimal issuer value. The graph on the right depicts the repayment amount X_D. *The plots show the values for an issuer financed with straight debt (thin solid line), for an issuer financed with the standard DCN contract (thick gray line) and for an issuer financed with the optimally designed DCN contract(dashed line). We compute the values using the parameters* $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, *and* $\alpha = 0.25$.



PPN contract surpasses that of the standard PPN contract with $\pi = 0.5$. The default probability of a high-risk issuer with $\delta = 1$ jumps from 5.1% up to 6.7%. This value is twice as high as the corresponding default probability of 3.3% with straight debt financing.

We conclude that allowing for optimal choice of the product parameters confirms our verdict on PPN financing by high-risk issuers. The issuer value can be increased at the expense of a considerably greater probability of default.

2.7.2 Discount notes

For DCN financing, the product parameter of choice is the maximum repayment X_D. The issuer simultaneously determines the optimal volume $\overline{D} \geq 0$. We present the results of the optimization in figure 2.11. The issuer value is shown on the left. The optimal maximum repayment amount X_D^* is plotted on the right. Both plots present the values for an issuer financed with the standard DCN contract with $X_D = 125$ (thick gray line) and the values for the optimally designed DCN contract (dashed line).

Figure 2.11 shows that the cap X_D^* of the optimally designed DCN contract increases monotonically with δ. For $\delta = 0.62$, the optimal cap is roughly equal to 125, which corresponds to the parameter of the standard DCN contract discussed in the previous sections. The maximum promised repayment is equal to 184 for the high risk issuer with $\delta = 1$.

The most remarkable outcome is that the issuer who uses the optimally designed DCN contract never defaults for any given risk weight δ. So the favorable characteristic already derived in section 2.6.2 is again observed. Consequently, the issuer value can be further increased.

2.7.3 Further products

In this section, we test the robustness of our results for two different product types. As a representative for products with discontinuous payoffs, we consider express notes (ENs). In addition, we analyze short notes (SNs), whose payoff decreases when the value of the underlying increases.

Formally, the promised payoff of an EN (see left-hand side of figure 2.12) is given by

$$CE_T = \begin{cases} (1+r_E) \cdot \overline{E} & \text{if } \widetilde{R}_T \geq X_E, \\ \min\left\{1, \frac{1}{X_L} \cdot \widetilde{R}_T\right\} \cdot \overline{E} & \text{if } \widetilde{R}_T < X_E. \end{cases} \tag{2.12}$$

Figure 2.12: Promised payoff of further products

The graph on the left shows the promised payoff CE_T of an express note with strike price X_E and coupon r_E. The investor incurs losses for values of the underlying below X_L. The graph on the right shows the promised payoff CS_T of a short note with strike price $X_S = R_0$ and upper cap X_M.

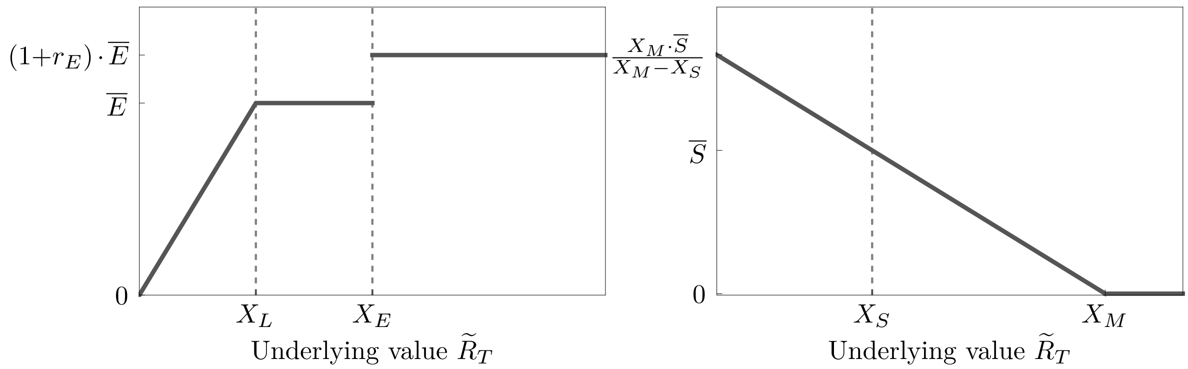
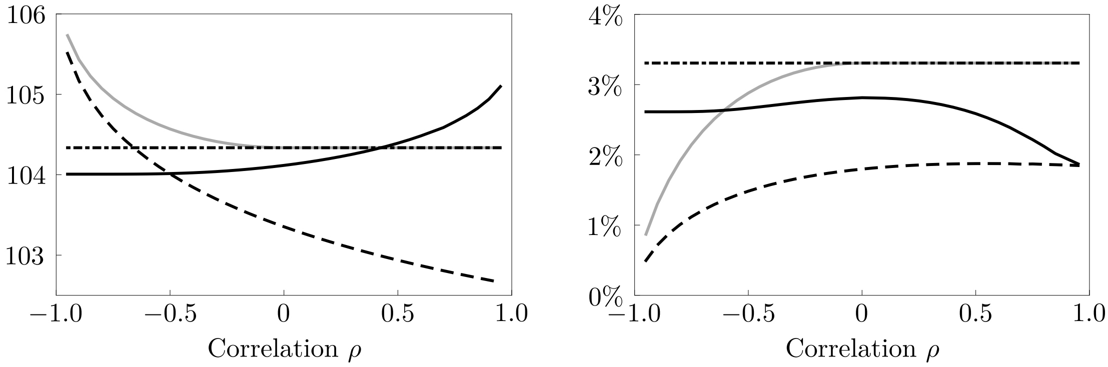


Figure 2.13: Issuer value and default probabilities of further products

The plot on the left shows the optimal issuer value V_0 for straight debt financing (dot-dashed line), for EN financing (solid black line), for SN financing (dashed line) and for a financing mix of short notes and straight debt (solid gray line) depending on the correlation ρ. The plot on the right shows the corresponding risk-neutral default probabilities. We compute the values using the model parameters $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_L = 50$, $X_E = 100$, $r_E = 30\%$, $X_S = 100$, and $X_M = 200$.



If the value of the underlying asset at maturity is above the lower threshold X_L, then the investor receives the nominal amount $\overline{E}$. Should the underlying value end up above the upper threshold $X_E > X_L$, the investor receives an additional coupon payment of r_E. Should the underlying asset fall below X_L, the investor incurs a loss. Express notes can be thought of as discount notes with strike price X_L and an additional coupon payment above the second strike X_E, where the promised payoff has a jump.

We represent the promised payoff of SNs (see right-hand side of figure 2.12) as

$$CS_T = \max\left\{\frac{X_M - \widetilde{R}_T}{X_M - X_S}, 0\right\} \cdot \overline{S}, \tag{2.13}$$

where the strike price X_S is usually set equal to the initial value R_0 of the underlying asset. The promised payoff of an SN is positive as long as $\widetilde{R}_T < X_M$. The investors get the maximum payoff $\frac{X_M}{X_M - X_S} \cdot \overline{S}$ when the value of the underlying drops to $\widetilde{R}_T = 0$.

Figure 2.13 shows the issuer value on the left and corresponding default probabilities on the right depending on the correlation ρ for four different scenarios: straight debt financing (dot-dashed line), financing with ENs (solid black line), financing with SNs (dashed line) and financing with a mix of straight debt and SNs (solid gray line).

The value of an issuer financed with ENs increases monotonically with the correlation ρ. The graph looks similar to the issuer value with DCN financing (see figure 2.6). Thus, the findings from section 2.5 are once more confirmed. For high correlations, the issuer can increase its value by financing with ENs. The default probability can be reduced for any correlation. In addition, an issuer financing with a mix of straight debt and ENs (not shown) adds a positive fraction of ENs to the financing mix for all positive correlations.

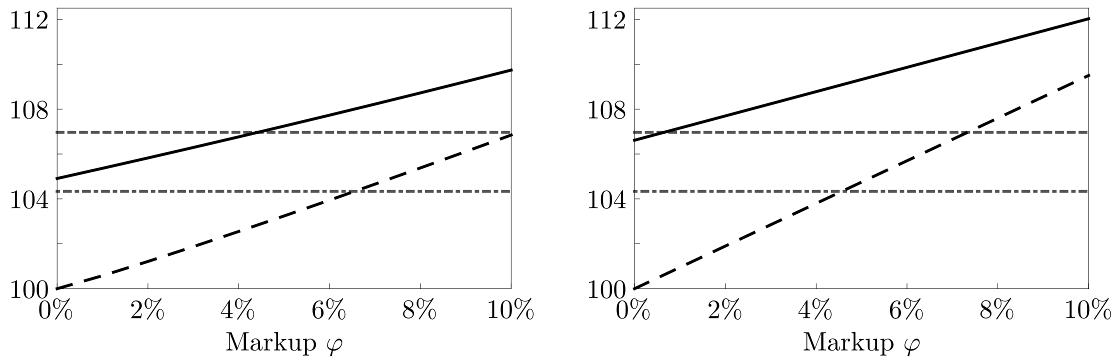
The results for SN financing reverse the results from section 2.5. Due to the negative relation between the SN payoff and the underlying, the issuer benefits from SNs when the correlation between the asset value return and the underlying asset's return is negative. The issuer value with SN financing decreases with the correlation. Issuers add SNs to a financing mix with straight debt for all negative correlations. In addition, the default probability can be significantly reduced for all correlations. Hence, SNs possess an insurance property similar to that of DCNs. We conclude that our results are robust to important variations on the payoff of RSPs.

2.7.4 Product complexity

An important empirical observation is that issuers sell RSPs to retail investors at a sizable markup. Stoimenov and Wilkens (2005) report an average markup at issuance of 3.9% for the German market. This markup increases with the complexity of the products. In

Figure 2.14: Issuer value with product markup

The plot on the left shows the optimal issuer value with PPN financing (solid line) depending on the product markup φ. The plot on the right shows the optimal issuer value with DCN financing (solid line). Both plots include the optimal issuer value with straight debt financing for a high risk issuer with $\delta = 1$ (dot-dashed horizontal line) and a low risk issuer with $\delta = 0$ (dashed horizontal line). Both plots also include the issuer value for PPN financing and DCN financing for a scenario without tax benefits (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.



a theoretical contribution, Carlin (2009) establishes a link between product complexity and the ability to generate profits from that particular product.

In this section, we test the robustness of our model with respect to this empirical observation. We incorporate the additional friction that the issuer is able to sell the RSP at a markup φ on the fair value. Such a markup comprises fees for sales, structuring, and depository. The markup φ is an upfront fee that investors have to pay at issuance. Hence, the fair value materializes directly after issuance and the market remains free of arbitrage opportunities. The markup directly increases the size of the special dividend to equity holders, which is equal to $(1+\varphi)\cdot D_0$.

Figure 2.14 shows the optimal issuer value (solid line) for PPN financing on the left and DCN financing on the right. We display a high-risk issuer with $\delta = 1$, since RSPs are used to the maximum extent by this issuer. For comparison, figure 2.14 also shows the issuer value under straight debt financing for a low-risk issuer with $\delta = 0$ (dashed horizontal line) and for a high-risk issuer with $\delta = 1$ (dot-dashed horizontal line). Obviously, both are independent of the product markup φ.

The optimal value of an issuer financed with RSPs nearly linearly increases with the product markup φ. For both product types, the issuer value is greater compared to the high-risk issuer with straight debt financing. For PPNs, a product markup of $\varphi > 4.4\%$ is required such that PPN financing with $\delta = 1$ is advantageous to straight debt financing when $\delta = 0$. Hence, the maximum value from section 2.5 is exceeded. For DCNs, a very low markup of at least $\varphi = 0.7\%$ is required such that DCN financing is beneficial.

We regard the product markup φ as a substitute for the tax benefit of debt. Figure 2.14 also shows the respective issuer value for a scenario without tax benefits (dashed line), i.e., $\tau = 0$. For both product types, the issuer value is nearly linearly increasing in the markup φ. We conclude that our results are robust to the specific implementation of the friction. However, the financing benefit must be linked to the outstanding volume of the RSP.

2.8 Conclusion

So far, the literature on retail structured products has focused on the profit maximizing behavior of the issuer. We contribute two new themes to this literature. First, we argue that RSPs are a valuable funding source for the issuer. Consequently, the investors in RSPs are to some extent exposed to the issuer's business risk. Second, we show that RSPs can be used for risk management purposes. The use of RSPs as a hedging instrument enables issuers to transfer risks outside the financial system. In this paper, we evaluate the conditions under which RSPs can indeed have a positive impact not only on the issuer value, but also on the default probability.

In the context of our model, we show that low-risk issuers still use straight debt financing, but high-risk issuers prefer RSP financing over straight debt. By holding the leverage ratio constant, high-risk issuers can increase the firm value and at the same time decrease the probability of default. Nevertheless, the issuer has an incentive to optimally adjust the leverage ratio and asset risk weight. Even when accounting for these optimal decisions, RSPs are added to the financing mix when the correlation between the issuer's assets and the underlying asset is positive and when the assets are risky. Issuers with high asset risk thereby increase the probability of default when issuing PPNs, but they reduce it by issuing DCNs. The results also hold when the issuer can optimally design the RSP. Furthermore, our results are also robust to the empirically observed friction of a markup on the RSP's fair value charged by the issuer.

Adding retail structured products to the financing mix is especially beneficial when the value of the issuer's assets strongly depends on the value of the underlying asset of the RSPs. The underlying asset can either be directly included in the asset portfolio as, for example, part of direct investments, or as inventories for trading. In addition, some other components of the asset portfolio might be highly correlated with the underlying asset. For example, the value of a loan provided by the bank is highly correlated with the value of the debtor's equity, since both claims can be thought of as claims contingent on the debtor's assets. Hence, we conclude that the issuer's asset portfolio can be decomposed into a component which depends on the underlying and residual component.

Our model centers around a hedging error caused by the mismatch between the payoffs of the issuer's assets and liabilities. Because of the high degree of customization of retail structured products, perfect hedges are often not feasible. In addition, hedging transactions only reduce the risk exposure of the aggregated financial sector if the counterpart sits outside the financial system. Retail investors are ideal counterparts of hedging transactions, since they arguably incur lower bankruptcy cost compared to financial institutions and because their small size limits contagion.

When a perfect hedge should indeed be feasible, the issuer can convert the liability from RSPs into a zero bond. As we show in our analysis, the issuer actually does not choose the perfect hedge when the asset portfolio is highly correlated to the underlying. In this case, RSPs offer advantageous features compared to straight debt. DCNs possess the property of a lower repayment when the issuer's asset value declines. PPNs generate a funding advantage over straight debt in return for sharing potential gains.

Appendix

2.A Valuation of claims

The option pricing theory developed by Black and Scholes (1973) and Merton (1973b) provides the framework for the pricing of the claims. To improve readability, we use the following short-hand notation throughout this section.

$$N_1(X) = N[d_1(X)], \tag{2.14}$$
$$N_{-1}(X) = N[-d_1(X)] = 1 - N_1(X), \tag{2.15}$$
$$N_2(X) = N[d_2(X)], \tag{2.16}$$
$$N_{-2}(X) = N[-d_2(X)] = 1 - N_2(X), \tag{2.17}$$

where X denotes the strike price and $N[y]$ denotes the standard normal cumulative distribution function. The terms d_1 and d_2 are defined as

$$d_1(X) = \frac{\ln\left(\frac{R_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{2.18}$$
$$d_2(X) = \frac{\ln\left(\frac{R_0}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1(X) - \sigma\sqrt{T}. \tag{2.19}$$

The values of European call options c_0 and put options p_0 with strike price X are given by

$$c_0(X) = R_0 N_1(X) - Xe^{-rT} N_2(X), \tag{2.20}$$
$$p_0(X) = Xe^{-rT} N_{-2}(X) - R_0 N_{-1}(X). \tag{2.21}$$

2.A.1 Principal-protected notes

We first examine the case of financing with bonds and principal-protected notes. There are three conditions determining whether the issuer defaults. First, if the risk-free component of the asset portfolio does not suffice to repay the debt's principal, i.e., $\overline{B} + \overline{P} > (1-\delta)F_T$, then the issuer defaults for small values of the underlying $R_T < X_1$ with

$$X_1 = \frac{\overline{B} + \overline{P} - (1-\delta)F_T}{\delta}. \tag{2.22}$$

This case obviously requires $\delta > 0$. For $\delta = 0$, the issuer defaults independent of the outcome of R_T.

Second, in case the issuer fails to settle the liability from the option embedded in the principal-protected note for some outcomes, i.e., $\delta - \frac{\pi}{X_P} \cdot \overline{P} < 0$, it defaults for values of the underlying $R_T > X_2$ with

$$X_2 = \frac{\overline{B} + (1-\pi)\overline{P} - (1-\delta)F_T}{\delta - \frac{\pi}{X_P} \cdot \overline{P}}. \tag{2.23}$$

Third, the issuer might as well default for values of R_T below X_2. This situation happens when the principal amount is high, i.e., $\overline{B} + \overline{P} > (1-\delta)F_T + \delta \cdot X_P$, and the participation rate is low with $\pi < \frac{\delta}{\overline{P}} \cdot X_P$.

The following table summarizes the resulting six possible scenarios. The second column shows for which realizations R_T of the risky asset the issuer defaults. Columns 3 to 5 show for which choice of parameters $\overline{B}$, $\overline{P}$ and δ the respective scenario occurs. The final column shows the risk-neutral probability of default of the issuer for each scenario.

Case i	Default	$\overline{B} + \overline{P}$	$\delta - \frac{\pi}{X_P} \cdot \overline{P}$	δ	pd
1	never	$\leq A_{0,T}$	≥ 0	≥ 0	0
2	$> X_2$	$\leq A_{0,T}$	< 0	≥ 0	$N_2(X_2)$
3	$< X_1$	$(> A_{0,T}) \wedge (\leq A_{X_P,T})$	≥ 0	> 0	$N_{-2}(X_1)$
4	$(< X_1) \wedge (> X_2)$	$(> A_{0,T}) \wedge (\leq A_{X_P,T})$	< 0	> 0	$N_2(X_2) + N_{-2}(X_1)$
5	$< X_2$	$> A_{X_P,T}$	> 0	> 0	$N_{-2}(X_2)$
6	always	$> A_{X_P,T}$	≤ 0	≥ 0	1

We denote the payoff of the assets for $R_T = 0$ as $A_{0,T} = (1-\delta)F_T$ and abbreviate the payoff of the assets for $R_T = X_P$ with $A_{X_P,T} = (1-\delta)F_T + \delta \cdot X_P$. In addition, we introduce the following short-hand notations:

$$g_1 = R_0 \cdot (N_1(X_P) - N_1(X_2)) - X_P \cdot e^{-rT} \cdot (N_2(X_P) - N_2(X_2)), \tag{2.24}$$

$$g_2 = \delta \cdot R_0 \cdot (N_1(X_2) - N_1(X_1)) + (1-\delta) \cdot F_0 \cdot (N_2(X_2) - N_2(X_1)), \tag{2.25}$$

$$g_3 = R_0 \cdot N_1(X_2) - X_P \cdot e^{-rT} \cdot N_2(X_2). \tag{2.26}$$

The total firm value of the issuer $V_0^i(\overline{B},\overline{P})$ for the respective case i is given by

$$
\begin{aligned}
V_0^1 &= A_0 + \tau(1-e^{-rT})\left(\left(\overline{B}+\overline{P}\right)e^{-rT} + \tfrac{\pi}{X_P}\overline{P}\cdot c_0(X_P)\right), && (2.27)\\
V_0^2 &= A_0 + \tau\left(1-e^{-rT}N_{-2}(X_2)\right)\left((\overline{B}+\overline{P})e^{-rT}N_{-2}(X_2) + \tfrac{\pi}{X_P}\overline{P}\cdot g_1\right)\\
&\quad - \left(\alpha+(1-\alpha)\tau e^{-rT}N_{-2}(X_2)\right)\left(\delta R_0 N_1(X_2) + (1-\delta)F_0 N_2(X_2)\right), && (2.28)\\
V_0^3 &= A_0 + \tau\left(1-e^{-rT}N_2(X_1)\right)\left((\overline{B}+\overline{P})e^{-rT}N_2(X_1) + \tfrac{\pi}{X_P}\overline{P}\cdot c_0(X_P)\right)\\
&\quad - \left(\alpha+(1-\alpha)\tau e^{-rT}N_2(X_1)\right)\left(\delta R_0 N_{-1}(X_1) + (1-\delta)F_0 N_{-2}(X_1)\right), && (2.29)\\
V_0^4 &= A_0 + \tau\left(1-e^{-rT}(N_2(X_1)-N_2(X_2))\right)\\
&\quad \cdot\left((\overline{B}+\overline{P})e^{-rT}(N_2(X_1)-N_2(X_2)) + \tfrac{\pi}{X_P}\overline{P}\cdot g_1\right)\\
&\quad - \left(\alpha+(1-\alpha)\tau e^{-rT}(N_2(X_1)-N_2(X_2))\right)\cdot g_2, && (2.30)\\
V_0^5 &= A_0 + \tau\left(1-e^{-rT}N_2(X_2)\right)\left((\overline{B}+\overline{P})e^{-rT}N_2(X_2) + \tfrac{\pi}{X_P}\overline{P}\cdot g_3\right)\\
&\quad - \left(\alpha+(1-\alpha)\tau e^{-rT}N_2(X_2)\right)\left(\delta R_0 N_{-1}(X_2) + (1-\delta)F_0 N_{-2}(X_2)\right), && (2.31)\\
V_0^6 &= A_0\cdot(1-\alpha). && (2.32)
\end{aligned}
$$

2.A.2 Discount notes

We examine the claim values for the issuer financed with bonds and discount notes. The issuer's payoff is characterized by two default thresholds. First, if the risk-free portion of the asset portfolio is exceeded by the minimum debt payment, i.e., $\overline{B}+\overline{D} > (1-\delta)F_T$, then the issuer defaults for low values of the underlying below the threshold $R_T < X_3$ with

$$X_3 = \frac{\overline{B}-(1-\delta)F_T}{\delta-\gamma\overline{D}}. \tag{2.33}$$

This case requires $\delta-\gamma\overline{D} > 0$.

The default boundary X_3 is relevant for a further scenario. If the bank issues an amount of discount notes exceeding the value of its investment in the risky asset, i.e., $\delta-\gamma\overline{D} < 0$, then the bank defaults for values above the threshold $R_T > X_3$.

Second, the issuer's liabilities are in any case limited to $\overline{B}+\overline{D}$. Thus, the issuer never defaults for values of the underlying $R_T > X_4$ with

$$X_4 = \frac{\overline{B}+\overline{D}-(1-\delta)F_T}{\delta}. \tag{2.34}$$

This case requires $\delta > 0$. Otherwise, the default boundaries become independent of R_T. The following table summarizes the resulting 6 possible scenarios. The second column shows for which realizations R_T of the risky asset the issuer defaults. Columns 3 to 5 show for which choice of parameters $\overline{B}$, $\overline{D}$ and δ the respective scenario occurs. The final column shows the risk-neutral probability of default of the issuer for each scenario.

Case i	Default	$\overline{B}$	$\overline{B}+\overline{D}$	δ	pd
1	never	$\leq A_{0,T}$	$\leq A_{X_D,T}$	≥ 0	0
2	$(< X_4) \wedge (> X_3)$	$\leq A_{0,T}$	$> A_{X_D,T}$	> 0	$N_2(X_4) + N_{-2}(X_3)$
3	$> X_3$	$\leq A_{0,T}$	$> A_{X_D,T}$	$= 0$	$N_2(X_3)$
4	$< X_3$	$> A_{0,T}$	$\leq A_{X_D,T}$	≥ 0	$N_{-2}(X_3)$
5	$< X_4$	$> A_{0,T}$	$> A_{X_D,T}$	> 0	$N_{-2}(X_4)$
6	always	$> A_{0,T}$	$> A_{X_D,T}$	$= 0$	1

We denote the payoff of the assets for $R_T = 0$ as $A_{0,T} = (1-\delta)F_T$ and abbreviate the payoff of the assets for $R_T = X_D$ with $A_{X_D,T} = (1-\delta)F_T + \delta \cdot X_D$. We introduce the following short-hand notations:

$$g_4 = \delta \cdot R_0 \cdot (N_1(X_3) - N_1(X_4)) + (1-\delta) \cdot F_0 \cdot (N_2(X_3) - N_2(X_4)), \tag{2.35}$$

$$g_5 = \delta \cdot R_0 \cdot N_{-1}(X_3) + (1-\delta) \cdot F_0 \cdot N_{-2}(X_3), \tag{2.36}$$

$$g_6 = \delta \cdot R_0 \cdot N_{-1}(X_4) + (1-\delta) \cdot F_0 \cdot N_{-2}(X_4), \tag{2.37}$$

$$g_7 = X_D \cdot e^{-rT} \cdot (N_2(X_3) - N_2(X_D)) - R_0 \cdot (N_1(X_3) - N_1(X_D)). \tag{2.38}$$

The total firm value of the issuer $V_0^i(\overline{B},\overline{D})$ for the respective case i is given by

$$V_0^1 = A_0 + \tau(1-e^{-rT})\left(\left(\overline{B}+\overline{D}\right)e^{-rT} - \gamma\overline{D}\cdot p_0(X_D)\right), \tag{2.39}$$

$$\begin{aligned} V_0^2 = A_0 + \tau &\left(1 - e^{-rT}\left(N_2(X_4) - N_2(X_3)\right)\right) \\ &\cdot \left((\overline{B}+\overline{D})e^{-rT}\left(N_2(X_4) - N_2(X_3)\right) - \gamma\overline{D}\cdot g_7\right) \\ &- \left(\alpha + (1-\alpha)\tau e^{-rT}\left(N_2(X_4) - N_2(X_3)\right)\right)\cdot g_4, \end{aligned} \tag{2.40}$$

$$\begin{aligned} V_0^3 = A_0 + \tau &\left(1 - e^{-rT}N_{-2}(X_3)\right)\left(\overline{B}e^{-rT}N_{-2}(X_3) + \gamma\overline{D}\cdot R_0 N_{-1}(X_3)\right) \\ &- \left(\alpha + (1-\alpha)\tau e^{-rT}N_{-2}(X_3)\right)\cdot F_0 N_2(X_3), \end{aligned} \tag{2.41}$$

$$\begin{aligned} V_0^4 = A_0 + \tau &\left(1 - e^{-rT}N_2(X_3)\right)\left(\left(\overline{B}+\overline{D}\right)e^{-rT}N_2(X_3) - \gamma\overline{D}\cdot g_7\right) \\ &- \left(\alpha + (1-\alpha)\tau e^{-rT}N_2(X_3)\right)\cdot g_5, \end{aligned} \tag{2.42}$$

$$\begin{aligned} V_0^5 = A_0 + \tau &\left(1 - e^{-rT}N_2(X_4)\right)\left(\left(\overline{B}+\overline{D}\right)e^{-rT}N_2(X_4)\right) \\ &- \left(\alpha + (1-\alpha)\tau e^{-rT}N_2(X_4)\right)\cdot g_6, \end{aligned} \tag{2.43}$$

$$V_0^6 = A_0 \cdot (1-\alpha). \tag{2.44}$$

2.B Proofs of propositions

2.B.1 High-risk issuer

PPN financing

We restrict the participation rate to the typical case of $\pi \leq 1$. Since we would like to compare PPN financing to straight debt financing, we consider points where the issuers have equal probability of default under both financing choices, i.e., $pd_B = pd_P$. Under this condition, we can express the issuer value $V_{P,0}$ and the value of the principal-protected note CP_0 as

$$CP_0(\overline{P}) = B_0(\overline{B} = X) + a, \tag{2.45}$$

$$V_{P,0}(\overline{P}) = V_{B,0}(\overline{B} = X) + a \cdot b, \tag{2.46}$$

with

$$X = \begin{cases} \overline{P} & \text{for } 0 < \overline{P} \leq X_P, \\ \frac{(1-\pi)\cdot\overline{P}}{1-\frac{\pi}{X_P}\cdot\overline{P}} & \text{for } X_P < \overline{P} \leq \frac{X_P}{\pi}, \end{cases} \tag{2.47}$$

$$a = \begin{cases} \frac{\pi}{X_P}\cdot\overline{P}\cdot c_0(X_P) & \text{for } 0 < \overline{P} \leq X_P, \\ \frac{\pi}{X_P}\cdot\overline{P}\cdot\Big(R_0 N_1(X) - X_P\cdot e^{-rT}N_2(X)\Big) & \text{for } X_P < \overline{P} \leq \frac{X_P}{\pi}, \end{cases} \tag{2.48}$$

$$b = \tau(1 - e^{-rT}N_2(X)). \tag{2.49}$$

We want to show for a given probability of default at $\overline{B} = X$ and $\pi > 0$ that

$$\begin{aligned} & \lambda_B < \lambda_P \\ \iff\quad & \frac{B_0}{V_{B,0}} < \frac{CP_0}{V_{P,0}} \\ \iff\quad & \frac{B_0}{V_{B,0}} < \frac{B_0 + a}{V_{B,0} + a\cdot b} \\ \iff\quad & b\cdot B_0 < V_{B,0}. \end{aligned}$$

The last relation is always true, since $b < 1$ and by definition $B_0 \leq V_{B,0}$. This directly proves proposition 2.1 for PPN financing.

Moreover, this result also proves proposition 2.2 for PPN financing. For each value of $\overline{B}$, we can find a corresponding point for PPN financing with equal probability of default, which produces a higher firm value $V_{P,0} = V_{B,0} + a\cdot b > V_{B,0}$ and also a higher leverage ratio $\lambda_P > \lambda_B$. Since this finding is true for any value of $\overline{B}$, the graph of $V_{P,0}$ has to be strictly above the $V_{B,0}$ graph for any attainable leverage ratio $\lambda < 1$.

Another way of showing this result is using the first derivatives of the issuer value $V_{P,0}$ and the leverage ratio λ_P with respect to the participation rate π. We note that straight debt financing can be represented by $\pi = 0$.

$$\frac{\partial V_{P,0}}{\partial \pi} = \frac{a \cdot b}{\pi} > 0, \tag{2.50}$$

$$\frac{\partial \lambda_P}{\partial \pi} = \frac{a}{\pi \cdot (V_{B,0} + a \cdot b)^2} \cdot (V_{B,0} - b \cdot B_0) > 0. \tag{2.51}$$

Both derivatives are always positive. Hence, as long as $\overline{P} \leq \frac{X_P}{\pi}$, an increase in the participation rate always creates value, but does not increase the bankruptcy cost, since $\frac{\partial pd_P}{\partial \pi} = 0$. PPN financing is strictly superior to straight debt financing.

The case of $\overline{P} > \frac{X_P}{\pi}$ is not of interest, since the issuer always defaults, i.e., $\lambda_P = 1$ and $V_{P,0} = CP_0 = (1-\alpha)R_0$. The leverage ratio of $\lambda_B = 1$ cannot be attained under straight debt financing. The above outlined proof holds analogously for PPN designs with $\pi > 1$ as long as $\overline{P} \leq \frac{X_P}{\pi}$.

DCN financing

The relevant risk-neutral default probabilities are given by

$$pd_B\left(\overline{B}\right) > 0, \tag{2.52}$$

$$pd_D\left(\overline{D}\right) = \begin{cases} 0 & \text{for } \overline{D} \leq X_D, \\ pd_B\left(\overline{B} = \overline{D}\right) & \text{for } \overline{D} > X_D. \end{cases} \tag{2.53}$$

In the case of $\overline{D} > X_D$, the debt claim values are also identical, i.e., $B_0(\overline{B}) = CD_0(\overline{D} = \overline{B})$. The issuer values and leverage ratios agree as well. Hence, the default probability with DCN financing is either zero or agrees with the corresponding probability under straight debt financing. This finding proves proposition 2.1 for discount notes.

