Fostering Mathematical Competences by Preparing for a Mathematical Competition

Dissertation
zur Erlangung des Doktorgrades
der Wirtschafts- und Sozialwissenschaftlichen Fakultät
der Eberhard Karls Universität Tübingen

vorgelegt von
Franziska Rebholz
aus Balingen

Tübingen
2017

Tag der mündlichen Prüfung: 10.01.2018

Dekan: Professor Dr. rer. soc. Josef Schmid

1. Gutachter: Prof. Dr. phil. Jessika Golle

2. Gutachter: Prof. Dr. phil. Kathleen Stürmer

Danksagung

Zuerst möchte ich mich bei meinen Betreuern Prof. Dr. Jessika Golle, Prof. Dr. Ulrich Trautwein und Prof. Dr. Korbinian Moeller bedanken. Sie haben mir die Möglichkeit gegeben, im Rahmen meiner Dissertation verschiedene Blickwinkel auf ein Thema zu bekommen, das mir sehr am Herzen liegt. Für die dabei immer förderliche Unterstützung, das schnelle und wertvolle Feedback in allen Phasen meiner Arbeit und ihre Geduld möchte ich mich besonders bedanken!

Außerdem möchte ich meiner zweiten Gutachterin Prof. Dr. Kathleen Stürmer herzlich für ihre Bereitschaft danken, die vorliegende Arbeit zu begutachten. Für das hilfreiche Feedback zum Konzept von „Fit für die Mathematik-Olympiade", das die Basis meiner Dissertation bildet, möchte ich mich außerdem bei Frau Prof. Dr. Marianne Nolte und Frau Prof. Dr. Wessolowski bedanken!

Während meiner gesamten Zeit als Doktorandin habe ich sehr von der interdisziplinären und kooperativen Arbeitsatmosphäre des Hector-Instituts für Empirische Bildungsforschung und auch der Graduiertenschule & Forschungsnetzwerk LEAD profitiert. Ein großes Dankeschön geht dabei an Dr. Evelin Herbein, Dr. Julia Schiefer, Dr. Maike Tibus und Dr. Stefanie Rösch. Ohne sie wäre weder der interdisziplinäre Blickwinkel auf die Förderung mathematischer Kompetenzen noch die Planung und Durchführung der Studien möglich gewesen. Für die Unterstützung auf den letzten Metern meiner Dissertation möchte ich außerdem Eike Wille danken.

Da die konkrete Durchführung der Studien nur in der Praxis erfolgen konnte, möchte ich mich außerdem ganz herzlich bei allen beteiligten Kursleiterinnen und Kursleiter, Geschäftsführerinnen und Geschäftsführern sowie allen Eltern und Kinder bedanken. Besonders möchte ich mich bei Ute Petry (geb. Planz) für ihre Unterstützung bei der Umsetzung des Kurskonzepts in die Praxis zu Beginn meiner Zeit als Doktorandin bedanken!

Ein dickes Dankeschön geht auch an meine Familie und an meine Freunde - danke, für Eure Geduld und Euer Verständnis! Liebe Pe und lieber Kalle, danke für Eure Unterstützung! Lieber Flori, danke für die richten Worte zur richtigen Zeit! Danke, dass ihr immer für mich da seid!

Abstract

Mathematical competences are important for mastering the problems that are encountered in a modern society that values knowledge. Such competences are relevant not only for mastering the mathematical problems encountered in school but also for managing everyday life. In practice, mathematical competences are required for finding solutions to society's major problems (e.g., the prediction of global warming). Mathematical competences are thereby assumed to be individual cognitive abilities and skills as well as the outcomes of learning processes. An individual is ascribed with sophisticated mathematical competences if he or she is able to come up with new mathematical problems by applying previously existing mathematical competences meaningfully.

Therewith, fostering mathematical competences is of major importance. Based on a cognitive-socio-constructive understanding of learning in mathematics, students need learning possibilities that lock in their individual potential. Several mechanisms and factors have been shown to drive the acquisition of mathematical competences. To foster mathematical competences, challenging learning opportunities are necessary. Especially for students who are already able to solve curriculum-based tasks. One extracurricular enrichment approach that has been suggested to challenge students are (domain-specific, mathematical) academic competitions. But, to ensure that these students will be able to master the challenging problems they will face in the competition, they must prepare appropriately to solve such problems. Therefore, and to protect them from negative experiences such as failure, corresponding training programs have been suggested and implemented in practice. Such training programs prepare students to participate in a specific academic competition.

Paper 1 reviews the appropriateness of academic competitions by summarizing the roles ascribed to academic competitions with regard to the promotion of gifted students. Using the example of the Mathematical Olympiad for elementary school students, a training program that considers the strengths and weaknesses of mathematically gifted elementary school students is introduced. The training was aimed at enhancing the performance in the Mathematical Olympiad as well as (process-based) mathematical competences.

The effectiveness of this particular training was examined in two empirical studies: In Paper 2, a quasi-experimental pre- and posttest design was used to investigate the effects of the training. Dependent variables were success in the Mathematical Olympiad, mathematical competences, and the motivation to do mathematics (i.e., math self-concept and value beliefs for mathematics). A total of 201 third- and fourth-grade students participated in this study. Positive

effects were found for third and fourth graders' performance in the Mathematical Olympiad, their mathematical competences, and the task-specific interest in mathematics of fourth-grade students.

In Paper 3, the effects of a training that was aimed at fostering process-based mathematical competences on cognitive factors were investigated in detail. Dependent variables were success in the Mathematical Olympiad, content- and process-based mathematical competences, as well as domain-general cognitive abilities. Results of a randomized controlled field trial with 97 students indicated significant effects of the training on process-based competences but also transfer effects on domain-general abilities.

In summary, this dissertation provides evidence for the positive influences of a training for an academic competition in mathematics on students' performance in the competition and, additionally, their mathematical competences. Based on the results of the studies, questions for further educational research with regard to trainings and academic competitions can be deduced. The findings suggest that the effectiveness of separate core components should be investigated more detailed. Further, some implications for educational practice are summarized.

Zusammenfassung

Zur Lösung von Problemen in der modernen digitalen Wissensgesellschaft sind elaborierte mathematische Kompetenzen erforderlich. Nicht nur für mathematische Probleme in der Schule oder zur Bewerkstelligung des Alltags sind mathematische Kompetenzen notwendig, sondern auch in ihrer praktischen Anwendung zur Lösung bedeutsamer gesellschaftlicher Probleme wie beispielsweise zur Vorhersage von Klimaveränderungen. Dabei stellen mathematische Kompetenzen sowohl eine individuelle kognitive Fähigkeit als auch das Ergebnis von Lernprozessen dar. Einer Person werden dann elaborierte mathematische Kompetenzen zugeschrieben, wenn sie neue mathematische Probleme durch die sinnvolle Anwendung bereits existierender mathematischer Kompetenzen lösen kann.

Damit kommt der Förderung mathematischer Kompetenzen eine Schlüsselrolle zu. Basierend auf einem kognitiv-sozio-konstruktiven Verständnis mathematischen Lernens benötigen Schülerinnen und Schüler zur Entwicklung mathematischer Kompetenzen Lerngelegenheiten, die an ihr individuelles Potential anknüpfen. Dieses Potential setzt sich zusammen aus kognitiven und nichtkognitiven Faktoren, welche sich in verschiedenen Forschungstraditionen wie empirischer Bildungsforschung, numerischer Kognitionsforschung und pädagogischer Psychologie als einflussreich für den Erwerb mathematischer Kompetenzen gezeigt haben. Bei der Förderung mathematischer Kompetenzen ist es deshalb das Ziel, herausfordernde, dem Potential der Schülerinnen und Schüler angemessene Lerngelegenheiten zu schaffen. Dies gilt beispielsweise auch und vor allem für Lernende, die curriculare Aufgaben bereits spielend lösen können. Ein möglicher Ansatz zur Förderung mathematischer Kompetenzen dieser mathematisch besonders begabten und hochbegabten Schülerinnen und Schüler stellt extracurriculares Enrichment dar. Eine Form des Enrichments bieten (domänen-spezifischen) Schülerwettbewerb. Um Schülerinnen und Schüler auf das Lösen der herausfordernden Aufgaben eines solchen Schülerwettbewerbs vorzubereiten und gleichzeitig ihre mathematischen Kompetenzen zu vertiefen, wird der begleitende Einsatz von Trainingsprogrammen für spezifische Wettbewerbe (z. B. akademische Olympiaden) vorgeschlagen. Gleichzeitig zielen diese Programme darauf, negative Erfahrungen wie Versagen abzupuffern.

In Paper 1 wird die Angemessenheit von Schülerwettbewerben in der Begabtenförderung hinterfragt indem Rollen wie beispielsweise Differenzierung, die Schülerwettbewerben in der Begabtenförderung zugeschrieben werden, zusammengefasst werden. Anschließend wird am Beispiel der Mathematik-Olympiade für die Grundschule ein Trainingsangebot vorgestellt, das die Stärken und Schwächen mathematisch begabter Grundschulkinder berücksichtigt. Das

Training zielte sowohl auf eine erfolgreiche Teilnahme an der Mathematik-Olympiade als auch auf die Förderung mathematischer (insbesondere prozessorientierter) Kompetenzen ab.

Die Effektivität des Trainings wurde in zwei empirischen Studien untersucht: In Paper 2 wurde ein quasi-experimentelles Prä-Posttest-Design genutzt, um die Effekte es Trainings zu untersuchen. Im Sinne einer ganzheitlichen Förderung mathematischer Kompetenzen wurden neben dem Erfolg in der Mathematik-Olympiade und den mathematischen Kompetenzen auch die motivationalen Variablen Selbstkonzept und Wertüberzeugungen für Mathematik als abhängige Variablen erfasst. Insgesamt nahmen 201 Dritt- und Viertklässler an dieser Studie teil. Im Vergleich zu den Kindern der Kontrollgruppe zeigten sich für die Dritt- und Viertklässler die das Training besucht hatten, positive Effekte für die Leistung in der Mathematik-Olympiade, ihren mathematischen Kompetenzen sowie positive Effekte für das aufgabenspezifische Interesse der Viertklässler.

In Paper 3 wurden die Effekte des Trainings hinsichtlich kognitiver Faktoren im Detail untersucht. Abhängige Variablen waren der Erfolg in der Mathematik-Olympiade, inhalts- und prozessbezogene mathematische Kompetenzen sowie domänen-übergreifende kognitive Fähigkeiten. Die Ergebnisse einer randomisierten Warte-Kontrollgruppen-Studie mit 97 Schülerinnen und Schüler deuten auf positive Effekte des Trainings hinsichtlich der prozessbezogenen Kompetenzen aber auch auf Transfereffekte für domänen-übergreifende kognitive Fähigkeiten hin.

Zusammenfassend zeigen sich im Rahmen dieser Dissertation damit Hinweise dafür, dass Trainingsangebote die Leistung in einem Schülerwettbewerb verbessern und die Schülerinnen und Schüler darüber hinaus ihr Lernpotential vergrößern können. Ausgehend von den Ergebnissen der Studien, werden Fragestellungen für weitere Forschung im Zusammenhang mit wettbewerbsbegleitenden Trainingangeboten abgeleitet. So sollte beispielsweise die Effektivität einzelner Kernkomponente künftig genauer untersucht werden. Abschließend werden Implikationen für die Praxis zusammengefasst.

Contents

1 **Introduction and Theoretical Framework** 3

1.1 The Concept of Mathematical Competences 8

1.2 Acquisition of Mathematical Competences 16

1.3 Cognitive Processes and Mechanisms 21

1.4 Noncognitive Factors Influencing Mathematical Competences 27

1.5 Mathematically Gifted Students – Characteristics and Needs 33

1.6 Research Questions of the Present Dissertation 39

2 PAPER 1 **Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade** 43

3 PAPER 2 **Getting Fit for the Mathematical Olympiad: Positive Effects on Achievement and Motivation?** 65

4 PAPER 3 **Training Process-Based Mathematical Competences – Exploring Effects on Domain-Specific Factors and Domain-General Cognitive Abilities** 101

5 **General Discussion** 136

5.1 Discussion of General Findings 137

5.2 Limitations and Strengths 143

5.3 Implications for Educational Practice and Further Research 148

6 **References** 152

1

Introduction and Theoretical Framework

1 Introduction and Theoretical Framework

In modern western society, mathematical competences are seen as key competences that are relevant not only in school but also for vocational success and for managing everyday life (e.g., Bruder, Hefendehl-Hebeker, Schmidt-Thieme, & Weigand, 2015; Butterworth, Varma, & Laurillard, 2011; Grønmo, Lindquist, Arora, & Mullis, 2015; Murnane, Willett, & Levy, 1995; National Council of Teachers of Mathematics, NCTM, 2000; Organisation for Economic Co-operation and Development, OECD, 2014; Ritchie & Bates, 2013; Schrader & Helmke, 2008). For example, estimating the expected costs during one's next shopping trip requires mathematical competences, and so does a rough guess about the amount of gas necessary to drive to grandmother's house or the ability to detect logical errors in a partner's explanation for being late (see e.g., Loos & Ziegler, 2015). The application of mathematical competences are even required for solving major social problems such as the prediction of global warming or the algorithms implemented in navigation devices as well.