This reasoning also proves proposition 2.2 for $\overline{D} > X_D$. In the remaining case of $\overline{D} \leq X_D$, we express the issuer value under DCN financing as

$$V_{0,D}(\lambda) = R_0 \cdot \frac{1}{1 - (1 - e^{-rT})\tau\lambda}. \tag{2.54}$$

An issuer financed only with straight debt defaults for realizations of the underlying asset $R_T < \overline{B}$. The resulting issuer value is given by

$$V_{0,B}(\lambda) = R_0 \cdot \frac{1-\alpha' N_{-1}(\overline{B})}{1-(1-e^{-rT}(1-N_{-2}(\overline{B})))\tau\lambda} \tag{2.55}$$

with $\alpha' = \alpha + \tau(1-\alpha)$ and $\tau \leq \alpha' \leq 1$. The term $N_{-2}(\overline{B})$ corresponds to the risk-neutral default probability pd_B. To simplify the expression, we use the relation $N_{-1}(\overline{B}) = N_{-2}(\overline{B}) - \varepsilon$ with $0 \leq \varepsilon \leq 1$. The issuer value now reads

$$V_{0,B}(\lambda) = R_0 \cdot \frac{1-\alpha' pd_B + \alpha'\varepsilon}{1-(1-e^{-rT})\tau\lambda - pd_B e^{-rT}\tau\lambda}. \tag{2.56}$$

For $pd_B = 0$, which also implies $\varepsilon = 0$, the issuer value $V_{0,B}$ under straight debt financing agrees with the issuer value $V_{0,D}$ under DCN financing. We inspect the derivative of the issuer value with respect to the default probability given by

$$\frac{\partial V_{0,B}(\lambda)}{\partial pd_B} = \frac{R_0}{(\ldots)^2} \cdot \Big(-\alpha' + (\alpha' + e^{-rT}(1-\alpha')))\tau\lambda + \alpha' e^{-rT}\tau\lambda\varepsilon\Big). \tag{2.57}$$

The term in brackets is negative for $\varepsilon = 0$. Proposition 2 requires this derivative to be negative. Hence, we need to impose a condition of the form

$$\varepsilon \leq \overline{\varepsilon} = \frac{\alpha' - (\alpha' + e^{-rT}(1-\alpha')))\tau\lambda}{\alpha' e^{-rT}\tau\lambda}. \tag{2.58}$$

Both numerator and denominator are positive and smaller than 1. The upper boundary $\overline{\varepsilon}$ increases with the bankruptcy cost α and decreases with the leverage ratio λ and the tax rate τ. This restriction puts an upper boundary on $\sigma\sqrt{T}$, since $N(d_1) = N(d_2 + \sigma\sqrt{T})$. The restriction is not binding for typical parameter choices.

2.B.2 Low-risk issuer

DCN financing

The risk-neutral probability of default of the issuer financed with bonds is $pd_B = 0$ for all attainable leverage ratios $\lambda < 1$. This directly proves proposition 2.3 since the default probability of an issuer financed with DCN is positive at least for some $\lambda < 1$.

The issuer value under straight debt financing is given by

$$V_{0,B}(\lambda) = F_0 \cdot \frac{1}{1-(1-e^{-rT})\tau\lambda}. \tag{2.59}$$

The maximum attainable leverage ratio without default is at $\overline{B} = F_0 \cdot e^{rT}$ with

$$\lambda_B{}^{\max} = \frac{1}{1+(1-e^{-rT})\tau}. \tag{2.60}$$

We need to consider two cases. In the first case with $\overline{D} \leq F_0 \cdot e^{rT}$, the issuer is not defaulting. The issuer value is given by

$$V_{0,D}(\lambda) = F_0 \cdot \frac{1}{1-(1-e^{-rT})\tau\lambda}, \tag{2.61}$$

which agrees with the issuer value $V_{0,B}$ under straight debt financing.

The maximum attainable leverage ratio without default is at $\overline{D} = F_0 \cdot e^{rT}$ with

$$\lambda_D{}^{\max} = \frac{1}{q+(1-e^{-rT})\tau}. \tag{2.62}$$

with $q = \frac{e^{-rT}}{e^{-rT} - \gamma p_0(X_D)}$. From $q > 1$ follows that $\lambda_D{}^{\max} < \lambda_B{}^{\max}$.

In the second case with $\overline{D} > F_0 \cdot e^{rT}$, the issuer defaults for realizations of the underlying asset above the threshold $X_3 = \frac{F_0 \cdot e^{rT}}{\gamma \overline{D}}$. The issuer value is given by

$$V_{0,D}(\lambda) = F_0 \cdot \frac{1 - pd_D \cdot \tau - \alpha(1-(\tau+(1-\tau)(1-pd_D)))}{1-(1-e^{rT}(1-pd_D))\tau\lambda}. \tag{2.63}$$

Since an increase in the bankruptcy cost α always leads to a decrease in the issuer value, i.e., $\frac{\partial V_{0,D}}{\partial \alpha} < 0$, we can consider the limiting case of $\alpha = 0$. The resulting claim value is given by

$$V_{0,D}(\lambda)\Big|_{\alpha=0} = F_0 \cdot \frac{1 - pd_D \cdot \tau}{1-(1-e^{rT})\tau\lambda - pd_D \cdot e^{-rT}\tau\lambda}. \tag{2.64}$$

For $pd_D = 0$, the issuer value agrees with the value $V_{0,B}$ under straight debt financing. We inspect the first derivative with respect to the default probability given by

$$\frac{\partial V_{0,D}(\lambda)}{\partial pd_D}\Bigg|_{\alpha=0} = -F_0 \cdot \frac{\tau(1-\lambda(e^{-rT}+\tau(1-e^{-rT})))}{(\ldots)^2}. \tag{2.65}$$

The numerator is always positive. Hence, an increase in the default probability always results in a loss of value even for the limiting case of $\alpha = 0$. This proves proposition 2.4 for DCN financing.

PPN financing

The risk-neutral probability of default of the issuer financed with bonds is $pd_B = 0$ for all attainable leverage ratios $\lambda < 1$. This relation directly proves proposition 2.3, since the default probability of an issuer financed with PPN is positive at least for some $\lambda < 1$.

We need to consider two cases. In the first case with $\overline{P} > F_T = F_0 \cdot e^{rT}$, the issuer always defaults. For the resulting leverage ratio of $\lambda = 1$, the issuer value is independent of the financing choice.

In the second case of $\overline{P} \leq F_T$, the issuer defaults for realizations of the underlying above the threshold $X_2 = \frac{X_T}{\pi \cdot \overline{P}}(F_T - (1-\pi)\overline{P})$. The corresponding claim value is given by

$$V_{0,P}(\lambda) = F_0 \cdot \frac{1 - pd_P \cdot \tau - \alpha(1 - (\tau + (1-\tau)(1 - pd_P)))}{1 - (1 - e^{rT}(1 - pd_P))\tau\lambda}. \tag{2.66}$$

The functional form is the same as for the issuer value under DCN financing from equation (2.63). Of course, the claim values are not the same, since pd_P and pd_D depend differently on the leverage ratio λ, but the above outlined proof for DCN financing with $\overline{D} > F_T$ is valid for all positive default probabilities. Hence, the same reasoning can be applied to prove proposition 2.4 for PPN financing.

2.C Comparative static analysis

Figure 2.15: Comparative static analysis for constant leverage

The graphs show the issuer value V_0 and the corresponding probability of default pd of a high-risk issuer (see section 2.4.1). We compute the base case (thick solid line) using the values $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$. The first column shows different ***product designs*** *for the PPN with $\pi = 0.25$ (dashed line) and $\pi = 1$ (dot-dashed line) as well as for the DCN with $X_D = 100$ (dashed line). The second column shows two alternative scenarios for the* ***volatility*** *of the underlying with $\sigma_R = 0.1$ (dashed line) and $\sigma_R = 0.3$ (dot-dashed line). The third column shows two alternative scenarios for the* ***frictions*** *with a tax rate of $\tau = 0.25$ (dashed line) and bankruptcy costs of $\alpha = 0.5$ (dot-dashed line).*

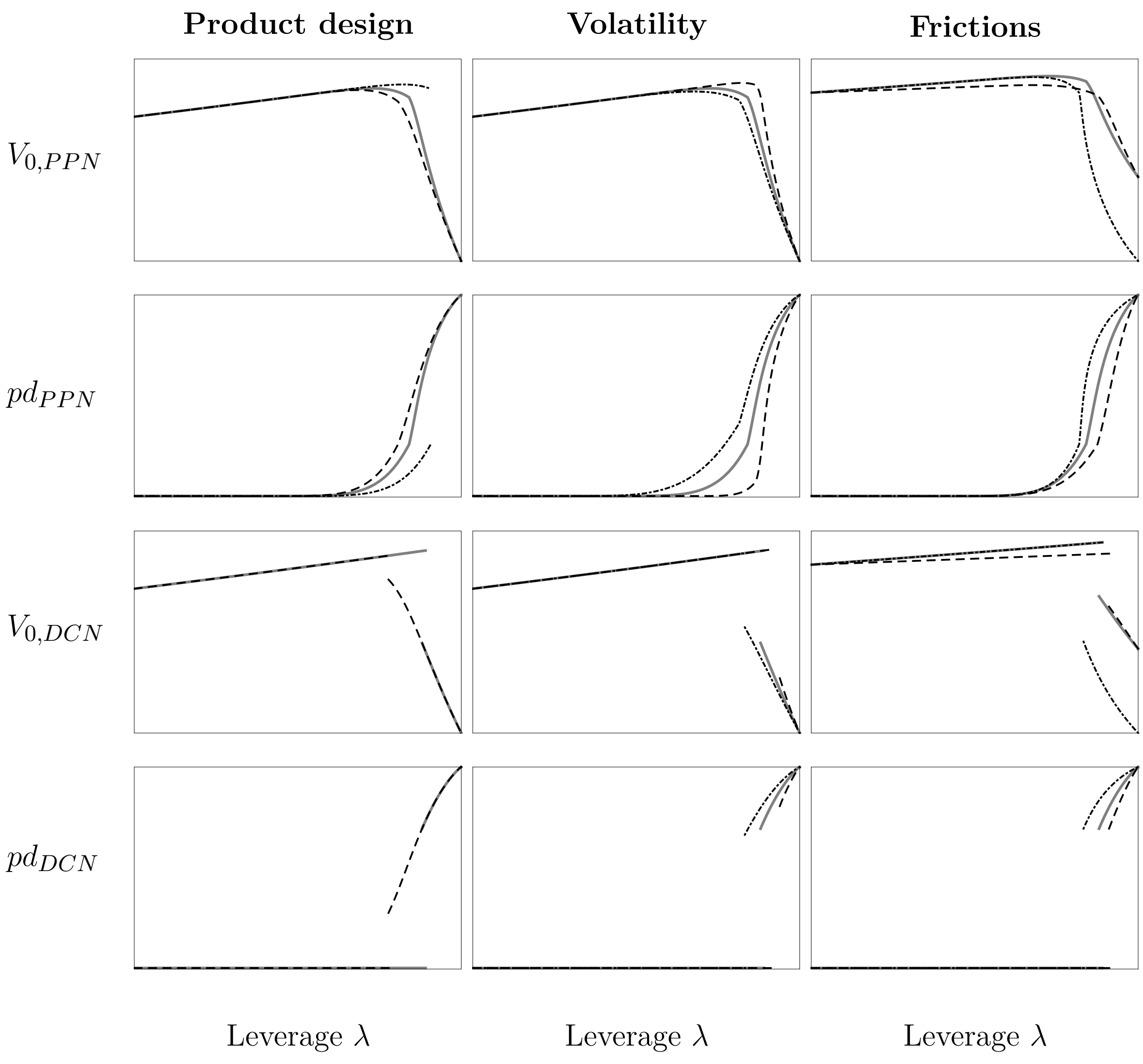


Figure 2.16: Comparative static analysis for optimal financing

The graphs show the issuer value V_0 and the corresponding probability of default pd. We compute the base case (thick solid line) using the values $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$. The first column shows different ***product designs*** *for the PPN with $\pi = 0.25$ (dashed line) and $\pi = 1$ (dot-dashed line) as well as for the DCN with $X_D = 100$ (dashed line) and $X_D = 150$ (dot-dashed line). The second column shows two alternative scenarios for the* ***volatility*** *of the underlying with $\sigma_R = 0.1$ (dashed line) and $\sigma_R = 0.3$ (dot-dashed line). The third column shows two alternative scenarios for the* ***frictions*** *with a tax rate of $\tau = 0.25$ (dashed line) and bankruptcy costs of $\alpha = 0.5$ (dot-dashed line).*

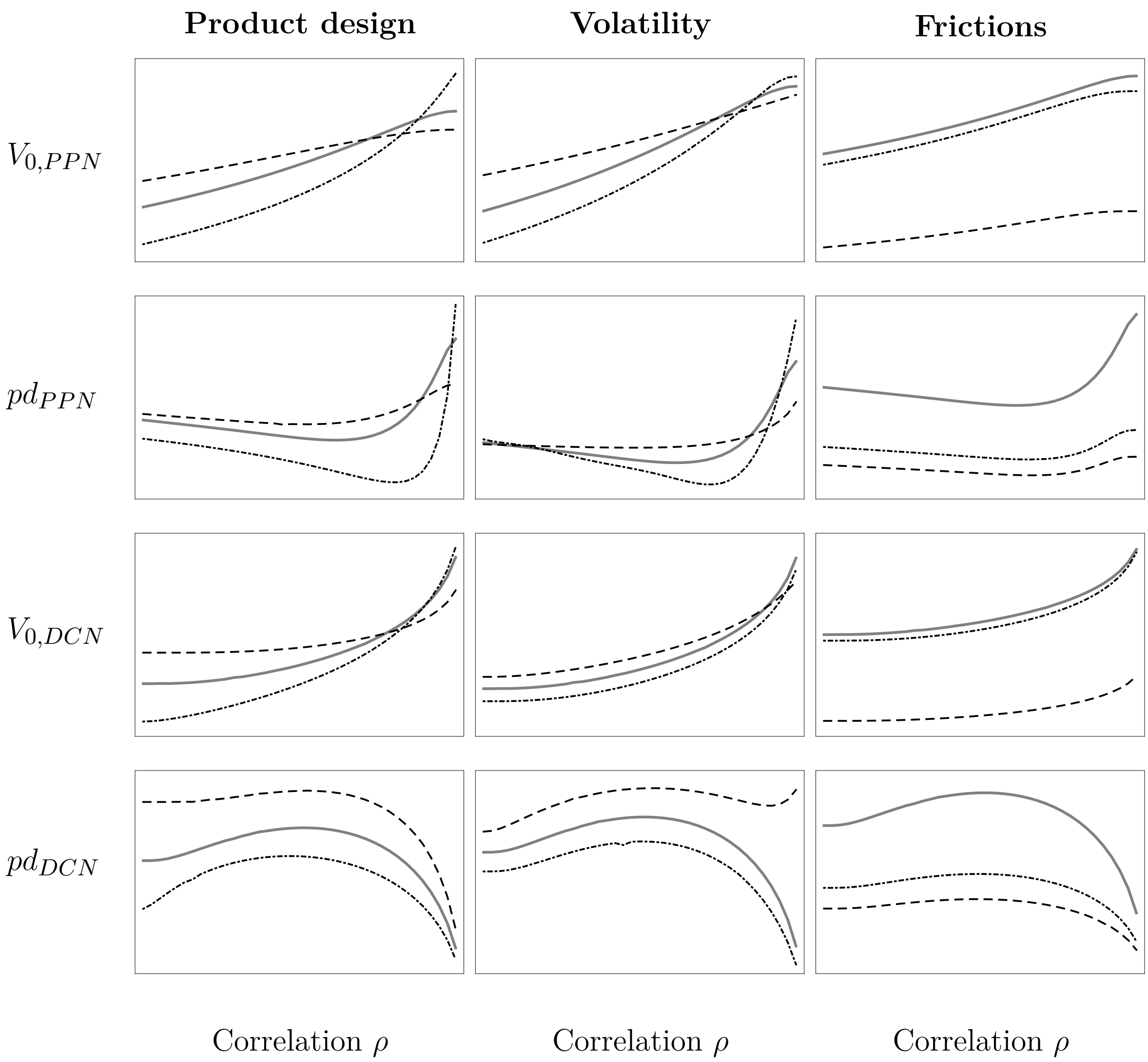
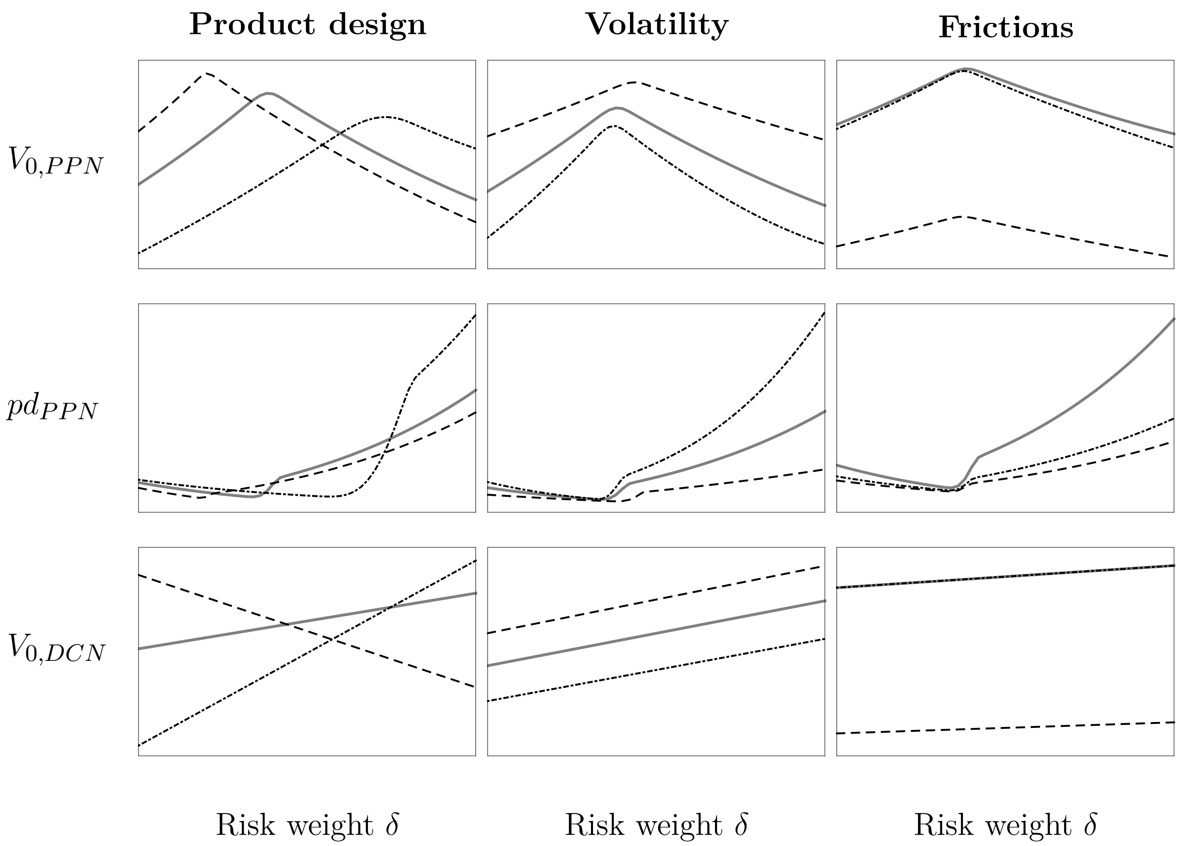


Figure 2.17: Comparative static analysis for optimal risk-taking

The graphs show the issuer value V_0 *and the corresponding probability of default pd. We compute the base case (thick solid line) using the values* $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ *and product parameters* $X_P = 100$, $\pi = 0.5$ *and* $X_D = 125$. *The first column shows different* ***product designs*** *for the PPN with* $\pi = 0.25$ *(dashed line) and* $\pi = 1$ *(dot-dashed line) as well as for the DCN with* $X_D = 100$ *(dashed line) and* $X_D = 150$ *(dot-dashed line). The second column shows two alternative scenarios for the* ***volatility*** *of the underlying with* $\sigma_R = 0.1$ *(dashed line) and* $\sigma_R = 0.3$ *(dot-dashed line). The third column shows two alternative scenarios for the* ***frictions*** *with a tax rate of* $\tau = 0.25$ *(dashed line) and bankruptcy costs of* $\alpha = 0.5$ *(dot-dashed line).*



Chapter 3

Contingent convertible debt†

3.1 Introduction

Contingent convertible (CoCo) bonds play an important role in the debate on banking stability. Not surprisingly, regulators favor this modern financing instrument for banks, because the idea of CoCo bonds is that they provide banks with additional capital in case they suffer from unfavorable economic conditions. Hence, the government and the taxpayer do not have to bail out distressed financial institutions any more. CoCo bonds enable banks and the capital market to take care of themselves. Furthermore, the fact that banks' liabilities vanish in case of financial distress seems to decrease the default probability at first glance. A potential reduction of the default probability would be a highly desirable property not only from the regulatory point of view.

Despite an intense public debate, rather few CoCo bonds have been issued until 2012. Since then, regulatory pressure — especially in Switzerland — has led to a wave of new issuances. Remarkably, a large portion of the issued contingend debt has a conversion ratio equal to zero. This feature economically means that CoCo bond holders are left with nothing when a conversion event occurs.

This anecdotal evidence raises three questions: (1) Why have banks been hesitant to issue CoCo bonds in the absence of regulatory pressure? (2) Why do banks choose a low or even zero conversion ratio? (3) What are the implications for the regulator from this issuance behavior?

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The objective of this chapter is to shed light on these three major questions regarding CoCo bonds and their issuances. Several answers to the first question can already be found in the literature. Many different aspects are discussed such as the risk-taking behavior induced by CoCos, the interaction between a credit crunch and CoCos, the incentives of capital market participants to manipulate prices in order to enforce or prevent a conversion, and the uniqueness of asset prices. We aim at surveying this broad literature and relating the main arguments for and against an issuance of CoCo bonds to each other.

In order to tackle the second and third question, we nest two aspects of the literature, namely the risk-taking incentives of banks and the likelihood of a credit crunch. Apparently, the design of CoCo bonds might drive the risk-taking and loan granting behavior of banks. The introduction of a continuous-time framework allows us to analyze the severity of these incentives for different CoCo bond designs. Risk taking is attractive for example for managers who are paid with stock options or for those who are interested in empire building if a higher risk is associated with a higher equity value.

The model explains that a high conversion ratio mitigates the bank's risk-taking incentives and simultaneously enhances its loan granting behavior. To accomplish this effect, the high conversion ratio has to be combined with a sufficiently high trigger level such that there is a wealth transfer from equity holders to debt holders at conversion. In addition, these two parameters should be higher for banks with a high leverage ratio. For this reason, regulators who advocate the use of CoCo bonds should also prescribe the specific product design to avoid distorted incentives.

The remainder of the chapter is organized as follows. Section 3.2 introduces the products and gives an overview of the issued CoCo bonds so far. Section 3.3 surveys the literature on CoCo bonds. In section 3.4, we introduce the model and show how CoCo bonds impact the risk-taking and loan-granting incentives of banks. Section 3.5 concludes.

3.2 Product overview

In this section, we shortly introduce how contingent capital works. CoCo bonds are issued either with a fixed maturity or in the form of consol bonds. They pay a coupon just like ordinary subordinated debt. In addition, CoCo bonds are equipped with a trigger mechanism. When the predefined trigger event occurs, the debt is either converted into equity or it is written down. There is no cash flow at conversion. The crucial difference to ordinary convertible bonds is that the conversion is mandatory in case of the trigger event. Flannery (2014) refers to this mechanism as pre-packaged reorganization.

The trigger event can be defined with respect to market based capital ratios or accounting data. In some instances, the conversion is also at the discretion of the regulatory authority. When the CoCo bonds convert into equity, the nominal amount of outstanding shares increases at conversion. The conversion ratio determines how many shares the CoCo bond holders receive. There are two different mechanisms for contracts which are written down. The face value can either be decreased by a predetermined amount. Alternatively, the amount by which the face value is reduced can be determined at conversion. In this case, the face value is reduced by the amount which is required to revoke the trigger condition. In some instances, the write-down is only temporary and the face value is restored when the bank recovers.

Llyods was the first bank to issue CoCo bonds in 2009. Only few banks followed until the issuance of CoCo contracts finally picked up in 2013. Nordal and Stefano (2014) report a total issuance volume of EUR 73.8 billion over the time period from January 2009 until June 2014. Their sample contains 102 contracts issued by 37 banks. All contracts have a trigger mechanism based on accounting ratios. In the following, we focus on the 10 biggest issuers which account for 80% of the volume.

Roughly 28% of issued CoCos convert into equity. Table 3.1 contains the key properties of these contracts. We estimate the wealth loss WL based on the approach of Berg and Kaserer (2014) with the following formula

$$WL = \frac{CR_t}{CR_0} \cdot \frac{n}{n+m} \cdot \frac{m \cdot S_0}{F} \tag{3.1}$$

where CR_t denotes the tier 1 capital ratio at which the CoCo converts, CR_0 is the current value of this ratio, m denotes the number of new shares given to the CoCo bond holders, n denotes the number of shares outstanding before conversion, S_0 denotes the current share price and F is the face value of the CoCo. We use data as of 31/12/2014 for time $t = 0$. Since the stochastic process of the capital ratio cannot be observed, we assume that the capital ratio moves in sync with the share price. Given this simplifying assumption, we compute a wealth loss between 33% and 98%. The wealth loss has been expressed in terms of face value, since quoted prices of CoCos are not readily available. However, if we assume that a CoCo is issued at par, then all CoCo bond holders incur significant losses in case of conversion.

The remaining 72% of issued CoCos, which possess a write down mechanism, are shown in table 3.2. Strictly speaking, write-down bonds are not convertible bonds. However, we include these products in the overview since they rely on the same trigger mechanism and induce similar incentives. Total-loss bonds are an extreme case of write-down bonds. The claims are written down to zero in case of a trigger event. In other words, contingent

Table 3.1: Overview of contingent convertible bonds

The table shows all CoCo bonds converting into equity of the 10 biggest issuers since 2009. The table reports the following information: (1) the issuer's name and consecutive number of issuance, (2) the issuance date, (3) the currency abbreviated with FX, (4) the notional in the issuance currency, (5) the coupon in percent, (6) the maturity of the bond in years at the time of issuance, and (7) the wealth loss WL in case of conversion. The symbol ∞ indicates when the CoCo is a consol bond. CR abbreviates the tier 1 capital ratio. We compute the wealth loss following the method proposed by Berg and Kaserer (2014) with data as of 31/12/2014.

Issuer	**Date**	**FX**	**Notional**	**Coupon**	**Years**	**Trigger**	**WL**
Lloyds 1	Nov-09	GBP	4.65bn	7.588-16.125%	10-23	$CR < 5\%$	38%
		USD	2.52bn	7.875-8.5%	11	$CR < 5\%$	38%
		JPY	37.00bn	6.75-8.07%	11-13	$CR < 5\%$	38%
		EUR	2.36bn	6.385-15%	10-11	$CR < 5\%$	38%
Credit Suisse 1	Feb-11	USD	2.0bn	7.875%	30	$CR < 7\%$	37%
Credit Suisse 2	Mar-12	CHF	0.75bn	7.125%	10	$CR < 7\%$	38%
Credit Suisse 3	Jul-12	USD	1.75bn	9.5%	∞	$CR < 7\%$	58%
BBVA 1	May-13	USD	1.5bn	9.0%	∞	$CR < 5.125\%$	77%
Credit Suisse 6	Oct-13	CHF	2.5bn	9.0%	∞	$CR < 7\%$	33%
		USD	3.5bn	9.5%	∞	$CR < 7\%$	33%
Barclays 3	Dec-13	EUR	1.0bn	8.0%	∞	$CR < 7\%$	63%
BBVA 2	Feb-14	EUR	1.5bn	7.0%	∞	$CR < 5.125\%$	71%
Lloyds 2	Apr-14	GBP	0.75bn	7.875%	∞	$CR < 7\%$	49%
		GBP	1.494bn	7.625%	∞	$CR < 7\%$	48%
		GBP	1.48bn	7.0%	∞	$CR < 7\%$	48%
		USD	0.75bn	6.375%	∞	$CR < 7\%$	52%
Barclays 4	Jun-14	USD	1.211bn	7.0%	∞	$CR < 7\%$	90%
		GBP	0.697bn	7.0%	∞	$CR < 7\%$	98%
Barclays 5	Jul-14	EUR	1.076bn	6.5%	∞	$CR < 7\%$	61%
BBVA 3	Feb-15	EUR	1.5bn	6.75%	∞	$CR < 5.125\%$	84%

Table 3.2: Overview of contingent write-down bonds

The table shows all contingent debt contracts with a write-down feature of the 10 biggest issuers since 2009. The table reports the following information: (1) the issuer's name and consecutive number of issuance, (2) the issuance date, (3) the currency abbreviated with FX, (4) the notional in the issuance currency, (5) the coupon in percent, (6) the maturity of the bond in years at the time of issuance, and (7) the write-down WD in percent of the face value. The symbol ∞ indicates when the CoCo is a consol bond. CR abbreviates the tier 1 capital ratio, ER denotes the equity ratio and SR denotes the solvency ratio.

A: Predetermined write-down

Issuer	Date	FX	Notional	Coupon	Years	Trigger	WD
Rabobank 1	Mar-10	EUR	1.25bn	6.875%	10	$ER < 7\%$	75%
UBS 1	Feb-12	USD	2.0bn	7.25%	10	$CR < 5\%$	100%
UBS 2	Aug-12	USD	2.0bn	7.625%	10	$CR < 5\%$	100%
Barclays 1	Nov-12	USD	3.0bn	7.625%	10	$CR < 7\%$	100%
KBC Groep 1	Jan-13	USD	1.0bn	8.0%	10	$CR < 7\%$	100%
Swiss Re	Mar-13	USD	0.75bn	6.375%	11.5	$SR < 125\%$	100%
Barclays 2	Apr-13	USD	1.0bn	7.75%	10	$CR < 7\%$	100%
UBS 3	May-13	USD	1.5bn	7.625%	10	$CR < 5\%$	100%
Credit Suisse 4	Aug-13	USD	2.5bn	6.5%	10	$CR < 5\%$	100%
Credit Agricole 1	Sep-13	USD	1.0bn	8.125%	20	$CR < 7\%$	100%
Credit Suisse 5	Sep-13	EUR	1.25bn	5.75%	12	$CR < 5\%$	100%
Credit Suisse 7	Dec-13	USD	2.25bn	7.5%	∞	$CR < 5.125\%$	100%
UBS 4	Feb-14	EUR	2.0bn	4.75%	12	$CR < 5\%$	100%
UBS 5	May-14	USD	2.5bn	5.125%	10	$CR < 5\%$	100%
Credit Suisse 8	Jun-14	USD	2.5bn	6.25%	∞	$CR < 5.125\%$	100%
UBS 6	Feb-15	EUR	1.0bn	5.75%	∞	$CR < 5.125\%$	100%
		USD	1.25bn	7.125%	∞	$CR < 7\%$	100%
		USD	1.25bn	7.0%	∞	$CR < 5.125\%$	100%

B: Discretionary write-down

Issuer	Date	FX	Notional	Coupon	Years	Trigger
Rabobank 2	Nov-11	USD	2.0bn	8.375%	∞	$CR < 8\%$
Rabobank 3	Nov-11	USD	2.0bn	8.4%	∞	$CR < 8\%$
Société Générale 1	Sep-13	USD	1.25bn	8.25%	∞	$CR < 5.125\%$
Société Générale 2	Dec-13	USD	0.973bn	7.875%	∞	$CR < 5.125\%$
Société Générale 3	Dec-13	USD	1.75bn	7.875%	∞	$CR < 5.125\%$
Credit Agricole 2	Jan-14	USD	1.75bn	7.875%	∞	$CR < 5.125\%$
KBC Groep 2	Mar-14	EUR	1.4bn	5.625%	∞	$CR < 5.125\%$
Credit Agricole 3	Apr-14	GBP	0.5bn	7.5%	∞	$CR < 5.125\%$
		EUR	1.0bn	6.5%	∞	$CR < 5.125\%$
Société Générale 4	Apr-14	EUR	1.0bn	6.75%	∞	$CR < 5.125\%$
Deutsche Bank 1	May-14	EUR	1.75bn	6.0%	∞	$CR < 5.125\%$
		USD	1.25bn	6.25%	∞	$CR < 5.125\%$
		GBP	0.65bn	7.125%	∞	$CR < 5.125\%$
Credit Agricole 4	Sep-14	USD	1.25bn	6.625%	∞	$CR < 5.125\%$
Deutsche Bank 2	Nov-14	USD	1.5bn	7.5%	∞	$CR < 5.125\%$
Rabobank 4	Jan-15	EUR	1.5bn	5.5%	∞	$CR < 7\%$

debt holders are left with nothing when the trigger event occurs. We can see from table 3.2 that roughly half of the CoCos with a write-down mechanism are total-loss bonds.

In all cases, bond holders incur a wealth loss. Holders of CoCos converting into equity incur this loss by receiving a share of the firm which is worth less than the value of the plain bond. The holders of write-down bonds incur the loss by a reduction of the face value. Total-loss bond holders are completely wiped out. In return, debt holders demand higher coupon payments well above those of ordinary subordinated debt (Nordal and Stefano, 2014).

Apparently, banks show a preference for low conversion ratios and a preference for issuing total-loss bonds. In section 3.4.2, we provide a rationale for this behavior and demonstrate the incentive effects induced by this particular design.

3.3 Recurring themes in the literature

Contingent capital has been mentioned in the literature as an effective tool to stabilize financial markets in distress. Flannery (2005, 2010) argues in favor of contingent capital since risk-taking costs are internalized rather than shifted towards tax payers in a public bail-out. Acharya et al. (2009) describe CoCo bonds as “clearly a good idea”. Furthermore, we find favorable mentions of these instruments in policy recommendations such as Stein (2004), Kaplan (2009), and Duffie (2009).

In the following, we highlight three issues, which have been in the focus of the recent literature. (See also Pazarbasioglu et al. (2011), Murphy et al. (2012), and Flannery (2014) for different takes on this topic.) First, we discuss risk-shifting incentives as a possible threat amplified by contingent capital. Second, we examine whether CoCo bonds can alleviate credit crunches. And third, we analyze to what extent CoCo bonds create incentives for either claim holder to manipulate prices and force a conversion.

3.3.1 Risk-shifting incentives

At first glance, contingent capital seems to be a universally beneficial financial instrument. In good states of the economy, the CoCo bond holders receive a coupon just like ordinary subordinated debt holders. In bad states of the economy, a potentially costly default is prevented by converting the CoCo bonds into new equity. However, it is exactly this feature which might have negative repercussions. The bank’s managers anticipate the conversion and take it into account when making their investment decisions. Consequently, the managers might be inclined to increase the riskiness of the bank's assets since the bank has additional downside protection provided by the CoCo bonds.

Straight debt is often argued to be an optimal financing contract since it causes an exchange of control rights in bad economic states, which equity holders prefer to avoid (Calomiris and Kahn, 1991; Flannery, 1994). This disciplining effect of straight debt is possibly weakened by CoCo bonds since the conversion mechanism postpones the transfer of control rights. Both Flannery (2005) and Pennacchi et al. (2014) already hint at this possible elimination of disciplining effects and recognize the existence of risk-shifting incentives.

In the following, we discuss the model of Koziol and Lawrenz (2012), who use a continuous-time framework in order to formally examine the effects of contingent capital on banks' risk-taking incentives. The bank's assets follow a Geometric Brownian motion. On the liabilities side, the bank takes government-insured deposits and issues debt, which can be either straight debt or contingent convertible debt. Both debt contracts are consol bonds with fixed coupon payments. The conversion trigger of the CoCo bonds is set such that conversion occurs at the time when the default threshold of the pure debt firm would be breached. The introduction of the classical trade-off between tax benefits of debt and bankruptcy costs allows to derive an optimal capital structure.

Koziol and Lawrenz (2012) consider two cases. In the first case, asset risk is contractible. If equity holders do not have discretion over the choice of risk, an issuance of CoCos increases the bank's debt capacity. This implies that the advantages of debt financing, such as tax shields, can be exploited to a larger extent under CoCo financing. At the same time, the default probability as well as the present value of distress costs are decreased by substituting straight debt with CoCo bonds. Hence, CoCo bonds are not only individually beneficial for the bank's equity holders but also socially optimal for the whole economy.

In the second case, contracts are incomplete in the sense that the bank is not able to credibly commit to a specific asset risk. As a consequence, CoCo bonds always distort risk-taking incentives if the claims are already part of the bank's capital structure. If the bank faces low financial constraints, it is always willing to increase the risk both with straight debt financing as well as with CoCo financing. If financial constraints are high, a bank financed with non-convertible debt prefers to reduce the asset risk. However, an issuer of CoCo bonds might still prefer to increase the asset risk. Hence, CoCo bonds have the potential to magnify risk-taking incentives, but never reduce them.

As savvy investors anticipate such a risk increase, they will demand a compensation. Tables 3.1 and 3.2 show that the issued CoCo bonds all pay a significantly higher coupon compared to straight debt. So are CoCo bonds still desirable from the equity holders' perspective? Two main effects impact the answer to this question. First, CoCo bonds always increase risk-taking incentives when compared to straight debt. This increases the probability of default and thus expected distress costs. Second, contingent capital relaxes financial constraints and enables banks to take advantage of tax benefits to a larger extent. While the former decreases the firm value, the latter increases it. If risk-shifting opportunities are low, a CoCo issuance might still result in a higher firm value. If risk-taking opportunities are high, however, the effect of relaxed financial constraints is overcompensated by higher expected distress costs and the firm value decreases.

In addition, Koziol and Lawrenz (2012) provide evidence that the issuance of CoCo bonds can simultaneously increase the firm value as well as the probability of default. This clearly undermines regulators' intentions to reduce risk-taking incentives of distressed financial institutions. In this case, the individually rational behavior of the bank has adverse, destabilizing effects on the whole financial system.

3.3.2 Procyclicality of lending and credit crunches

Contingent convertibles are foremost discussed in the context of bank stability. However, their use might also be able to mitigate another important problem in the financial sector: procyclical lending and credit crunches. (See Ivashina and Scharfstein (2010) for recent empirical evidence in the context of the 2008 financial crisis.)

Intuitively, when the state of the economy worsens, the risks of banks increase, e.g., market volatility is soaring and non-performing loans are accumulating. In such a situation, banks have two options to reduce their risks. They can either sell or hedge some of their risky investments or they can constrain new business. Since the first is usually difficult and expensive during times of economic crisis, banks regularly stick to the latter. In addition, shrinking the bank's assets is optimal for equity holders, since the benefits from injecting capital into the bank primarily accrue to debt holders. This results in procyclical lending behavior and even credit crunches, when banks fully cease lending to new customers.

How can contingent convertibles help? When the driving motive for banks not to grant new loans in bad times is the fear of financial distress, any financial instrument which reduces default risk or lowers costs associated to situations of financial distress also helps regarding the credit crunch issue. Hence, contingent capital is an obvious candidate for the solution of this problem.

In the following, we introduce the model of Crummenerl et al. (2014), who analyze a regulated financial institution in a world without taxes and bankruptcy costs. The bank inherits a risky loan portfolio, whose payoff depends on the realization of the future state of the economy as either good (with probability p) or bad (with probability $1-p$). The bank is considering to grant an additional (uncorrelated) risky loan. In doing so, the bank needs to take into account that it might have to recapitalize in the future to meet a Basel-type regulatory capital requirement.

Furthermore, the model incorporates adjustment costs in case the bank needs to reduce its debt ratio in the future due to the regulatory constraint. These costs can be interpreted as increased search or marketing costs. They represent the fact that banks cannot readily finance themselves in times of financial distress. Importantly, the adjustment costs do not occur when banks convert their outstanding CoCo bonds into new equity. The CoCo bonds have already been issued in $t = 0$ and, thus, there is no need for additional search or marketing effort when conversion occurs.

The important result from Modigliani and Miller (1958) tells us that the loan decision is independent of the economic state in a frictionless world, i.e., when recapitalization is always available at fair terms. The adjustment costs now link the loan decision to the economic outlook. Banks have an incentive not to grant a loan today if they thereby reduce the likelihood of expensive capital structure adjustments tomorrow. A credit crunch occurs when the probability $1-p$ of the bad state of the economy is high and when the bank is highly levered.

In this setup, CoCo bonds are a tool to avoid costly recapitalization. If the conversion of the CoCo bonds is on fair terms and if a sufficient amount of CoCo bonds is available, banks prefer to convert the CoCo bonds when the regulatory constraint is breached. In this case, the bank never incurs the adjustment cost and consequently always provides the additional loan. The credit crunch is successfully mitigated.

An important caveat to this finding is that it presumes that CoCo bonds are already issued by the bank, i.e., the issuance decision is exogenous. The key question is now whether banks want to issue them in the first place.

Crummenerl et al. (2014) model the issuance of CoCos in the following way: The bank has an outstanding amount of debt. If it decides to issue CoCo bonds, it first has to buy back outstanding debt at fair terms. This amount is replaced by contingent capital, which is priced to have zero net present value (NPV). As a consequence, the nominal amount of debt remains unchanged. It is also ensured that the amount of issued CoCo bonds is sufficient to avoid a costly capital increase in the future. Hence, the above result holds and banks always grant the additional loan when CoCo bonds are available. The bank has two further choices. It can decide against CoCo bonds and still grant the loan. Or it can decide against CoCos and against the loan, i.e., a credit crunch occurs.

There are three rationales driving the decision of the bank:

Adjustment costs: The bank saves expected adjustment costs in the case of CoCo financing if there is a positive probability of a capital increase.

Debt repurchase: Banks issuing CoCos have to buy back debt in $t = 0$ when the price is likely to be higher than it would be in $t = 1$ in times of distress. For banks issuing an additional loan, the expected necessary reduction of the debt level is higher if regulatory capital requirements are breached due to the higher risk-weighted assets.

Risk-shifting: An additional loan decision increases the equity value due to the higher overall risk of the bank's loan portfolio.

The avoidance of adjustment costs benefits the issuance of CoCos, while the risk-shifting incentives favor an affirmative loan decision. The costly debt repurchase is to the disadvantage of both, since the repurchase takes place based on expectations. The bank's decision depends on the outlook on the future state of the economy, i.e., the probability p, and the current debt level of the bank. For very low debt levels, the bank affirms the loan and no contingent capital is required. There are some debt levels for which the bank would not have granted the loan, but now decides to issue CoCos. In these instances, a credit crunch is successfully mitigated. This result contrasts the finding of Albul et al. (2013), who argue that banks are never willing to issue contingent capital voluntarily. However, when the debt level further increases, the bank is not willing to issue CoCos and does not grant the loan.

In addition, the benefits of CoCo bonds vanish with an increase in the probability p of the good state, i.e., when banks have an optimistic view of the economic development. Hence, banks are not issuing CoCo bonds in good times or when a bubble potentially occurs, which can both be interpreted as a high expectation of the probability p.

In summary, CoCo bonds increase the debt capacity of the financial system and they are a well desired instrument in bad states of the economy. Despite these benefits, banks are not willing to issue contingent capital in good times. These findings put regulators in a dilemma. If they consider contingent capital as an appropriate instrument to prevent credit crunches, they have to prescribe a mandatory issuance of these claims. This comes at a cost, which has to be borne by the bank's owners.

3.3.3 Incentives to force a conversion

The conversion trigger mechanism plays a crucial role in the design of contingent capital. As we have seen in section 3.2, most of the CoCo issues so far have triggers based on accounting numbers. These accounting numbers seem to be a good choice: they are compiled based on common rules, they are not distorted by potentially irrational market movements and they are frequently monitored by regulators. The most common measure used to trigger the conversion of CoCo bonds is the tier 1 capital ratio, which relates the bank's core book equity to its risk-weighted assets.

However, accounting numbers have two major disadvantages. First, they are backward looking. Thus, they include information about the economic prospects of the bank only to the extent to which this information can be inferred from the bank's past performance. Second, the management does have some discretion, e.g., over how and when to account for impairment losses. Needless to say, the books can also be manipulated by the management, like in the cases of Enron and Lehman Brothers. As a consequence of these two disadvantages, conversion might happen too late.

The alternative to using accounting numbers is to resort to market based triggers. Since the asset value process is not directly observable, the only available measure is the bank's share price. When markets are efficient, the share price appropriately reflects all available information on the future prospects of the bank. However, if the trigger mechanism is based on the share price, which in turn is influenced by the time and terms of conversion, distortions in the pricing of the bank's shares might arise. This issue is highlighted by Sundaresan and Wang (2015) as well as Albul et al. (2013).

In the following, we introduce a simple example from Sundaresan and Wang (2015) to illustrate the impact of market based triggers on equilibrium pricing. Assuming that conversion can only occur at maturity, they consider the conditions for conversion and no-conversion of the CoCo bonds. The CoCo bonds are not converted if the asset value A after all payments to non-convertible debt holders is higher than the conversion threshold K plus the coupon payment c to CoCo bond holders, i.e., $A > K + c$. When the CoCo bonds are converted, the CoCo bond holders receive m new shares. The number of shares prior to conversion is denoted by n. After conversion, the bank is unlevered and the share price should be below the conversion threshold, which is equivalent to $A \leq \frac{n+m}{n} \cdot K$.

Sundaresan and Wang (2015) distinguish two cases. In the first case, there is a wealth transfer at conversion towards the debt holders, i.e., $\frac{m}{n} \cdot K > c$. As a result, the two above conditions can be met at the same time for some asset values. This leads to two equilibrium prices, which depend on the beliefs of investors. In one equilibrium, all investors believe that conversion does not occur. In the second equilibrium, all investors believe that conversion occurs, which causes the equity value to hit the trigger threshold. Sundaresan and Wang (2015) generalize this result in a continuous-time setting and show that multiple equity values are possible well before a potential conversion.

In the second case, there is a wealth transfer at conversion from debt holders to equity holders, i.e., $\frac{m}{n} \cdot K < c$. In this case, there exists a range of asset values for which both of the above conditions are simultaneously not met. This result is caused by the wealth transfer to equity holders, which causes the share price to rise at conversion. However, if the share price is close to trigger level, the expected increase after conversion lifts the share price above the trigger threshold and conversion is prevented. The consequence is that an equilibrium share price does not exist at all for some asset values.

Apparently, the issue that the equilibrium share price either has multiple solutions or no solution at all is caused by the wealth transfer at conversion. Albul et al. (2013) propose a constant adjustment of conversion ratios in order to ensure that the market value of the debt claim is at any point in time equal to the market value of received shares at conversion. However, this approach is difficult to implement. Sundaresan and Wang (2015) propose a continuous adjustment of the coupon to the rate of short-term risky bank obligations. Thus, the market value of the debt claim remains close to par and a conversion ratio can be determined upon CoCo issuance. This ensures that CoCo holders receive the equivalent of their bond market value (par) in shares when the (equity-)trigger is breached.

The avoidance of multiple equilibrium prices is important since they give rise to manipulation incentives for claim holders. Albul et al. (2013) show that equity holders have incentives to drive down the share price and force a conversion if conversion ratios are sufficiently low. For example, managers might distribute false negative information to lower the price. The opposite is true if there is a wealth transfer from equity holders to CoCo bond holders at conversion. In line with Duffie (2009) and McDonald (2013), the authors argue that CoCo bond holders might engage into short-selling activities in order to trigger a conversion and benefit when the fair share price is restored. If wealth transfers cannot be avoided completely, McDonald (2013) proposes to retire the outstanding CoCo bonds gradually and randomly to limit the gains of manipulations.

We have shown that the practical implementation of the trigger mechanism is crucial and might potentially lead to distorted equilibrium prices and manipulation incentives. The key determinant of these incentives is the wealth transfer at conversion, which depends on the conversion ratio as well as the conversion threshold.

3.4 Design of contingent convertible debt

We have highlighted three issues related to CoCo bonds. First, CoCo bonds induce risk-shifting incentives for managers. Second, we have shown that it might not be optimal for banks to issue CoCo bonds in good times even though they help to mitigate a credit crunch in bad times. And third, situations might arise in which claim holders have an incentive to manipulate prices. All three issues are related to the specific design of the CoCo bonds.

Despite these concerns, we observe a wave of issuances of CoCo bonds since 2013. Some of those CoCos have been issued voluntarily by banks in the aftermath of the financial crisis, i.e., in a severe state of the economy. And others have been issued more recently to fulfill prospective regulatory requirements, e.g., in Switzerland. Many of these recent issues have been total-loss bonds, i.e., they have a conversion ratio of zero. We now pursue the question why banks are choosing this particular design and what the regulatory implications from this behavior are.

In the following, we introduce a simple continuous-time framework of a bank financed with CoCo bonds and equity. We analyze how the product design of the CoCo bonds, i.e., the choice of the conversion ratio and the trigger level, impact the severity of the risk-taking incentives and credit crunch effects caused by CoCo bonds.

3.4.1 Model framework

We consider a bank with assets V, which follow a diffusion process of the form

$$\frac{\mathrm{d}V}{V} = \mu \ \mathrm{d}t + \sigma \ \mathrm{d}z, \tag{3.2}$$

where μ denotes the expected return of the assets, σ is the volatility of asset returns and z is a standard Wiener process.

The bank has two claims outstanding: contingent convertible debt and equity. As long as the bank is solvent, the CoCo bond holders receive an instantaneous coupon payment of c. In case the trigger event occurs, i.e., the asset value decreases below the conversion threshold $VC < V$, the CoCo bonds are converted into equity and the bank continues as an unlevered entity. At conversion, the CoCo bond holders receive a share γ of the equity and the old equity holders retain a share $1-\gamma$ of the bank's equity. The capital structure is determined at time $t=0$ and remains static thereafter.

Applying the pricing approach of Leland (1994), the CoCo bond value D at time $t=0$ amounts to

$$D = \frac{c}{r} + \theta \cdot \left(\gamma \cdot VC - \frac{c}{r}\right). \tag{3.3}$$

The first term $\frac{c}{r}$ gives the value of a risk-free consol bond with coupon payments c, where r is the risk-free rate. The second term gives the value of the conversion feature. When conversion occurs, the bond holders get a fraction $\gamma \in [0,1]$ of the assets, which have value VC at the time of conversion. The bond holders lose the coupon payments. Hence, $\gamma \cdot VC - \frac{c}{r}$ corresponds to the wealth transfer from equity holders to bond holders at the time of conversion.

The factor

$$\theta = \left(\frac{V}{VC}\right)^{-\frac{2r}{\sigma^2}} \tag{3.4}$$

corresponds to the present value at $t = 0$ of 1 EUR paid at the time of conversion.

We can rewrite equation (3.3) as

$$D = (1-\theta)\cdot\frac{c}{r} + \theta\cdot\gamma\cdot VC. \tag{3.5}$$

The discount factor θ is a measure for the conversion risk. The CoCo value is equal to the value $\frac{c}{r}$ of the cash flow without conversion weighted with the factor $1-\theta$ plus the value $\gamma \cdot VC$ of the received assets at conversion weighted with the factor θ.

Since we do not consider tax-benefits of debt and bankruptcy costs, the equity value S corresponds to

$$S = V - D. \tag{3.6}$$

To improve comparability between banks with different CoCo designs, we keep the leverage ratio fixed, i.e., the CoCo value D is constant across different product designs. For a given CoCo design represented by γ and VC, the coupon rate $\bar{c}$ is determined as

$$\bar{c} = (D - \theta\cdot\gamma\cdot VC)\cdot\frac{r}{1-\theta}. \tag{3.7}$$

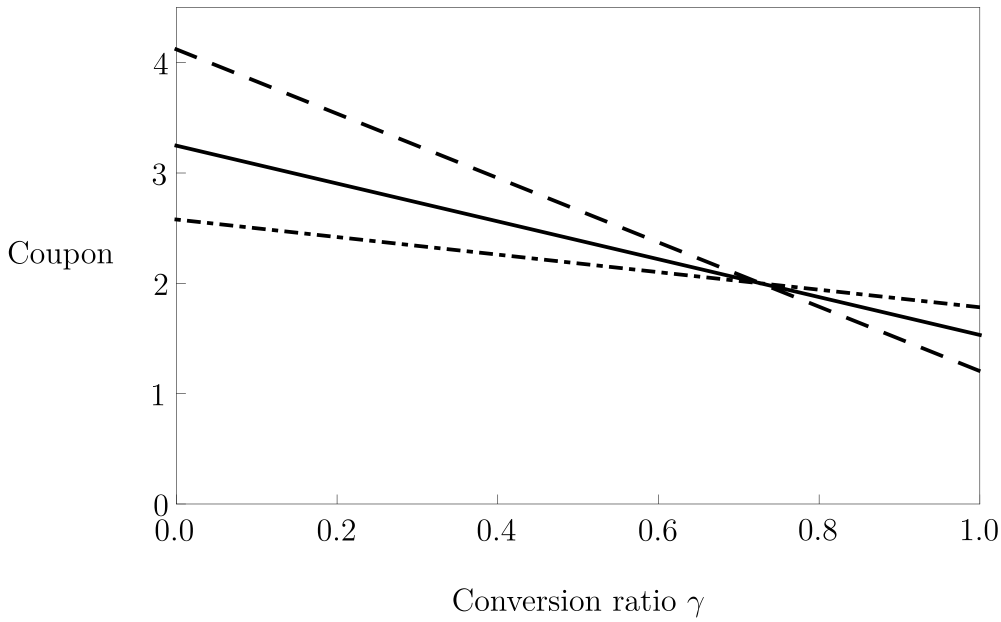
The coupon payment strictly increases with the bank's leverage ratio and strictly decreases with the conversion ratio γ. Figure 3.1 plots the coupon depending on the conversion ratio γ for three different volatility levels. The three lines cross each other at $\widehat{\gamma} = \frac{D}{VC}$, which implies that there is no wealth transfer between the claim holders for this particular conversion ratio, i.e., the debt holders demand the same coupon independent of the volatility. For values of $\gamma < \widehat{\gamma}$, there is a wealth transfer from debt holders to equity holders at conversion. Consequently, a higher volatility, which makes conversion more likely, is compensated by a higher coupon payment. In the opposite case when $\gamma > \widehat{\gamma}$, the bond holders benefit from conversion and a higher volatility results in a lower coupon payment.

3.4.2 Asset substitution problem

In the first step of our analysis, we evaluate the risk-shifting incentives of the equity holders. We have to distinguish between two fundamental cases: a bank before and after the issuance of CoCo bonds. In our model, we assume complete and frictionless markets. If all products are fairly priced, CoCo bond holders anticipate the behavior of banks and price the claims accordingly. Hence, a market friction is required to explain the issuance of CoCo bonds.

Figure 3.1: Coupon for different volatility levels

The plot shows the instantaneous coupon payment c for a debt value $D = 40$ and a trigger level of $VC = 55$ depending on the conversion ratio γ for three different volatility levels of $\sigma = 0.20$ (dotted line), $\sigma = 0.25$ (solid line), and $\sigma = 0.30$ (dashed line). The remaining parameters are $V = 100$ and $r = 0.05$.



Of course there are reasons why the bank's management might want to deviate from the optimal policy for the shareholders. This can, for example, be the case when the bank managers are paid with stock options and thus benefit from an increase of the bank's business risk. Furthermore, managers might be interested in empire building and acquire more risky and even unprofitable businesses.

However, we abstract from these issues for the remainder of this analysis. We assume that the CoCo bonds have already been issued and that the bank's managers can alter the riskiness of the assets, i.e., risk is not contractible by the bond holders.

We examine the resulting incentive effects of different CoCo bond designs, which crucially depend on the associated wealth transfer at conversion. We summarize our first result in the following proposition. (See appendix 3.A for proof.)

Proposition 3.1 (Risk-shifting)
Equity holders have an incentive to undertake risk-shifting whenever there is a wealth transfer from CoCo bond holders to equity holders at conversion, i.e., $\gamma \cdot VC - \frac{c}{r} < 0$.

Intuitively, when there is a wealth transfer from CoCo bond holders to equity holders, the equity holders benefit from conversion. Hence, it becomes worthwhile for equity holders to increase the likelihood of conversion by investing in more risky projects. In the opposite case, conversion corresponds to a penalty for equity holders. Hence, they prefer to avoid conversion and reduce risk.

When we hold the leverage ratio constant across different CoCo bond designs, the wealth transfer is to the benefit of equity holders when $\gamma < \widehat{\gamma}$ and it is to the benefit of the CoCo bond holders when $\gamma > \widehat{\gamma}$. We can conclude that the low or even zero conversion ratio of the recently issued CoCo bonds amplify the risk-shifting incentives of equity holders. The case of $\gamma = 0$ corresponds to the total-loss bond design discussed in section 3.2.

We further demonstrate this result by analyzing the classical asset substitution problem. Initially, the bank's assets have volatility σ_l. The bank can accept a new project with net present value ΔV, which changes the total asset volatility to a higher volatility $\sigma_h > \sigma_l$. The asset substitution problem occurs if the equity holders are willing to accept negative NPV projects, i.e., $\Delta V < 0$, under the new risk environment σ_h.

We are now solving for the critical change ΔV^* in the asset value, such that the equity holders achieve exactly the same equity value as under the low risk environment, i.e.,

$$S(V, \sigma_l) = S(V + \Delta V^*, \sigma_h). \tag{3.8}$$

Again, the occurrence of the asset substitution problem crucially depends on the wealth transfer at conversion. (See appendix 3.A for proof.)

Proposition 3.2 (Asset substitution problem)
The asset substitution problem occurs whenever there is a wealth transfer from CoCo bond holders to equity holders at conversion, i.e., $\gamma \cdot VC - \frac{c}{r} < 0$.

We illustrate this result numerically in figure 3.2. The graph shows the critical value change ΔV^* for three different volatility levels σ_h depending on the conversion ratio γ. All points represent CoCos with the same value D and the same conversion threshold VC. In line with our previous finding, we see that the critical value change is negative for conversion ratios below the threshold $\widehat{\gamma}$. The critical value change monotonically increases with the conversion ratio and is positive for values above $\widehat{\gamma}$. The effect is more pronounced, i.e., the curves are steeper, for high values of σ_h.

We have shown that the wealth transfer at conversion determines the incentives for equity holders to increase risk and to invest in new projects. So far, we have analyzed the impact of the conversion ratio, but the wealth transfer is also impacted by the trigger level, which by definition coincides with the value of the assets at conversion. A higher trigger level thus implies a shift of wealth towards the debt holders.

The trigger level also changes incentives through another channel. The likelihood of conversion increases ceteris paribus with the trigger level, i.e., $\frac{\partial \theta}{\partial VC} > 0$. Whenever wealth is transferred to debt holders at conversion, e.g., for high conversion ratios, equity holders dislike high trigger levels since they make a conversion more likely. Whenever the wealth transfer is to the benefit of the equity holders, e.g., for low conversion ratios, the effect of the trigger level is ambiguous. Equity holders benefit from a higher likelihood of conversion, but higher trigger levels also reduce the benefit when conversion occurs.

Before we continue the analysis of the asset substitution problem, we review the general risk-shifting incentive of equity holders. When again holding the leverage ratio $\frac{D}{V}$ constant, we can rewrite the condition regarding the wealth transfer from proposition 3.1. There is a transfer from CoCo bond holders to equity holders when $\gamma \cdot \frac{VC}{V} < \frac{D}{V}$, i.e., when the value promised to CoCo bond holders at conversion in percentage of today's asset value is smaller than the current leverage ratio of the bank. This is a convenient reformulation of the condition, since the leverage ratio of a bank can easily be observed.

In the following, we examine the role of the trigger level in the asset substitution problem for a bank with a low conversion ratio of $\gamma = 0.25$ and a bank with a high conversion ratio of $\gamma = 0.75$. Figure 3.3 shows the critical asset value change ΔV^* depending on the trigger level as percentage of initial assets, i.e., $\frac{VC}{V}$. The claims are priced such that the leverage ratio $\frac{D}{V}$ is held constant.

We first discuss the asset substitution for the bank with the low conversion ratio of $\gamma = 0.25$, which is pictured on the left of figure 3.3. Consistent with our previous finding,

Figure 3.2: Asset substitution and conversion ratio

The plot shows the critical asset value change ΔV^* for a debt value $D = 40$ and a trigger level of $VC = 55$ depending on the conversion ratio γ for three opportunities to increase the asset risk from $\sigma_l = 0.25$ to $\sigma_h = 0.30$ (dotted line), $\sigma_h = 0.35$ (solid line), or $\sigma_h = 0.40$ (dashed line). The remaining parameters are $V = 100$ and $r = 0.05$.

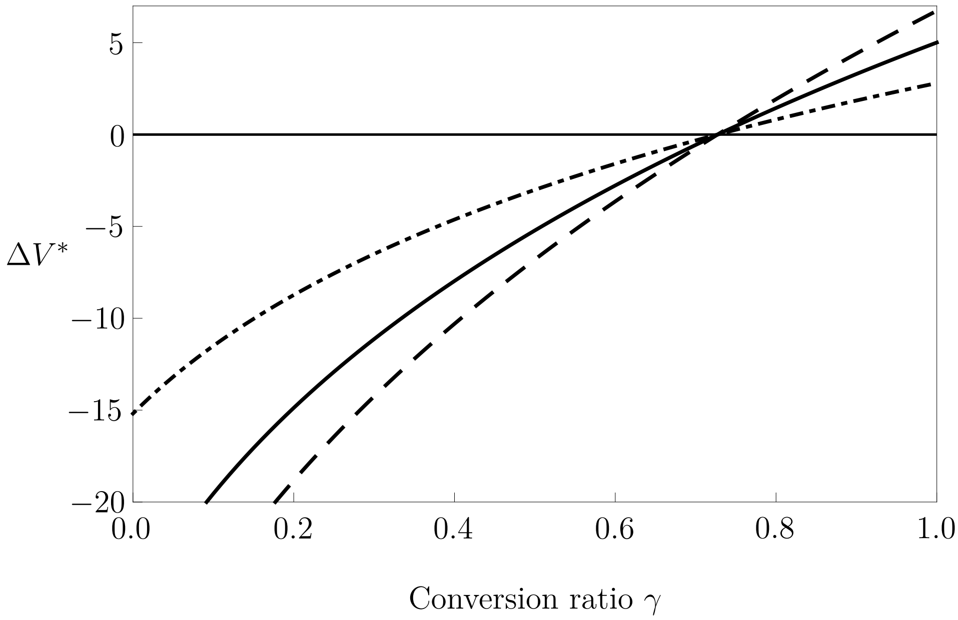
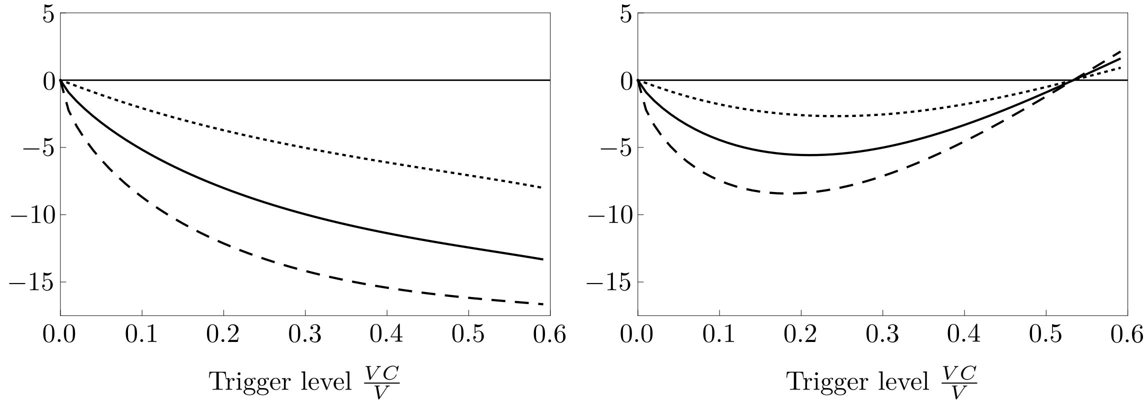


Figure 3.3: Asset substitution and trigger level

The plot shows the critical asset value change ΔV^* for a debt value $D = 40$ and two conversion ratios of $\gamma = 0.25$ on the left and $\gamma = 0.75$ on the right. The critical asset value is plotted depending on the relative trigger level $\frac{VC}{V}$ for three opportunities to increase the asset risk from $\sigma_l = 0.25$ to $\sigma_h = 0.30$ (dotted line), $\sigma_h = 0.35$ (solid line), or $\sigma_h = 0.40$ (dashed line). The remaining parameters are $V = 100$ and $r = 0.05$.



the critical NPV is negative. It monotonically decreases with the trigger level. The low conversion ratio ensures that the wealth transfer remains to the benefit of the equity holders even when the trigger level increases. At the same time, the likelihood of conversion increases as well. In sum, a higher trigger level worsens the asset substitution problem.

We observe a different pattern for the bank with the high conversion ratio of $\gamma = 0.75$, which is pictured on the right of figure 3.3. For low trigger levels, equity holders still have the incentive to engage in asset substitution. The wealth transfer benefits the equity holders and conversion is very unlikely. For high trigger levels, the critical asset value increases with the trigger level. It becomes positive for values of VC above $\frac{D}{\gamma}$, for which the wealth transfer changes to the benefit of the CoCo bond holders.

Regulators are interested in stabilizing financial markets in distressed situations and providing the economy with necessary liquidity when it is most needed. Contingent capital is often praised as being the magic remedy in financial downturns. We show that this view has to be taken with caution.

Our results show that the impact of CoCo bonds on risk-taking incentives strongly depend on the conversion ratio and the associated wealth transfer. If banks are inclined to increase their risk, they benefit from low conversion ratios, which imply a wealth transfer from bond holders when conversion occurs. An increase of the asset volatility makes a conversion more likely. Hence, equity holders have strong incentives to force a conversion.