Thereby, mathematical competences involve more than the simple mastering of even complex calculations. In line with some authors who have suggested that mathematics is the science of patterns and structures (e.g., Devlin, 1996, 2003, 2004; Wittmann, 2005, July), mathematical situations include all situations involving abstract theoretical quantities and qualities as well as their relationships (e.g., Grebe, 2013). Thereby, mathematical competences are seen as the outcomes of learning processes in the field of mathematics that show up in the successful mastering of mathematical problems (e.g., Chomsky, 1968; Klieme, 2004; Leuders, 2011; Niss & Højgaard, 2011).

In focusing on how the individual student learns mathematics (e.g., Beck, Guldimann, & Zutavern, 1991), an understanding of the cognitive mechanisms that enable the student to successfully solve mathematical problems is crucial (e.g., Georges, Hoffmann, & Schiltz, 2017). Hence, mathematical competences are also needed to capture the characteristics of a multidimensional construct that involves all of the individual requirements that are necessary to deal with mathematical problems (e.g., Leuders, 2014; Weinert, 2001a, 2001b, 2001c). Regarding these requirements, many domain-general and domain-specific cognitive (e.g., intelligence and prior mathematical competences) as well as noncognitive factors (e.g., social background and motivation) have been shown to be associated with mathematical competences (see e.g., Alcock et al., 2016; Cerda et al., 2015; Fleischer, Koeppen, Kenk, Klieme, & Leutner, 2013; Fuchs et al., 2010; Klieme, 2004, Klieme, Eichler et al., 2008; Köller, 2010; LeFevre, 2016; Passolunghi & Lanfranchi, 2012; Schenke, Rutherford, Lam, & Bailey, 2016; Schneider, Küspert, & Krajewski, 2016; Schrader & Helmke, 2008; Sella, Sader, Lolliot, & Cohen Kadosh, 2016; Sullivan, Frank, & Barner, 2016; Träff, 2013; Wang, Halberda, & Feigenson, 2017).

In particular, cognitive abilities that are correlated with, explain, or predict mathematical competences have frequently been examined. Thereby, domain-general cognitive abilities (e.g., intelligence or working memory) are assumed to also influence competences in domains other than mathematics (e.g., science, language). For instance, Neisser and colleagues (1996) reported a moderate correlation between intelligence and school grades.[1] Further, Kriegbaum, Jansen, and Spinath (2015) conducted a study in which intelligence was found to explain variance in students' competences but mostly in students' mathematical competence (Kriegbaum et al., 2015). But, domain-specific abilities such as the ability to understand number magnitude or counting were found to be important too (Dehaene, 1992; Krajewski & Schneider, 2009a, 2009b; Schneider et al., 2016; von Aster & Shalev, 2007; Winkelmann, Robitzsch, Stanat, & Köller, 2012). Especially the interplay of several domain-general and domain-specific cognitive abilities has been suggested to influence the development of mathematical competences (Alcock et al., 2016; Fuchs et al., 2010; LeFevre, 2016; Sullivan et al., 2016; Träff, 2013). Hence, for instance, in a study by Kunter and Voss (2013) using COACTIV data, amongst other prior competences, cognitive ability and reading literacy predicted mathematical competences on an individual level 1 year later.

Looking at the acquisition of mathematical competences, motivation for mathematics (i.e., a domain-specific noncognitive factor) must not be neglected (Cerda et al., 2015; Kriegbaum et al., 2015; Kriegbaum & Spinath, 2016; Murayama, Pekrun, Lichtenfeld, & Vom Hofe, 2013; Neisser et al., 1996). For instance, Kriegbaum and colleagues (2015) found that motivational constructs (i.e., math self-concept, self-efficacy, interest, and goal orientations) predicted mathematical competences 1 year later even when they controlled for pretest differences based on PISA-I-PLUS data (teenagers). In a longitudinal study, Murayama and colleagues (2013) reported that (intrinsic) motivation predicted growth in mathematical competences across a period of 5 years.

Drawing a more holistic picture of mathematical competences, the interplay of cognitive and noncognitive factors has been found to influence mathematical competences as well. For

[1] Within the framework of this dissertation, school grades were used as the in-school measure of competences. However, it should be noted that grades are supposed to be influenced by teachers' expectations or the achievement level of the respective grade. Therefore, they have weaknesses in terms of objectivity and reliability. This limitation should be kept in mind by readers.

instance, Kriegbaum and Spinath (2016) reported results of a study in which the relation between parents' SES and students' mathematical competences was mediated by intelligence and motivation. Controlling for motivation, a study by Murayama and colleagues (2013) even reported that intelligence did not predict growth in mathematical competences. Going further, domain-general abilities tend to be a necessary but not sufficient factor for influencing mathematical competences (Neisser et al., 1996, for a deeper discussion regarding intelligence, see e.g., Arvey et al., 1994).

Based on a cognitive-socio-constructive understanding of learning (e.g., Beck et al., 1991; Kunter & Trautwein, 2013), providing learning opportunities that lock in individual requirements and center on students' needs (e.g., Oelkers & Reusser, 2008) is an inherent part of fostering mathematical competences in many educational systems around the world (see e.g., Bruder et al., 2015; Edwards, Nichols, & Sharpe, 1972; Kilpatrick, Swafford, & Findell, 2001; Kultusministerkonferenz, KMK, 2004a, 2004b; NCTM, 2000; Niss & Højgaard, 2011). But, as indicated by many large-scale assessments and frequently reported by teachers, there are individual differences in the requirements (i.e., the learning potential based on domain-general and domain-specific cognitive and noncognitive factors) and the manifestation of mathematical competences (e.g., Bos, Wendt, Köller, & Selter, 2012; OECD, 2004, 2007, 2016; Stanat, Pant, Böhme, & Richter, 2012; Wendt, Bos et al., 2016). Indeed, results from the PISA and TIMS studies have indicated that fewer students belong to the group of top performers. In TIMSS 2007, 2011, and 2015, there were between 5% and 6% of German students who reached the competency level that is supposed to reflect very sophisticated curricularly demanded mathematical competences (Wendt, Bos et al., 2016, results of other large-scale studies in which German students participated indicated comparable results, see Bos et al., 2012; OECD, 2004, 2007, 2016; Stanat et al., 2012). Thus, it is not surprising that there are elementary school students who are already able to solve curriculum-based mathematical problems and tasks (e.g., Koshy, Ernest, & Casey, 2009).

To give the top-performing students the opportunity to deploy their mathematical potential and to increase their mathematical competences, these students also need appropriate learning environments that challenge them (e.g., Diezmann & Watters, 2001; Koshy et al., 2009; Subotnik, Olszewski-Kubilius, & Worrell, 2011). Especially for the students with really high potential (i.e., gifted and talented students), several in- and out-of-school approaches that are aimed at accelerating or enriching their learning environments have been shown to markedly enhance their potential (Kulik & Kulik, 1987; Lubinski & Benbow, 2006; Steenbergen-Hu & Moon, 2010).

One opportunity that offers complex situations for applying and acquiring mathematical competences outside of school curricula is domain-specific academic competitions (Abernathy & Vineyard, 2001; Bicknell, 2008; Callahan, Hunsaker, Adams, Moore, & Bland, 1995; Fauser, Messner, Beutel, & Tetzlaff, 2007; Oswald, Hanisch, & Hager, 2005; Riley & Karnes, 1998). However, this special kind of enrichment program comes with a competitive environment (Bicknell, 2008; Wirt, 2011) that is supposed to negatively affect students' motivation (e.g., decreasing self-concept; Marsh, & Parker, 1984) or trigger stress or self-doubt (Clinkenbeard, 1989). Therefore, and to prepare students to be able to master the demands of the challenging tasks that are presented in such competitions, some authors have suggested that academic competitions be combined with trainings to prepare students to participate in such competitions (Cropper, 1998; Fauser et al., 2007; Kießwetter, 2013; Oswald et al., 2005; Ozturk & Debelak, 2008a, 2008b).

But, are academic competitions even an appropriate tool for fostering mathematical competences? Can trainings capture the gist of the matter? Are they successful in boosting positive expectations and counterbalancing the negative influences of academic competitions? The present dissertation is aimed at answering these questions by first reviewing the role of academic competitions in fostering gifted elementary school students by using the example of the Mathematical Olympiad. Second, a mathematical training that was developed under the assumption that it prepares students for the requirements of the Mathematical Olympiad is introduced and evaluated in two empirical studies. Thereby, three research questions are addressed. First, the appropriateness of academic competitions in fostering mathematical competences is examined, and the necessity of pedagogical accomplishment is explained. Second, effects of the training on achievement and motivational aspects are examined by taking a close look at social comparison processes. Third, the effects on cognitive factors caused by a training that was aimed at fostering process-based mathematical competences are explored in detail.

The present dissertation has the following structure: In the introduction chapter, the three research questions are embedded in a broader framework. Before proceeding, some vocabulary words are clarified by characterizing the concept of mathematical competences from an educational and social science perspective (1.1). In Chapter 1.2, considerations regarding the acquisition of mathematical competences (i.e., learning mathematics) are summarized. Subsequently, to explain cognitive mechanisms, domain-general and domain-specific cognitive factors that are supposed to enable the acquisition of mathematical and especial numerical competences are delineated (1.3). In Chapter 1.4, the noncognitive factors that are supposed to influence mathematical competences are also summarized. In Chapter 1.5, the characteristics and needs of mathematically gifted students are derived. In a last step, the three research questions are described (1.6).

Subsequently, Papers 1 to 3 are enclosed. Chapter 2 (*Paper 1*) summarizes expectations of academic competitions and the framework of a training that was designed to prepare students to be able to meet the requirements of the Mathematical Olympiad. Afterwards, effects of the training on the development of achievement and motivational variables are examined, and the results of a quasi-experimental empirical study are presented (Chapter 3, *Paper 2*). Chapter 4 (*Paper 3*) presents the results of a randomized controlled field trial exploring differential effects of the training on cognitive factors. Chapter 5 of the present dissertation contains an overall discussion of the three papers, considerations regarding the effects of the training, and implications for practice and further research.

Overall, the contents of the present dissertation are from different research fields. Research in social and educational science, mathematics education, and educational psychology is considered in addition to developmental psychology and motivational research. Thus, the present dissertation claims to offer an interdisciplinary approach to the holistic fostering of mathematical competences. Nevertheless, the dissertation focuses on the students' perspective. In particular, the combination of cognitive and motivational aspects means that mathematical competences can be understood as a learning potential on the individual level. Thereby, the aspect of teachers as the ones who mainly influence students' learning environments is more or less ignored. As the training was part of a German enrichment program—namely, the Hector Children's Academy Program (for more information, see Rothenbusch, Zettler, Voss, Lösch, & Trautwein, 2016)—the literature in both German and English needed to be considered because the participants of the training were part of the German education system, and there is a huge community of German *Fachdidaktik* whose literature is mainly published in German.

1.1 The Concept of Mathematical Competences

In this chapter, some vocabulary with regard to the concept of competences and especially the operationalization of mathematical competences is clarified. Therefore, considerations and suggestions from social and educational science as well as empirical studies are considered.

1.1.1 The concept of competences

In social and education science, the concept of competences is used to describe a multi-dimensional construct that considers cognitive but also motivational, social, emotional, or volitional abilities that enable the reasonable use of solutions in different domain-specific situations in a functional, methodological, and activity-orientated way (Chomsky, 1968; Klieme, 2004; Klieme & Hartig, 2008; Klieme, Hartig et al., 2008, 2008; Leuders, 2014; Simonton, 2003; Weinert, 1999, 2001a, 2001c; Winkelmann et al., 2012). Competences are classified as outcomes that enable a person to reasonably handle the increasing complexity of a modern society that values knowledge (Chomsky, 1968; Klieme, 2004; Klieme, Hartig et al., 2008; KMK, 2004a, 2004b; NCTM, 2000; Niss & Højgaard, 2011; OECD, 1999, 2004, 2014). Thereby, competences are classified as domain-specific dispositions of available or learnable skills and abilities that render a person able to solve problems in certain and perhaps complex situations (Klieme, 2004; Klieme & Hartig, 2008; Weinert, 1999, 2001a, 2001c). In addition, competences are used to describe learning outcomes in terms of a person’s success in facing tasks, problems, and situations by using abilities and knowledge in realistic contexts (Bruder et al., 2015; Chomsky, 1968; Kilpatrick et al., 2001; Klieme, 2004; Klieme & Hartig, 2008; Köller & Parchmann, 2012; NCTM, 2000; Niss & Højgaard, 2011; Weinert, 1999, 2001a, 2001b). Thus, the concept of competences is a homonym: On the one hand, it describes an individual’s learning potential in terms of skills and abilities. On the other hand, it specifies the outcomes of learning processes.