From the regulatory point of view, this effect is rather undesired. As we have shown, the risk-taking incentives diminish if the conversion ratio increases. For high conversion ratios, the asset substitution problem is fully mitigated and banks have incentives to reduce risk. Therefore, regulators should clearly prefer high conversion ratios with regard to the stability of the financial system.

We also conclude that the interaction between the two product parameters needs to be taken into account to mitigate the asset substitution problem. Both the conversion ratio and the trigger level should be sufficiently high, such that there is a wealth transfer from equity holders to CoCo bond holders at conversion. In particular, the critical trigger level increases with the leverage ratio of the bank. Therefore, highly levered banks should issue CoCo bonds with higher trigger levels compared to banks with low debt ratios.

Given the product parameters are fixed, equity holders can potentially increase the leverage of the bank, which again could make asset substitution worthwhile. This could be prevented either by a covenant of the CoCo bond or by a regulatory restriction of the bank's leverage, as recently proposed by the Dodd-Frank Act in the US.

3.4.3 Debt overhang problem

In the second step of our analysis, we focus on the loan granting behavior of banks. We have already discussed in section 3.3.2 that CoCo bonds can help to mitigate a credit crunch. In the following, we examine how the product design choices influence loan-granting incentives. Again, our results show that loan granting behavior highly depends on the wealth transfer at conversion.[5]

We consider the classical debt overhang problem. The equity holders are considering an out-of-pocket investment at time $t=0$, which can be interpreted as granting an additional loan. The investment requires an upfront payment of I and increases the asset value by $(1+y)\cdot I$, where y denotes the return of the investment. The asset risk remains unchanged. We determine the critical required return y^* on the investment such that equity holders are indifferent between holding the amount I in cash and injecting the money into the bank to finance the additional loan, i.e., we solve the condition

$$S(V)+I=S(V+(1+y^*)\cdot I)\,. \tag{3.9}$$

We find that also the loan granting behavior depends on the wealth transfer at conversion. (See appendix 3.A for proof.)

Proposition 3.3 (Loan granting)
A credit crunch occurs whenever there is a wealth transfer from CoCo bond holders to equity holders at conversion, i.e., $\gamma\cdot VC-\frac{c}{r}<0$.

The bank is only willing to grant loans with a return above y^*. A positive critical return y^* implies that loans with a low but positive NPV, i.e., with return $y\in(0,y^*)$, which should be granted from the social planner's perspective, are not approved by the bank. Hence, a credit crunch occurs. In contrast, critical returns below zero indicate that equity holders are willing to accept loans which decrease the asset value. Arguably, it is not desirable from the social planner's perspective that negative NPV loans are granted. However, the bank provides sufficient liquidity to the financial system and a credit crunch does not occur.

Figure 3.4 demonstrates the effects on lending behavior for three different volatility levels. The plot shows the critical return y^* depending on the conversion ratio γ. The required critical return is positive for low conversion ratios and monotonically decreases with γ.

[5] In contrast, Crummenerl et al. (2014) focus on one specific CoCo bond design. The conversion ratio is determined in $t=0$ such that the expected value of the CoCo bond is the same with or without the conversion feature.

The critical return is zero for $\widehat{\gamma} = \frac{D}{VC}$ and negative for high conversion ratios. The effect is more pronounced, i.e., the curves are steeper, for high asset volatilities σ.

The key observation is that CoCo bonds with low conversion ratios exacerbate liquidity dry outs. Again, the critical factor is the wealth transfer from equity holders to debt holders at conversion. For low conversion ratios below $\widehat{\gamma}$, the associated wealth transfer is negative. Hence, it is not in the interest of equity holders to grow the assets and thereby reduce the likelihood of conversion. Consequently, they only take on additional investments which offer a high rate of return.

The opposite is true for high conversion ratios above $\widehat{\gamma}$. In this case, the wealth transfer is to the benefit of the CoCo bond holders and the equity holders prefer to avoid conversion. Hence, they inject cash into the bank to grow the asset value and thereby decrease the likelihood of conversion. Intuitively, they prefer to lose a small amount of value today rather than losing a large amount of value at conversion. Hence, they are even willing to undertake negative NPV projects.

Again, we also analyze the role of the trigger level regarding the credit crunch issue. Figure 3.5 shows the critical required return for a bank with a low conversion ratio of $\gamma = 0.25$ and a bank with a high conversion ratio of $\gamma = 0.75$. All claims are priced such that the leverage ratio $\frac{D}{V}$ is held constant.

In the low conversion rate scenario, which is pictured on the left of figure 3.5, the credit crunch problem always occurs and worsens with an increase in the trigger level. The critical return increases monotonically with the trigger level. Equity holders always benefit from conversion. The likelihood of conversion increases with the trigger level, which equity holders like. Hence, they are not willing to grow the balance sheet of the bank, which would increase the distance to the trigger level, and demand high returns of new investments to be compensated for the lower likelihood of conversion.

We observe a different pattern for the high conversion rate scenario, which is pictured on the right of figure 3.5. The credit crunch still occurs for low conversion ratios, since the wealth transfer is to the benefit of the equity holders. With an increase of the trigger level, the likelihood of conversion increases, which equity holders like. Hence, the critical return increases with the trigger level. But at the same time, an increase in the trigger level reduces the wealth transfer, which causes the critical return to decline when the trigger level is further increased. For trigger levels above $\frac{D}{\gamma}$, the wealth transfer switches to the benefit of debt holders. In this case, the equity holders dislike conversion and, thus, prefer to grow the balance sheet. A credit crunch is successfully mitigated.

We conclude that CoCo bonds with high conversion ratios prevent a credit crunch and should be advocated by regulators. This finding is also in line with our results on the

Figure 3.4: Loan granting and conversion ratio

The plot shows the required critical return y^* for a debt value $D = 40$ and a trigger level of $VC = 55$ depending on the conversion ratio γ for three different volatility levels of $\sigma = 0.20$ (dotted line), $\sigma = 0.25$ (solid line), and $\sigma = 0.30$ (dashed line). The remaining parameters are $V = 100$ and $r = 0.05$.

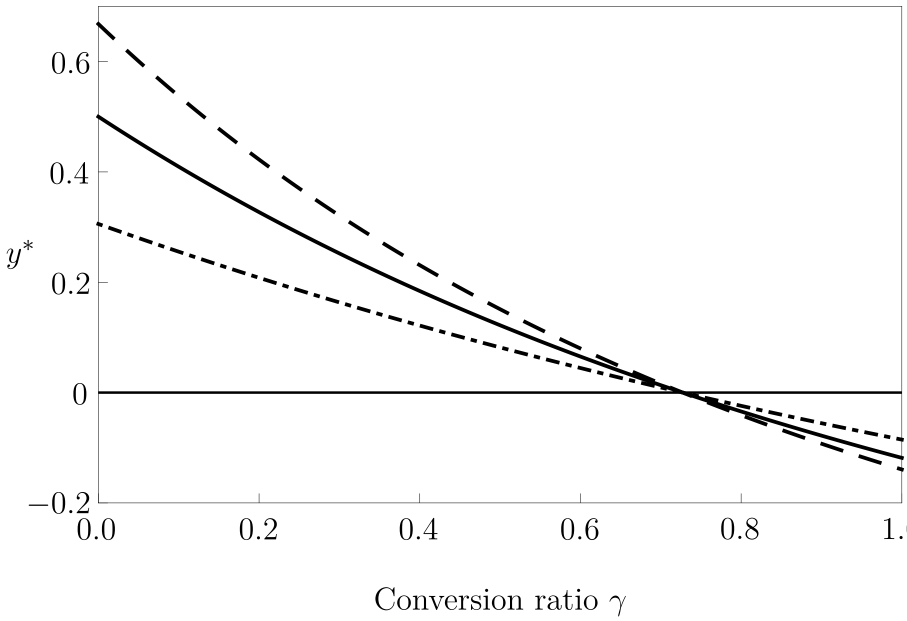
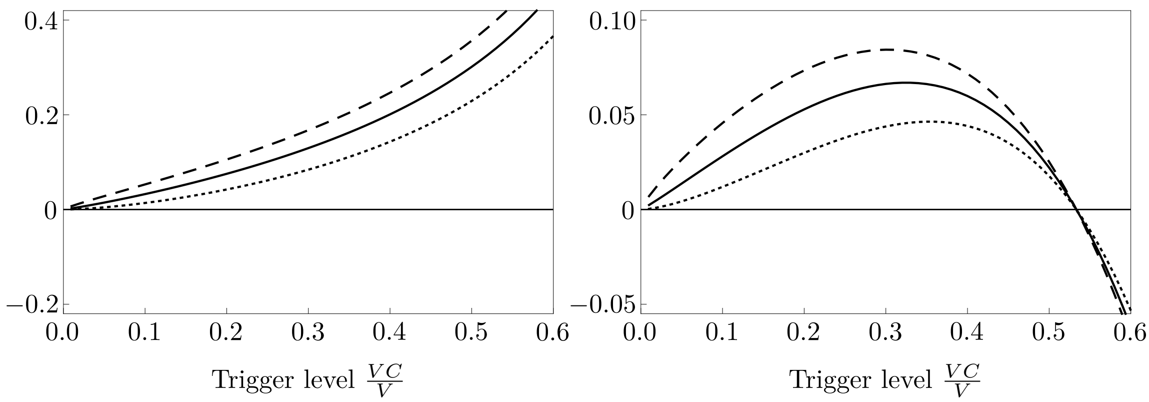


Figure 3.5: Loan granting and trigger level

The plot shows the required critical return y^* for a debt value $D = 40$ and two conversion ratios of $\gamma = 0.25$ on the left and $\gamma = 0.75$ on the right. The critical return is plotted depending on the relative trigger level $\frac{VC}{V}$ for three different volatility levels of $\sigma = 0.20$ (dotted line), $\sigma = 0.25$ (solid line), and $\sigma = 0.30$ (dashed line). The remaining parameters are $V = 100$ and $r = 0.05$.



risk-shifting issue. A wealth transfer from equity holders to bond holders at conversion is the key feature, which a CoCo bond should possess from the regulatory perspective. This penalizing effect of conversion mitigates not only the risk-shifting incentive of equity holders but also ensures the liquidity supply of the financial system.

This finding is especially striking given the empirical evidence that most of the recent issues of contingent debt are total-loss bonds, i.e., $\gamma = 0$. At first glance, these products have a very favorable property from the regulatory point of view, since the debt completely vanishes in the event of default. Hence, banks with total-loss bonds cannot default. However, it is desirable for equity holders to trigger the event which causes the wipe out of debt holders. The result is an odd situation, in which a credit crunch occurs even though the bank is not subject to default risk.

From a social planner's perspective, it is desirable that positive NPV projects are always financed. Hence, the social planner should design the CoCo bond such that the associated wealth transfer is equal to zero, i.e., $\gamma \cdot VC = \frac{c}{r}$. This mitigates the credit crunch problem, but also ensures that negative NPV projects are not undertaken. We have also shown in the previous section that the asset substitution problem does not occur when there is no wealth transfer at conversion.

3.5 Conclusion

The financial crisis emerging in 2008 illustrated the need for a more stable banking system and gave rise to the idea of contingent convertible debt. These novel financing instruments seem to be a universal remedy at first glance, since they prevent bankruptcy and keep banks alive in times of financial crisis. We review the literature on CoCo bonds and explore different rationales why banks might be reluctant to issue CoCo bonds. First, shareholders might want to avoid risk-shifting incentives for managers. Second, we have shown that it might not be optimal for banks to issue CoCo bonds in good times even though they help to mitigate a credit crunch in bad times. And third, shareholders might want to avoid situations in which claim holders have an incentive to manipulate the share price of the bank to enforce or prevent a conversion. Nevertheless, we observe a waive of new issuances — mainly due to regulatory pressure — in the recent years.

We also observe that the majority of the more recent issues of CoCo bonds were so called total-loss bonds, i.e., they have a conversion ratio of zero. We introduce a simple continuous-time framework to investigate why banks prefer this particular CoCo bond design. In specific, we look at the choice of the conversion ratio and the trigger level as well as the interaction between these two. We find that whenever there is a wealth transfer at conversion to the benefit of the equity holders, e.g., when the conversion ratio is very low, the equity holders have an incentive to engage in risk-shifting behavior. We conclude that the CoCo bonds might have been issued for this purpose in the first place, since they are a good tool to eliminate the downside risk for the bank. We show that CoCo bonds which are designed to have a wealth transfer to equity holders at conversion also cause a reduction of credit supply, since the bank is not willing to finance all positive NPV projects.

We finally discuss the regulatory implications regarding the design of CoCo bonds. We find that a regulator, who is concerned with risk-shifting and who wants to prevent credit crunches, should advocate a CoCo bond design which ensures a punitive or no wealth transfer at conversion. This implies that the conversion ratio as well as the trigger level should be sufficiently high. In addition, the leverage of the bank needs to be taken into account. The conversion feature should be designed stricter for banks with high leverage ratios. These findings are especially relevant for regulators, who plan to oblige banks to issue CoCo bonds. Recent examples from Switzerland suggest that banks facing a mandatory introduction are choosing the product parameters to their advantage. Hence, mandatory introduction rules should also prescribe the specific product design to make the CoCo bonds work.

Appendix

3.A Proofs of propositions

Risk-shifting (proposition 3.1)

Proof. The first derivative of the equity value with respect to the asset volatility is

$$\frac{\partial S}{\partial \sigma} = -\left(\gamma \cdot VC - \frac{c}{r}\right) \cdot \frac{4\theta r}{\sigma^3} \cdot \ln\left(\frac{V}{VC}\right). \tag{3.10}$$

The second and third terms are always positive, since $VC < V$ by definition. The first term in brackets corresponds to the wealth transfer to debt holders at conversion and determines the sign of the derivative. If the wealth transfer is positive, the derivative is negative and equity holders have an incentive to avoid conversion. In the opposite case, i.e., when the wealth transfer benefits equity holders, the derivative becomes positive and equity holders benefit from risk-shifting.

If we assume that the debt level is fixed and plug in equation (3.7) for the coupon, the derivative (3.10) simplifies to

$$\frac{\partial S}{\partial \sigma} = -\left(\gamma \cdot VC - D\right) \cdot \frac{r}{1-\theta}. \tag{3.11}$$

Again, the term in brackets determines the sign of the derivative. This term is positive for high conversion ratios above the threshold $\widehat{\gamma} = \frac{D}{VC}$ and negative for low conversion ratios below this threshold. ■

Asset substitution (proposition 3.2)

Proof. We have to consider two cases. In the first case, we assume that the wealth transfer is to the benefit of equity holders, i.e., $\gamma \cdot VC - \frac{c}{r} < 0$. So we know from proposition 3.1 that an investment with zero net present value, i.e., $\Delta V = 0$, which increases the asset volatility to σ_h, is resulting in a higher equity value.

We next inspect the first derivative of the equity value with respect to the asset value V, which is continuous and given by

$$\frac{\partial S(\sigma_h)}{\partial V} = 1 - \left(\gamma \cdot VC - \frac{c}{r}\right) \cdot \frac{\partial \theta}{\partial V}. \tag{3.12}$$

We can see from the derivative that two effects impact the equity value. First, a higher asset value directly goes into the pocket of equity holders, since the value of coupon payments is unchanged. Second, the likelihood of conversion is reduced, which is to the disadvantage of equity holders. The derivative is positive, when the first effect outweighs the second, i.e., for low absolute wealth transfers with $\gamma \cdot VC - \frac{c}{r} \geq \frac{\sigma^2 V}{2r\theta}$. In this case, the critical NPV is always negative and at least some negative NPV projects are undertaken. When the absolute wealth transfer surpasses the threshold, i.e., for $\gamma \cdot VC - \frac{c}{r} < \frac{\sigma^2 V}{2r\theta}$, then the derivative becomes negative. In this case, *all* negative NPV projects are undertaken. The wealth transfer at conversion is so large that the equity holders want to increase the likelihood of conversion by reducing the asset value.

In the second case, we assume that the wealth transfer is to the benefit of CoCo bond holders, i.e., $\gamma \cdot VC - \frac{c}{r} > 0$. We know from proposition 3.1 that an investment with zero net present value, i.e., $\Delta V = 0$, which increases the asset volatility to σ_h, is now resulting in a lower equity value. The derivative (3.12) is now always positive. Hence, equity holders always want to grow the asset value to make conversion more unlikely. Negative NPV projects are never undertaken. ■

Loan granting (proposition 3.3)

Proof. We can rewrite condition (3.9) in the following form

$$y^* = \frac{1}{I} \cdot \left(\gamma \cdot VC - \frac{c}{r}\right) \cdot \left(\theta' - \theta\right) \tag{3.13}$$

where θ' denotes the discount factor after the investment, i.e., when the assets of the firm have increased to $V + (1+y) \cdot I$. The term $\theta' - \theta$ is always negative for all admissible values of y. Hence, the wealth transfer determines the sign of the critical return y^*. When the wealth transfer is to the benefit of equity holders, i.e., $\gamma \cdot VC - \frac{c}{r} < 0$, the critical required return is positive. Hence, some positive NPV projects are not financed and a credit crunch occurs. In the opposite case, when the wealth transfer is to the benefit of CoCo holders, i.e., when $\gamma \cdot VC - \frac{c}{r} > 0$, the critical required return is negative. Hence, also negative NPV projects are financed and a credit crunch is mitigated. ■

Chapter 4

Synthesis (part I)

In the frictionless world of Modigliani and Miller (1958), there is no need for financial institutions. Investors possess full information, have unrestricted access to all markets and can trade without transaction costs. The economic rationale for banks arises from frictions. Banks can reduce monitoring and search costs (Leland and Pyle, 1977; Diamond, 1984). Banks can provide cheap market access and liquidity. Banks can slice and dice cash flows of different size, maturity and risk to match the preferences of investors. (See Hellwig (1991), Bhattacharya and Thakor (1993), and Freixas and Rochet (2008) for an overview of the related literature.) For example, retail investors are often not capable of producing the desired payoff of an investment product themselves because of limited market access, short-selling restrictions, and transaction costs. The offering of retail structured products by large financial institutions addresses these frictions. Apparently, retail investors are willing to pay a high profit margin for this service (Wilkens et al., 2003; Stoimenov and Wilkens, 2005).

Despite all these benefits, the financial system itself is the origin of serious frictions. Most importantly, banks are subject to considerable bankruptcy costs. The default of Lehman Brothers, a highly levered investment bank and also prominent issuer of retail structured products, provides ample evidence for the complexity and issues of resolving distressed financial institutions. Furthermore, the fear of contagion surrounding the events in 2008 led to a decline of the stock market, dry out of the interbank lending market, and subsequent recession in the US. Therefore, bankruptcy costs have to play an important role in the analysis of debt contracts of financial institutions such as retail structured products. There is no need to incorporate distress costs in the context of contingent debt, since default never occurs in the model described in chapter 3. However, the avoidance of bankruptcy costs is the leading motive for the issuance of CoCo bonds in the first place.

The second important friction is the tax deductibility of interest payments. The Economist, a weekly newspaper, recently labeled this subsidy of debt as “the great distortion" and "a dangerous flaw" (May 16th 2015, print edition). The forgone tax revenues are estimated with USD 1,235 billion for the year 2007 alone. However, this was before the sharp decline in interest rates succeeding the financial crisis. But even today, the cost of the subsidy accounts for as much as 2% of GDP in the US. Consequently, it is sensible to include this sizable friction into a model of the capital structure, as for example, in the context of retail structured products in chapter 2. The model also proves robust with respect to the introduction of other frictions which benefit the issuance of structured products, for example, the empirically observed product markup.

The tax benefit of debt also plays a role in the issuance of CoCo bonds. Avdjiev et al. (2013) reports that 64% of issued CoCo bonds enjoy a tax deduction of coupon payments. In addition, Flannery (2014) observes that issuers try to avoid CoCo designs which do not qualify for tax deductibility, such as the issuance in the form of preferred shares.

In the context of these frictions, the analysis of chapter 2 shows that retail structured products have several advantages. The issuer can acquire additional funds, which the retail investors would otherwise invest directly into the capital market or into mutual funds. In addition, banks can use the products for the purpose of risk management. However, the issuance of retail structured products might also be the source of additional risks for the bank. Gorton and Metrick (2012) and Cochrane (2014) consider the recent financial crisis as a run on short-term bank liabilities, for example, repos. Similarly, investors can withdraw their funds from retail structured products at any time, since the issuers also act as market makers. This kind of run on retail structured products is special, since the origin of the run is a decline in the value of the underlying asset and, thus, not in the influence of the issuing financial institution.

All the analyzed debt contracts have in common that they create serious incentive problems, for example, risk-shifting behavior. As the analysis in chapter 3 shows, the incentive problems are aggravated by CoCo bonds which transfer wealth from debt holders to equity holders at conversion. Notably, not one single contract of those surveyed in section 3.2 provides a wealth transfer from equity holders to debt holders. Contingent debt with a write-down feature can actually never produce such a wealth transfer. As a consequence, the issuing banks have to pay hefty coupons to compensate the debt holders for the expected loss.

To address this issue, Bulow and Klemperer (2015) propose a new type of contingent debt called equity recource notes (ERNs). The trigger mechanism of ERNs is based on the share price. When the share price drops below the trigger level at a coupon date, the issuer makes the coupon payments with new shares instead of cash. The number of issued new shares is determined such that their value matches exactly the omitted cash payment. Bulow and Klemperer (2015) show that this design solves many of the issues of traditional CoCos. In particular, there is no wealth transfer at conversion and the ERNs create counter-cyclical incentives to provide new loans.

Besides the described incentive problems, which mainly concern the relation between equity and debt holders, the issuance of contingent debt has also severe consequences for the relation between equity holders and managers. Most importantly, the current managers of the bank are entrenched when CoCo bonds are converted (French et al., 2010). Hence, the important mechanism that managers of distressed firms are replaced in case of bankruptcy is suspended. Furthermore, the incentive contract of managers need to incorporate the risk-taking incentives induced by CoCo bonds. For example, Baas (2014) derives the optimal compensation contract in the same setting as in chapter 3. Flannery (2014) predicts a reduction of risk-taking incentives when managers are obliged to hold CoCo bonds themselves.

In addition to analyzing risk-taking incentives, academics produced a myriad of regulatory recommendations to improve the financial system since the crisis unfolded in the fall of 2008. Many of the proposed rules have made it into legislation, for example, as part of the Dodd-Frank Act of 2010. The main focus of these recommendations is that banks need to increase their equity ratios (French et al., 2010). Hence, the issuance of contingent debt, which converts into equity in distressed times, is well in line with these proposals. In addition, the Dodd-Frank Act introduced a restriction of banks' leverage ratios. The finding that retail structured products increase the stability of a high risk issuer when the leverage ratio remains constant is in support of this regulatory measure.

However, there are also critical voices which regard the taken steps as insufficient (Admati and Hellwig, 2013). The financial crisis demonstrated that the regulatory mechanisms in place failed, but these are reenforced by the recent reforms. Put metaphorically, the old medicine did not work, so regulators simply drink more of it instead of trying a new — potentially better — one. Along these lines, Cochrane (2014) makes the proposal that banks should cease to issue fixed value short-term claims, such as deposits. As a consequence, bank runs are eliminated. He also proposes to create special vehicles which buy shares of banks and issue debt claims instead of the banks. The advantage of this construct is that the special vehicles can be resolved easily in case of bankruptcy.

In summary, the regulatory supervision and attention has increased sharply after the financial crisis. However, it needs to be seen whether the modified regulatory scheme will succeed to prevent future crisis or whether the critical thoughts of Admati and Hellwig (2013) or Cochrane (2014) should have received more attention.

Part II

Asset Pricing and Derivatives

Chapter 5

Low volatility puzzle and beta contraction‡

5.1 Introduction

According to the capital asset pricing model (CAPM) developed by Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966), an investor purchasing two different portfolios, which have the same beta factor, should expect the same return from both positions. However, empirical research shows that a levered position of stocks with low return volatility historically generated higher returns than stocks with high return volatility, even though both positions carry the same systematic risk as measured by the beta factor (Haugen and Heins, 1975; Blitz and Van Vliet, 2007). We show that neither frictions (Black, 1972; Boehme et al., 2009; Baker et al., 2011; Frazzini and Pedersen, 2014) nor exotic investor preferences (Barberis and Huang, 2008; Han and Kumar, 2013) are required to explain the low volatility puzzle. A simple extension of the CAPM framework incorporating stochastic correlations is sufficient to bring the — at first glance contradictory — empirical evidence in line with classical asset pricing theory.

Our analysis proceeds in two major steps. First, we show that the beta factors of low and high volatility assets behave differently. When correlations increase, the beta factor of low volatility assets increases, while the beta factor of high volatility assets decreases. We refer to this behavior as beta contraction. Since correlation shocks also increase the volatility of the market portfolio, the systematic risk borne by an investor of low volatility assets is more sensitive to changes in correlations compared to high volatility assets.

‡This chapter is based on the working paper *"The Risk of Low Volatility Stocks: A Theoretical Explanation for an Empirical Puzzle"* (Crummenerl, 2015).

In the second step, we evaluate the pricing consequences of this mechanism. In equilibrium, investors anticipate the higher sensitivity of low volatility stocks and demand a premium compared to the expected return predicted by the standard CAPM. In contrast, high volatility stocks offer a lower expected return than predicted. We calibrate the model to a standard set of parameters and show that the effect is robust and sizable. A zero-beta portfolio, which is constructed by purchasing a levered position of low volatility assets and a short position in high volatility assets, is generating a risk premium between 0.72% and 2.46% per year relative to the standard CAPM risk premium.

Our findings analogously apply to two related empirical puzzles. Since assets with low volatility usually also have a low beta factor, the empirical puzzle is also obtained by comparing the returns of stocks with low and high beta factors (Black et al., 1972; Asness et al., 2012; Frazzini and Pedersen, 2014). Furthermore, we can regard the idiosyncratic return volatility of an asset. Any pricing relevant factor, for example, the sensitivity of an asset to correlation shocks, which is not accounted for in the benchmark asset pricing model, appears in the idiosyncratic volatility term. Ang et al. (2006) are the first to document that high idiosyncratic volatility is related to low historical returns. In summary, there are three ways to measure the risk of an asset. In the context of our model, all three perspectives agree.

Our work is further motivated by the empirical literature on correlation risk. Goetzmann et al. (2005) show that correlations vary significantly over time and are therefore stochastic in nature. In addition, Ang and Chen (2002), Longin and Solnik (2001), and Hong et al. (2007) provide evidence that increases in correlations frequently occur in market downturns. Hence, the beta factor of low volatility assets tends to increase exactly in times when investors do not want to have a high systematic risk exposure. At the same time, high beta assets provide a hedge against correlation increases. This evidence is further underpinned by Krishnan et al. (2009) and Driessen et al. (2009), who find that that correlation risk empirically carries a significant market price. However, these studies fail to detect the systematic pattern of beta factor movements in response to correlation shocks.

The aforementioned asset pricing puzzles are well documented for stocks. We show that the price deviation compared to the standard CAPM carries over to other types of securities, i.e., derivatives and risky corporate bonds. For this purpose, we derive the prices of European options in the equilibrium setting with stochastic correlations. Based on this pricing framework, we formulate testable empirical hypothesis on (1) the cross-section of beta factors, (2) the beta factors of sorted portfolios, and (3) the beta factors of different types of securities issued by the same firm.

The remainder of the chapter is structured as follows. The relevant literature is discussed in section 5.2. In section 5.3, we develop an illustrative portfolio setting and analyze the impact of correlation shocks on both the beta factor and the systematic risk. In section 5.4, we incorporate stochastic correlations into an equilibrium pricing model and derive return differences relative to the standard CAPM pricing formula. The prices of European options and a structural model of the firm are developed in section 5.5. We outline an empirical design based on this structural model in section 5.6. Section 5.7 concludes. Proofs and technical developments are in the appendix.

5.2 Literature review

Our work is related to two strands of the literature. First, there is a broad empirical literature focusing on the detection of asset pricing puzzles. The typical line of argument of these studies is to suggest a factor, which is not captured by either the standard CAPM or the model of Fama and French (1992, 1993), and to show that this factor has price implications for the cross-section of expected returns either using portfolio sorts or Fama and MacBeth (1973) regressions.

The particular factor of interest in the context of this work is the return volatility. The empirical fact that stocks with low volatility offer a higher risk-adjusted return compared to stocks with high volatility has first been documented by Haugen and Heins (1975). More recent empirical evidence is provided by Blitz and Van Vliet (2007) and Li et al. (2014).

Two further empirical puzzles are closely related to this low volatility puzzle. We can decompose the total return volatility into a systematic risk component and an idiosyncratic component. Stocks with low volatility typically exhibit also a low beta factor. And similarly to low volatility stocks, also stocks with low beta factors have higher risk-adjusted returns compared to high beta stocks (Black et al., 1972; Asness et al., 2012). In other words, the empirical security market line is flatter than theory predicts.

In case there is a factor, which is pricing relevant but not included in the asset pricing model, it should empirically appear in the idiosyncratic volatility component. Ang et al. (2006) report a staggering 106 basis points per month excess return of a low idiosyncratic volatility portfolio over a high idiosyncratic volatility portfolio, while both carry the same systematic risk. The effect is confirmed by many other studies such as Zhang (2006), Ang et al. (2009), Jiang et al. (2009), Guo and Savickas (2010), Chen and Gallmeyer (2010), and Huang et al. (2011). However, the low idiosyncratic volatility puzzle has turned out to be much more sensitive to changes in the applied empirical methods (Fu, 2009; Bali and Cakici, 2008; Huang et al., 2011).

Some studies also try to identify fundamental factors, e.g., volatility of fundamental cash flows (Irvine and Pontiff, 2009), growth options of the firm (Barinov, 2011) or intensity of product market competition (Gaspar and Massa, 2006), which are correlated to or predicted by idiosyncratic volatility. Therefore, the low idiosyncratic volatility puzzle might be caused by these factors being priced in the cross section of returns.

In summary, this empirical strand of the literature has established solid evidence that the low volatility puzzle is persistent and of considerable magnitude. The resulting risk premium is negative, i.e., stocks with higher volatility have on average lower risk-adjusted returns. However, these studies lack an explanation for why these factors should be uncorrelated to market risk and thus end up in the idiosyncratic volatility term. Hence, we have to consider a second strand of the literature, which attempts to provide theoretical explanations for the puzzle. Blitz et al. (2014) review this literature in great detail. We identify two lines of reasoning.

The first line of reasoning is to propose frictions, which do not originate from the firm, but cause a shift of demand from low volatility stocks to high volatility stocks. In the context of such limitations to arbitrage, Black (1972) and Frazzini and Pedersen (2014) analyze leverage and margin constraints, while Boehme et al. (2009) discuss short-selling restrictions. Baker et al. (2011) propose investment restrictions, e.g., caused by the use of benchmarks for portfolio managers. For example, if mutual funds can only invest a restricted amount into risky stocks but want to increase the portfolio's systematic risk exposure, high volatility stocks are better suited for this purpose than low volatility stocks.

The second line of reasoning considers investor preferences. Barberis and Huang (2008) mention the higher skewness of high volatility stocks as an attractive characteristic in the context of the cumulative prospect theory proposed by Tversky and Kahneman (1992). Han and Kumar (2013) find related empirical evidence for such volatility seeking retail investors.

Our work differs from the aforementioned literature in several aspects. First, we do not introduce an additional risk-factor besides market risk. Even though we consider two sources of uncertainty, the result is a one-factor model with varying exposures to this single risk factor. Second, we do not require market frictions. And third, we do not consider exotic investor preferences. Hence, we offer a parsimonious explanation for the low volatility puzzle, which brings classical asset pricing theory in line with the empirical evidence.

5.3 Illustrative portfolio setting

The objective of this section is to provide an intuitive understanding for the key mechanism of our model. To identify this mechanism, it is sufficient to focus on the risk characteristics of assets and to abstract from the implications on the asset's return. Hence, we first analyze the consequences of correlation changes for a single asset within a simple portfolio setting. In particular, we evaluate whether a low volatility asset is impacted differently than a high volatility asset when correlations rise.

5.3.1 Framework

Our framework bases on a classical portfolio context with N assets. The return of each asset has identical correlation $\rho \geq 0$ with the return of any other asset. Asset i has return volatility σ_i. We think of the remaining $N-1$ assets as a representative group of homogeneous assets with identical return volatility σ_{-i}. For tractability reasons, we assume that all N assets have the same weight in the market portfolio. Asset i is a low volatility asset when $\sigma_i < \sigma_{-i}$ and it is a high volatility asset when $\sigma_i > \sigma_{-i}$.

We compute several risk properties of asset i and the market portfolio m. All these representations result from the linearity and additivity properties of covariance and from the relationship between covariance and correlation.

The variance σ_m^2 of the return of the market portfolio amounts to

$$\sigma_m^2 = \frac{1}{N^2}\left(\sigma_i^2 + (N-1)\sigma_{-i}^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(N-2)\sigma_{-i}^2\rho\right). \tag{5.1}$$

Accordingly, we can represent the beta factor β_i as

$$\begin{aligned}\beta_i &= \frac{cov\left(r_i, r_m\right)}{\sigma_m^2} \\ &= \frac{N\cdot\left(\sigma_i^2 + (N-1)\sigma_i\sigma_{-i}\rho\right)}{\sigma_i^2 + (N-1)\sigma_{-i}^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(N-2)\sigma_{-i}^2\rho}.\end{aligned} \tag{5.2}$$

Furthermore, we measure idiosyncratic volatility ε_i as the standard deviation of the return residuals net of systematic risk according to the standard CAPM pricing, i.e.,

$$\begin{aligned}\varepsilon_i^2 &= \sigma_i^2 - \beta_i^2\cdot\sigma_m^2 \\ &= \sigma_i^2 - \frac{\left(\sigma_i^2 + (N-1)\sigma_i\sigma_{-i}\rho\right)^2}{\sigma_i^2 + (N-1)\sigma_{-i}^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(N-2)\sigma_{-i}^2\rho}.\end{aligned} \tag{5.3}$$

In the empirical literature, idiosyncratic volatility is frequently defined as the residual of a time-series regression of the asset's return on the market portfolio return and the factors for the size effect and value effect derived by Fama and French (1993). Since we do not want to mix theoretical frameworks, we prefer the consistent definition of idiosyncratic volatility in the context of our model.

The question how a higher volatility σ_i of asset i affects its beta factor β_i is not trivial, because both the covariance $cov(r_i, r_m)$ in the numerator and the variance σ_m^2 of the market portfolio return in the denominator of representation (5.2) increase with the volatility σ_i. Similarly, the effect of the asset volatility σ_i on the idiosyncratic volatility ε_i is not obvious. We summarize the relationship in the following two propositions. (See appendix 5.A for proofs.)

Proposition 5.1 (Beta factor and volatility)
The beta factor β_i strictly increases with the volatility σ_i of the considered asset.

Proposition 5.2 (Idiosyncratic volatility and volatility)
The idiosyncratic volatility ε_i strictly increases with the volatility σ_i of the considered asset.

In the context of our theoretical analysis, a low (high) total volatility is always equivalent to both a low (high) idiosyncratic volatility and a low (high) beta factor. Thus, low volatility assets have a low beta factor $\beta_i < 1$, while high volatility assets have a high beta factor $\beta_i > 1$. For the remainder of our analysis, we will only use the terms low and high volatility, but still have in mind that they can also be interpreted as low (high) idiosyncratic volatility and low (high) systematic volatility, i.e., low (high) beta factors, respectively.

5.3.2 Sensitivity of beta factor

In the following, we follow the typical CAPM notion and think of an investor holding a well diversified portfolio, which only consists of the market portfolio and a risk-free asset. The relevant risk of a single asset for this particular investor originates only from the risk contribution to the market portfolio, which is measured by the systematic risk $\beta_i \cdot \sigma_m$. The asset-specific risk is fully eliminated when holding a well diversified portfolio and has therefore no impact on the risk borne by the investor.

We consider an investor who — in line with classical portfolio theory and the standard CAPM — naively regards the correlation ρ as deterministic. This is the *ex ante* view. However, the true correlation realizing *ex post* might differ from the ex ante belief of the investor. We denote this case, in which the ex ante presumed correlation differs from the ex post correlation, as a correlation shock.

In the first step, we analyze the impact of correlation shocks on the beta factor β_i. The key finding is summarized in the following proposition. (See appendix 5.A for proof.)

Proposition 5.3 (Beta factor and correlation)
The beta factor β_i of a low (high) volatility asset with $\sigma_i < \sigma_{-i}$ ($\sigma_i > \sigma_{-i}$) increases (decreases) with the correlation ρ.

The property described in proposition (5.3) is illustrated in figure 5.1. It depicts the beta factor β_i as a function of the correlation ρ for both the case in which asset i is a low volatility asset (solid line) and the case in which asset i is a high volatility asset (dashed line). The beta factor of the high volatility asset declines from 1.8 for low correlations to 1.35 for correlations close to one. The beta factor of the low volatility asset increases from 0.3 for low correlations to 0.55 for high correlations. Since the beta factors of low and high volatility stocks move closer to each other in response to a correlation shock, we denote this behavior as beta contraction.

Even though not at the core of our analysis, we also want to analyze what happens to the beta factor when the average asset volatility increases. In contrast to proposition 5.1, we consider the case when the volatility of all N assets change simultaneously by the same amount $\Delta\sigma$. The result is summarized in the following proposition. (See appendix 5.A for proof.)

Proposition 5.4 (Beta factor and average volatility)
The beta factor β_i of a low (high) volatility asset with $\sigma_i < \sigma_{-i}$ ($\sigma_i > \sigma_{-i}$) increases (decreases) when all asset volatilities simultaneously increase.

A numerical example for this property is presented in figure 5.2. The plot shows the effect on the beta factor β_i of a simultaneous change of all asset volatilities by the same amount $\Delta\sigma$. The pattern is similar to figure 5.1. The beta factor of the low volatility asset (solid line) strictly increases. For the case of $\Delta\sigma = -0.1$, the low volatility asset has a resulting volatility of $\sigma_i = 0$. Hence, it is risk-free and the beta factor becomes zero. The beta factor increases up to 0.7 for an upward shift of all volatilities by 10 percentage points. In contrast, the beta factor of the high volatility asset (dashed line) is strictly decreasing from 1.8 to 1.3.

Figures 5.1 and 5.2 underline the important characteristic of high volatility assets compared to low volatility assets. The exposure to the market portfolio, captured by the beta factor β_i, declines when a correlation shock or a volatility shock occurs for high volatility assets but rises for low volatility assets.

Figure 5.1: Beta factor depending on correlation

The graph shows the beta β_i of a low volatility asset (solid line) with $\sigma_i = 0.1$ and of a high volatility asset (dashed line) with $\sigma_i = 0.3$ depending on the correlation ρ between each pair of assets. The remaining parameters are $\sigma_{-i} = 0.2$ and $N = 5$.

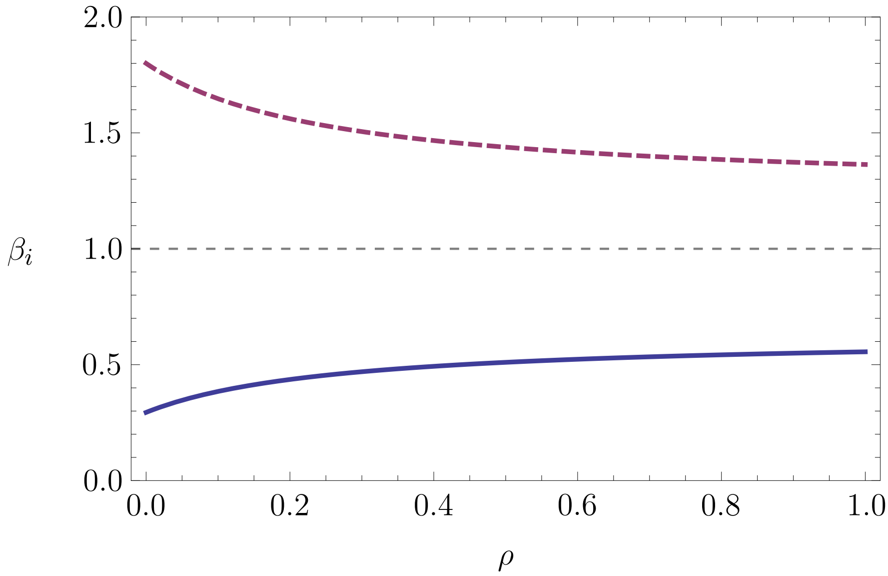
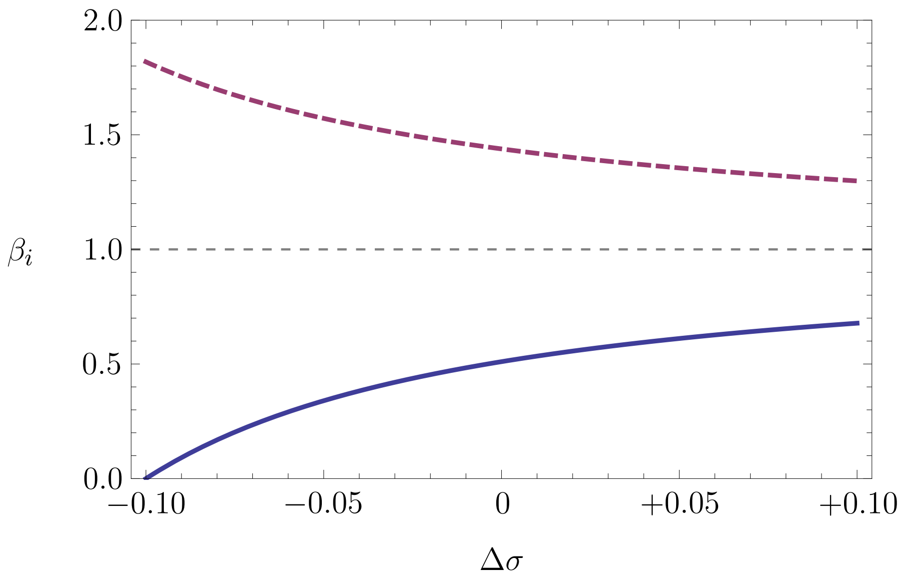


Figure 5.2: Beta factor depending on average volatility

The graph shows the beta factor β_i of a low volatility asset (solid line) with initial volatility $\sigma_i = 0.1$ and of a high volatility asset (dashed line) with initial volatility $\sigma_i = 0.3$ when all asset volatilities simultaneously change by the amount $\Delta\sigma$. The remaining parameters are $\sigma_{-i} = 0.2$ and $N = 5$.



5.3.3 Sensitivity of systematic risk exposure

So far, we have analyzed the impact of a correlation shock on the beta factor in isolation. However, since a correlation shock also implies a higher risk σ_m of the market portfolio return, we need to consider the combined effect on the systematic risk borne by the investor.

In the second step, we analyze the systematic risk $\beta_i \cdot \sigma_m$ of the considered asset i, i.e., the only risk for which investors obtain an extra reward in a classical CAPM world. We summarize our finding in the following proposition.

Proposition 5.5 (Systematic risk and correlation)
The increase of the systematic risk $\beta_i \cdot \sigma_m$ in percentage terms for a rise of the correlation ρ is higher (lower) for low (high) volatility assets compared to that of the market portfolio.

Proposition 5.5 is a direct consequence of the property derived in proposition 5.3. The market portfolio has a beta factor equal to one regardless of the correlation and its return volatility is strictly increasing with ρ. The percentage increase of a low volatility asset's systematic risk exposure $\beta_i \cdot \sigma_m$ when a correlation shock occurs is higher than for the market portfolio, because both the volatility σ_m and the beta factor β_i increase with ρ. In contrast, the beta factor of a high volatility asset declines with ρ. Therefore, the percentage increase of the systematic risk $\beta_i \cdot \sigma_m$ of a high volatility asset is lower than that of the market portfolio.

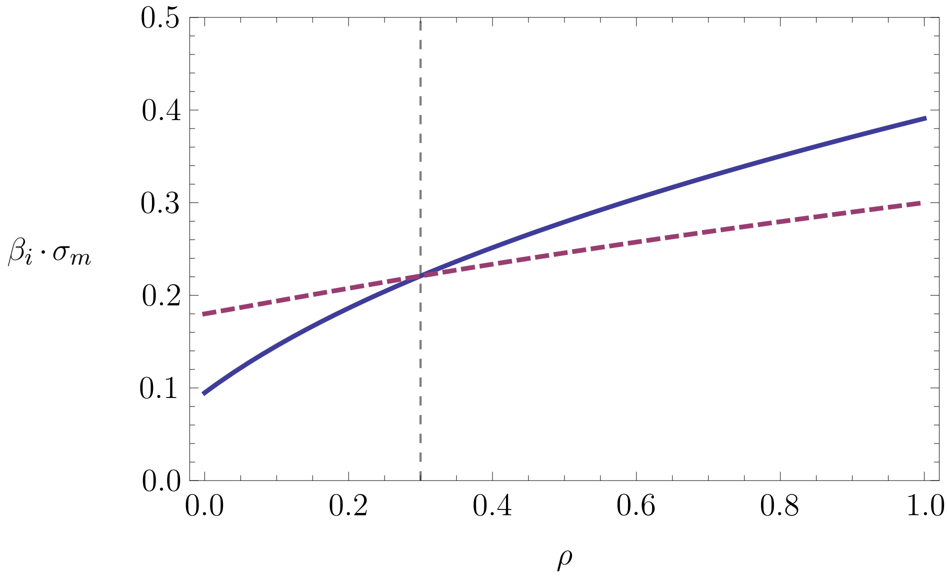
We further illustrate this result by comparing two investments, which are constructed such that they have the same systematic risk exposure. In a CAPM world, such two investments with identical beta factors should provide the same expected return. Hence, from the investor's perspective, both positions must be equally attractive marginal investments.

The first investment is an unlevered position in a high volatility asset with beta factor β_i^h. The second investment is a levered position in a low volatility asset with beta factor β_i^l. To bring this second investment to the same systematic risk exposure as the first investment, we need to create a portfolio investing of a fraction $\psi = \beta_i^h / \beta_i^l$ in the low volatility asset and a fraction $1 - \psi$ in the risk-free asset. The latter corresponds to a credit, since $\psi > 1$. Since the risk-free asset has a beta factor of zero, the portfolio beta is equal to $\psi \cdot \beta_i^l + (1 - \psi) \cdot 0 = \beta_i^h$.

In figure 5.3, we plot the systematic risk exposure for these two investments. The portfolio weight $\psi = 3.9$ has been determined such that the systematic risk is the same for both investments for an initial correlation of $\rho = 0.3$. The systematic risk of both the low volatility asset (solid line) and the high volatility asset (dashed line) increases with the

Figure 5.3: Systematic risk depending on correlation

The graph shows the systematic risk $\psi \cdot \beta_i^l \cdot \sigma_m$ of a levered low volatility asset (solid line) with $\sigma_i^l = 0.1$ in comparison to the systematic risk $\beta_i^h \cdot \sigma_m$ of an unlevered high volatility asset (dashed line) with $\sigma_i^h = 0.3$ depending on the correlation ρ between each pair of assets. The low volatility asset is levered by the factor $\psi = 3.91$, such that it bears the same systematic risk as the high volatility asset for a correlation of $\rho = 0.3$. The remaining parameters are: $\sigma_{-i} = 0.2$ and $N = 5$.



correlation ρ. Importantly, the sensitivity of the systematic risk $\beta_i \cdot \sigma_m$ for a correlation shock is lower in the case of a high volatility asset. In particular, the systematic risk for a low volatility asset varies from 0.09 to 0.39, while the corresponding range for the high volatility asset is narrower, i.e., between 0.18 to 0.30.

In this section, we have compared two investments, which carry the same systematic risk exposure. We have shown that the pricing relevant risk measures crucially depend on the ex post realized correlation and they do so in a systematic manner. In particular, the risk properties of a low volatility asset react differently to changes in correlations compared to those of a high volatility asset. As a consequence, a levered position in a low volatility asset behaves differently than a high volatility asset, even though both positions carry the same systematic risk.

5.4 Equilibrium model

Since investors only care for the ex post borne systematic risk, the analysis of the previous section reveals the advantage of a high volatility asset compared to a levered investment into a low volatility asset. Correlation shocks more strongly impact the relevant risk of a low volatility asset than that of a high volatility asset. In other words, high volatility assets provide a better protection against increases of systematic risk than low volatility assets.

The standard CAPM, which regards correlations as deterministic, predicts that investments with the same systematic risk exposure should have the same expected return. However, this major implication of the standard CAPM does no longer hold when we allow for changes of the correlation ρ between the returns of any two assets. To evaluate the pricing consequences of beta contraction, we incorporate stochastic correlations into an equilibrium pricing model. Our objective is to obtain endogenous effects on market prices and expected returns when investors anticipate and correctly account for the effect of changing correlations.

5.4.1 Framework

The setup exhibits several parallels to the one described in section 5.3. The discrete-time, one-period economy has N assets. The asset j has a payoff $\widetilde{Y}_j$ at the end of the considered period with mean equal to one. The current price of the asset is denoted by Y_{0j}. The market portfolio contains all assets. Thus, it has an expected payoff equal to N. We denote the payoff of the market portfolio with $\widetilde{M}$ and its price with M_0.

We again single out the asset i, whose volatility δ_i of the terminal payoff is different from the other assets. The remaining $N-1$ assets have payoffs with volatility δ_{-i}. As before, we call asset i a low volatility asset when $\delta_i < \delta_{-i}$, and we call it a high volatility asset when $\delta_i > \delta_{-i}$.

The returns of asset j and the market portfolio are denoted with r_j and r_m, respectively. The corresponding expected returns are labeled as μ_j and μ_m, respectively. Moreover, a risk-free asset exists, which yields a return equal to r_f. In this setup, payoffs and their distributions are exogenous, while prices, returns and expected returns are endogenous. (See appendix 5.B.1 for detailed representations of these items.)

For pricing purposes, we regard a representative investor having a Bernoulli utility function $u(z)$ in terminal wealth z. We make use of the well known pricing relationship in discrete-time financial economics between the expected return μ_i of a single asset and the expectation μ_m of the return of the market portfolio. (See appendix 5.B.2 for a detailed derivation.)

$$\mu_i = r_f + \left(\mu_m - r_f\right) \cdot \frac{cov\left(r_i,\ u'(\widetilde{M})\right)}{cov\left(r_m,\ u'(\widetilde{M})\right)}. \tag{5.4}$$

The fundamental relationship (5.4) only requires the existence of a representative investor and knowledge of the distribution of the asset payoffs $\widetilde{Y}_i$. When imposing special assumptions for the utility function $u(z)$ and the distribution of the asset payoffs, the CAPM pricing equation can be obtained, but it does not hold in general. We choose a utility function $u(z)$ with constant absolute risk aversion

$$u(z) = -\frac{1}{\lambda}\exp\left(-\lambda \cdot z\right). \tag{5.5}$$

We finally introduce the distribution assumptions for the two sources of uncertainty, i.e., for the asset payoffs and the correlations. The correlation between each pair of two different assets' payoffs is ρ_k. To keep the model as simple as possible, we consider only two states $k \in \{u, d\}$, with $\rho_u > \rho_d$. State u has probability π and state d has probability $1-\pi$. In other words, either all pairs of asset payoffs have a high correlation ρ_u or a low correlation ρ_d. The asset payoffs in both states u and d are multivariate normally distributed.

Notably, that all assumptions of the CAPM world are satisfied in this setup when correlations are non-stochastic. Thus, the only reason for potential pricing deviations must come from stochastic correlations.

Now we have all the required ingredients to derive the pricing equation with stochastic correlations. We obtain a pricing equation for asset i relative to the pricing of the market portfolio. We take the price M_0 or equivalently the expected return $\mu_m > r_f$ of the market portfolio as given and focus on the endogenous expected return μ_i of asset i.[1] Applying the law of total covariance and Stein's lemma to expression (5.4), we obtain the following tractable representation for the expected return. (See appendix 5.B.3 for a comprehensive derivation.)

$$\mu_i = r_f + \left(\mu_m - r_f\right) \cdot \frac{M_0}{Y_{0i}} \cdot \frac{\pi \cdot em(\rho_u) \cdot ci(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot ci(\rho_d)}{\pi \cdot em(\rho_u) \cdot vm(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot vm(\rho_d)} \tag{5.6}$$

with

$$ci(\rho) := \delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho, \tag{5.7}$$

$$vm(\rho) := \delta_i^2 + (N-1) \cdot \delta_{-i}^2 + (N-1)\, \delta_{-i} \cdot (2 \cdot \delta_i + (N-2)\, \delta_{-i})\, \rho, \tag{5.8}$$

$$em(\rho) := \lambda \cdot \exp\left(-\lambda \cdot N + \tfrac{1}{2} \lambda^2 \cdot vm(\rho)\right). \tag{5.9}$$

The term $ci(\rho)$ denotes the covariance between the payoff $\widetilde{Y}_i$ of asset i and the market portfolio $\widetilde{M}$ conditional on the correlation ρ. The term $vm(\rho)$ is a short-hand for the conditional variance of the market portfolio payoff $\widetilde{M}$ depending on ρ. And the term $em(\rho)$ is the expectation of $-u''(\widetilde{M})$ conditional on ρ.

To identify pricing deviations from the standard CAPM prediction, we compute the related expected return μ_i of asset i when the traditional CAPM pricing equation is applied to those asset prices arising from the equilibrium model with stochastic correlation.

Due to the additional correlation risk, the standard CAPM pricing is no longer valid. Consequently, the expected return $\widehat{\mu}_i$ predicted by the standard CAPM differs from the true expected return in equilibrium. For given market prices Y_{0i} and M_0, the expected return of asset i predicted by the standard CAPM amounts to

$$\widehat{\mu}_i = r_f + \left(\mu_m - r_f\right) \cdot \frac{M_0}{Y_{0i}} \cdot \frac{\pi \cdot ci\,(\rho_u) + (1-\pi) \cdot ci\,(\rho_d)}{\pi \cdot vm\,(\rho_u) + (1-\pi) \cdot vm\,(\rho_d)}. \tag{5.10}$$

Representation (5.6) for the true return expectation differs from the CAPM prediction (5.10) in that it has an additional weighting term $em(\rho_k)$ for the covariance and variance terms for given correlations ρ_k in state k. The impact of this weighting term em is examined more detail in the following section.

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[1] With further assumptions, such as the knowledge of initial wealth, one might also obtain the endogenous value of the market portfolio. This procedure would, however, require the numerical evaluation of an additional equation and disguise the view on the important pricing difference between the true model with stochastic correlations and the standard CAPM pricing formula.

5.4.2 Effect of stochastic correlations on equilibrium pricing

The key objective of this work is to derive pricing implications from incorporating stochastic correlations in an equilibrium framework. In the previous section, we have derived representation (5.6) for the expected return μ_i in the true model with stochastic correlations. We have also derived representation (5.10) for the expected return $\widehat{\mu}_i$, which results from applying the well-known CAPM formula without correlation risk on the prices resulting from the true model. In this section, we analyze under which conditions the true expected return μ_i in the model with correlation risk is higher or lower than the expected return $\widehat{\mu}_i$ predicted by the standard CAPM pricing formula. The result is summarized in the following proposition. (See appendix 5.B.4 for proof.)

Proposition 5.6 (Expected return and volatility)
Low volatility assets ($\delta_i < \delta_{-i}$) have an expected return above the CAPM prediction, i.e., $\mu_i > \widehat{\mu}_i$, while high volatility assets ($\delta_i > \delta_{-i}$) have a return expectation below the CAPM prediction, i.e., $\mu_i < \widehat{\mu}_i$. There is no price deviation in the special case of identical volatility ($\delta_i = \delta_{-i}$).

As a result, we have a simple relationship for price deviations compared to the CAPM prediction which originates from the relation between the volatility δ_i of the considered asset i relative to the volatility δ_{-i} of the other assets. This finding is consistent with empirical observations that low volatility stocks yield higher risk-adjusted returns than high volatility stocks. Since in the real world, the correlation must be considered as stochastic rather than deterministic, our model provides a meaningful explanation for an — at first glance — empirical mispricing of the standard CAPM.

In the following, we provide an economic intuition as well as a technical reasoning for this key finding. The proposition generally confirms the intuition gained from the analysis of beta factors in the previous section 5.3. We have shown that the ex post realized systematic risk of low volatility assets is more sensitive to a correlation increase than that of high volatility assets. Investors dislike ex post increases in realized market risk. Hence, the higher sensitivity of low volatility assets must be compensated in the form of an additional premium compared to the standard CAPM prediction.

When comparing representations (5.6) and (5.10) for the expected return under both regimes, it is apparent that the price deviation is rooted in the additional weighting term em. This weigh is monotonically increasing with the correlation, i.e., $\partial em/\partial\rho > 0$. In consequence, the investor puts a higher emphasis on the state in which correlations are high. Since market risk as measured by vm is also increasing with the correlation, investors appreciate the fact that the beta factor of high volatility assets is decreasing. Hence, high volatility assets provide a hedge against increases in the correlation and subsequent increases of the market risk. Hence, high volatility assets offer a lower expected return compared to low volatility assets.

Following this technical line of thought, we can generalize our result for a larger class of utility functions. Given that asset payoffs are jointly normally distributed, which is a condition for the application of Stein's lemma, all utility functions for which marginal utility evaluated at the market portfolio declines with increases in the correlation, i.e., $\partial \mathbb{E}\left(u''(M)\right)/\partial\rho < 0$, reproduce the finding from proposition 5.6.

Notably, the findings do not hold for the well known quadratic utility representation. In this case, the second derivative of the utility function, and thus also the weighting term em, is constant. The resulting expected return in both models is the same. Consequently, if one believes that the low volatility puzzle is caused by stochastic correlations, one has to dismiss the notion that investors' utility has a quadratic functional form.

We have already highlighted in proposition 5.1 that beta factors also contract in response to simultaneous shocks on all volatilities. In addition, market volatility is also increasing when the average volatility increases. Even though our model does not explicitly consider stochastic volatility, we can evaluate the pricing consequences of ex post increases in the average volatility by inspecting the weighting term em. Similarly to the effect of correlation increases, the weight also increases with the volatility level, i.e., $\partial em/\partial\delta > 0$. (See appendix 5.B.3 for proof.) Hence, investors also overweight states of the world with high volatility levels. We conclude that the effect described in proposition 5.6 is magnified when considering also the volatility as stochastic.

We have assumed the simplest possible distribution for the correlations. But since the pattern described in proposition 5.6 does not depend on the probability π, we can generalize the findings to any arbitrary discrete distribution. When we regard a correlation distribution with a total number of states K, we can easily see that the numerator in equation (5.6) becomes $\sum_{k=1}^{K} \pi_k \cdot em(\rho_k) \cdot ci(\rho_k)$. The corresponding denominator is given by $\sum_{k=1}^{K} \pi_k \cdot em(\rho_k) \cdot vm(\rho_k)$. Since the general additive form of equation (5.6) is preserved and the pattern originates from the weighting term $em(\rho_k)$, which increases in the correlation, we can conclude that the low volatility puzzle is robust with respect to the distribution assumption. Needless to say, the magnitude of the effect does depend on the specific functional form of the distribution.

5.4.3 Model calibration

We have established that there is a price deviation between the model incorporating stochastic correlations and the prediction of the standard CAPM. In this subsection, we analyze the magnitude of the observed price deviation of stocks explained by our equilibrium model compared to the standard CAPM.

We estimate the correlation distribution from daily return data of S&P 500 stocks between January 1998 and June 2012 using the following simple method. We first calculate the mean correlation ρ_t at each point in time. The average over time of these mean correlations is denoted as μ_ρ. We next calculate the average of all mean correlations above μ_ρ and the average of all mean correlations below μ_ρ, which we use as estimates for ρ_u and ρ_d, respectively. Finally, the probability π is determined such that the expected value equals the empirical average, i.e., $\mathbb{E}(\rho) = \mu_\rho$. This leads to the following relationship

$$\pi = \frac{\mu_\rho - \rho_d}{\rho_u - \rho_d}. \tag{5.11}$$

The estimates of the correlation distribution, which are robust with respect to the used estimation window, are presented in table 5.1. The estimate for the probability π varies between 0.425 and 0.444. The estimate deviates from 0.5 since the empirical correlation distribution is slightly skewed. The correlation in the down state ranges from 0.188 to 0.205, while it is between 0.422 and 0.445 for the up state. The correlation distribution is designed to preserve the empirical mean. But also the standard deviation $Std(\rho)$ with values between 0.113 and 0.127 is very close to the empirically observed standard deviation of 0.137.

In the following, we use the correlation estimates based on the 1 month estimation window, i.e., $\pi = 0.425$, $\rho_u = 0.445$ and $\rho_d = 0.188$. The parameters are summarized in table 5.2. We consider $N = 5$ assets, which we — in line with the empirical literature (Ang et al., 2006; 2009) — interpret as 5 portfolios of stocks.

For the market risk premium, we adopt the long-term average $\mu_m - r_f = 0.06$ determined by Welch (2000) for short investment horizons. The risk-free rate of $r_f = 0.05$ roughly corresponds to the long-term average 3-month Treasury Bill rate of 5.13% between 1962 and 2012.

We choose $\delta_{-i} = 0.20$ approximately matching the historical volatility of 0.214 of the S&P 500 index. In the following, we use $\delta_i^l = 0.1$ and $\delta_i^h = 0.4$ to illustrate the effects for low volatility and high volatility assets, respectively.

Table 5.1: Distribution of correlations

The table shows estimates for the parameters π, ρ_u and ρ_d of the correlation distribution and the corresponding standard errors SE of the estimation. The four columns show the results for rolling estimation windows for the correlation of 1 month, 3 months, 6 months and 1 year. All estimates marked by *** are statistically different from zero at the 1% significance level. The table also reports the expected value $\mathbb{E}(\rho)$ and standard deviation $Std(\rho)$ of the correlation distribution based on the estimates for π, ρ_u and ρ_d.

	Estimation window			
	1 month	**3 months**	**6 months**	**1 year**
Observations	3759	3715	3651	3520
π	0.425 ***	0.444 ***	0.432 ***	0.429 ***
$SE(\pi)$	0.010	0.010	0.010	0.010
ρ_u	0.445 ***	0.422 ***	0.433 ***	0.442 ***
$SE(\rho_u)$	0.003	0.003	0.003	-0.120
ρ_d	0.188 ***	0.194 ***	0.201 ***	0.205 ***
$SE(\rho_d)$	0.001	0.001	0.001	0.123
$\mathbb{E}(\rho)$	0.297	0.295	0.301	0.307
$Std(\rho)$	0.127	0.113	0.125	0.117

Table 5.2: Parameters for base case

The table shows the parameter choices for the base case calibration. The values of the correlation distribution correspond to the estimates using the 1 month estimation window in table 5.1.

Parameter	**Symbol**	**Value**
Probability	π	0.425
Correlation up state	ρ_u	0.445
Correlation down state	ρ_d	0.188
Number of assets	N	5
Market risk premium	$r_m - r_f$	0.06
Risk-free rate	r_f	0.05
Volatility	δ_{-i}	0.20
Risk aversion parameter	λ	5

Investors are characterized by the risk-aversion parameter $\lambda = 5$, which is in the lower range of option implied risk aversion parameters estimated from S&P 500 index options by Bliss and Panigirtzoglou (2004). The resulting relative risk aversion coefficient in our model is 23.6. We admit that this value appears to be quite high. However, Cochrane and Hansen (1992) find that a relative risk aversion of greater than 35 is required to explain the equity premium puzzle, which is well above our choice.[2]

To obtain numerical results, we assume an expected return of the market portfolio μ_m, which implicitly determines the price $M_0 = \frac{1}{1+\mu_m}$. Using $Y_{0i} = \frac{1}{1+\mu_i}$, we evaluate equations (5.6) and (5.10) for μ_i and $\widehat{\mu}_i$, respectively, and compute the spread. Figure 5.4 plots the difference between the expected return μ_i of the true model with stochastic correlations and the standard CAPM prediction $\widehat{\mu}_i$ as a function of the asset volatility δ_i.

As figure 5.4 shows, the return spread first increases with the volatility δ_i and then declines. Clearly, for a volatility δ_i close to zero, the risk premia under both representations are close to zero, so that the difference approaches zero as well. In line with proposition 5.6, the return spread is also zero when the volatility of all assets agrees, i.e., for $\delta_i = \delta_{-i}$. For high volatility stocks, the negative return spread widens with increasing volatility δ_i.

The magnitude of the effect is also of economic significance. Assuming the assets are priced using the standard CAPM model not accounting for correlation risk, an investor knowing the true model could earn a significant positive amount by shorting the high volatility stock and buying a levered low volatility stock, such that both stocks bear the same systematic risk. For example, such a zero-beta portfolio consisting of a short position in a high volatility stock with $\delta_i^h = 0.4$ and a short position in a low volatility stock with $\delta_i^l = 0.1$, which is levered by the factor $\psi = \beta_i^h / \beta_i^l = 4.29$, theoretically generates an excess premium of $\mu_0 = 144$ basis points (bp) per year. We denote this scenario as *base case* in the following.