With the competence approach, the former content-driven curricula were changed to outcomes that can be described concretely in terms of what students should learn and what teachers should teach (Bernholt, Neumann, & Nentwig, 2012; Köller & Parchmann, 2012). In the German educational system, for instance, these outcomes are described in terms of content and performance standards, informed by the literature (Köller & Parchmann, 2012). Nevertheless, competences are still an ambiguous construct (for a critical review, see e.g., Schecker, 2012) that combines aspects of learning, achievement, and performance in a respective domain. Within the scope of this dissertation, the term competences is used to describe all forms of academic achievement or performance that identify both an individual disposition and a learning outcome. In line with Klieme and Hartig (2008), interindividual differences in academic performance are perceived as an actual conversion of people's competences (Klieme, Hartig et al., 2008).

1.1.2 The operationalization of competences

In psychological research traditions, the concept of competences comes primarily from a pragmatic-functional perspective that concentrates on the cognitive aspect (Klieme & Hartig, 2008). Given the assumption that cognitive abilities contribute to outcomes, the concept of competences is part of the characteristic of a psychological construct that could be operationalized and therewith assessed by considering tasks that reflect the requirements of real life (see e.g., Klieme & Hartig, 2008; Köller, 2010; Leuders, 2014; Weinert, 2001a, 2001c). Therefore, there is a need for (a) the development or formulation of a theoretical model that is based on the characterization of contents and structures of respective competences informed by the literature, (b) a psychometric model, (c) a statistical model that describes the mathematical relations between latent variables, and (d) diagnostic assessment and an empirical examination (Hartig, 2008; Klieme & Leutner, 2006; Köller & Parchmann, 2012; Leuders, 2014; Niss & Højgaard, 2011). Thus, sufficiently formulated theoretical models—based in general on pedagogical and didactical considerations—enable the empirical measurement of inter- and intrapersonal differences in competences via a look at people's performance in certain contexts (Klieme & Hartig, 2008). Such theoretical models of domain-specific competences are seen from either the perspective of a structure of cognitive processes for acquiring competences—resulting in competence structure models—or from the perspective of concentrating on the complexity of tasks, resulting in competence level models (Fleischer et al., 2013; Leuders, 2014; Webb, Day, & Romberg, 1988; Wilson, 1992). Based on the different ideas for formulating theoretical models of competences, the psychometric models also vary from uni- to multidimensional continuous or categorical variables (Leuders, 2014).

On the one hand, competence level models are based on a priori disjoint categorical levels of competence and enable differentiated information about individual differences for each category. Such competence level models offer the opportunity to qualitatively describe criteria detailing the requirements that an individual is able to manage according to his or her development in the respective category (see Fleischer et al., 2013). For example, Bayrhuber, Leuders, Bruder, and Wirtz (2010) developed and empirically evaluated a four-dimensional competence model that described the competence of problem solving with functions. In this cross-sectional study of $N = 872$ seventh- and eighth-grade students, typical competence profiles of eighth graders were established on the basis of the four-dimensional model of this competence by computing latent class analyses[2] (Bayrhuber et al., 2010).

On the other hand, competence structure models focus on the cognitive processes necessary to cope with the requirements in a certain part of a domain (see Fleischer et al., 2013; Hartig & Klieme, 2006). Thereby, the reference point for defining subdimensions ranges across cognitive processes (see e.g., Hartig & Jude, 2008), variable types of tasks and problems (see e.g., Leutner, Fleischer, Wirth, Greiff, & Funke, 2012), psychological constructs (e.g., the understanding of science for assessing science competences, see Schiefer, 2017), different curricular contents (see, e.g., Winkelmann et al., 2012; Winkelmann & Robitzsch, 2009), and different formats of representation and problems (see e.g., Bayrhuber et al., 2010). Nevertheless, there are approaches—for example, in PISA, TIMSS, or the German National Assessment Studies conducted by the IQB—that can be applied to create clusters of such continuous variables and to define competence proficiency levels depending on people's general mathematical competences (see, e.g., Bos, 2008; IQB, 2008; Köller & Parchmann, 2012; OECD, 2016; Reiss, Roppelt, Haag, Pant, & Köller, 2012; Reiss & Winkelmann, 2009; Wendt, Bos et al., 2016). Combining the categorical aspect of competence level models and the considerations that need to be made about cognitive processes in order to understand the structure of competences, for example, Kunina-Habenicht, Rupp, and Wilhelm (2009) examined a multidimensional competence model to assess individual profiles of arithmetic competence. In this cross-sectional study of $N = 464$ elementary school students, seven latent classes were examined to describe students' arithmetic competence, separating the four basic arithmetic skills and a modeling skill that was embedded in the basic arithmetic skills (Kunina-Habenicht et al., 2009).

[2] For the seventh graders, no competence profiles indicating strengths or weaknesses on the different competences were identified (Bayrhuber et al., 2010).

1.1.3 Operationalization of mathematical competences

One of the most prominent approaches that has been used to operationalize people's performance in mathematics is the concept of *mathematical literacy*, which is assessed in the OECD's Programme for International Student Assessment (PISA):

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen (OECD, 1999, p. 41).

According to the OECD (1999), mathematical literacy is influenced by different aspects such as mathematical competences that describe (a) general concepts for working mathematically (e.g., problem solving) as well as (b) mathematical contents (e.g., mathematical big ideas). According to considerations about the concept of competences (see 1.1.1), mathematical literacy can also be considered mathematical competences (for more information about the concept of mathematical literacy referring to the concept of competences, see, e.g., Weinert, 1999, 2001a).

To assess mathematical competences in IEA's (International Association for the Evaluation of Educational Achievement) *Trends in International Mathematics and Science Studies* (TIMSS), again, the contents and the cognitive dimensions were differentiated (see, e.g., Grønmo et al., 2015; Wendt, Bos et al., 2016). For example, for the fourth graders, number, geometric shapes/measures, and data display (i.e., contents) as well as knowing, applying, and reasoning (i.e., cognitive performance standards) were assessed for nine different types of tasks (see e.g., Bos, 2008; Grønmo et al., 2015; Selter, Walther, Wessel, & Wendt, 2016).

Just as mathematical competences has been operationalized in large-scale assessments such as PISA or TIMSS, many education systems nowadays employ competence models that differentiate between more general and more content-specific competences (e.g., in Canada, Germany, the USA, or Denmark, see Klieme et al., 2003; NCTM, 2000; Niss & Højgaard, 2011). For example, the competence scheme implemented in the German educational standards[3] for mathematics[4] (KMK, 2004b) or the U.S. *Principles and Standards for School Mathematics* (NCTM, 2000) differentiate between content- and process-based competences (see, e.g., Bloom, 1976; Blum, 2012; Köller, 2010; KMK, 2004b, NCTM, 2000). Thereby, process-based competences involve broader, cognitive operations in terms of the more general aspects of mathematics including strategies and methods (Köller, 2010; KMK, 2004b; NCTM, 2000; Winkelmann et al., 2012). Content-based competences embrace specific mathematical contents that are oriented toward a mathematical view of everyday life and embrace arithmetic, geometry, algebra, and stochastics (see Blum, 2012; Freudenthal, 1986; Köller, 2010).

The idea of separating mathematical competences into content- and process-based competences was examined in a few empirical studies. For example, Klieme, Neubrand, and Lüdtke (2001), Blum and colleagues (2004), and Brunner, Krauss, and Martignon (2011) reported very high correlations for both the different content-based competences and the different process-based competences based on analyses from PISA 2000 and 2003. Also, Klieme, Artelt and colleagues (2010) reported very high correlations between content- and process-based competences. Already based on data from TIMSS in the 1990s, Köller (1998) conducted a factor analysis that revealed six content-based competences (at this particular point in time called dimensions) but also indicated one common factor. Nevertheless, there is still no consensus about the subcompetences of content- and process-based competences. For example, in the German educational standards for elementary school students, five process- and five content-based competences have been suggested (KMK; 2004b). Köller (2010) identified five content-based but six process-based competences, and the educational standards of the German state of Baden-Württemberg supposed four content-based but five process-based competences (Ministerium für Kultus, Jugend und Sport Baden-Württemberg, 2016).

For elementary school students, studies by Winkelmann and Robitzsch (2009) as well as Winkelmann and colleagues (2012) tested for an analytical differentiation of the five content-based and six process-based competences supposed by the German National Assessment conducted by the IQB. Based on an overall N = 16,000 third- and fourth-grade students and items that were attributed a priori to two content-based and three process-based competences, results

[3] Nowadays, many education systems are based on educational standards (see, Bernholt et al., 2012).

[4] According to Köller (2010), this competence scheme is based on (a) Bloom (1976), who developed a taxonomy to describe cognitively oriented educational objectives, (b) considerations of the OECD’s operationalization of mathematical literacy, (c) NCTM’s (2000) *Principles and Standards for School Mathematics*, and (d) Freudenthal (1986) and Winter (1995).

indicated a five-factor model for the content-based competences. Besides didactical considerations, these competences could be separated but thus showed high correlations and a large proportion of shared variance. The process-based competences were not separable (Winkelmann et al., 2012). Therewith, it is not surprising that in German National Assessments, mathematical competences have been assessed by items that refer only to the content-based competences (see e.g., Richter et al., 2012).

Results of large-scale studies in education have consistently focused on public interest, as competences that were used a outcomes of learning processes in one educational system are used to measure the quality of learning opportunities and the success of the respective educational system (see e.g., Haag & Roppelt, 2012; Klieme, Hartig et al., 2008). In Germany, national (e.g., German National Assessment in 2011) and international large-scale studies (e.g., IGLU-E 2001 and 2006, TIMSS 2007, 2011, and 2015) have been conducted to assess the mathematical competences of elementary school students. Across all such studies, the mathematical competences of German elementary school students have been located in the middle of the spectrum and have been found to be more or less stable (Bos, 2008; Selter et al., 2016, 2012). In addition, German elementary school students appear to be quite homogenous as nearly 90% of the students have shown moderate mathematical competences (competence levels II, III, and IV, see Bos, 2008; Selter et al., 2012, 2016; Stanat et al., 2012).

1.1.4 Gender differences in mathematical competences

In recent decades, girls have outperformed boys in academic competences. For example, in Germany, nearly 38% of all girls reached the highest educational achievements (Abitur) in contrast to approximately 30% of all boys (Stanat et al., 2012). Independent of domain (e.g., language, science, mathematics), girls showed better grades than boys (Voyer & Voyer, 2014). Nevertheless, in their meta-analysis, Voyer and Voyer (2014) found that the advantages that girls had in grades were smallest in mathematics compared with other domains. When standardized competence tests—for example, in large-scale studies such as PISA—have been used to look at gender differences in mathematics, in general, boys have been found to do better (e.g., Benbow, 1988; Brunner et al., 2011; Grebe, 2013; Leder & Forgasz, 2008; Liu & Wilson, 2009; Liu, Wilson, & Paek, 2008; Voyer & Voyer, 2014).

Nevertheless, studies that have examined gender differences in mathematical competences have revealed quite an inconsistent pattern (see e.g., Böhme & Roppelt, 2012; Hyde, 2005). For elementary school students, Hyde, Fennema, and Lamon (1990) reported no gender differences in their meta-analysis but found a small gender gap beginning in the teenage years.

For young adolescents, gender differences in mathematical competences have consistently been reported (e.g., in PISA studies; for more details, see e.g., Winkelmann, van den Heuvel-Panhuizen, & Robitzsch, 2008). However, German elementary schools students' mathematical competences have been found to be significantly higher for boys than for girls in the TIMS studies (albeit just a bit; Böhme & Roppelt, 2012; Brehl, Wendt, & Bos, 2012; Wendt, Steinmayr, & Kasper, 2016) and the German National Assessment (Stanat et al., 2012). Also, Winkelmann and colleagues (2008) and Winkelmann and van den Heuvel-Panhuizen (2009) reported better global and content-based competences for boys than for girls. In their studies, the authors analyzed data from the Evaluation of the Standards in Mathematics in Primary School (ESMaP)—a study conducted by the Institute for Educational Progress (IQB) at Humboldt University, Berlin, Germany in connection with the PIRLS 2006 study—and additionally reported overall gender differences in favor of boys (Winkelmann et al., 2008; Winkelmann & Robitzsch, 2009).

Even when differences in intelligence have been controlled for, the gap between boys' and girls' mathematical competences has increased (Brunner et al., 2011; Brunner, Krauss, & Kunter, 2008). In their studies using PISA 2000 data, in order to analyze gender differences, the authors used nested-factor models in which they assumed that intelligence and mathematical competences independently explained differences. They also used standard models with which they attempted to explain differences only with the measure of mathematical competences. Their results revealed small gender differences when the standard models were used and large gender differences when the nested-factor models were used (Brunner et al., 2008; Brunner et al., 2011).

Looking at girls' and boys' distributions of high and low achievers in the TIMSS or the German National Assessment, girls were overrepresented at the lowest competence level, and more boys than girls belonged to the top performers at the highest competence level (e.g., Brehl et al., 2012, 2012; Schneider et al., 2016; Stanat et al., 2012; Wendt, Steinmayr et al., 2016). Overall, boys have tended to show greater variability in mathematical competences (ranging from the very lowest to the top levels) than girls (see e.g., Hyde, Lindberg, Linn, Ellis, & Williams, 2008).

But, in line with the decreasing gender gap in mathematical competences (see (Brehl et al., 2012; Hanna, 2000; Wendt, Steinmayr et al., 2016), the results of TIMSS 2015 indicated no such difference for girls and boys in their competence level distributions for the first time in Germany (Wendt, Steinmayr et al., 2016). One might even speculate that the decreasing gender gap reported in recent decades is perhaps confounded by the claim made in mathematics education that gender differences have now been balanced in mathematical competences (see, e.g., Brunner et al., 2011). Perhaps the gender differences in mathematical competences can be explained by differences in girls' and boys' choices of educational courses (see, e.g., Hyde et al., 2008; Hyde, 2016).

In addition, the gender differences in mathematical competences vary across different mathematical competences (Liu et al., 2008; Liu & Wilson, 2009). For example, many studies have revealed that boys show better competences in problem solving and in the competences necessary to deal with geometrical tasks, whereas girls are better at arithmetic (e.g., Benbow, 1988; Brehl et al., 2012; Geary, Saults, Liu, & Hoard, 2000; Hyde et al., 1990; Hyde, 2005; Köller & Klieme, 2000; Liu & Wilson, 2009; Schneider et al., 2016; Walther, Schwippert, Lankes, & Stubbe, 2008). For example, in data from the PIRLS/IGLU study, boys showed higher mathematical competences in solving new problems, but girls were better at applying routine strategies (Walther et al., 2008, the same pattern was observed by, e.g., Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Even for high-achieving students, Kell, Lubinski, and Benbow (2013) reported differences in mathematical reasoning competences for boys and girls. It is interesting that these differences predicted educational (inorganic vs. organic disciplines) and occupational outcomes (career-focused vs. a more balanced life; see Kell et al., 2013).

Overall, boys have tended to show slightly higher mathematical competences than girls (e.g., Hyde et al., 1990; Hyde, 2005; Hyde et al., 2008; Hyde, 2016), regardless of whether these were caused by differences in boys' and girls' cognitive abilities (e.g., in spatial cognition or intelligence; see Geary et al., 2000) or whether they were determined by culture (see e.g., Grebe, 2013).

1.2 Acquisition of Mathematical Competences

Competences are skills that are supposed to develop over many years (Klieme, 2004; Klieme, Hartig et al., 2008). Education that is geared toward the acquisition of competences tends to focus on outcomes and to classify learning as an accumulating process that concentrates on the ability to cope with the requirements of different tasks and situations (see e.g., KMK, 2004a). Therewith, students can be said to have developed domain-specific competences if they can (a) apply their skills and choose appropriate solutions for dealing with specific situations, (b) access the necessary knowledge, (c) understand central relationships, and (d) access the skills, abilities, and previous experiences necessary for action (see KMK, 2004a). Congruent with the activity-oriented approach for measuring competences (see 1.1.1), their acquisition is also thought to be an active process, mediated through learning (Henningsen & Stein, 1997; Weinert, 2001a).

1.2.1 Learning mathematics and acquiring mathematical competences

Learning mathematics (i.e., building new mathematical competences) is supposed to be an active, self-regulated, constructive, hierarchical, and social process (see e.g., Bransford, Brown, & Cocking, 2000; Collins, Brown, & Newman, 1989; Franke, Kazemi, & Battey, 2007; Hasemann, Gasteiger, & Padberg, 2014; Robins & Mayer, 1993): In line with a cognitive socio-constructivist understanding of learning, individual learning processes are centered to understand the acquisition of mathematical competences (e.g., see e.g., Beck et al., 1991; Kunter & Trautwein, 2013). Thus, prior mathematical competences provide a meaningful framework for acquiring new mathematical competences while solving mathematical problems (see e.g., Hasemann et al., 2014; Robins & Mayer, 1993; Schneider et al., 2016). Seidel and Shavelson (2007) describe learning as

> … a set of constructive processes in which the individual student (alone or socially) builds, activates, elaborates, and organizes knowledge structures. From this conception of learning, it follows that teaching should maximize the opportunity for students to engage in activities that promote higher order learning. (Seidel & Shavelson, 2007, p. 459)

Thus, learning mathematics and therewith acquiring mathematical competences is characterized by understanding mathematical circumstances (Deal & Wismer, 2010). Understanding mathematical circumstances is catalyzed by the ability to recognize and use patterns and structures (Nolte, 2013b). Hence, according to several experts, mathematics has been characterized as the science of patterns (Devlin, 1997; 2003; 2004; Wittmann, 2005). These patterns can be found everywhere in everyday life, whereby many situations can be classified as mathematical situations. Besides the obvious mathematical problems (e.g., basic arithmetic), this implies that mathematical problems do not necessarily have to deal with numbers (e.g., logical and geometric problems are also supposed to be mathematical). Understanding mathematical concepts—and, therewith, acquiring mathematical competences—requires abilities that are broader than being able to calculate (i.e., numerical competences) and that support, for instance, the abilities to form abstract representations or to recognize patterns and structures (e.g., Nolte, 2013b; Primi, Ferrão, & Almeida, 2010). Already before entering school—before getting in touch with prearranged formal learning—young children “explore patterns, shapes, and spatial relations; compare magnitudes; and count objects” (Clements & Sarama, 2007, p. 462), show interest, and show the potential to acquire and apply sophisticated basic mathematical competences. Learning mathematics and acquiring mathematical competences is therewith assumed to be the outcome of applying mathematical competences in problems that require complex cognitive processes such as reasoning (Diezmann & Watters, 2001; Franke et al., 2007; Kunter & Voss, 2011; McAllister & Plourde, 2008).

1.2.2 The interplay of content- and process-based competences

In line with the assumption that knowledge in mathematical concepts facilitates learning procedures and vice versa (e.g., Rittle-Johnson & Siegler; Schneider, Rittle-Johnson, & Star, 2011), content- and process-based competences are supposed to be necessary for a person to be able to cope with specific mathematical situations (Bloom, 1976; Blum, 2012; Köller, 2010; Winkelmann & Robitzsch, 2009). In looking at mathematical problems, every mathematical problem, task, or situation is assumed to be characterized by three different aspects that fit into the following three-dimensional taxonomy (Blum, 2012; see Figure 1).

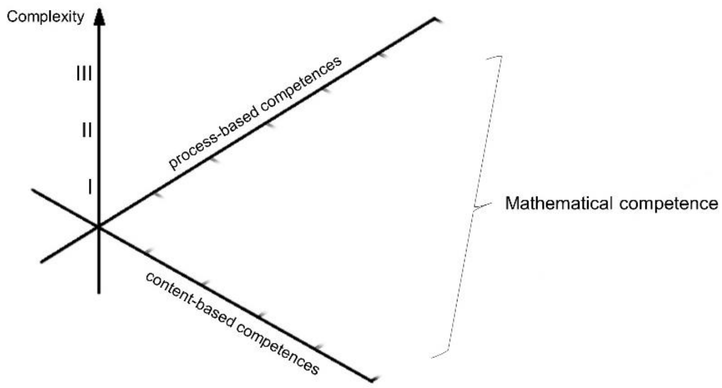


Figure 1. Schematic illustration of a mathematical competence scheme combining the approaches of the German educational standards, TIMSS, and German National Assessment Studies. Both content- and process-based competences and complexity are supposed to span a vector space (based on Bloom, 1976; Blum, 2012; Roppelt & Reiss, 2012, Köller, 2010; KMK; 2004b).

For example, a simple word problem ("Kati (K) has seven drops, Jan (J) has three less. How many drops does Jan have?") is supposed to require process- and content-based competences. In a first step, problem solving and modeling (i.e., process-based) competences are necessary to transfer the word problem into an appropriate calculation ("J = 7 – 3"). In a second step, arithmetical competences (i.e., subtraction) are essential to obtain a solution (J = 4), and again, process-based competences support the formulation of an answer ("Jan has four drops"). Thus, every mathematical problem is supposed to require several different content- and process-based mathematical competences. Complexity is classified according to students' age and the sophistication of the necessary mathematical competences. Considering the complexity, mathematical problems are systematically assigned to a combination of the three dimensions. Nevertheless, clearly classifying mathematical tasks to one or more content- or process-based competences is challenging. Thus, Niss and Højgaard (2011) even went so far as to assume that the different mathematical competences are so closely related that "they form a continuum of overlapping clusters" (p. 9). Nevertheless, different mathematical problems concentrate on different content- and process-based competences (for further information about the classification of one mathematical problem to different content- and process-based competences, see e.g., Winkelmann et al., 2012; Winkelmann & Robitzsch, 2009).

Based on the characteristic of competences as an outcome of learning and as acquired by learning, the assumption that later mathematical competences are based on prior mathematical competences seems obvious (e.g., Watts et al., 2015, see also Rittle-Johnson & Siegler; Schneider et al., 2011). This assumption has been corroborated by several studies that have indicated a relation between students' early and later mathematical competences (e.g., Bailey, Siegler, &

Geary, 2014; Cerda et al., 2015; Duncan et al., 2007; Watts et al., 2015). For example, in their longitudinal study, Duncan and colleagues (2007) predicted later mathematical competences while controlling for nearly 80 variables (e.g., general cognitive abilities, family background, or socio-emotional skills). Their results indicated, amongst others, that prior mathematical competences were the strongest predictor of later mathematical competences. Using PISA-I-PLUS data (German sample of PISA 2003 assessed again in 2004), prior mathematical competences explained the largest portion of later mathematical competence (Kriegbaum et al., 2015). It is interesting that Bailey, Watts, Littlefield, and Geary (2014) reported that the predictive strength of prior mathematical competences on later mathematical competences is more or less independent of the time span between the assessments of prior and later mathematical competences. Further, in a study by Bailey and colleagues (2016), preexisting differences in mathematical competences even explained about 70% of the control group's ability to catch up to the intervention group, which participated in a successful (and effective) intervention (fadeout effect). In their study, Bailey and colleagues (2016) examined whether the fadeout effect was caused by a lack of challenges in the learning of the participants in the intervention group with more sophisticated mathematical competences or whether preexisting differences could explain the fadeout. In their study, they matched the control and intervention group participants after the intervention, which revealed differences on the pretest but also in the long run (Bailey et al., 2016). Also in a longitudinal study examining nearly 200 Finish elementary school students, Aunola, Leskinen, Lerkkanen, and Nurmi (2004) reported that early mathematical competences before entering school predicted later mathematical competences in second grade, indicated by a gap in Grade 2 between students with higher and lower early mathematical competences.

Prior mathematical competences as a domain-specific cognitive factor have been shown to predict later mathematical competences (cf. the Matthew effect; for an explanation regarding this effect, see, e.g., Ditton & Krüsken, 2009; Merton, 1968). This led the authors to consider whether the acquisition of mathematical competences is a hierarchical process of which arithmetical competences form the basis (e.g., Schneider et al., 2016). Some studies have incorporated this consideration of a hierarchical process: For example, in a study by Georges and colleagues (2017), general mathematical competences were more strongly related to arithmetical competences in younger students than in adults. On the basis of these results, the authors concluded that different strategies seem to be necessary to solve the same problems for different developmental steps of mathematical problems (Georges et al., 2017). Thus, some authors have even assumed that the mathematical competences that are necessary for coping with less complex problems might turn into an automatic process through which a person can develop more sophisticated mathematical competences (e.g., Grabner et al., 2007; Schneider et al., 2016). Therewith, in particular, the interplay between content- and process-based competences (i.e., the interplay between domain-specific knowledge and applications of appropriate strategies) is supposed to drive the acquisition of new and more sophisticated mathematical competences that enable a person to cope with more complex mathematical demands.