5.4.4 Comparative static analysis

The main result that low volatility stocks earn an additional risk premium and that high volatility stocks exhibit a negative return spread compared to the standard CAPM prediction is quite robust to changes in the model parameters. We first test the sensitivity with respect to the crucial correlation distribution by comparing three different scenarios, which are depicted in figure 5.5. The solid line always represents the base case. Table 5.3

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[2]Cochrane (2005) requires an even higher value above 50 for the relative risk aversion to explain the equity premium puzzle and notes that it must be equal to a staggering value of 250 to explain the correlation puzzle, i.e., the weak correlation between stock returns and consumption.

summarizes the scenarios and also reports the premium μ_0 in basis points per year of a zero-beta portfolio constructed as described above.

First, we consider parallel shifts of the correlation in both states u and d (see plot A in figure 5.5). The pattern is robust to such parallel shifts. The return spread is slightly more pronounced for downward shifts. The premium μ_0 of the zero-beta portfolio increases to 176.0 bp for a downward shift in correlation of 0.05 and decreases to 120.3 bp for an upward shift of the same magnitude.

Second, we consider different spreads $\rho_u - \rho_d$ while keeping the probability π constant (see plot B in figure 5.5). In line with intuition, the pattern is more pronounced for high spreads than for low spreads. The premium μ_0 of the zero-beta portfolio obviously disappears for the extreme case of $\rho_u - \rho_d = 0$. In our concrete example, the zero-beta portfolio earns 211.4 bp in case of an increase in the spread $\rho_u - \rho_d$ by 0.05 and 71.5 bp in case of a decrease by the same magnitude. However, the standard deviation of the correlation distribution decreases to 0.078 in the latter case, which is well below the empirically observed standard deviation of 0.137. Thus, we believe that the real world correlation distribution provides sufficient variation, which is in line with the empirical findings of Goetzmann et al. (2005).

Third, we consider how skewed correlation distributions affect the results (see plot C in figure 5.5) as opposed to our almost symmetric distribution of the base case. We know from Ang and Chen (2002), Longin and Solnik (2001), and Hong et al. (2007) that correlations increase sharply in market downturns which usually coincide with rare events such as recessions or financial crises. We keep the average correlation μ_ρ constant, while decreasing the probability π for the upward deviation ρ_u, which is larger in magnitude. Thereby, we evaluate a skewed distribution with $\pi = 0.10$ and $\rho_u = 0.6$ as well as a rare disaster-like distribution with $\pi = 0.01$ and $\rho_u = 0.7$. In both cases, the observed pattern is much more pronounced especially for high volatility stocks. The resulting zero-beta portfolio premium μ_0 jumps to 246.1 bp and 234.1 bp, respectively. Hence, skewness significantly magnifies the price deviation between the true model and the standard CAPM prediction.

We conclude that the economic significance of the zero-beta portfolio return is robust to many plausible parameter choices for the correlation distribution. The predicted pattern is especially sensitive to the spread $\rho_u - \rho_d$ and to the skewness of the distribution. We derive values between 71.5 bp and 246.1 bp for the premium μ_0 of the zero-beta trading strategy.

The results are also robust to changes in the remaining model parameters. Table 5.4 summarizes the scenarios for the different parameters and reports the premium μ_0 of the

Figure 5.4: Expected return difference to CAPM prediction

The graph shows the return difference $\mu_i - \widehat{\mu}_i$ of an asset i in basis points depending on its payoff volatility δ_i. Further parameter values are given in table 5.2.

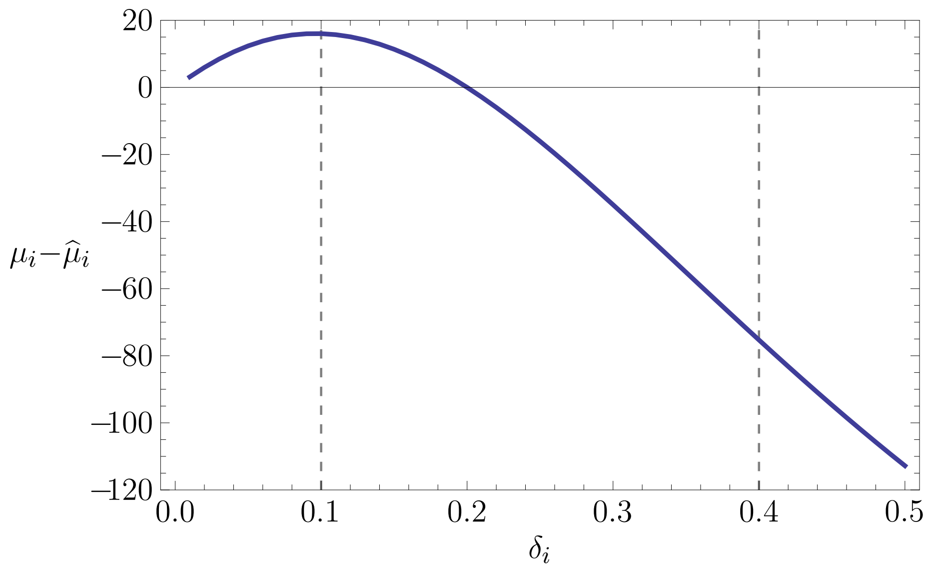


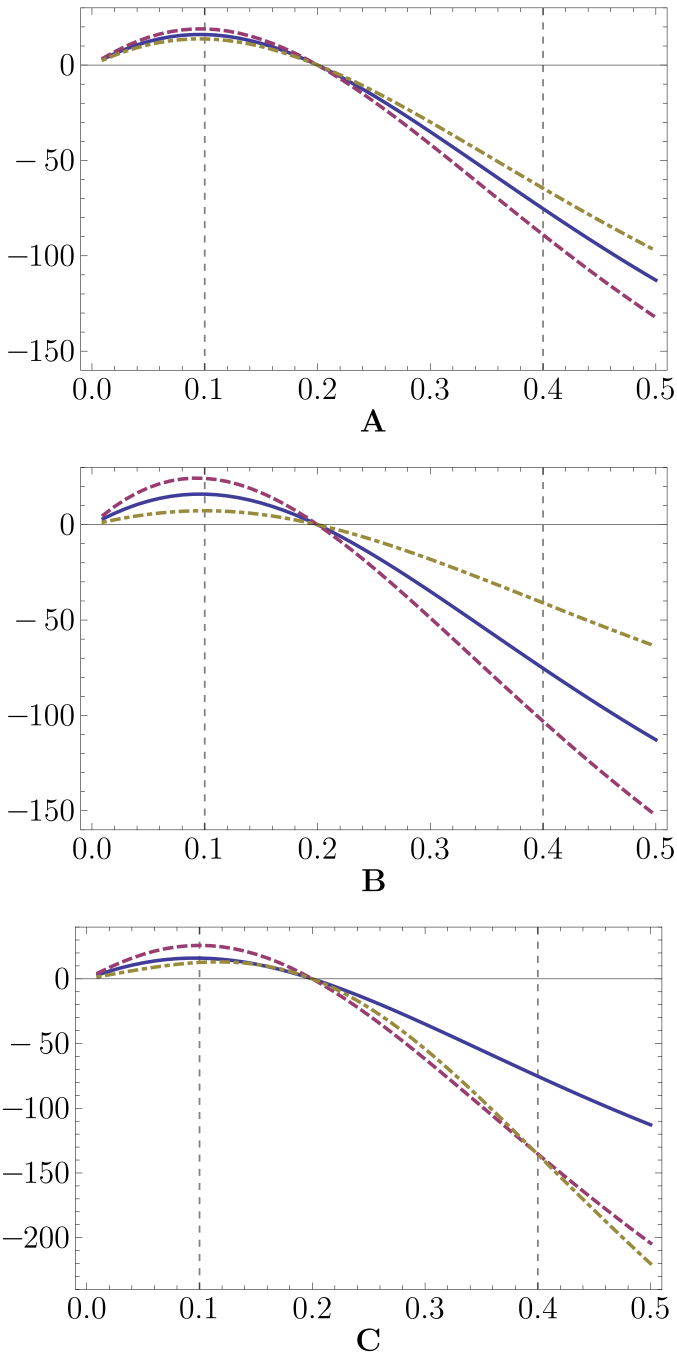
Table 5.3: Correlation distribution scenarios

The table shows different scenarios for the correlation distribution parameters π, ρ_u and ρ_d as well as the expected value $\mathbb{E}(\rho)$ and the standard deviation $Std(\rho)$ of the resulting distribution. The last column reports the premium μ_0 in basis points per year of a zero-beta portfolio consisting of a levered long position in the low volatility stock with $\delta_i^l = 0.1$ and a short position in the high volatility stock with $\delta_i^h = 0.4$.

	π	ρ_u	ρ_d	$\mathbb{E}(\rho)$	$Std(\rho)$	μ_0
Base case	0.425	0.445	0.188	0.297	0.127	144.0
Upward shift	0.425	0.495	0.238	0.347	0.127	120.3
Downward shift	0.425	0.395	0.138	0.247	0.127	176.0
Tighter spread	0.425	0.395	0.238	0.305	0.078	71.5
Widened spread	0.425	0.495	0.138	0.290	0.176	211.4
Skewed distribution	0.100	0.600	0.250	0.285	0.105	246.1
Disaster-like distribution	0.010	0.700	0.29	0.294	0.041	234.1

Figure 5.5: Return differences for various correlation scenarios

The plots show the return difference $\mu_i - \widehat{\mu}_i$ in basis points per year depending on the return volatility δ_i of asset i. The solid line in all plots represents the base case with parameter values from table 5.2. The remaining lines represent deviations with regard to the correlation assumptions. ***Plot A:*** *downward shift of correlation levels to $\rho_u = 0.395$, $\rho_d = 0.138$ (dashed line) and upward shift of correlation levels to $\rho_u = 0.495$, $\rho_d = 0.238$ (dot-dashed line).* ***Plot B:*** *widened spread of correlation distribution to $\rho_u - \rho_d = 0.357$ (dashed line) and tightened spread of correlation distribution to $\rho_u - \rho_d = 0.157$ (dot-dashed line).* ***Plot C:*** *skewed distribution with $\pi = 0.10$, $\rho_u = 0.60$, $\rho_d = 0.25$ (dashed line) and disaster-like distribution with $\pi = 0.01$, $\rho_u = 0.70$, $\rho_d = 0.29$ (dot-dashed line).*



zero-beta portfolio in basis points. The corresponding graphs, whose general pattern is robust to all changes, can be found in figure 5.7 in the appendix.

We first evaluate the effect of an increase in the number of considered assets N. The observed pattern is less pronounced for increasing N and basically disappears for very large N. The premium of the zero-beta portfolio decreases to 139.7 bp for N=10 and further to 91.5 for N=20. The reason for this is the simplifying assumption that there are $N-1$ identical assets in the economy and only one single assets i which deviates. Of course, the contribution of this single deviation to the market risk σ_m is tiny for a large number of assets N. We prefer the interpretation of N as portfolios, which also permits the empirical test of the pattern for large numbers of stocks.

We next test how the market properties impact the results. It is apparent from the pricing equations (5.6) and (5.10) that the market risk premium $\mu_m - r_f$ magnifies the observed return spread. An increase by 0.02 leads to premium of the zero-beta portfolio of 197.8 bp, while a decrease by the same amount lowers the premium to 93.3 bp.

We find see that the results are only marginally impacted by the risk-free rate r_f. We consider changes in the risk-free rate to either $r_f = 0$ or $r_f = 0.1$. The premium of the zero-beta trading strategy only changes slightly by 6.9 bp compared to the base case in both directions.

An increase in the volatility δ_{-i} of the $N-1$ other stocks shifts the graph in figure 5.4 to the right. However, the performance of the zero-beta portfolio only changes marginally.

In contrast, the results are very sensitive to changes in the risk aversion parameter λ. Intuitively, the lower the risk aversion, the less pronounced the pattern. In the extreme case of a risk-neutral investor, the difference between the expected returns according to the two models disappears, since the investor is indifferent to any source of risk. Interestingly, the pattern does only change marginally for a value of λ above 5 given the chosen set of parameters. We test for cases of $\lambda = 3$ and $\lambda = 10$, which leads to a premium of the zero-beta trading strategy of 78.9 bp and 166.8 bp, respectively.

In summary, our findings are robust to different plausible parameter choices. The return deviation disappears only in extreme cases with almost no correlation risk or risk-neutral investors. An investor using a zero-beta trading strategy could earn 144.0 bp in our base case. The derived bandwidth for the premium μ_0 of this zero-beta portfolio ranges from 71.5 bp up to 246.1 bp. Thus, the results are also of substantial economic significance.

Table 5.4: Scenarios for further parameters

The table shows different scenarios for the market risk premium $\mu_m - r_f$, the payoff volatility δ_{-i} of the $N-1$ remaining assets, the risk aversion parameter λ and the risk-free rate r_f. The last column reports the premium μ_0 in basis points per year of a zero-beta portfolio consisting of a levered long position in the low volatility stock with $\delta_i^l = 0.1$ and a short position in the high volatility stock with $\delta_i^h = 0.4$.

	N	$\mu_m - r_f$	r_f	δ_{-i}	λ	μ_0
Base case	5	0.06	0.20	5	0.05	144.0
Number of assets N	10					139.7
	20					91.5
Market risk premium $\mu_m - r_f$		0.08				197.8
		0.04				93.3
Risk-free rate r_f			0.00			137.2
			0.10			150.9
Volatility δ_{-i}				0.25		130.2
				0.15		144.8
Risk aversion parameter λ					10	166.8
					3	78.9

5.5 Structural model

We have derived a theoretical model in section 5.4 which is well capable of explaining the low volatility puzzle. However, it is still unclear to what extent other explanations contribute to the observed empirical pattern. To answer this question, we need to develop a research design to distinguish the different forces at work. The objective of this chapter is to build the methodological basis for this empirical research design.

It has to be noted that derivatives are in zero-net supply. The options are priced such that no investor will demand them in equilibrium. Hence, they are not included in the market portfolio and investors hold a mix of the market portfolio and the risk-free asset.

This chapter is structured as follows. First, we derive the price of European call and put options in section 5.5.1. We also analyze the difference between the true option prices and the prices predicted by the standard CAPM. In section 5.5.2, these pricing formulae are used to derive a structural model to consistently value different claims issued by the same firm.

5.5.1 Pricing of European options

We recall the equilibrium setting from the previous section 5.4. There is one asset i with a different volatility than all other $N-1$ assets. In this section, we derive the price of European options written on this particular underlying asset i.

The payoff $\widetilde{C}_i$ of a call option with strike price X and the payoff $\widetilde{P}_i$ of a put option with the same strike price X are given by

$$\widetilde{C}_i = \max\left\{\widetilde{Y}_i - X,\ 0\right\}, \tag{5.12}$$

$$\widetilde{P}_i = \max\left\{X - \widetilde{Y}_i,\ 0\right\}. \tag{5.13}$$

Given the distribution assumption from the previous section, the payoff of the underlying asset $\widetilde{Y}_i$ can become negative with a positive probability. Hence, a negative strike price would be feasible. Even though the following valuation formulae are valid for any arbitrary strike price, it makes sense to restrict the analysis to positive exercise prices $X > 0$.

We first derive the call price and later use the put-call-parity to price the put option. The fundamental pricing equation (5.4), which is valid for any arbitrary payoff distribution and all well defined utility functions, can be rewritten in terms of prices. (See appendix

5.C.1 for details.) The call price C_{0i} equals

$$C_{0i} = \frac{1}{1+r_f} \cdot \left(\mathbb{E}[\widetilde{C}_i] - \frac{cov\left(\widetilde{C}_i, u'(\widetilde{M})\right)}{cov\left(\widetilde{M}, u'(\widetilde{M})\right)} \cdot M_0 \cdot (\mu_m - r_f) \right) \tag{5.14}$$

Intuitively, the price of an asset is equal to the expected payoff discounted at the risk-free rate minus an adjustment term. The price is reduced by the adjustment term when the covariance between the asset payoff and the marginal utility is positive, i.e., whenever the beta factor of the asset is positive.

In the next step, we make use of the distribution assumptions for the asset payoffs and correlations from section 5.4.1. In addition, we recall the assumption for the representative investor's utility function (see equation 5.5). Applying Stein's lemma results in the following call price. (See appendix 5.C.2 for a detailed derivation.)

$$C_{0i} = \frac{1}{1+r_f} \cdot \left(\mathbb{E}[\widetilde{C}_i] - \frac{\pi \cdot em(\rho_u) \cdot cc(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot cc(\rho_d)}{\pi \cdot em(\rho_u) \cdot vm(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot vm(\rho_d)} \cdot mp \right) \tag{5.15}$$

with

$$\mathbb{E}[\widetilde{C}_i] = \delta_i \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right) - (X-1) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right), \tag{5.16}$$

$$cc(\rho) = \left(\delta_i^2 + (N-1)\delta_i\delta_{-i}\rho\right) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right), \tag{5.17}$$

$$vm(\rho) = \delta_i^2 + (N-1)\delta_{-i}^2 + 2(N-1)\delta_i\delta_{-i}\rho + (N-1)(N-2)\delta_{-i}^2\rho, \tag{5.18}$$

$$em(\rho) = \lambda \cdot \exp\left(-\lambda \cdot N + \frac{1}{2}\lambda^2 \cdot vm(\rho)\right), \tag{5.19}$$

$$mp = M_0 \cdot (\mu_m - r_f) = \tfrac{N}{1+\mu_m} \cdot (\mu_m - r_f), \tag{5.20}$$

where φ denotes the probability density function of the standard normal distribution and Φ denotes the corresponding cumulative distribution function. As in the previous section, vm is a shorthand notation for the market variance, cc abbreviates the covariance between the call payoff and the market portfolio, and em is the weighting term. The shorthand mp denotes the market risk premium in dollar terms.

For comparison, the call price $\widehat{C}_{0i}$ in the standard CAPM, i.e., when using the expected correlation for pricing, is given by

$$\widehat{C}_{0i} = \frac{1}{1+r_f} \cdot \left(\mathbb{E}[\widetilde{C}_i] - \frac{\pi \cdot cc(\rho_u) + (1-\pi) \cdot cc(\rho_d)}{\pi \cdot vm(\rho_u) + (1-\pi) \cdot vm(\rho_d)} \cdot mp \right). \tag{5.21}$$

Similarly to expression (5.10) for the expected return of the underlying asset, the weighting term em drops out of the formula.

Given the different representations for the call price in the correlation risk model (equation 5.15) and the standard CAPM prediction (equation 5.21), it is apparent that the call prices in the two models are different. This difference is illustrated in figure 5.6 using the parameters from the base case calibration from section 5.4.3. The plot shows the price difference $C_{0i} - \widehat{C}_{0i}$ in percent of the true call price C_{0i} for three different strike prices depending on the volatility δ_i of the underlying asset. The solid line represents an in-the-money call (with $X = 0.9$). The dashed line depicts the values for an at-the-money call (i.e., $X = 1.0$). The dot-dashed line shows an out-of-the-money call (with $X = 1.1$).

The graphs show that the price of call options on low volatility assets as predicted by the standard CAPM is lower compared to the price in the correlation risk model. The opposite is true for call options on high volatility assets.

We have already stated in proposition 5.6 that low volatility assets have a higher risk-adjusted return compared to the prediction by the standard CAPM. This higher return corresponds to a lower price. Similarly, call options on low volatility assets exhibit prices below the standard CAPM prediction. Evidently, this observation raises the question whether the price deviation of the call option is resulting from the price deviation of the underlying asset or whether the call option itself has a property which causes and potentially magnifies the effect. The answer to this question is given by the following proposition. (See appendix 5.C.3 for proof.)

Proposition 5.7 (Call pricing)
The price of a European call option written on an underlying asset with low volatility ($\delta_i < \delta_{-i}$) is lower than predicted by the standard CAPM, i.e., $C_{0i} < \widehat{C}_{0i}$. The European call option on an underlying asset with high volatility ($\delta_i > \delta_{-i}$) is more expensive than predicted by the standard CAPM, i.e., $C_{0i} > \widehat{C}_{0i}$. The price deviation stems solely from the different prices of the underlying asset.

To understand this result, we rearrange the expression for the call price, such that it has the following form

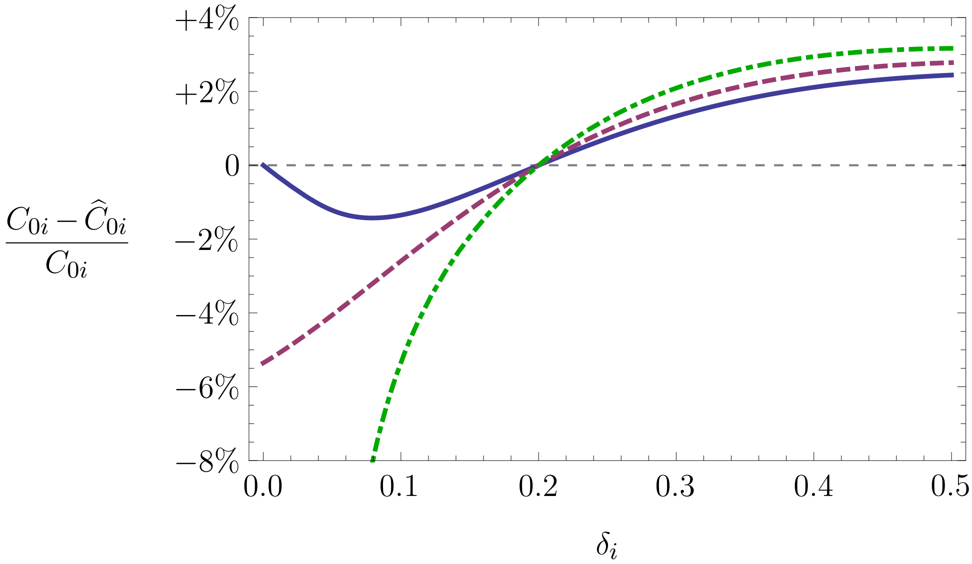
$$C_{0i} = Y_{0i} \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right) - \frac{X}{1+r_f} \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right) + \frac{\delta_i}{(1+r_f)} \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right). \tag{5.22}$$

This expression of the call price resembles similarities to the well known option pricing of Black and Scholes (1973). The call price is a function of only five variables: the underlying asset value Y_{0i}, the underlying asset's volatility δ_i, the risk free rate r_f, the strike price X and the time to maturity, which is standardized to 1 in this setting for simplicity.

Notably, the weighting term *em* is only contained in the price of the underlying asset Y_{0i}. Hence, equation (5.22) is valid for both the call price in the correlation risk model as well

Figure 5.6: Difference between call prices

The plot shows the price difference $C_{0i} - \widehat{C}_{0i}$ in percent of the true call price C_{0i} for three different strike prices $X = 0.9$ (solid line), $X = 1.0$ (dashed line), and $X = 1.1$ (dot-dashed line) depending on the volatility δ_i of the underlying asset. The further parameter values are given in table 5.2.



as the call price in the standard CAPM. Since the weighting term and, thus, also the correlation do not influence any other component of the call price, we conclude that the price deviation of the call originates only from the price deviation of the underlying asset.

Next, we repeat the derivation for put options. The price of the put option can easily be obtained by applying the put-call parity. The resulting put price is

$$P_{0i} = -Y_{0i} \cdot \Phi\left(\tfrac{X-1}{\delta_i}\right) + \frac{X}{1+r_f} \cdot \Phi\left(\tfrac{X-1}{\delta_i}\right) + \frac{\delta_i}{(1+r_f)} \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right). \tag{5.23}$$

In analogy to proposition 5.7, we can summarize the findings regarding put options in the following proposition. (See appendix 5.C.3 for proof.)

Proposition 5.8 (Put pricing)
The price of a European put option written on an underlying asset with low volatility ($\delta_i < \delta_{-i}$) is higher than predicted by the standard CAPM, i.e., $P_{0i} > \widehat{P}_{0i}$. The European put option on an underlying asset with high volatility ($\delta_i > \delta_{-i}$) is cheaper than predicted by the standard CAPM, i.e., $P_{0i} < \widehat{P}_{0i}$. The price deviation stems solely from the different prices of the underlying asset.

5.5.2 Pricing of equity and risky debt

The derived option pricing framework allows to build a structural model as proposed by Merton (1974). In this section, we derive the values of equity and risky corporate debt and examine the features of this structural model.

In the following, we assume that the firm's assets correspond to the specific asset i which was in the focus of our analysis so far. Hence, the value of the firm's asset is equal to Y_{0i} and the assets have volatility δ_i. The firm has debt with face value F which matures at the same time as we receive the cash flow from the assets.

The equity position is equivalent to a call option on the firm's assets. The resulting equity value E_0 is given by

$$E_0 = Y_{0i} \cdot \Phi\left(-\tfrac{F-1}{\delta_i}\right) - \frac{F}{1+r_f} \cdot \Phi\left(-\tfrac{F-1}{\delta_i}\right) + \frac{\delta_i}{(1+r_f)} \cdot \varphi\left(\tfrac{F-1}{\delta_i}\right). \tag{5.24}$$

The debt claim can be decomposed into a risk-free bond with face value F and a short put with strike price F. The resulting risky debt value D_0 is given by

$$D_0 = Y_{0i} \cdot \Phi\left(\tfrac{F-1}{\delta_i}\right) + \frac{F}{1+r_f} \cdot \Phi\left(-\tfrac{F-1}{\delta_i}\right) - \frac{\delta_i}{(1+r_f)} \cdot \varphi\left(\tfrac{F-1}{\delta_i}\right). \tag{5.25}$$

The structural model in this equilibrium setting possesses the key features of the original Merton (1974) model. We summarize two important properties in the following proposition. (See appendix 5.C.3 for proof.)

Proposition 5.9 (Structural model)
Both equity holders and debt holders effectively hold a fraction of the firm's asset risk. The irrelevance theorem of Modigliani and Miller (1958) holds.

5.6 Testable hypotheses

In the equilibrium model presented in section 5.4, the change in the risk exposure is induced by changes in the correlation. Even though the model includes only one period, we can conclude that beta factors vary over time as correlations move over time. (See Santos and Veronesi (2004) and Armstrong et al. (2013) for empirical evidence on time-varying beta factors.) Moreover, the discovered mechanism of beta contraction implies that the beta factors move in a *systematic* manner. The objective of this section is to develop an empirical research design. We summarize the predictions in three hypotheses.

We first consider the cross-section of beta factors. While the mean of the beta factors of all traded securities should always be close to one, the dispersion of the beta factors should vary over time. The dispersion can be inspected using histograms or by introducing a quantitative measure. We propose the standard deviation of the beta factors at one point in time as an appropriate measure.

Hypothesis 5.1 (Cross-section of beta factors)
The dispersion of beta factors should vary over time. In particular, the dispersion should decrease when market risk and correlations increase. In contrast, the cross-section of beta factors should be more dispersed in times of low market risk and low correlations.

The market risk is usually measured using the standard deviation of the returns of the market portfolio. Alternatively, we can resort to the implied volatility of traded index options, which is reported by the volatility index VIX. Furthermore, credit spreads also contain information on the market risk premium.

The empirical literature on asset pricing puzzles usually proceeds by sorting stocks into portfolios based on a specific property of the stocks, for example, based on the volatility (Haugen and Heins, 1975; Blitz and Van Vliet, 2007), idiosyncratic volatility (Ang et al., 2006; 2009), or beta factor (Frazzini and Pedersen, 2014; Asness et al., 2012). The creation of portfolios has two advantages. First, the desired property, for example, identifying stocks with low volatility, is held constant over time due to frequent readjustments of the portfolios. Second, idiosyncratic risk is eliminated by holding a well diversified portfolio. This implies that other factors, which might also determine expected returns, but are uncorrelated to the property of interest, i.e., volatility in our case, are randomly distributed across portfolios and do not distort the results.

It is already established in this empirical literature that portfolios of low volatility stocks have higher risk-adjusted returns compared to portfolios of high volatility stocks, which is in line with the key result of our model. However, we can make additional predictions on the behavior of these sorted portfolios.

Hypothesis 5.2 (Beta factors of sorted portfolios)
The beta factor of a portfolio containing low volatility stocks should increase when market risk and correlations increase. In contrast, the beta factor of a portfolio containing high volatility stocks should decrease when market risk and correlations increase. Hence, the beta factors of these two portfolios should be negatively correlated.

The development of the structural model in section 5.5 is not an end in itself. It can be used to distinguish two competing explanations for the low volatility anomaly. Both explanations have in common that they produce a contraction of beta factors. However, the origin of the beta contraction is different.

The first explanation is provided by the correlation risk model from section 5.4. Beta factors contract in response to a correlation shock. The second explanation developed by Frazzini and Pedersen (2014) relies on a funding friction of levered investors. When the funding restriction is binding, levered investors are adjusting their portfolios from low beta securities to high beta securities to maintain the desired level of idiosyncrativ risk. Hence, there is a surplus demand for high beta securities in times when the funding restriction is binding.

Notably, the cash flows from the different types of assets in Frazzini and Petersen's model remain unchanged. The effect is caused by different demand for the individual securities which depends on the securities' beta factor. In contrast, a correlation increase has cash flow consequences, which in turn change the risk profile of the firms' assets. The structural model helps to evaluate how the these cash flow consequences are transmitted to the different types of securities issued by the firm.

Hypothesis 5.3 (Beta factors of different securities)
The correlation risk model predicts that the beta factors of all of a firm's outstanding securities move in the same direction when correlations increase. In contrast, the friction

based model predicts that beta factors of all of the firm's outstanding securities move closer to one as a reaction to a funding shock.

We first review the prediction of the correlation risk model about what happens to the beta factors of the respective securities in the event of beta contraction. According to the structural model developed in section 5.5, all claim holders of the firm effectively possess a positive fraction of the firm's assets. So when the assets of the firm have a low volatility and the asset's beta factor increases in response to a correlation shock, the beta factors of *all* of the firm's claims increases as well. Conversely, when the firm's assets have a high volatility and the beta factor decreases after a correlation shock, the beta factors of equity and debt decrease too. Hence, the direction of the predicted effect is determined by whether the firm's assets have a low or high volatility.

In contrast, the direction of the effect in the friction based model of Frazzini and Pedersen (2014) depends on whether the individual security has a low or high risk. The difference between the prediction by the two competing models can best be demonstrated by considering a firm whose assets have an average volatility, i.e., $\delta_i = \delta_{-i}$ in our model. The correlation risk model predicts that the beta factor of the assets and all of the firm's claims should remain unaffected. However, the equity claim has a higher volatility and the debt claim has a lower volatility than the firm's assets. The same relation is true for the beta factors of equity and debt. So the friction based model predicts that the beta factor of debt increases and the beta factor of equity decreases in response to a shock with respect to the funding restriction.

Notably, the source of the shocks in the two models are different. Both models do not make any statement on the economic origin of the shocks. Hence, it also has to be empirically tested in what way these two shocks are related and whether they occur at the same time.

5.7 Conclusion

We study the impact of stochastic correlations on the risk characteristics of low and high volatility assets. We find that the beta factors of low and high volatility assets react differently to correlation increases: the beta of a low volatility asset rises whereas the beta of a high volatility asset falls. As a consequence, the systematic risk carried by a low volatility asset is found to be more sensitive towards correlation changes in comparison to a high volatility asset.

In the second step of our analysis, we incorporate stochastic correlations in an equilibrium model and derive the pricing consequences of this systematic movement of beta factors. As a result, low volatility assets provide investors with a higher expected return compared to what the standard CAPM predicts. In contrast, high volatility assets offer lower expected returns compared to the standard CAPM benchmark. We calibrate the model to a standard set of parameters and derive the premium of a zero-beta portfolio consisting of a levered long position in low volatility assets and short position in high volatility assets. The comparative static analysis shows that the effect is robust to changes in the key parameters.

Finally, we derive the prices of European options in this equilibrium setting. We find that the price deviation compared to the standard CAPM carries over to option prices, but stems only from the different prices of the underlying asset. The development of a structural model of the firm is the basis for three proposed empirical tests.

The central contribution of this work is a theoretical explanation of the low volatility puzzle. The model calibration shows that the magnitude of the effect is sizable. Furthermore, the model is parsimonious, for example, compared to the approach of Frazzini and Pedersen (2014), since it does not require a friction to explain the puzzle. Thus, stochastic correlations can be considered as a major cause of the low volatility puzzle. This evidence is further supported by recent empirical findings that correlation risk carries a significant market price (Krishnan et al., 2009; Driessen et al., 2009).

Our results apply analogously to the idiosyncratic risk puzzle. Since true idiosyncratic risk is — by definition — not accounted for in the standard CAPM, the effects derived in our theoretical analysis appear as part of the empirically determined idiosyncratic risk in the data. We hold the total volatility of an asset constant in our model. Thus, an ex post increase in the systematic risk is equivalent to a decrease in the asset's idiosyncratic risk. Assets with low volatility, whose beta factor is increasing in response to a correlation shock, thus have lower idiosyncratic volatility than predicted. The sensitivity to changes in correlations is higher for those assets with low idiosyncratic volatility, which coincides with low total volatility.