1.3 Cognitive Processes and Mechanisms

Students' outcomes are based on the understanding that learning mathematics is equal to the acquisition of competences, and learning is characterized as successfully meeting domain-, situation-, and demand-specific requirements (KMK, 2004a). Within this focus on competences, an understanding of the cognitive mechanisms that enable people to successfully solve mathematical problems is crucial (e.g., Georges et al., 2017). As explained in Chapter 1.2, mathematical competences appear to depend on prior mathematical competences in a complex circular manner. Thus, it is necessary to ask which factors are associated with and influence them.

1.3.1 The role of domain-general cognitive abilities

Domain-general cognitive factors are assumed to influence educational success not only in one but also in several domains (Schneider et al., 2016). With regard to mathematical competences, much research has been devoted to examining the influences of domain-general cognitive abilities on mathematical competences (e.g., Clark, Pritchard, & Woodward, 2010; Welsh, Nix, Blair, Bierman, & Nelson, 2010). In some studies, the speed of information processing (e.g., Fuchs et al., 2010; Passolunghi & Lanfranchi, 2012; Träff, 2013), executive functions (e.g., Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014; Träff, 2013), early language competences (i.e., phonological abilities, e.g., Bradley & Bryant, 1985; Passolunghi & Lanfranchi, 2012; Szűcs et al., 2014), and spatial abilities (e.g., Szűcs et al., 2014; Wai, Lubinski, & Benbow, 2009) have been revealed to predict competences in several domains such as language, science, and mathematics.

However, the most frequently investigated domain-general cognitive abilities are intelligence and working memory. For instance, in a study of Singaporean students by Lee, Ng, Ng, and Lim (2004), mathematical competences were positively correlated with working memory, intelligence, and reading competences. In particular, the extent to which working memory predicted mathematical competences (i.e., solving word problems) was mediated by reading competences and intelligence (Lee et al., 2004). Intelligence as the ability to acquire and apply knowledge and skills, to learn effectively, to think logically and abstractly, and to solve (new) problems is one of the most frequently examined constructs for determining competences in school; it is believed to be a consequence of competencies as well (Arvey et al., 1994; Gottfredson & Deary, 2004; Hasselhorn & Gold, 2017; Neisser et al., 1996; Roberts & Lipnevich, 2012).[5] For example, in a study by Deary, Strand, Smith, and Fernandes (2007), intelligence and end-of-school competences (i.e., performance on exams) showed moderate to large correlations, but the highest correlation was identified between intelligence and mathematical competence. Using German samples, for instance, in the Munich SCHOLASTIK study, intelligence and grades in mathematics revealed a moderate correlation too (Bullock & Ziegler, 1997). These findings were corroborated in a study by Spinath, Freudenthaler, and Neubauer (2010), who conducted a study that indicated that intelligence was the strongest predictor of competences in all domains but especially of mathematical competences. Further, intelligence, which was assessed at the age of 11, explained 59% of the variance in mathematical competences at the age of 16 (Spinath et al., 2010). These results were again corroborated by a recent study by Kriegbaum and Spinath (2016) who found stable high correlations for mathematical competences and intelligence at two time points (PISA-I-PLUS data). For the cross-sectional PISA 2003 sample, Kriegbaum and colleagues (2015) reported that intelligence explained the largest proportion of mathematical competences. In particular, the knowledge-independent construct of fluid intelligence was found to be an important predictor of mathematical competences (Floyd, Evans, & McGrew, 2003; Geary & Moore, 2016; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Primi et al., 2010; Taub, Keith, Floyd, & McGrew, 2008). In a study by Primi and colleagues (2010), individuals with higher fluid intelligence revealed a faster increase in mathematical competences. The authors tried to explain their results through an influence of intelligence on reasoning abilities, an understanding of mathematical concepts, and problem solving (Primi et al., 2010).

Like intelligence, the relevance of working memory[6] for mathematical competences is quite noncontroversial and has also been examined in several studies (see e.g., Bull & Lee, 2014; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; de Smedt et al., 2009; Navarro et al., 2011; Schneider et al., 2016; van der Ven, Klaiber, & van der Maas, 2016; van der Ven, van der Maas, Straatemeier, & Jansen, 2013). For instance, in a study by Navarro and colleagues (2011), the authors aimed to predict arithmetical competences with working memory, inhibitory processes, and phonological awareness. In particular, working memory was revealed to be an important predictor of students' mathematical competences (Navarro et

[5]In general, there is a large debate on how to define, conceptualize, and measure intelligence and the influence of intelligence on people's lives. For more information about the construct of intelligence and its influence on people's lives, see, for example, Arvey et al. (1994); Baltes, Staudinger, and Lindenberger (1999), Ceci (1991); Neisser et al. (1996), Roberts and Lipnevich (2012).

[6] For more information about the cognitive system that is supposed to temporarily store information and keep it available for executive processes, see Baddeley (1986), Baddeley and Hitch (1974).

al., 2011). More specifically, several studies examined the correlation between mathematical competences and a visuospatial part of working memory. For example, van der Ven and colleagues (2016) examined whether early mathematical competences (i.e., the ability to transcode numbers, basic arithmetic skills) were predicted by working memory. In their cross-sectional studies with about 26,000 students from preschool to sixth grade, they found that working memory was correlated with early mathematical competences (i.e., transcoding numbers and adding; van der Ven et al., 2016).

1.2.3 Domain-specific cognitive abilities - using the example of numerical cognition

Assuming a hierarchical acquisition of mathematical competences, arithmetical competences are commonly classified as the most basic part of mathematical competences (e.g., Georges et al., 2017; Krajewski & Schneider, 2009a, 2009b; Schneider et al., 2016; Thompson, Nuerk, Moeller, & Kadosh, 2013). Consequently, much research has been devoted to examining the development of arithmetical competences and the factors that influence these competences (e.g., Dehaene, 1992, 2011; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Krajewski & Schneider, 2009a, 2009b; LeFevre et al., 2010; LeFevre, 2016; Siegler & Lortie-Forgues, 2014; von Aster & Shalev, 2007). In numerical cognition research in which the cognitive development of numerical abilities has been examined, the assumption is that domain-general cognitive abilities (e.g., intelligence, working memory) as well as number-specific abilities (e.g., understanding number magnitude) influence mathematical competences (e.g., Alcock et al., 2016; Fuchs et al., 2010; Passolunghi & Lanfranchi, 2012; Sella et al., 2016; Sullivan et al., 2016; Thompson et al., 2013; Träff, 2013).

One domain-specific ability that is supposed to influence early arithmetical competences (e.g., understanding the concept of magnitudes or numbers) is the ability to estimate numbers on a number line in space (number line estimation; e.g., Siegler & Opfer, 2003). Several studies have examined the relation between early arithmetical competences and this number representation (e.g., Georges et al., 2017; Link, Nuerk, & Moeller, 2014; Siegler & Opfer, 2003, for further studies, see also Booth & Siegler, 2006, 2008; Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011; Laski & Siegler, 2007; Link, Moeller, Huber, Fischer, & Nuerk, 2013; Siegler & Booth, 2004). For example, in a study by Link and colleagues (2014), the accuracy (i.e., the percentage of absolute error) in estimating numbers on a bounded number line was correlated with adding and subtracting. Thus, the mental representation of numbers is assumed to be "the most basic level of numerical cognition upon which all other (more complex) numerical and mathematical thinking builds" (Thompson et al., 2013, p. 325). This hierarchy was corroborated by a more recent study: Georges and colleagues (2017) examined the relation between the quality with which numbers were mapped in space (i.e., mental number line) and different facets of mathematical competences (i.e., arithmetic and visuo-spatial competences) of elementary school students in Luxembourg. The results indicated that, especially for young students, arithmetic competences were related to number-space mapping, whereas visuo-spatial competence—necessary to solve more complex mathematical problems that did not involve numbers—were not related to the quality with which numbers were mapped in space (Georges et al., 2017).

At first glance, spatial abilities are obviously necessary for dealing with geometrical problems, but spatial abilities have also been suggested to influence students' early arithmetical competence and especially numerical representation, which is in turn supposed to support numerical abilities. For example, in a study by Thompson and colleagues (2013) on university students, mental rotation ability was significantly correlated with the accuracy of mapping numbers on the mental number line. The authors supposed that higher mental rotation led to more sophisticated number representation, the "mental organization and framework within which information about the cognitive concept of numbers is stored" (Thompson et al., 2013, p. 325). Especially for younger students, Gunderson, Ramirez, Beilock, and Levine (2012) reported a study in which spatial skills even predicted elementary school children's early arithmetical competences.

Several models have been developed to explain the development of early arithmetical competences (Cipolotti & Butterworth, 1995; Dehaene, 1992; Krajewski, 2008; Noel & Seron, 1993; von Aster & Shalev, 2007). For instance, Krajewski (2008) suggested a model of number-magnitude understanding that was based on empirical studies and reflected results from large-scale studies. At the first level of the model, infants are able to distinguish quantities, know numbers, and repeat an exact numerical order. At the second level, three-year-old children acquire competences in recognizing relations (many, some, little) and become aware of Arabic numbers. Later (Level 3,about preschool age) children link magnitudes and numbers to be able to do basic arithmetic (for further information, see Krajewski, 2008; Krajewski & Schneider, 2009a, 2009b; Schneider et al., 2016). Similar competences are considered in the model of mastering numbers by von Aster and Shalev (2007), who additionally considered brain locations and an increasing working memory capacity. Cerebral functions were considered in more detail in the Triple-Code-Model by Dehaene (1992). According to this model, three cardinal representations (visual Arabic number form, analog magnitude representation, and auditory verbal word frame) are supposed to interact and drive early numerical competences. In addition, domain-general abilities (e.g., executive control and working memory) have been suggested to be especially involved when people need to solve more complex mathematical problems (for more information, see e.g., Dehaene, 1992; Dehaene & Cohen, 1995; Klein et al., 2016; Schneider et al., 2016).

But it looks as though it is the interplay of domain-general and domain-specific cognitive abilities in particular that drives mathematical competences (see e.g., Navarro et al., 2011). For instance, Passolunghi and Lanfranchi (2012) found a positive effect of domain-general abilities (e.g., working memory and processing speed) on domain-specific numerical abilities (e.g., magnitude comparison, seriation, use of number words). But, they also found positive effects of both domain-general (i.e., working memory, processing speed) and domain-specific numerical abilities (i.e., magnitude comparison, classification, general understanding of numbers) on later mathematical competences.

1.2.4 The interplay of domain-general and domain-specific cognitive abilities

Regarding the interplay of domain-general and domain-specific cognitive abilities, a complex interdependence has been observed. For instance, Sullivan and colleagues (2016) reported a study in which domain-general factors (e.g., general fluid intelligence and working memory) were revealed to be even better at predicting differences in mathematical competences than domain-specific numerical factors were (i.e., Approximate Number System and dot estimation). In line with this finding, Träff (2013) and Fuchs and colleagues (2010) reported that domain-general cognitive abilities were especially good predictors of the mathematical competences that are necessary for more complex tasks (i.e., word problems). In a study by Bailey, Watts and colleagues (2014), individual differences in students' later mathematical competences were more likely to depend on stable domain-general factors (e.g., domain-general cognitive abilities, reading competences, or family background) rather than simply on prior mathematical competences. In their study, Bailey and colleagues (2014) examined whether time-varying state effects or stable trait effects explained individual differences in mathematical competences. Their results indicated that the trait effects mostly accounted for the longitudinal stability of mathematical competences (Bailey, Watts et al., 2014). Considering domain-general cognitive abilities to be (stable) trait effects and domain-general abilities to be (time-varying) state effects, the results of Bailey and colleagues (2014) were corroborated by a recent study by Sullivan and colleagues (2016). In their longitudinal study, they observed that domain-general cognitive abilities such as intelligence and working memory were better predictors of differences in mathematical competences than domain-specific abilities were (Sullivan et al., 2016). Also, in studies conducted by Brunner and colleagues (2008, 2011) in which gender differences in mathematical competences were examined, mathematical competences depended on intelligence and specific math factors.

In line with the model with the largest influence by Dehaene (1992), it is especially the mathematical competences that are necessary to solve more complex mathematical problems that most likely depend to a larger extent on domain-general than on domain-specific cognitive abilities (excluding prior mathematical competences). For example, in a longitudinal study, Fuchs and colleagues (2010) assessed whether domain-general cognitive abilities (e.g., nonverbal problem solving, executive function, working memory) and domain-general abilities (i.e., performance on the Number Set Test; see Geary, Bailey, & Hoard, 2009) could be used to predict mathematical competences in calculations and the solving of mathematical word problems. Their results indicated that domain-specific factors were associated with both competences in solving word problems and in calculations, whereas domain-general cognitive abilities reliably predicted competences only in word problems (Fuchs et al., 2010). This dependence of the necessary domain-specific and domain-general cognitive abilities on the complexity of the mathematical problem was also corroborated by a recent study by Träff (2013). In his longitudinal study, Träff (2013) also observed that domain-specific factors (e.g., dot counting) predicted the mathematical competences necessary to manage complex (i.e., word problems) and basic (i.e., arithmetic fact retrieval) mathematical problems. Further, he observed that domain-general cognitive abilities predicted the mathematical competences necessary to deal with problems that were more complex than arithmetic fact retrieval (Träff, 2013). Overall, both domain-general and domain-specific cognitive abilities were found to contribute to mathematical competences.

1.4 Noncognitive Factors Influencing Mathematical Competences

Mathematical competences are important for coping with the requirements of Western societies. To understand how mathematical competences can be fostered, it is necessary to understand which further factors are correlated with, explain, or predict mathematical competences. Besides the cognitive factors that were summarized in Chapters 1.2 and 1.3, noncognitive factors such as social background, emotions, and motivation have also been shown to be relevant with regard to the acquisition of mathematical competences (see e.g., Murayama et al., 2013; Pinxten, Marsh, Fraine, van den Noortgate, & van Damme, 2014; Schukajlow, Rakoczy, & Pekrun, 2017; Sirin, 2005). In the following, distal domain-general factors and rather domain-specific factors that have been shown to be predictive of later mathematical competences will be summarized.

1.4.1 Distal Domain-General Factors

To acquire mathematical competences, domain-general noncognitive factors are relevant. For example, it is well-known from previous research that social background predicts mathematical competences (see e.g., Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; OECD, 1999, 2004, 2007, 2016; Sirin, 2005). Even though social background was operationalized in different ways in these studies (e.g., socioeconomic status, SES) and either grades or standardized competence tests were used to assess domain-specific academic competences, all studies revealed a positive association between parents' social background and students' competences, and this finding was independent of domain. For example, Sirin's (2005) meta-analysis indicated a moderate relationship between academic competences and social background. Similarly, the international data from PISA 2003 as well as PISA 2006 showed moderate correlations between mathematical competences and parents' SES (OECD, 2007). Considering only the German sample from the PISA 2003 data, Kriegbaum and Spinath (2016) reported a moderate correlation between parents' SES and students' mathematical competences. However, based on PISA and TIMSS data and compared with other countries, the association between parents' SES and competences in the German sample was above the international average (see e.g., Stubbe, Schwippert, & Wendt, 2016).

In recent research, domain-general (noncognitive) aspects of personality were also examined to determine whether they influence and predict academic competences (e.g., Poropat, 2009; Spinath et al., 2010). For instance, in his meta-analysis, Poropat (2009) reported significant correlations between conscientiousness, agreeableness, and openness with academic competences. His results even indicated that correlations between academic competences and conscientiousness were independent from domain-general cognitive abilities (i.e., intelligence). In looking at only mathematical competences, results from a study by Spinath and colleagues (2010) indicated, amongst others, that conscientiousness was significantly correlated with grades in mathematics.

1.4.2 Motivation

Motivation refers to a construct that considers all motives that lead to certain actions that facilitate some behaviors and prohibit others with the attempt to reach a future goal (Deci & Ryan, 1993; Kleinginna & Kleinginna, 1981). These reasons for behaving in a certain way in a specific situation are assumed to be a mental condition for the long-lasting development of competences (Klieme & Hartig, 2008; Middleton & Spanias, 1999). In contrast to social background or personality, which have been observed to be correlated with competences in general, motivation has to be classified as a domain-specific construct (e.g., Wigfield, 1997). Independent of causal mechanisms, (domain-specific) motivation is classified as a significant predictor of (domain-specific) academic competences (e.g., Kriegbaum et al., 2015; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Möller, Pohlmann, Köller, & Marsh, 2009; Vecchione, Alessandri, & Marsicano, 2014). Many studies have investigated the influence of different motivational factors on academic, and especially, on mathematical competences (see e.g., Cerda et al., 2015; Dörner & Güss, 2013; Kriegbaum et al., 2015; Lee et al., 2004; Marsh et al., 2005; Murayama et al., 2013; Musu-Gillette, Wigfield, Harring, & Eccles, 2015; Navarro et al., 2011; Navarro et al., 2012; Steinmayr, Wirthwein, & Schöne, 2014; Suárez-Álvarez, Fernández-Alonso, & Muñiz, 2014).

The expectancy-value theory (EVT) of achievement-related choices offers a broad model that describes the influence of motivation on competences. Thereby, different central constructs of motivation (i.e., expectancy and value beliefs) are considered to predict competences (Eccles et al., 1983): A person's (subjective) expectations of performance ("Can I do this?") and his/her personal value attributed to the specific tasks ("Why do I want to do this?") are directly related to academic competences (Atkinson, 1957; Eccles et al., 1983; Wigfield & Eccles, 1992; Wigfield & Eccles, 2000; Wigfield, Tonks, & Klauda, 2009). In several empirical studies, expectancy (i.e., competence beliefs) and value beliefs (i.e., interest, cost, as well as attainment and utility value) have been found to predict competences (see e.g., Eccles & Wigfield, 1995; Marsh & Martin, 2011; Trautwein et al., 2012; Trautwein, Lüdtke, Marsh, Köller, & Baumert, 2006). Whereas empirical studies have indicated the importance of both factors, in the following, only findings regarding competence beliefs are summarized. But the associations and processes that have been documented between students' value beliefs and competences are similar to those found between students' self-concept and competences (Marsh et al., 2008; Trautwein et al., 2006).

The importance of competence beliefs using the example of self-concept

Competence beliefs have been deemed the most prominent motivational factor for predicting and explaining performance-related behavior (Arens, Yeung, Craven, & Hasselhorn, 2011; Lüdtke, Köller, Artelt, Stanat, & Baumert, 2002; Marsh & Yeung, 1997; Praetorius, Kastens, Hartig, & Lipowsky, 2016). Thereby, competence beliefs have been conceptualized in different ways in various motivational theories. For example, self-concept or self-efficacy which differ in their reference point (task or domain) or their time orientation (future or previous, see e.g., Bong & Skaalvik, 2003). But, independent from their operationalization, competence beliefs are supposed to have positive influences on effort and persistence and should therewith result in higher competences (see Wigfield et al., 2015, see also Abramson, Seligman, & Teasdale, 1978; Bandura & Jourden, 1991; Marsh et al., 2005).

With regard to mathematical competences, domain-specific self-concept has been reported to show important associations with mathematical competences (e.g., Marsh, 2014). Thereby, self-concept is a construct that refers to "…a person's perception of himself. These perceptions are formed through his experience with his environment,[…] described as: organized, multifaceted, hierarchical, stable, developmental, evaluative, differentiable" (Shavelson, Hubner, & Stanton, 1976, p. 411). Therewith, Shavelson and colleagues (1976) described self-concept as a multidimensional construct for which nonacademic and academic facets differentiate even further between domain-specific subfacets. Regarding the importance of self-concept for academic competences, Hansford and Hattie (1982) had already reported a meta-analysis of 128 studies that examined the relationship between various self-measures (e.g., self, self-concept, self-esteem) and measures of performance/achievement. On average, math self-concept showed—like the self in general—small to moderate positive correlations with mathematical competences (Hansford & Hattie, 1982).

The interdependence of self-concept and competences

When questioning the causal ordering between self-concept and competences, the answer is quite similar to the circular interdependence of cognitive abilities being both a determinant and a consequence of competences (e.g., Trautwein & Möller, 2016) because academic self-concept and academic competences “are mutually reinforcing, each leading to gains in the other” (Marsh & Martin, 2011, p. 73, see also Marsh & Craven, 2006). On the one hand, it seems plausible that high competences in one domain positively influence domain-specific self-concept (skill-development model). On the other hand, it also seems plausible that high domain-specific self-concept could enhance domain-specific competences (self-enhancement model), perhaps catalyzed by a more elaborated learning effort caused by higher motivation (e.g., Guay, Marsh, & Boivin, 2003). However, “prior self-concept affects subsequent achievement and prior achievement affects subsequent self-concept” (Guay et al., 2003, p. 124). Thus, as reported, for example, by Marsh, Byrne, and Yeung (1999), a reciprocal effect model that combines skill-development with the self-enhancement model seems quite plausible and has been corroborated by many studies.

Associations between self-concept and competences have been investigated in different developmental ages. As early as elementary school, a reciprocal interrelation has been reported for students. For instance, in a multicohort longitudinal study (three cohorts each assessed at three measurement points) by Guay and colleagues (2003), developmental trends between (domain-specific) academic competences and corresponding domain-specific self-concept were examined in elementary school. Their results indicated that the association between self-concept and a person’s competences increased with age, and the model with the best fit for explaining the causal ordering was the reciprocal effects model (Guay et al., 2003). Overall, Guay and colleagues’ (2003) study revealed that the association of competences and self-concept is relevant even for elementary school students.

But, which processes influence the development of self-concept? Research has revealed that the low correlations between self-concept and external indicators of people’s competence such as cognitive abilities in very young children (Marsh, 1989, 1990) increase with age, and self-evaluations become more differentiated (Guay et al., 2003; Wigfield & Karpathian, 1991, 1991). For instance, in the study by Guay and colleagues (2003), the reliability of elementary school students’ self-concept increased with age (second to sixth grade). Thus, to an increasing degree in school, feedback is supposed to influence the development of domain-specific self-concept through social and dimensional comparison processes (e.g., Ehm, 2014; Lüdtke et al., 2002). Dimensional comparison processes in which students compare their competences across different domains/subjects are summarized in the concept of the *Internal/External frame of reference model* (I/E model; see e.g., Ehm, Nagler, Lindberg, & Hasselhorn, 2014; Marsh, 1986). Social comparison processes in which students compare their competences with peers lead to the *big-fish-little-pond effect* (BFLPE; see e.g., Marsh, 1987; Marsh & Parker, 1984).

Both processes are summarized in the following.

In the I/E model, both external (interindividual, e.g., classmates) and internal (intraindividual, e.g., based on prior feedback, feedback in different domains) comparison processes are assumed to influence people's domain-specific self-concept (Ehm, 2014; Ehm et al., 2014). Whereas domain-specific competences (e.g., language and mathematics) and corresponding self-concept in language and mathematics have shown positive correlations, competences and contrasting (for mathematical competences, language self-concept, and vice versa) self-concepts have revealed negative associations, although the self-concepts in the contrasting domains tend to be rather uncorrelated (Marsh, 1986). Such patterns of results have been observed in several empirical studies (e.g., Ehm, 2014; Ehm et al., 2014; Marsh, 1986; Möller et al., 2009; Möller & Köller, 2001; Möller, Streblow, Pohlmann, & Köller, 2006) and have also indicated the multidimensionality of self-concept (for more information about the construct, see e.g., Bong & Skaalvik, 2003; Marsh, 2014; Shavelson et al., 1976).

However, not only do students compare their competences across domains, but they also compare their competences with the competences of other students. Of two students showing the same individual competences, the one in the higher achieving environment will be likely to report a lower self-concept than the one in the lower achieving environment (for more information, see e.g., Marsh, 1987; Marsh et al., 2008; Marsh & Hau, 2003; Marsh & Parker, 1984; for a rather critical review, see Dai & Rinn, 2008). For instance, Marsh, Chessor, Craven, and Roche (1995) reported two studies in which students who were grouped in a higher achieving environment (i.e., gifted and talented classes) showed lower academic self-concept than students who experienced a lower achieving reference group (i.e., regular classes). Even for elementary school students, the average ability level of classmates showed significant influences on students' academic self-concept, and such upward comparison processes were observed (e.g., Kastens, Gabriel, & Lipowsky, 2013; Lüdtke et al., 2002).

Centering on the individual processes that influence the development of self-concept must not be viewed in isolation in models such as the EVT, the I/E model, or the BFLPE. More likely, a combination of all processes (social, dimensional, temporal comparisons) may explain people's self-concepts. In predicting course choice, for instance, Nagy and colleagues (2007, 2008) combined the EVT and the I/E model. Their results indicated that prior achievement predicted self-concept, which again predicted course choice mediated by interest (for each, positively in the same domain, negatively in the opposite domain; for more information, see Nagy et al., 2008; Nagy, Trautwein, Baumert, Köller, & Garrett, 2007).