Appendix

5.A Proofs of propositions for portfolio setting

Beta factor and volatility (proposition 5.1)

Proof. The derivative of the beta factor β_i with respect to the volatility σ_i of the return given by

$$\frac{\partial \beta_i}{\partial \sigma_i} = \frac{N \cdot (N-1)\sigma_{-i}\left(\rho\sigma_i^2 + \left(2\sigma_i\sigma_{-i} + (N-1)\rho\sigma_{-i}^2\right)(1+(N-2)\rho)\right)}{\left(\sigma_i^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(1+(N-2)\rho)\sigma_{-i}^2\right)^2} > 0 \tag{5.26}$$

is always positive. Hence, sorting all assets according to their beta factor gives the same ranking as sorting the asset according to their return volatility. ■

Idiosyncratic volatility and volatility (proposition 5.2)

Proof. The squared idiosyncratic volatility ε_i^2 is defined as

$$\varepsilon_i^2 = Var\left(r_i - (r_f + \beta_i(r_m - r_f))\right) \tag{5.27}$$

$$= Var(r_i - \beta_i r_m) \tag{5.28}$$

$$= \sigma_i^2 + \beta_i^2\sigma_m^2 - 2\beta_i \cdot cov(r_i, r_m) \tag{5.29}$$

$$= \sigma_i^2 - \beta_i^2\sigma_m^2. \tag{5.30}$$

We plug in the expression from equations (5.1) and (5.2). Taking the first derivative with respect to σ_i yields

$$\frac{\partial \varepsilon_i^2}{\partial \sigma_i} = \frac{2(n-1)^2(1-\rho)(1+(n-1)\rho)\sigma_i\sigma_{-i}^3(\sigma_{-i} + \rho(\sigma_i + (n-2)\sigma_{-i}))}{(\sigma_i^2 + 2(n-1)\rho\sigma_i\sigma_{-i} + (n-1)(1+(n-2)\rho)\sigma_{-i}^2)^2} > 0 \tag{5.31}$$

which is always positive. Hence, sorting a portfolio using idiosyncratic volatility gives exactly the same sort as using total volatility. ■

Beta factor and correlation (proposition 5.3)

Proof. Making use of equation (5.2) and taking the derivative of β_i with respect to the correlation ρ, we obtain:

$$\frac{\partial \beta_i}{\partial \rho} = (\sigma_{-i} - \sigma_i) \cdot \frac{N \cdot (N-1)\sigma_i \sigma_{-i} (\sigma_i + (N-1)\sigma_{-i})}{\left(\sigma_i^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(1+(N-2)\rho)\sigma_{-i}^2\right)^2}. \tag{5.32}$$

Since the fraction on the right-hand-side in representation (5.32) is positive, we only need to regard the sign of the first factor $(\sigma_{-i} - \sigma_i)$ in order to conclude about the sign of the derivative $\partial \beta_i / \partial \rho$. Thus, the derivative is positive for a low volatility asset, i.e., $\sigma_i < \sigma_{-i}$, and negative for a high volatility asset, i.e., for $\sigma_i > \sigma_{-i}$. ■

Beta factor and average volatility (proposition 5.4)

Proof. Starting from expression (5.2), we replace σ_i with the term $\sigma_i + \Delta\sigma$ and σ_{-i} with the term $\sigma_{-i} + \Delta\sigma$. We then compute the first derivative with respect to $\Delta\sigma$ and evaluate it at $\Delta\sigma = 0$. The sign of the resulting expression

$$\begin{aligned}\left.\frac{\partial \beta_i}{\partial(\Delta\sigma)}\right|_{\Delta\sigma=0} &= (\sigma_{-i} - \sigma_i) \cdot N \cdot (N-1) \\ &\quad \cdot \frac{\left(2\sigma_i\sigma_{-i}(1+(N-2)\rho) + (N-1)\sigma_{-i}^2\rho(1+(N-2)\rho) + \sigma_i^2\rho\right)}{\left(\sigma_i^2 + 2(N-1)\sigma_i\sigma_{-i}\rho + (N-1)(1+(N-2)\rho)\sigma_{-i}^2\right)^2}\end{aligned} \tag{5.33}$$

again depends on the term $(\sigma_{-i} - \sigma_i)$, since all other factors are positive. The derivative is positive for a low volatility asset, i.e., $\sigma_i < \sigma_{-i}$, and negative for a high volatility asset, i.e., for $\sigma_i > \sigma_{-i}$. ■

5.B Derivation of expected returns

5.B.1 Notation

We focus on asset i which has payoff $\widetilde{Y}_i$ and current price Y_{0i}. We denote the standard deviation of the terminal payoff as

$$\delta_i := Std\left(\widetilde{Y}_i\right). \tag{5.34}$$

The $N-1$ other assets have a standard deviation of the terminal payoff equal to

$$\delta_{-i} := Std\left(\widetilde{Y}_j\right), \text{ for all } j \neq i. \tag{5.35}$$

The current price of the market portfolio is defined as

$$M_0 := \sum_{j=1}^{N} Y_{0j} \tag{5.36}$$

with a payoff equal to

$$\widetilde{M} := \sum_{j=1}^{N} \widetilde{Y}_j. \tag{5.37}$$

In our notation, r_j and r_m denote the return of asset j and the market portfolio, respectively. The returns can be represented by

$$r_j = \frac{\widetilde{Y}_j}{Y_{0j}} - 1, \tag{5.38}$$

$$r_m = \frac{\widetilde{M}}{M_0} - 1. \tag{5.39}$$

The corresponding expected returns μ_j of asset j and μ_m of the market portfolio are

$$\mu_j = \frac{1}{Y_{0j}} - 1, \tag{5.40}$$

$$\mu_m = \frac{N}{M_0} - 1. \tag{5.41}$$

5.B.2 Fundamental pricing relation

The following derivation of the fundamental pricing equation is based on Huang and Litzenberger (1988). We consider the one-period optimization problem of a representative investor with a time-additive and state-independent utility function $u(c_t)$ in consumption c_t at time t. The investor consumes a fraction of his endowment at time $t = 0$. The remaining fraction is invested. The payoff from the investment is consumed at time $t = 1$.

The resulting pricing kernel m is given by

$$m = \frac{u'(c_1)}{u'(c_0)}. \tag{5.42}$$

The risk-free asset pays 1 unit of consumption in all states. Hence, we can express the risk-free rate r_f as

$$\frac{1}{1+r_f} \equiv \mathbb{E}\left[m \cdot 1\right]. \tag{5.43}$$

The price Y_{0j} of any risky asset j with payoff $\widetilde{Y}_j$ is given by

$$Y_{j0} = \mathbb{E}\left[m \cdot \widetilde{Y}_j\right]. \tag{5.44}$$

This expression can be modified to get the expected excess return

$$\begin{aligned}
& Y_{0j} = \mathbb{E}\left[m \cdot \widetilde{Y}_j\right] && (5.45)\\
\Rightarrow \quad & 1 = \mathbb{E}\left[m \cdot \frac{\widetilde{Y}_j}{Y_{0i}}\right] && (5.46)\\
\Rightarrow \quad & 1 = cov\left(m, \frac{\widetilde{Y}_j}{Y_{0j}}\right) + \mathbb{E}\left[m\right] \cdot \mathbb{E}\left[\frac{\widetilde{Y}_j}{Y_{0j}}\right] && (5.47)\\
\Rightarrow \quad & \frac{1}{1+r_f} \cdot \mathbb{E}\left[\frac{\widetilde{Y}_j}{Y_{0j}}\right] = 1 - cov\left(m, \frac{\widetilde{Y}_j}{Y_{0j}}\right) && (5.48)\\
\Rightarrow \quad & \mathbb{E}\left[\frac{\widetilde{Y}_j}{Y_{0j}}\right] - (1+r_f) = -(1+r_f) \cdot cov\left(m, \frac{\widetilde{Y}_j}{Y_{0j}} - 1\right) && (5.49)\\
\Rightarrow \quad & \mathbb{E}\left[r_j - r_f\right] = -(1+r_f) \cdot cov\left(r_j, m\right). && (5.50)
\end{aligned}$$

The same holds for the market portfolio

$$\mathbb{E}\left[r_m - r_f\right] = -(1+r_f) \cdot cov\left(r_m, m\right). \tag{5.51}$$

Substituting in the expression for the expected return of asset j

$$\mathbb{E}\left[r_i - r_f\right] = \frac{cov\left(r_i, m\right)}{cov\left(r_m, m\right)} \cdot \mathbb{E}\left[r_m - r_f\right]. \tag{5.52}$$

Consumption at time $t = 1$ is equal to the payoff of the market portfolio

$$\mathbb{E}\left[r_i - r_f\right] = \frac{cov\left(r_i, u'(\widetilde{M})\right)}{cov\left(r_m, u'(\widetilde{M})\right)} \cdot \mathbb{E}\left[r_m - r_f\right]. \tag{5.53}$$

This result holds for any well-behaved utility function and any distribution of payoffs.

5.B.3 Equilibrium pricing with stochastic correlations

We further evaluate the right-hand side in equation (5.4). From the law of total covariance and the representation of returns with payoffs, we can write

$$\mu_i = r_f + \left(\mu_m - r_f\right) \cdot \frac{\pi \cdot cov\left(r_i, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(r_i, u'(\widetilde{M}) \middle| \rho_d\right)}{\pi \cdot cov\left(r_m, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(r_m, u'(\widetilde{M}) \middle| \rho_d\right)} \tag{5.54}$$

$$= r_f + \left(\mu_m - r_f\right) \cdot \frac{\pi \cdot cov\left(\frac{\widetilde{Y}_i}{Y_{0i}}, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(\frac{\widetilde{Y}_i}{Y_{0i}}, u'(\widetilde{M}) \middle| \rho_d\right)}{\pi \cdot cov\left(\frac{\widetilde{M}}{M_0}, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(\frac{\widetilde{M}}{M_0}, u'(\widetilde{M}) \middle| \rho_d\right)} \tag{5.55}$$

$$= r_f + \left(\mu_m - r_f\right) \cdot \frac{M_0}{Y_{0i}} \cdot \frac{\pi \cdot cov\left(\widetilde{Y}_i, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(\widetilde{Y}_i, u'(\widetilde{M}) \middle| \rho_d\right)}{\pi \cdot cov\left(\widetilde{M}, u'(\widetilde{M}) \middle| \rho_u\right) + (1-\pi) \cdot cov\left(\widetilde{M}, u'(\widetilde{M}) \middle| \rho_d\right)} \tag{5.56}$$

As a result of multivariate-normally distributed payoffs for a given correlation ρ, we can apply Stein's lemma to evaluate the conditional covariance terms

$$cov\left(\widetilde{Y}_i, u'(\widetilde{M}) \middle| \rho\right) = \mathbb{E}\left(u''(\widetilde{M}) \middle| \rho\right) \cdot cov\left(\widetilde{Y}_i, \widetilde{M} \middle| \rho\right) \tag{5.57}$$

$$= -em(\rho) \cdot ci(\rho), \tag{5.58}$$

$$cov\left(\widetilde{M}, u'(\widetilde{M}) \middle| \rho\right) = \mathbb{E}\left(u''(\widetilde{M}) \middle| \rho\right) \cdot cov\left(\widetilde{M}, \widetilde{M} \middle| \rho\right) \tag{5.59}$$

$$= \mathbb{E}\left(u''\left(\widetilde{M}\right) \middle| \rho\right) \cdot var\left(\widetilde{M} \middle| \rho\right) \tag{5.60}$$

$$= -em(\rho) \cdot vm(\rho). \tag{5.61}$$

The covariance between the payoffs of asset i and the market portfolio is

$$ci(\rho) := cov\left(\widetilde{Y}_i, \widetilde{M} \middle| \rho\right) \tag{5.62}$$

$$= cov\left(\widetilde{Y}_i, \sum_{j=1}^{N} \widetilde{Y}_j \middle| \rho\right) \tag{5.63}$$

$$= \delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho. \tag{5.64}$$

The variance of the market portfolio payoff is given by

$$vm(\rho) := var\left(\widetilde{M} \middle| \rho\right) \tag{5.65}$$

$$= \delta_i^2 + (N-1) \cdot \delta_{-i}^2 + 2 \cdot (N-1) \cdot \delta_i \delta_{-i} \rho + 2 \cdot \frac{1}{2} (N-1)(N-2) \delta_{-i}^2 \rho \tag{5.66}$$

$$= \delta_i^2 + (N-1) \cdot \delta_{-i}^2 + (N-1) \delta_{-i} \cdot (2 \cdot \delta_i + (N-2) \delta_{-i}) \rho. \tag{5.67}$$

Finally, the weighting term is given by

$$\begin{aligned}
em(\rho) &:= -\mathbb{E}\left(u''(\widetilde{M})\middle|\rho\right) && (5.68)\\
&= -\mathbb{E}\left(-\lambda\cdot\exp(-\lambda\cdot\widetilde{M})\right) && (5.69)\\
&= \lambda\cdot\exp\left(-\lambda\cdot\mathbb{E}\left(\widetilde{M}\right)+\frac{1}{2}\lambda^2\cdot var\left(\widetilde{M}\middle|\rho\right)\right) && (5.70)\\
&= \lambda\cdot\exp\left(-\lambda\cdot N+\frac{1}{2}\lambda^2\cdot vm(\rho)\right). && (5.71)
\end{aligned}$$

The step from (5.69) to (5.70) follows from the normal distribution of the market portfolio payoff. The weighting term is monotonically increasing in the correlation, since

$$\frac{\partial em(\rho)}{\partial\rho}=\frac{1}{2}\lambda^3(N-1)\delta_{-i}(2\delta_i+\delta_{-i}(N-2))\cdot\exp(...)>0. \qquad (5.72)$$

The derivative of the weighting term with respect to a simultaneous shift $\Delta\delta$ in all asset volatilities is given by

$$\left.\frac{\partial em(\rho)}{\partial(\Delta\delta)}\right|_{\Delta\delta=0}=\lambda^3((N-1)\rho+1)(\delta_i+\delta_{-i}(N-1))\cdot\exp(...). \qquad (5.73)$$

This derivative is positive for all correlations $\rho>-\frac{1}{N-1}$, which is true by assumption.

5.B.4 Proofs of propositions

Expected return and volatility (proposition 5.6)

Proof. To evaluate whether the true expected return μ_i is above the CAPM prediction $\widehat{\mu}_i$, we compare the true risk premium $\frac{\pi\cdot em(\rho_u)\cdot ci(\rho_u)+(1-\pi)\cdot em(\rho_d)\cdot ci(\rho_d)}{\pi\cdot em(\rho_u)\cdot vm(\rho_u)+(1-\pi)\cdot em(\rho_d)\cdot vm(\rho_d)}$ from equation (5.6) to the risk premium $\frac{\pi\cdot ci(\rho_u)+(1-\pi)\cdot ci(\rho_d)}{\pi\cdot vm(\rho_u)+(1-\pi)\cdot vm(\rho_d)}$ predicted by the standard CAPM from equation (5.10).

Both terms have the following structure

$$\frac{w_1\cdot ci(\rho_u)+w_2\cdot ci(\rho_d)}{w_1\cdot vm(\rho_u)+w_2\cdot vm(\rho_d)}, \qquad (5.74)$$

where w_1 and w_2 are positive weights. These weights w_1 and w_2 differ for the true model and the risk premium predicted by the standard CAPM.

According to the mean value theorem, we can therefore write the following helpful inequality for the crucial risk premiums

$$\frac{ci(\rho_u)}{vm(\rho_u)} \leq \frac{w_1 \cdot ci(\rho_u) + w_2 \cdot ci(\rho_d)}{w_1 \cdot vm(\rho_u) + w_2 \cdot vm(\rho_d)} \leq \frac{ci(\rho_d)}{vm(\rho_d)}, \quad \text{given } \frac{ci(\rho_u)}{vm(\rho_u)} \leq \frac{ci(\rho_d)}{vm(\rho_d)}, \tag{5.75}$$

$$\frac{ci(\rho_u)}{vm(\rho_u)} \geq \frac{w_1 \cdot ci(\rho_u) + w_2 \cdot ci(\rho_d)}{w_1 \cdot vm(\rho_u) + w_2 \cdot vm(\rho_d)} \geq \frac{ci(\rho_d)}{vm(\rho_d)}, \quad \text{given } \frac{ci(\rho_u)}{vm(\rho_u)} \geq \frac{ci(\rho_d)}{vm(\rho_d)}. \tag{5.76}$$

A further important property is that the risk premium $\frac{w_1 \cdot ci(\rho_u) + w_2 \cdot ci(\rho_d)}{w_1 \cdot vm(\rho_u) + w_2 \cdot vm(\rho_d)}$ tends to $\frac{ci(\rho_u)}{vm(\rho_u)}$ the higher the weight w_1 relative to w_2 is. Technically speaking, the difference $\frac{ci(\rho_u)}{vm(\rho_u)} - \frac{w_1 \cdot ci(\rho_u) + w_2 \cdot ci(\rho_d)}{w_1 \cdot vm(\rho_u) + w_2 \cdot vm(\rho_d)}$ has a lower absolute value, the higher $\frac{w_1}{w_1 + w_2}$ is for given values of $ci(\rho_u)$, $ci(\rho_d)$, $vm(\rho_u)$ and $vm(\rho_d)$.

In the case of $\delta_i = \delta_{-i}$, the ratio $\frac{ci(\rho)}{vm(\rho)}$ simplifies to $\frac{1}{N}$ regardless of the correlation ρ. Hence inequalities (5.75) and (5.76) show that the relevant terms for the risk premium in both cases, the true and the CAPM prediction, must equal $\frac{1}{N}$ and are therefore identical. For this reason, we have $\mu_i = \widehat{\mu}_i$ whenever $\delta_i = \delta_{-i}$ holds.

For volatility $\delta_i \neq \delta_{-i}$, the ratios $\frac{ci(\rho)}{vm(\rho)}$ might depend on the correlation ρ. The inequality $\frac{ci(\rho_u)}{vm(\rho_u)} > \frac{ci(\rho_d)}{vm(\rho_d)}$ simplifies to

$$\frac{ci(\rho_u)}{vm(\rho_u)} > \frac{ci(\rho_d)}{vm(\rho_d)} \tag{5.77}$$

$$\iff \frac{\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_u}{\delta_i^2 + (N-1) \cdot \delta_{-i}^2 + (N-1)\delta_{-i} \cdot (2 \cdot \delta_i + (N-2)\delta_{-i})\rho_u} > \dots$$

$$\frac{\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_d}{\delta_i^2 + (N-1) \cdot \delta_{-i}^2 + (N-1)\delta_{-i} \cdot (2 \cdot \delta_i + (N-2)\delta_{-i})\rho_d} \tag{5.78}$$

$$\iff 1 + \frac{(N-1) \cdot \delta_{-i}^2 + (N-1)\delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_u}{\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_u} < \dots$$

$$1 + \frac{(N-1) \cdot \delta_{-i}^2 + (N-1)\delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_d}{\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_d} \tag{5.79}$$

$$\iff (\delta_{-i}^2 + \delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_u)(\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_d) < \dots$$

$$(\delta_{-i}^2 + \delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_d)(\delta_i^2 + (N-1) \cdot \delta_i \delta_{-i} \rho_u) \tag{5.80}$$

$$\iff \delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_u \cdot \delta_i^2 + \delta_{-i}^2 \cdot (N-1) \cdot \delta_i \delta_{-i} \rho_d < \dots$$

$$\delta_{-i} \cdot (\delta_i + (N-2)\delta_{-i})\rho_d \cdot \delta_i^2 + \delta_{-i}^2 \cdot (N-1) \cdot \delta_i \delta_{-i} \rho_u \tag{5.81}$$

$$\iff (\delta_i + (N-2)\delta_{-i})(\rho_u - \rho_d) \cdot \delta_i < \delta_{-i}^2 \cdot (N-1)(\rho_u - \rho_d) \tag{5.82}$$

$$\iff \delta_i^2 + (N-2)\delta_{-i}\delta_i - \delta_{-i}^2 \cdot (N-1) < 0 \tag{5.83}$$

$$\iff (\delta_i - \delta_{-i})(\delta_i + (N-1)\delta_{-i}) < 0 \tag{5.84}$$

$$\iff \delta_i < \delta_{-i} \tag{5.85}$$

Since $\frac{ci(\rho_u)}{vm(\rho_u)} > \frac{ci(\rho_d)}{vm(\rho_d)}$ if and only if $\delta_i < \delta_{-i}$, we only need to compare the ratio of the weights $\frac{w_1}{w_1+w_2}$ for the true model and the CAPM prediction to see which risk premium term is higher. In case of the true model the ratio $\frac{w_1}{w_1+w_2}$ amounts to

$$\frac{\pi \cdot em(\rho_u)}{\pi \cdot em(\rho_u) + (1-\pi) \cdot em(\rho_d)} \tag{5.86}$$

and can be written as

$$\frac{\pi}{\pi + (1-\pi)} \cdot \frac{\pi \cdot em(\rho_u) + (1-\pi) \cdot em(\rho_d)}{\pi \cdot em(\rho_u) + (1-\pi) \cdot em(\rho_d)}. \tag{5.87}$$

In the representation of the CAPM prediction the weight ratio is simply

$$\frac{\pi}{\pi + (1-\pi)}. \tag{5.88}$$

For this reason, we can conclude that the weight ratio $\frac{w_1}{w_1+w_2}$ in the true model always exceeds that in the CAPM predicition. This is because the second factor in equation (5.87) on the right-hand side is always above 1

$$\frac{\pi \cdot em(\rho_u) + (1-\pi) \cdot em(\rho_d)}{\pi \cdot em(\rho_u) + (1-\pi) \cdot em(\rho_d)} > 1 \tag{5.89}$$

$$\iff \quad em(\rho_u) > em(\rho_d). \tag{5.90}$$

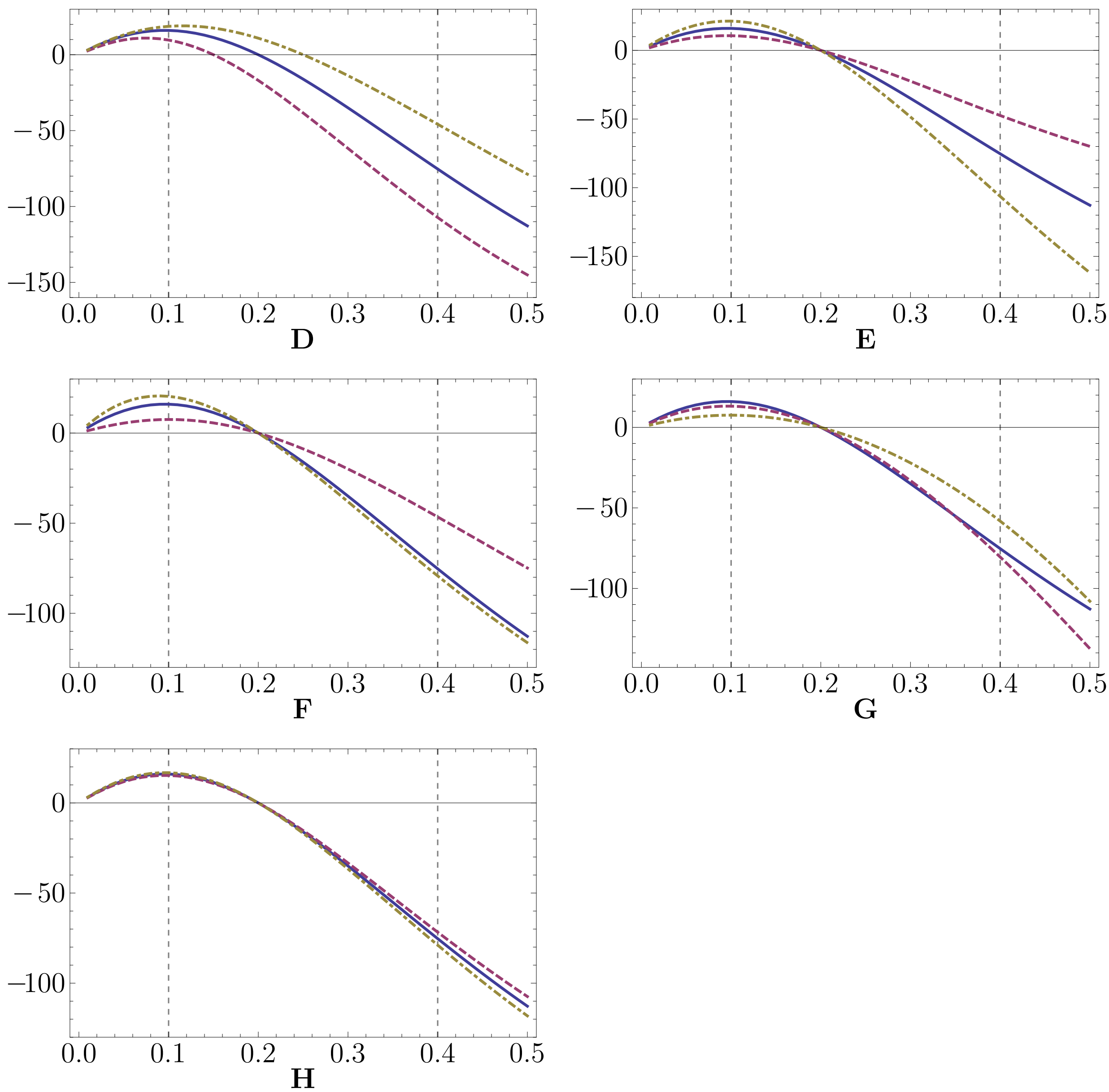
The validity of the fact that $em(\rho)$ is increasing in ρ directly follows from the fact that $vm(\rho)$ is increasing in ρ.

As a result, we can now conclude that the risk premium term of the true model is always closer to $\frac{ci(\rho_u)}{vm(\rho_u)}$ and likewise more distant to $\frac{ci(\rho_d)}{vm(\rho_d)}$ than in the representation of the CAPM prediction. Since $\frac{ci(\rho_u)}{vm(\rho_u)}$ is higher than $\frac{ci(\rho_d)}{vm(\rho_d)}$ if and only if a low volatility asset is considered, i.e., $\delta_i < \delta_{-i}$, we see why low volatility assets have positive excess returns relative to the CAPM prediction and high volatility assets exhibit negative excess returns relative to the CAPM prediction. ■

5.B.5 Comparative static analysis

Figure 5.7: Comparative static analysis for further parameters

The plots show the return difference $\mu_i - \widehat{\mu}_i$ in basis points per year depending on the volatility δ_i of asset i. The solid line in all plots represents the base case with parameter values as in table 5.2. The remaining lines represent deviations with regard to one set of parameters. ***Plot D:*** *volatility of remaining assets $\delta_{-i} = 0.15$ (dashed line) and $\delta_{-i} = 0.25$ (dot-dashed line).* ***Plot E:*** *market risk premium $\mu_m - r_f = 0.04$ (dashed line) and $\mu_m - r_f = 0.08$ (dot-dashed line).* ***Plot F:*** *risk-aversion $\lambda = 3$ (dashed line) and $\lambda = 10$ (dot-dashed line).* ***Plot G:*** *number of assets $N = 10$ (dashed line) and $N = 20$ (dot-dashed line).* ***Plot H:*** *risk-free rate $r_f = 0$ (dashed line) and $r_f = 0.1$ (dot-dashed line).*



5.C Derivation of derivatives prices

5.C.1 Fundamental pricing relation

The return of the call option is defined as

$$r_{ci} = \frac{\widetilde{C}_i}{C_{0i}} - 1 \tag{5.91}$$

Applying the fundamental pricing relation derived in appendix 5.B.2 gives

$$\mathbb{E}\left[r_{ci} - r_f\right] = \frac{cov\left(r_{ci}, u'(\widetilde{M})\right)}{Cov\left(r_m, u'(\widetilde{M})\right)} \cdot \mathbb{E}\left[r_m - r_f\right] \tag{5.92}$$

$$\Rightarrow \quad \frac{\mathbb{E}\left[\widetilde{C}_i\right]}{C_{0i}} - 1 - r_f = \frac{\frac{1}{C_{0i}} \cdot Cov\left(\widetilde{C}_i, u'(\widetilde{M})\right)}{\frac{1}{M_0} \cdot Cov\left(\widetilde{M}, u'(\widetilde{M})\right)} \cdot \mathbb{E}\left[r_m - r_f\right] \tag{5.93}$$

$$\Rightarrow \quad \mathbb{E}\left[\widetilde{C}_i\right] - (1 + r_f) \cdot C_{0i} = \frac{Cov\left(\widetilde{C}_i, u'(\widetilde{M})\right)}{Cov\left(\widetilde{M}, u'(\widetilde{M})\right)} \cdot M_0 \cdot \mathbb{E}\left[r_m - r_f\right] \tag{5.94}$$

$$\Rightarrow \quad C_{0i} = \frac{1}{1 + r_f} \cdot \left(\mathbb{E}\left[\widetilde{C}_i\right] - \frac{Cov\left(\widetilde{C}_i, u'(\widetilde{M})\right)}{Cov\left(\widetilde{M}, u'(\widetilde{M})\right)} \cdot M_0 \cdot \mathbb{E}\left[r_m - r_f\right]\right) \tag{5.95}$$

Again, this result holds for any arbitrary payoff and all well-behaved utility functions.

5.C.2 Pricing of European options

In the next step, we make use of the distribution assumptions for payoffs and correlations as well as of the utlity function for the representative investor. From the law of total covariance and Stein's lemma follows that the true call price can be expressed as

$$C_{0i} = \frac{1}{1+r_f} \cdot \left(\mathbb{E}[\widetilde{C}_i] - \frac{\pi \cdot em(\rho_u) \cdot cc(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot cc(\rho_d)}{\pi \cdot em(\rho_u) \cdot vm(\rho_u) + (1-\pi) \cdot em(\rho_d) \cdot vm(\rho_d)} \cdot mp\right). \tag{5.96}$$

The call price predicted by the standard CAPM is given by

$$\widehat{C}_{0i} = \frac{1}{1 + r_f} \cdot \left(\mathbb{E}\left[\widetilde{C}_i\right] - \frac{\pi \cdot cc(\rho_u) + (1-\pi) \cdot cc(\rho_d)}{\pi \cdot vm(\rho_u) + (1-\pi) \cdot vm(\rho_d)} \cdot mp\right). \tag{5.97}$$

The expected payoff of the call is

$$\begin{aligned}
\mathbb{E}[\widetilde{C}_i] &= \mathbb{E}\left[\max\left\{\widetilde{Y}_i - X, 0\right\}\right] && (5.98)\\
&= \int_{-\infty}^{+\infty} \max\{Y_i - X, 0\} \cdot f_i(Y_i)\ \mathrm{d}Y_i && (5.99)\\
&= \frac{1}{\sqrt{2\pi}} \int_{\frac{X-1}{\delta_i}}^{+\infty} (\delta_i \cdot z + 1 - X) \cdot \mathrm{e}^{-\frac{1}{2}z^2} \mathrm{d}z && (5.100)\\
&= \delta_i \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{X-1}{\delta_i}} z \cdot \mathrm{e}^{-\frac{1}{2}z^2} \mathrm{d}z + (1 - X) \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{X-1}{\delta_i}} \mathrm{e}^{-\frac{1}{2}z^2} \mathrm{d}z && (5.101)\\
&= \delta_i \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right) + (1 - X) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right), && (5.102)
\end{aligned}$$

where φ denotes the probability density function of the standard normal distribution and Φ denotes the corresponding cumulative distribution function.