Finally, what about gender differences in math self-concept? Especially for German elementary school students, there were gender differences in mathematical competences (see 1.1.3). For instance, in the TIMS studies, German elementary school boys showed higher mathematical competences than girls. In the same sample, both genders showed quite high mathematical self-concepts, but boys' math self-concept was higher with a medium effect size (d_{2015} = .36, d_{2011} = .40; see Brehl et al., 2012; Wendt, Steinmayr et al., 2016). Equivalent findings in which boys showed higher math self-concept than girls were reported in several empirical studies conducted in countries other than Germany (see e.g., Eccles, Wigfield, Harold, & Blumenfeld, 1993; Nagy et al., 2010; Sax, Kanny, Riggers-Piehl, Whang, & Paulson, 2015; Steinmayr et al., 2014; Steinmayr & Spinath, 2008; Wigfield et al., 1997; Wigfield & Eccles, 1994). Using the EVT to explain gender differences in mathematical competences and choices related to mathematics (and the related STEM subjects) for values in mathematics, an inconsistent pattern was revealed: Some studies also indicated that boys' values were higher than girls' (e.g., Marsh et al., 2005), whereas some indicated no differences (e.g., Wigfield et al., 1997, for an overview, see e.g., Gaspard, 2015).

1.5 Mathematically Gifted Students – Characteristics and Needs

The previous chapter summarized the concept of mathematical competences and factors that are assumed to influence the acquisition of mathematical competences. This current chapter now takes a close look at students who have much more sophisticated mathematical competences than their same-aged peers. First, their common characteristics are summarized, and second, an idea about how to foster mathematical competences of such students is introduced.

1.5.1 The concept of mathematical giftedness

Although all students are supposed to and can acquire mathematical competences (e.g., Lee & Ginsburg, 2009), there are interindividual differences in mathematical competences (see e.g., Bos et al., 2003; Bos et al., 2012; OECD, 2004, 2006, 2016; Stanat et al., 2012; Wendt, Bos et al., 2016). Some students show extraordinary, above-average strengths in mathematical competences and are, for example, already able to solve curriculum-based tasks (Koshy et al., 2009; Ziegler, 2008). In general, students who are expected to have the potential to show extraordinary competences are classified as gifted[7] (Subotnik et al., 2011; Ziegler, 2008).

According to modern models that are used to explain giftedness, giftedness is—like competences—described as a multidimensional development process that is necessarily based on extraordinary domain-general cognitive abilities (Arvey et al., 1994; Heller, 1993; Heller, Mönks, Subotnik, & Sternberg, 2000; Neisser et al., 1996; Sternberg, 2011; Subotnik et al., 2011). Considering only domain-general cognitive ability would be too one-sided, as domain-general cognitive abilities are also assumed to be (a) a potential and (b) influenced by environment (e.g., social background, stimuli). Moreover, Arvey and colleagues (1994) described the observation that, with regard to educational success, other factors besides intelligence had an influence (see also Ziegler, 2008). Giftedness is assumed to result from the complex interplay between intelligence and, for instance, motivation, creativity, spatial ability, family background, social or practical skills, or personality (Heller, 1993; Heller et al., 2000; Kell et al., 2013; Mönks & Mason, 2000; Subotnik et al., 2011; Wai et al., 2009; Ziegler, 2008). According to Sternberg and Zhang (1995), gifted students are supposed to fulfill five criteria (i.e., excellence, rarity, demonstrability, productivity, and a value criterion) in at least one domain (Sternberg, 2011; Ziegler, 2008).

Students who show potential in terms of mathematics are classified as mathematically gifted (e.g., Bicknell, 2008). This domain-specific giftedness is also supposed to necessarily

[7] In general, the terms gifted, highly gifted, and talented are used interchangeably (Ziegler (2008).

depend on high domain-general cognitive abilities that are supplemented by cognitive and non-cognitive sophisticated domain-specific factors (cf. Landerl, Bevan, & Butterworth, 2004; Stern, 1998, 2017). For example, Diezmann and Watters (2016) reported the observation that these students show an outstanding motivation for mathematics.

To do justice to these students, much research has been devoted to examining the characteristics that distinguish mathematically gifted students from their same-aged peers who do not exhibit these strengths. For example, Koshy and colleagues (2009, p. 215) described mathematical giftedness as "…the quality of being able to do mathematics, that is, being able to perform mathematical tasks and to utilize mathematical knowledge effectively..." (p. 215). Mathematically gifted students are supposed to demonstrate mathematical thinking that is qualitatively different from the thinking of their peers, for example, by showing quite early intense mathematical curiosity and demonstrating an understanding of all things related to quantity (Deal & Wismer, 2010, 55ff; Koshy et al., 2009). In mathematical learning processes, mathematically gifted students are further assumed to follow complex lines of thoughts (i.e., reasoning), to detect mathematical patterns and structures, and to demonstrate a higher level of logical thinking about spatial, numerical, or symbolical relationships (Deal, & Wismer, 2010; Koshy, Ernest, & Casey, 2009; Leikin, 2010; Diezmann, & Watters, 2002). All these strengths in mathematical competences are supposed to support the acquisition of mathematical competences (Deal, & Wismer, 2010; McAllister, & Plourde, 2008).

1.5.2 Promoting mathematically gifted students

Although mathematically gifted students are ascribed as having the potential to contribute meaningful solutions to the problems of modern society (Diezmann & Watters, 2001; Koshy et al., 2009), there are also some challenges when working with such students. Indeed, mathematically gifted students show more sophisticated mathematical competences than their same-aged peers and, in particular, strengths in solving new mathematical problems. Nevertheless, Bezold (2012) noted some weaknesses in such students' abilities to build and justify hypotheses. Similar observations were reported by Bardy and Hrzán (2010), who further reported mathematically gifted students' weaknesses in justifying solutions and writing them down. Also, although these students were able to recognize and sometimes use mathematical patterns and structures, verbalizing these findings was observed to be problematic (Käpnick, 1998). Assuming the differentiation of mathematical competences in process- and content-based mathematical competences, mathematically gifted students' unbalanced development of these two facets were obviously able to account for process-based competences (e.g., problem solving and arguing; see Deal & Wismer, 2010; McAllister & Plourde, 2008). To ensure that mathematically gifted students do not lose their enthusiasm for mathematics (e.g., McAllister & Plourde, 2008), these students need appropriate learning opportunities that will lock in their potential (cf. Beck et al., 1991; Kunter & Trautwein, 2013).

To do justice to the complex needs of mathematically gifted students, several measures were recommended (see Ziegler, 2008): (a) acceleration, (b) enrichment, (c) pull-out programs, and (d) ability grouping (e.g., special classes or schools; Stumpf, 2011). Several studies have examined the effectiveness of these measures. For instance, acceleration—moving more quickly through the curriculum (Ziegler, 2008)—was revealed to have positive effects on students' development in several studies (e.g., Kulik & Kulik, 1987; Steenbergen-Hu & Moon, 2010). Further, enrichment programs—more specific and more detailed learning opportunities (Stumpf, 2011; Ziegler, 2008)—indicated positive effects in enhancing students' competences (e.g., Aljughaiman & Ayoub, 2012 Kulik & Kulik, 1987; Reis & Renzulli, 2010; Vaughn, Feldhusen, & Asher, 1991) but also seemed promising for increasing students' interest and motivation in a particular domain (Petersen & Wulff, 2017; Stake & Mares, 2001).

Beneficial learning environments for mathematically gifted students

To tap their individual potential, students need appropriate learning environments. For preschool students, for example, Niklas and Schneider (2012) reported that home numeracy environments in early childhood are needed to influence the later development of mathematical competence. In a study by Blums, Belsky, Grimm, and Chen (2016), the results of structural equation models indicated that students' early environment was indeed predicted by mother's education, but amongst others, mathematical competences were mediated by executive functions and language. Thus, stimuli and input from a mathematically enriching environment predict mathematical competences (e.g., Clements & Sarama, 2011; Schneider et al., 2016). Thereby, education is understood as a learning environment that facilitates intelligent and meaningful learning opportunities (Kunter & Voss, 2011, 2013).

Education is expected to provide opportunities to convey the "necessary knowledge, skills, abilities, and what else it needs to solve particular problems or answer particular questions" (Neumann, Bernholt, & Nentwig, 2012, p. 507; Kunter & Voss, 2011). Therewith, learning mathematics is not supposed to correspond with the acquisition of an overarching knowledge base but is supposed to be an application of what has been learned (Neumann et al., 2012). But, what should education look like if it is to be able to increase individuals' likelihood of applying their learning? Intuitively, one might think about methods and organizational or social forms of education. Therefore, much research has been devoted to developing several combinations of teaching methods and organizational and social forms of education (for further information, see e.g., Meyer, 2014, 2016). But, empirical research has shown that learning success is explained to a greater extent by the cognitive teaching-learning processes that depend on teaching quality (see e.g., Kunter & Trautwein, 2013; Seidel & Shavelson, 2007; Veenman, Kenter, & Post, 2010). Three dimensions of teaching quality have been shown to be important for supporting students' learning: (a) classroom management, (b) cognitive activation, and (c) individual learning support (see e.g., Baumert et al., 2013; Klieme, 2006; Kunter & Trautwein, 2013; Kunter & Voss, 2011). Classroom management embraces all actions and strategies that support a trouble-free education and maximizes study time (i.e., time on task; see Seidel & Shavelson, 2007). Individual learning support describes all forms of teacher-student interactions that support students' understanding, and cognitive activation refers to the intellectual demands necessary to actively perform learning processes (see e.g., Kunter & Voss, 2011). Focusing on the individual and following up on the cognitive-constructivist understanding of learning, the latter two provide opportunities to tie in with prior competences for acquiring new competences (e.g., Köller & Parchmann, 2012).

From a students' perspective, the most common idea for cognitively activating students is the idea of giving them the opportunity to solve challenging tasks (Diezmann & Watters, 2001; Henningsen & Stein, 1997; Kunter & Trautwein, 2013; Kunter & Voss, 2011). In mathematics, such challenging problems should provide opportunities to explore and give students the possibility to "… explain, clarify and revise their mathematical ideas and problem constructions" (Deal & Wismer, 2010; Diezmann & Watters, 2001, p. 7; McAllister & Plourde, 2008). Therewith, challenging tasks are supposed to trigger students to actively deal with mathematical themes and provide them with opportunities to search for mathematical patterns and structures (Henningsen & Stein, 1997; Wittmann, 2005, July). Thus, challenging tasks are supposed to support students' acquisition of competences by allowing deeper processing (Klieme & Rakoczy, 2008). In particular, word problems that (a) are meaningful and relevant to the students, (b) allow for individual definitions of (sub-)questions, and (c) focus on reasoning and communication are supposed to trigger solution processes that transcend looking for keywords or bringing together all of the relevant and irrelevant numbers presented in the problem (Bransford et al., 2012). Concentrating on process-based mathematical competences rather than on content-based mathematical competences, these criteria are supposed to be implemented in open tasks that enable students to apply different approaches to solve the problem (Bardy & Hrzán, 2010).

Competitions as special challenges for mathematically gifted students

As mathematically gifted students usually easily solve curriculum-based tasks, there is the risk that these fast-paced learners will not be given enough cognitively activating and challenging tasks to reach their mathematical potential (e.g., Rotigel & Fello, 2004; Rotigel & Fello, 2016). Hence, to exploit their potential, gifted students in general need challenging environments that enable qualitatively high learning experiences (Diezmann & Watters, 2001; McAllister & Plourde, 2008; Reis & Renzulli, 2010; Subotnik et al., 2011; Ziegler, 2008). However, schools—and therefore teachers—have a limited amount of time to devote to each individual (Diezmann & Watters, 2000; Petersen & Wulff, 2017). Thus, (mathematically) gifted students often get a raw deal (Reis & Renzulli, 2010) in formal education in relation to their potential. Thus, specific measures are necessary to foster the competences of these students (Sternberg, 2011) and give them the opportunity to live up to their potential (Stumpf, 2011). In terms of enrichment measures, for example, in-school approaches such as extra lessons or workshops as well as out-of-school approaches such as summer schools or academic competitions have been suggested (Bicknell, 2008; Höffler, Bonin, & Parchmann, 2017; Petersen & Wulff, 2017).

Academic competitions—for more information about the characteristics of (good) academic competitions, see Forrester (2010) as well as Petersen and Wulff (2017, p. 3)—are assumed to be a good way to challenge and foster students' competences (Ozturk & Debelak, 2008a, 2008b; Petersen & Wulff, 2017) and therewith to nurture their potential (Pyrt, 2000). Furthermore, academic competitions are supposed to balance cognitive and noncognitive (e.g., motivational) enhancement (Petersen & Wulff, 2017). Hence, academic competitions can provide a platform from which to evaluate one's own performance and compare it with others (Goldstein & Wagner, 1993) in terms of mastering challenging tasks (Höffler et al., 2017; Ozturk & Debelak, 2008a, 2008b). Therewith, academic competitions are even supposed to enhance students' competence in terms of a holistic understanding of competences including noncognitive and cognitive factors (e.g., motivation and the cognitive aspect of competences) by aiming to develop students' competences. Such competitions are assumed to motivate students to be engaged in the competition's domain even beyond participation (Höffler et al., 2017).