The abbreviation cc denotes the covariance between the payoffs of the call option and the market portfolio, which can be derived as follows

$$\begin{aligned}
cc(\rho) &= Cov(\widetilde{C}_i, \widetilde{M}|\rho) && (5.103)\\
&= \mathbb{E}[\widetilde{C}_i \cdot \widetilde{M}] - \mathbb{E}[\widetilde{C}_i] \cdot \mathbb{E}[\widetilde{M}] && (5.104)\\
&= \mathbb{E}\left[\max\left\{\widetilde{Y}_i - X, 0\right\} \cdot \left(\widetilde{Y}_i + \sum_{j=1}^{N-1} \widetilde{Y}_j\right)\right] - N \cdot \mathbb{E}[\widetilde{C}_i] && (5.105)\\
&= \mathbb{E}\left[\max\left\{\widetilde{Y}_i - X, 0\right\} \cdot \widetilde{Y}_i\right] + (N-1) \cdot \mathbb{E}\left[\max\left\{\widetilde{Y}_i - X, 0\right\} \cdot \widetilde{Y}_j\right] - N \cdot \mathbb{E}[\widetilde{C}_i] && (5.106)\\
&= \int_{-\infty}^{+\infty} \max\{Y_i - X, 0\} \cdot Y_i \cdot f_i(Y_i)\ \mathrm{d}Y_i\\
&\quad + (N-1) \int_{-\infty}^{+\infty} \max\{Y_i - X, 0\} \cdot Y_j \cdot f_{ij}(Y_i, Y_j)\ \mathrm{d}Y_i \mathrm{d}Y_j - N \cdot \mathbb{E}[\widetilde{C}_i] && (5.107)\\
&= \delta_i \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right) + (1 - X + \delta_i^2) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right)\\
&\quad + (N-1) \cdot (\delta_i \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right) + (1 - X + \delta_i \delta_{-i} \rho) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right))\\
&\quad - N \cdot \left(\delta_i \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right) + (1 - X) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right)\right) && (5.108)\\
&= \left(\delta_i^2 + (N-1)\delta_i \delta_{-i} \rho\right) \cdot \Phi\left(-\tfrac{X-1}{\delta_i}\right). && (5.109)
\end{aligned}$$

As before, vm is an abbreviation for the variance of the market portfolio payoff and em is the weighting term. The shorthand mp denotes the market risk premium expressed in terms of units of payoff, i.e.,

$$mp = M_0 \cdot \mathbb{E}[r_m - r_f] = \frac{N}{1 + \mu_m} \cdot (\mu_m - r_f). \tag{5.110}$$

5.C.3 Proofs of propositions

Call pricing (proposition 5.7)

Proof. The fundamental pricing relation (5.14) is also valid for the underlying asset

$$Y_{0i} = \frac{1}{1+r_f} \cdot \Big(\mathbb{E}[\widetilde{Y}_i] - \beta_i \cdot mp\Big) \tag{5.111}$$

where β_i denotes the risk exposure in either model. We can solve for this risk premium

$$\beta_i = \frac{1 - Y_{0i} \cdot (1+r_f)}{mp} \tag{5.112}$$

and substitute it into the formula for the call price

$$\begin{aligned} C_{0i} &= \frac{1}{1+r_f} \cdot \Big(\mathbb{E}[\widetilde{C}_i] - \beta_i \cdot \Phi\Big(-\tfrac{X-1}{\delta_i}\Big) \cdot mp\Big) && (5.113)\\ &= \frac{1}{1+r_f} \cdot \Big(\mathbb{E}[\widetilde{C}_i] - \Big(1 - Y_{0i} \cdot (1+r_f)\Big) \cdot \Phi\Big(-\tfrac{X-1}{\delta_i}\Big)\Big) && (5.114)\\ &= Y_{0i} \cdot \Phi\Big(-\tfrac{X-1}{\delta_i}\Big) - \frac{X}{1+r_f} \cdot \Phi\Big(-\tfrac{X-1}{\delta_i}\Big) + \frac{\delta_i}{1+r_f} \cdot \varphi\Big(\tfrac{X-1}{\delta_i}\Big). && (5.115) \end{aligned}$$

Hence, the call price is a function $C_{0i} = f\Big(Y_{0i}, \delta_i, X, r_f\Big)$ of the underlying asset's price, its volatility, the strike price and the risk-free rate. In addition, both the time to maturity and the expected payoff of the underlying asset, which have been normalized to one, influence the call price. The call price does not depend on the properties of the other assets, i.e., not on N, δ_{-i} or ρ. Thus, the price deviation of the call option stems only from the difference in the underlying asset's price in the two models.

From proposition 5.6 follows, that the low volatility asset has a higher expected return compared to the CAPM prediction. A higher expected return corresponds to a lower true price as predicted by the standard CAPM. Hence, also the call price is lower than predicted. Conversely, the call price is higher for high volatility assets. There is no difference between the call prices in both models for $\delta_i = \delta_{-i}$. Also, when we regard Y_{0i} as exogenous, for example, because it is observable on the stock exchange, both models should yield the same call price. ■

Put pricing (proposition 5.8)

Proof. From the put-call-parity follows

$$P_{0i} = \frac{X}{1+r_f} + C_{0i} - Y_{0i} \tag{5.116}$$

$$= \frac{X}{1+r_f} \cdot \Phi\left(\tfrac{X-1}{\delta_i}\right) - Y_{0i} \cdot \Phi\left(\tfrac{X-1}{\delta_i}\right) + \frac{\delta_i}{(1+r_f)} \cdot \varphi\left(\tfrac{X-1}{\delta_i}\right). \tag{5.117}$$

Similarly to the call price, the put price does not depend on N, δ_{-i} or ρ. So the only source of a price deviation to the standard CAPM prediction is the price of the underlying asset Y_{0i}. ■

Structural model (proposition 5.9)

(i) Both equity holders and debt holders effectively hold a fraction of the firm's asset risk.

Proof. The risk-sharing between equity and debt holders is straightforward, since the asset value Y_{0i} is influencing both values. Even though it is not feasible to duplicate the payoffs of the options in our model, we can apply the economic intuition of the duplication portfolio to interpret the claim values of equity and debt. The equity value contains the term $\Phi(-a) \cdot Y_{0i}$, while the debt value contains the term $\Phi(a) \cdot Y_{0i}$ with $a = \frac{F-1}{\delta_i}$. Since $\Phi(-a) + \Phi(a) = 1$, we can conclude that both claim holders hold a fraction of the firm's risk. This is in line with the traditional interpretation of Merton (1974) that equity holders hold a fraction of the firm's assets financed with a risk-free credit and that debt holders hold the remaining fraction of the firm's assets and a risk-free investment. ■

(ii) The irrelevance theorem of Modigliani and Miller (1958) holds.

Proof. The sum of equity value and debt value is

$$E_0 + D_0 = Y_{0i} = const. \tag{5.118}$$

and does not depend on the face value of debt F. Hence, the capital structure choice does not influence the total firm value. ■

Chapter 6

Synthesis (part II)

In his presidential address to the American Finance Association, Cochrane (2011) asserts that much of the variation in stock prices does not stem — as one would presume — from variation in expected cash flows, but rather from variation in discount rates. Following this idea, a new generation of asset pricing models emerged which produce time-varying risk premia, for example, the habit model (Campbell and Cochrane, 1999), the long-run risk model (Bansal and Yaron, 2004) or disaster risk models (Rietz, 1988; Barro, 2006; Gabaix, 2008). Empirical tests show that these models are indeed capable of explaining the equity premium puzzle.

However, not all of the variation in discount rates results from variation in the market risk premium. In addition, the exposure to the market risk, which is usually expressed using the beta factor, varies over time as well. Empirical evidence on time varying risk exposures is provided, for example, by Santos and Veronesi (2004), Adrian and Franzoni (2009), Armstrong et al. (2013), and Malamud and Vilkov (2015). Jagannathan and Wang (1996) incorporate this idea into a theoretical model. They assume that the standard CAPM holds conditionally, i.e., every period, but beta factors and the risk premium may change from one period to another. An unconditional linear two-factor model emerges, in which the second factor represents a risk premium for beta instability. Jagannathan and Wang (1996) conclude that their model empirically performs substantially better than the static CAPM. (See also Lettau and Ludvigson (2001) and Santos and Veronesi (2006) for further empirical tests.) However, Lewellen and Nagel (2006) show that the conditional CAPM does not explain asset pricing puzzles like, for example, the value effect.

The correlation risk model presented in chapter 5 differs from the conditional CAPM of Jagannathan and Wang (1996) in several aspects. First, the change in the risk exposure is caused by changes in the correlation (or average volatility). Hence, it is a key *result* that beta factors move over time — and not an assumption. In contrast, the origin of the time-variation in beta factors is not specified in the conditional CAPM. Second, the discovered mechanism of beta contraction implies that beta factors move in a *systematic* manner. The different beta sensitivity of low volatility assets and high volatility assets is not discussed by Jagannathan and Wang (1996). Furthermore, the correlation risk model from chapter 5 also differs from the intertemporal CAPM of Merton (1973a), since there is only one risk factor, namely the market portfolio.

There is plenty of empirical evidence that correlations change over time and especially in market downturns (Ang and Chen, 2002; Longin and Solnik, 2001; Goetzmann et al., 2005; Hong et al., 2007). However, it remains unclear whether to regard correlations as state variables in the sense of Merton (1973a) or whether correlations and their time variation are the result of other economic forces at work. Given there is such a fundamental state variable, then the observed market decline and the simultaneous jump in correlations could both be triggered by a shock to this state variable. Consequently, times in which beta contraction can be observed should coincide with times of bad economic conditions. Similarly, Frazzini and Pedersen (2014) observe beta contraction in times of high funding spreads of financial institutions. Since funding is usually more constrained during recessions, the observed beta contraction might as well be caused by this economic state variable underlying the recession. Hence, there is need for further research to disentangle the different forces at work. The test hypotheses formulated in section 5.6 offer a good starting point.

The simplifying assumption from chapter 5 that the correlation between each pair of assets is the same is obviously not met in reality. Thus, the sensitivity of each asset's beta factor with respect to changes in correlations or other sources of uncertainty is most likely different. Nevertheless, the pattern of beta contraction has immense practical relevance. The starting point for the following three examples is that the crucial sensitivity can be identified empirically.

First, the estimation of beta factors could be improved. It is already common practice to adjust the estimate of the beta factor towards one according to the method of Vasicek (1973). The rationale for this adjustment is to correct for an estimation error, since the unconditional estimate, i.e., without having any piece of information about the firm, should equal the average of all beta factors, which is one by definition. Even though the rationale is different to the mechanism developed in chapter 5, the direction of the adjustment is the same. However, the adjustment according to the Vasicek method is crude and the same for all assets. For example, Bloomberg reports an adjusted beta which is calculated as two thirds times the beta estimated from a time-series regression plus one third times one. Following the idea of beta contraction from chapter 5, one could determine the actual sensitivity of each asset's beta factor with respect to changes in correlations from the data. The group of assets with a high sensitivity should receive a larger adjustment than the group of assets with low beta sensitivity.

Second, the variability of the beta factors also plays an important role in the design of trading strategies. As an example, lets consider an active investor, i.e., an investor not holding the market portfolio, who wants to maintain a predefined level of systematic risk. Since beta factors change over time, the portfolio's systematic risk changes as well. Consequently, the investor needs to adjust the portfolio and incurs transaction costs. Alternatively, the investor could determine the sensitivity of beta factors and invest only in assets with a relatively stable — or persistent — beta factor. Doing so, the investor can avoid costly readjustments of the portfolio.

Transaction costs also play an important role for the question why asset pricing anomalies are not exploited and, thus, eliminated by investors (Frazzini et al., 2012). Hence, assets offering a cost advantage should therefore exhibit a surplus demand by hedge funds and the like, which should in turn drive up their prices and lower their expected returns (Lenz, 2014).

Third, the mechanism of beta contraction needs to be taken into account when evaluating the performance of portfolio managers. There is anecdotal evidence that many asset management firms offer products investing in low volatility stocks. Due to the popularity of the low volatility strategy, the financial data provider Standard & Poors created an index mapping the performance of a portfolio of 100 out of the S&P 500 stocks with the lowest volatility. Since the risk of low volatility stocks is not accounted for in the typical linear performance measures, such as Sharpe ratio or Treynor ratio, the reported risk-adjusted returns of these products and indeces are artificially inflated. The mechanism behind the low volatility puzzle benefits this scam. The disadvantage of low volatility stocks becomes only visible when correlations jump up, which empirically coincides with sharp market downturns. Hence, portfolio managers can enjoy an extra return compared to their competitors in good times and can attribute all losses in bad times to the overall market decline.

Part III

Attachments

Bibliography

Acharya, V. V., T. F. Cooley, M. Richardson, and I. Walter (2009): "*Real time solutions for US financial reform*", **Working Paper**, Stern Working Group on Financial Reform, New York University.

Admati, A. R. and M. Hellwig (2013): "*The bankers' new clothes: What's wrong with banking and what to do about it*", 1st ed., Princeton University Press.

Adrian, T. and F. Franzoni (2009): "*Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM*", **Journal of Empirical Finance 16 (4)**, pp. 537–556.

Akerlof, G. A. (1970): "*The market for lemons: Quality uncertainty and the market mechanism*", **The Quarterly Journal of Economics 84 (3)**, pp. 488–500.

Albul, B., D. M. Jaffee, and A. Tchistyi (2013): "*Contingent convertible bonds and capital structure decisions*", **Working Paper**, University of California at Berkeley.

Ang, A. and J. Chen (2002): "*Asymmetric correlations of equity portfolios*", **Journal of Financial Economics 63 (3)**, pp. 443–494.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006): "*The cross section of volatility and expected returns*", **The Journal of Finance 61 (1)**, pp. 259–299.

——— (2009): "*High idiosyncratic volatility and low returns: International and further US evidence*", **Journal of Financial Economics 91 (1)**, pp. 1–23.

Armstrong, C. S., S. Banerjee, and C. Corona (2013): "*Factor-loading uncertainty and expected returns*", **Review of Financial Studies 26 (1)**, pp. 158–207.

Asness, C. S., A. Frazzini, and L. H. Pedersen (2012): "*Leverage aversion and risk parity*", **Financial Analysts Journal 68 (1)**, pp. 47–59.

Avdjiev, S., A. V. Kartasheva, and B. Bogdanova (2013): "*Cocos: a primer*", **BIS Quarterly Review**, pp. 43–56.

Baas, E. (2014): "*Agency conflicts related to contingent convertible bonds*", **Master Thesis**, University of Tübingen.

Baker, M., B. Bradley, and J. Wurgler (2011): "*Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly*", **Financial Analysts Journal 67 (1)**, pp. 40–54.

Bali, T. G. and N. Cakici (2008): "*Idiosyncratic volatility and the cross section of expected returns*", **Journal of Financial and Quantitative Analysis 43 (1)**, pp. 29.

Bansal, R. and A. Yaron (2004): "*Risks for the long run: A potential resolution of asset pricing puzzles*", **The Journal of Finance 59 (4)**, pp. 1481–1509.

Banz, R. W. (1981): "*The relationship between return and market value of common stocks*", **Journal of Financial Economics 9 (1)**, pp. 3–18.

Barberis, N and M. H. Huang (2008): "*Stocks as lotteries: The implications of probability weighting for security prices*", **American Economic Review 98 (5)**, pp. 2066–2100.

Barinov, A. (2011): "*Idiosyncratic volatility, growth options, and the cross-section of returns*", **Working Paper**, University of Rochester.

Barro, R. J. (2006): "*Rare disasters and asset markets in the twentieth century*", **The Quarterly Journal of Economics 121 (3)**, pp. 823–866.

Basu, S. (1983): "*The relationship between earnings' yield, market value and return for NYSE common stocks: Further evidence*", **Journal of Financial Economics 12 (1)**, pp. 129–156.

Baule, R., O. Entrop, and M. Wilkens (2008): "*Credit risk and bank margins in structured financial products: Evidence from the German secondary market for discount certificates*", **Journal of Futures Markets 28 (4)**, pp. 376–397.

Baule, Rainer (2011): "*The order flow of discount certificates and issuer pricing behavior*", **Journal of Banking & Finance 35 (11)**, pp. 3120–3133.

Baxter, N. D. (1967): "*Leverage, risk of ruin and the cost of capital*", **The Journal of Finance 22 (3)**, pp. 395–403.

Berg, T. and C. Kaserer (2014): "*Does contingent capital induce excessive risk-taking?*", **Journal of Financial Intermediation**, forthcoming.

Bhattacharya, S. and A. V. Thakor (1993): "*Contemporary banking theory*", **Journal of Financial Intermediation 3 (1)**, pp. 2–50.

Black, F. (1972): "*Capital market equilibrium with restricted borrowing*", **Journal of Business 45 (3)**, pp. 444–455.

Black, F., M.C. Jensen, and M. Scholes (1972): "*The capital asset pricing model: some empirical tests*", in: Studies in the theory of capital markets, edited by Jensen, M. C., Praeger, pp. 79–121.

Black, F. and M. Scholes (1973): "*The pricing of options and corporate liabilities*", **Journal of Political Economy 81 (3)**, pp. 637–654.

Bliss, R. R. and N. Panigirtzoglou (2004): "*Option-implied risk aversion estimates*", **The Journal of Finance 59 (1)**, pp. 407–446.

Blitz, D., E. G. Falkenstein, and P. Van Vliet (2014): "*Explanations for the volatility effect: An overview based on the CAPM assumptions*", **Journal of Portfolio Management 40 (3)**, pp. 61–76.

Blitz, D. and P. Van Vliet (2007): "*The volatility effect: Lower risk without lower return*", **Journal of Portfolio Management 34 (1)**, pp. 102–113.

Boehme, R. D., B. R. Danielsen, P. Kumar, and S. M. Sorescu (2009): "*Idiosyncratic risk and the cross-section of stock returns: Merton (1987) meets Miller (1977)*", **Journal of Financial Markets 12 (3)**, pp. 438–468.

Brennan, M. J. and E. S. Schwartz (1978): "*Corporate income taxes, valuation, and the problem of optimal capital structure*", **Journal of Business 51 (1)**, pp. 103–114.

Breuer, W. and A. Perst (2007): "*Retail banking and behavioral financial engineering: The case of structured products*", **Journal of Banking & Finance 31 (3)**, pp. 827–844.

Bulow, J. and P. Klemperer (2015): "*Equity recourse notes: Creating counter-cyclical bank capital*", **Economic Journal**, forthcoming.

Calomiris, C. W. and C. M. Kahn (1991): "*The role of demandable debt in structuring optimal banking arrangements*", **The American Economic Review 81 (3)**, pp. 497–513.

Campbell, J. and J. Cochrane (1999): "*By force of habit: A consumption based explanation of aggregate stock market behavior*", **Journal of Political Economy 107 (2)**, pp. 205–251.

Carlin, B. I. (2009): "*Strategic price complexity in retail financial markets*", **Journal of Financial Economics 91 (3)**, pp. 278–287.

Chen, Z. and M. Gallmeyer (2010): "*The volatility of liquidity, idiosyncratic risk, and asset returns*", **Working Paper**, University of Virginia.

Cochrane, J. H. (2005): "*Asset pricing*", 2nd ed., Princeton University Press.

——— (2011): "*Presidential address: Discount rates*", **The Journal of Finance 66 (4)**, pp. 1047–1108.

——— (2014): "*Toward a run-free financial system*", **Working Paper**, University of Chicago.

Cochrane, J. H. and L. P. Hansen (1992): "*Asset pricing explorations for macroeconomics*", in: NBER Macroeconomics Annual 1992, volume 7, MIT Press, pp. 115–182.

Crummenerl, M. (2015): "*The risk of low volatility stocks: A theoretical explanation for an empirical puzzle*", **Working Paper**, University of Tübingen.

Crummenerl, M., K. Heldt, and C. Koziol (2014): "*Contingent capital makes credit crunches less likely – but do banks want to have it?*", **Review of Managerial Science 8 (2)**, pp. 175–196.

Crummenerl, M. and C. Koziol (2014): "*Bank financing with structured products – how to make contingent convertibles work*", **Die Unternehmung: Swiss Journal of Business Research and Practice 68 (2)**, pp. 108–128.

——— (2015): "*From wall street to main street – how banks can offload their asset risk onto retail investors*", **Schmalenbach Business Review (sbr) 67 (3)**, pp. 1–44.

Decamps, J.-P., J.-C. Rochet, and B. Roger (2004): "*The three pillars of Basel II: optimizing the mix*", **Journal of Financial Intermediation 13 (2)**, pp. 132–155.

Diamond, D. W. (1984): "*Financial intermediation and delegated monitoring*", **The Review of Economic Studies 51 (3)**, pp. 393–414.

Driessen, J., P. J. Maenhout, and G. Vilkov (2009): "*The price of correlation risk: Evidence from equity options*", **The Journal of Finance 64 (3)**, pp. 1377–1406.

Duffie, D. (2009): "*Contractual methods for out-of-court restructuring of systemically important financial institutions*", **Working Paper**, Stanford University.

Entrop, O., H. Scholz, and M. Wilkens (2009): "*The price-setting behavior of banks: An analysis of open-end leverage certificates on the German market*", **Journal of Banking & Finance 33 (5)**, pp. 874–882.

Fama, E. F. (1991): "*Efficient capital markets: II*", **The Journal of Finance 46 (5)**, pp. 1575–1617.

Fama, E. F. and K. R. French (1992): "*The cross-section of expected stock returns*", **The Journal of Finance 47 (2)**, pp. 427–465.

——— (1993): "*Common risk factors in the returns on stocks and bonds*", **Journal of Financial Economics 33 (1)**, pp. 3–56.

——— (1996): "*Multifactor explanations of asset pricing anomalies*", **The Journal of Finance 51 (1)**, pp. 55–84.

Fama, E. F. and J. D. MacBeth (1973): "*Risk, return, and equilibrium: Empirical tests*", **The Journal of Political Economy 81 (3)**, pp. 607–636.

Fischer, E., R. Heinkel, and J. Zechner (1989): "*Dynamic capital structure choice: Theory and tests*", **The Journal of Finance 44 (1)**, pp. 19–40.

Flannery, M. J. (1994): "*Debt maturity and the deadweight cost of leverage: Optimally financing banking firms*", **The American Economic Review 84 (1)**, pp. 320–331.

——— (2005): "*No pain, no gain? Effecting market discipline via reverse convertible debentures*", in: Capital adequacy beyond basel, edited by Scott, H. S., Oxford University Press, pp. 171–196.

——— (2010): "*Stabilizing large financial institutions with contingent capital certificates*", **Working Paper**, University of Florida.

——— (2014): "*Contingent capital instruments for large financial institutions: A review of the literature*", **Annual Review of Financial Economics 6 (1)**, pp. 225–240.

Frazzini, A., R. Israel, and T. J. Moskowitz (2012): "*Trading costs of asset pricing anomalies*", **Working Paper**, University of Chicago.

Frazzini, A. and L. Pedersen (2014): "*Betting against beta*", **Journal of Financial Economics 111 (1)**, pp. 1–25.

Freixas, X. and J.-C. Rochet (2008): "*Microeconomics of banking*", 2nd ed., MIT Press.

French, K. R., M. N. Baily, J. Y. Campbell, J. H. Cochrane, D. W. Diamond, D. Duffie, A. K. Kashyap, F. S. Mishkin, R. G. Rajan, D. S. Scharfstein, R. J. Shiller, H. S. Shin, M. J. Slaughter, J. C. Stein, and R. M. Stulz (2010): "*The squam lake report: Fixing the financial system*, Princeton University Press.

Fu, F. (2009): "*Idiosyncratic risk and the cross-section of expected stock returns*", **Journal of Financial Economics 91 (1)**, pp. 24–37.

Gabaix, Xavier (2008): "*Variable rare disasters: A tractable theory of ten puzzles in macro-finance*", **The American Economic Review 98 (2)**, pp. 64–67.

Gaspar, J.-M. and M. Massa (2006): "*Idiosyncratic volatility and product market competition*", **The Journal of Business 79 (6)**, pp. 3125–3152.

Goetzmann, W. N., L. Li, and K. G. Rouwenhorst (2005): "*Long-term global market correlations*", **Journal of Business 78 (1)**, pp. 1–38.

Goldstein, R., N. Ju, and H. Leland (2001): "*An EBIT-based model of dynamic capital structure*", **The Journal of Business 74 (4)**, pp. 483–512.

Gorton, G. and A. Metrick (2012): "*Securitized banking and the run on repo*", **Journal of Financial Economics 104 (3)**, pp. 425–451.

Guo, H. and R. Savickas (2010): "*Relation between time-series and cross-sectional effects of idiosyncratic variance on stock returns*", **Journal of Banking & Finance 34 (7)**, pp. 1637–1649.

Han, B. and A. Kumar (2013): "*Speculative retail trading and asset prices*", **Journal of Financial and Quantitative Analysis 48 (2)**, pp. 377–404.

Harvey, C. R., Y. Liu, and H. Zhu (2015): "*... and the cross-section of expected returns*", **Working Paper**, National Bureau of Economic Research.

Haugen, R. A. and A. J. Heins (1975): "*Risk and the rate of return on financial assets: Some old wine in new bottles*", **Journal of Financial and Quantitative Analysis 10 (5)**, pp. 775–784.

Hellwig, M. (1991): "*Banking, financial intermediation and corporate finance*", in: European financial integration, edited by Giovannini, A. and C. Mayer, Cambridge University Press, pp. 35–63.

Henderson, B. J. and N. D. Pearson (2011): "*The dark side of financial innovation: A case study of the pricing of a retail financial product*", **Journal of Financial Economics 100 (2)**, pp. 227–247.

Hong, Y., J. Tu, and G. Zhou (2007): "*Asymmetries in stock returns: Statistical tests and economic evaluation*", **Review of Financial Studies 20 (5)**, pp. 1547–1581.

Huang, C. and R. H. Litzenberger (1988): "*Foundations for financial economics*", 1st ed., Elsevier Science Publishing.

Huang, W., Q. Liu, S. G. Rhee, and L. Zhang (2011): "*Another look at idiosyncratic volatility and expected returns*", **Journal of Investment Management 9 (4)**, pp. 26–51.

Irvine, P. J. and J. Pontiff (2009): "*Idiosyncratic return volatility, cash flows, and product market competition*", **Review of Financial Studies 22 (3)**, pp. 1149–1177.

Ivashina, V. and D. Scharfstein (2010): "*Bank lending during the financial crisis of 2008*", **Journal of Financial Economics 97 (3)**, pp. 319–338.

Jagannathan, R. and Z. Wang (1996): "*The conditional CAPM and the cross-section of expected returns*", **The Journal of Finance 51 (1)**, pp. 3–53.

Jensen, M. C. and W. H. Meckling (1976): "*Theory of the firm: Managerial behavior, agency costs and ownership structure*", **Journal of Financial Economics 3 (4)**, pp. 305–360.

Jiang, G. J., D. Xu, and T. Yao (2009): "*The information content of idiosyncratic volatility*", **Journal of Financial and Quantitative Analysis 44 (1)**, pp. 1–28.

Kaplan, S. N. (2009): "*Should banker pay be regulated?*", **The Economists' Voice 6 (11)**.

Koziol, C. and J. Lawrenz (2012): "*Contingent convertibles. solving or seeding the next banking crisis?*", **Journal of Banking & Finance 36 (1)**, pp. 90–104.

Kraus, A. and R. H. Litzenberger (1973): "*A state-preference model of optimal financial leverage*", **The Journal of Finance 28 (4)**, pp. 911–922.

Krishnan, C., R. Petkova, and P. Ritchken (2009): "*Correlation risk*", **Journal of Empirical Finance 16 (3)**, pp. 353–367.

Leland, H. E. (1994): "*Corporate debt value, bond covenants, and optimal capital structure*", **The Journal of Finance 49 (4)**, pp. 1213–1252.

Leland, H. E. and D. H. Pyle (1977): "*Informational asymmetries, financial structure, and financial intermediation*", **The Journal of Finance 32 (2)**, pp. 371–387.

Leland, H. E. and K. B. Toft (1996): "*Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads*", **The Journal of Finance 51 (3)**, pp. 987–1019.

Lenz, M. (2014): "*Factor loading persistence and expected returns*", **Working Paper**, University of Mannheim.

Lettau, M. and S. Ludvigson (2001): "*Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying*", **Journal of Political Economy 109 (6)**, pp. 1238–1287.

Levy, H. (1978): "*Equilibrium in an imperfect market: A constraint on the number of securities in the portfolio*", **The American Economic Review 68 (4)**, pp. 643–658.

Lewellen, J. and S. Nagel (2006): "*The conditional CAPM does not explain asset-pricing anomalies*", **Journal of Financial Economics 82 (2)**, pp. 289–314.

Li, X., R. N. Sullivan, and L. García-Feijóo (2014): "*The low-volatility anomaly: Market evidence on systematic risk versus mispricing*", **Financial Analysts Journal**, forthcoming.

Lintner, J. (1965): "*The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets*", **The Review of Economics and Statistics 47 (1)**, pp. 13–37.

Longin, F. and B. Solnik (2001): "*Extreme correlation of international equity markets*", **The Journal of Finance 56 (2)**, pp. 649–676.

Malamud, S. and G. Vilkov (2015): "*Betting against betas and their factor structure*", **Working Paper**, Swiss Finance Institute.

Markowitz, H. (1952): "*Portfolio selection*", **The Journal of Finance 7 (1)**, pp. 77–91.

McDonald, R. L. (2013): "*Contingent capital with a dual price trigger*", **Journal of Financial Stability 9 (2)**, pp. 230–241.

Merton, R. C. (1973): "*An intertemporal capital asset pricing model*", **Econometrica 41 (5)**, pp. 867–887.

——— (1973): "*A rational theory of option pricing*", **Bell Journal of Economics and Management Science 4 (1)**, pp. 141–183.

——— (1974): "*On the pricing of corporate debt: The risk structure of interest rates*", **The Journal of Finance 29 (2)**, pp. 449–470.

——— (1987): "*A simple model of capital market equilibrium with incomplete information*", **The Journal of Finance 42 (3)**, pp. 483–510.

Modigliani, F. and M. H. Miller (1958): "*The cost of capital, corporation finance and the theory of investment*", **American Economic Review 48 (3)**, pp. 261–297.

——— (1963): "*Corporate income taxes and the cost of capital: a correction*", **American Economic Review 53 (3)**, pp. 433–443.

Mossin, J. (1966): "*Equilibrium in a capital asset market*", **Econometrica 34 (4)**, pp. 768–783.

Murphy, G., M. Walsh, and M. Willison (2012): "*Precautionary contingent capital*", **Financial Stability Paper 16**, Bank of England.

Nordal, K. B. and N. Stefano (2014): "*Contingent convertible bonds (cocos) issued by European banks*", **Staff Memo 19**, Norges Bank.

Pazarbasioglu, C., J. Zhou, V. Le Leslé, and M. Moore (2011): "*Contingent capital: economic rationale and design features*", **Staff Discussion Note SDN/11/01**, International Monetary Fund.

Pennacchi, G., T. Vermaelen, and C. Wolff (2014): "*Contingent capital: The case of COERCs*", **Journal of Financial and Quantitative Analysis 49 (3)**, pp. 541–574.

Rietz, Thomas A (1988): "*The equity risk premium a solution*", **Journal of Monetary Economics 22 (1)**, pp. 117–131.

Robichek, A. A. and S. C. Myers (1966): "*Problems in the theory of optimal capital structure*", **Journal of Financial and Quantitative Analysis 1 (2)**, pp. 1–35.

Santos, T. and P. Veronesi (2004): "*Conditional betas*", **Working Paper**, National Bureau of Economic Research.

——— (2006): "*Labor income and predictable stock returns*", **Review of Financial Studies 19 (1)**, pp. 1–44.

Scott Jr., H., J (1976): "*A theory of optimal capital structure*", **The Bell Journal of Economics 7 (1)**, pp. 33–54.

Sharpe, W. F (1964): "*Capital asset prices: A theory of market equilibrium under conditions of risk*", **The Journal of Finance 19 (3)**, pp. 425–442.

Spence, M. (1973): "*Job market signaling*", **The Quarterly Journal of Economics 87 (3)**, pp. 355–374.

Stein, J. C. (2004): "*Commentary on 'Rebalancing the three pillars of Basel II' by J. C. Rochet*", **Economic Policy Review 10 (2)**, Federal Reserve Bank of New York, pp. 27–30.

Stigler, G. J. (1961): "*The economics of information*", **The Journal of Political Economy 69 (3)**, pp. 213–225.

Stoimenov, P. A. and S. Wilkens (2005): "*Are structured products 'fairly' priced? An analysis of the German market for equity linked instruments*", **Journal of Banking and Finance 29 (12)**, pp. 2971–2993.

Sundaresan, S. and Z. Wang (2015): "*On the design of contingent capital with a market trigger*", **The Journal of Finance 70 (2)**, pp. 881–920.

Tversky, A. and D. Kahneman (1992): "*Advances in prospect theory: Cumulative representation of uncertainty*", **Journal of Risk and Uncertainty 5 (4)**, pp. 297–323.

Vasicek, O. A. (1973): "*A note on using cross-sectional information in bayesian estimation of security betas*", **The Journal of Finance 28 (5)**, pp. 1233–1239.

Warner, J. B. (1977): "*Bankruptcy costs: Some evidence*", **The Journal of Finance 32 (2)**, pp. 337–347.

Welch, I. (2000): "*Views of financial economists on the equity premium and on professional controversies*", **The Journal of Business 73 (4)**, pp. 501–537.

Wilkens, S., C. Erner, and K. Röder (2003): "*The pricing of structured products in Germany*", **The Journal of Derivatives 11 (1)**, pp. 55–69.

Williamson, O. E. (1981): "*The economics of organization: The transaction cost approach*", **American Journal of Sociology 87 (3)**, pp. 548–577.

Zhang, X. (2006): "*Information uncertainty and stock returns*", **The Journal of Finance 61 (1)**, pp. 105–137.