One of the most prominent academic competitions around the world is the academic Olympiad whose tasks are classified as challenging (Campbell, Wagner, & Walberg, 2000; Campbell & Walberg, 2010; Olson, 2005; Petersen & Wulff, 2017). This special form of academic competition is provided in many countries and is characterized by an international level in which the top performers of the participating countries compete (for an overview of the procedure of the Olympiads in Biology, Chemistry, Mathematics, Physics, and Junior Science, see Petersen & Wulff, 2017, p. 5). But, for students who step up to this plate, they should be ready to master the challenges of the competition (cf. Pajares & Schunk, 2002). Otherwise, they will not only miss the chance to enhance their mathematical competences by applying prior mathematical competences, but also, their beliefs about their own competences are vulnerable (Höffler et al., 2017). Based on such considerations and a demand for continuous and systematic enrichment programs (deliberate practice; see e.g., Subotnik et al., 2011; Ziegler, 2008), the preparation to solve challenging tasks has to be part of an enrichment program, too (Bicknell, 2008). Some authors (e.g., Cropper, 1998; Ozturk & Debelak, 2008a) have suggested that participation in an academic (mathematical) competition should be combined with a corresponding pedagogical training.

1.6 Research Questions of the Present Dissertation

On the basis of the need for sophisticated mathematical competences to deal with the requirements and to be able to solve the problems of a modern western knowledge society, the present dissertation examined how mathematical competences can be fostered. Thereby, students who tend to belong to the group of high-achieving students in mathematics were in the focus. Furthermore, as early fostering of mathematical competences can increase the likelihood of achieving expertise and not losing enthusiasm for mathematics (e.g., Johnson, 1983; Johnson, 1990; McAllister & Plourde, 2008), the present dissertation focused on elementary school students.

In a first step, domain-specific and domain-general factors who are supposed and examined to influence the development of mathematical competences were delineated. Thereby, one way to enrich the learning environments of students who are already able to solve curriculum-based tasks are academic competitions (see Chapter 1.5.2). Every year, parents and teachers encourage such students to participate in a domain-specific competition (e.g., Fauser et al., 2007), and an increase has been observed in the number of students interested in participating in academic competitions (Petersen & Wulff, 2017). But, do academic competitions affect students' development? In Paper 1 (*Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade*), the appropriateness of learning environments provided by academic competitions as a way to enrich gifted students was examined in detail. In particular, the roles of academic competitions in fostering gifted students were explored on the basis of the literature. Also, the need for training measures (i.e., training courses) that are aimed at preparing students to participate in an academic competition was delineated. Using the example of the Mathematical Olympiad for elementary school students, the requirements and challenges of this particular competition were presented. As there is the danger that the challenges provided by academic competitions are not common for the mathematically gifted students who are used to being successful in mastering mathematical problems (Kießwetter, 2013; Nolte, 2013b), the core components of the training "Getting fit for the Mathematical Olympiad" which consider strengths and weaknesses of mathematically gifted students were introduced.

Providing more challenging tasks, the training is supposed to trigger a more intense way to deal with mathematical problems and, therewith, allows a deeper application of mathematical competences. Thus, positive effects on performance in the academic competition (i.e., the Mathematical Olympiad) and on the mathematical competences of the students who attended the training were expected in comparison with the students who did not attend it. However, although such a training is also assumed to reflect the competitive aspect of the competition (e.g., Cropper, 1998), offering a training for a mathematical competition poses a new problem: The average level of ability should be higher than what the participant is used to encountering in class (Bicknell, 2008; Ozturk & Debelak, 2008b; Riley & Karnes, 1998). On the one hand, being part of the selected high-achieving group may result in positive feelings such as pride (i.e., an assimilation effect or *Basking-in-reflected-glory-effect*, e.g., Marsh, Kong, & Hau, 2000). In turn, one would expect positive effects on participants' motivation (e.g., Rinn, 2007). On the other hand, however, the BFLPE (see Chapter 1.3.2) suggests the opposite (for a study reporting negative effects on self-concept for students in a gifted program in Israel, see Zeidner & Schleyer, 1999). Motivation might even decrease when students compare their own performance with the performances of other high-achieving students (e.g., Dai & Rinn, 2008; Marsh et al., 2000; Marsh et al., 2008; Marsh & Parker, 1984). In sum, as for merely participating in an academic competition, participating in a training could lead to increases (for the successful students) or decreases (for the unsuccessful ones) in (domain-specific) motivation (Höffler et al., 2017).

Paper 2 (*Getting Fit for the Mathematical Olympiad: Positive Effects on Achievement and Motivation?*) investigated the effectiveness of the training "Getting fit for the Mathematical Olympiad" with respect to mathematical competences in general and motivational factors using a quasi-experimental pre- and posttest design in a natural setting (Shadish, Cook, & Campbell, 2002). Both students who attended the training "Getting fit for the Mathematical Olympiad" and a control group participated in the study. Based on the deeper engagement in solving challenging mathematical problems and tasks, positive effects on performance in the Mathematical Olympiad and on mathematical competences for students who attended the training in comparison with students who did not were expected. On the other hand, the training was offered to third- and fourth-grade students together, which may have led to different social comparison processes for each age group. Thus, different effects on motivational factors (i.e., math self-concept and value beliefs) were expected for the two age groups.

Based on the finding that solving more complex mathematical problems requires process- rather than content-based mathematical competences (Fuchs et al., 2010; Sullivan et al., 2016; Träff, 2013), Chapter 1.2), Paper 3 (*Training Process-Based Mathematical Competences – Exploring Effects on Domain-Specific Factors and Domain-General Cognitive Abilities*) examined the cognitive aspects of "Getting fit for the Mathematical Olympiad" in detail. A randomized waitlist control group design (Friedman, Furberg, & DeMets, 2010) was used to examine whether the training that specifically focused on process-based competences had differential effects on domain-specific factors (i.e., domain-specific content-based mathematical competences) and domain-general cognitive abilities besides effects on process-based competences. The process-based training was expected to have only a small effect on tasks that require only basic arithmetical competences (e.g., adding). Tasks requiring more process-based competences (e.g., supplementary tasks) were expected to be influenced to a greater extent. As domain-general cognitive abilities are supposed to have more influence on more complex problems (Fuchs et al., 2010; Krajewski & Schneider, 2009a, 2009b; Sullivan et al., 2016; Träff, 2013; von Aster & Shalev, 2007), the study explored whether enhancing process-based competences would have an influence on domain-general cognitive abilities.

The two empirical studies included in the present dissertation were conducted in two different school years (i.e., 2014/2015 and 2015/2016). The framework was delivered by the Hector Children's Academy Program (HCAP), an extracurricular enrichment measure for elementary school students in the German state of Baden-Württemberg (for more information, see Rothenbusch et al., 2016). In this program, "Getting fit for the Mathematical Olympiad" was developed and evaluated. After pilot-testing in 2013/2014, the training was offered by different course instructors who were given information about the core components of the training, scripted manuals, and master copies of all materials to be able to teach the training (for more information about the procedure, see Herbein, 2016).

2

Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade

Rebholz, F., & Golle, J. (2017). Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade [Fostering mathematical skills in elementary school – the role of academic competions using the example of the Mathematical Olympiad]. In Trautwein, U. & Hasselhorn, M. (Ed.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente.* (pp. 213–228). Göttingen: Hogrefe.

Zusammenfassung

National und international gibt es eine Vielzahl akademischer Schülerwettbewerbe. Diese können verschiedene Funktionen haben: Enrichment und Differenzierung, Zusammenarbeit mit Peers fördern, kompetitive Umwelt herstellen, Motivatoren bzw. Anreize setzen, Identifikation und Diagnose. Im folgenden Kapitel werden diese Funktionen im Hinblick auf die Begabtenförderung beschrieben. Dabei werden sowohl die Erwartungen seitens der Autoren an Wettbewerbe als auch empirische Ergebnisse berücksichtigt. Die häufig sehr komplexen Aufgaben von Schülerwettbewerben stellen eine ideale intellektuelle Herausforderung für Begabte dar. Damit bieten Wettbewerbe eine Lernumwelt, in der auch Begabte ihre Fähigkeiten vertiefen und weiterentwickeln können. Ein Bespiel für intellektuell anspruchsvolle Wettbewerbe sind die Schüler-Olympiaden (Mathematik, Chemie, Physik und Biologie). Am Beispiel der Mathematik-Olympiade für die Grundschule wird ein Wettbewerb konkret vorgestellt. Um den Teilnehmerinnen und Teilnehmern der Mathematik-Olympiade eine Wettbewerbsteilnahme in begleitetem Rahmen zu ermöglichen, wurde ein Vorbereitungskurs, zugeschnitten auf die Stärken und Bedürfnisse mathematisch Begabter, entwickelt. Dieser Kurs, der offene Aufgaben und kooperatives Arbeiten als Ausgleich zum kompetitiven Setting von Wettbewerben fokussiert, wird in diesem Beitrag in Kombination mit ersten empirischen Ergebnissen zur Wirksamkeit des Kursangebots vorgestellt. Ausgehend von diesem Beispiel werden die Rolle von Schülerwettbewerben im Ganztagsangebot andiskutiert und Ideen zur flächendeckenderen Implementierung von Wettbewerben vorgestellt.

Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade

Schülerwettbewerbe in der Begabtenförderung

Wettbewerbe haben in Schulen und in der Öffentlichkeit einen hohen Stellenwert (Oswald et al., 2005). Es gibt eine Fülle mehr oder weniger bekannter nationaler und internationaler akademischer Wettbewerbe, wie z.B. *Schüler experimentieren* und *Jugend forscht* oder die *Olympiaden* (Mathematik, Physik, Biologie, Chemie). Dabei unterscheiden sich die Wettbewerbsformate erheblich und reichen von Individual- über Gruppen- zu Klassenwettbewerben mit Klausuren, Projekten und Diskussionen. Auch in den Aufgabenausrichtungen unterscheiden sich die Wettbewerbe von *Ausführen und Darbieten* (z.B. Jugend musiziert) über *Entdecken und Herausfordern* (z.B. Mathematik-Olympiade) bis hin zu *Erfinden und Konstruieren* (z.B. Schüler experimentieren/Jugend forscht). Unabhängig von der Aufgabenausrichtung und dem Wettbewerbsformat sollen Wettbewerbe möglichst viele Schülerinnen und Schüler (SuS) mithilfe von herausfordernden Aufgaben für einen bestimmten Inhaltsbereich begeistern und motivieren (Fauser et al., 2007; Oswald et al., 2005).

Nach Fauser und Kollegen (2007) sowie Ozturk und Debelak (2008a) sollen Wettbewerbe via verschiedener Funktionen unterschiedliche Einflüsse auf die akademische und persönliche Entwicklung von Teilnehmerinnen und Teilnehmern von Schülerwettbewerben haben. Vor allem bei langfristig angelegten Wettbewerben erwarten die Autoren als Resultate der Teilnahme unter anderem ein vertieftes Aufgabenverständnis oder das Lernen von Selbstdisziplin und das Erkennen des Zusammenhangs zwischen Arbeit und Erfolg (Fauser et al., 2007; Ozturk & Debelak, 2008a). Die Anzahl empirischer Studien zur Überprüfung kurz- sowie langfristiger Effekte von Wettbewerben ist allerdings relativ niedrig; hauptsächlich retrospektive Befragungen ehemals erfolgreicher Teilnehmerinnen und Teilnehmer akademischer Olympiaden wurden durchgeführt (Campbell & Verna, 2010; Campbell & Walberg, 2010; Lengfelder & Heller, 2002; Oswald et al., 2005). Lediglich am Leibniz-Institut für die Pädagogik der Naturwissenschaften und Mathematik (IPN) in Kiel gibt es derzeit Projekte in Forschungslinie 4 „Wissenschaftskommunikation und extracurriculare Förderung“, die die Datenlage substantiell zu verbessern versprechen (http://www.ipn.uni-kiel.de/de/forschung/forschungslinien /forschungslinie-4). Im Folgenden werden die von Bicknell (2008), Fauser und Kollegen (2007), Ozturk und Debelak (2008a, 2008b) sowie Peters and Sieve (2013) erwarteten und in den Befragungen ermittelten Funktionen von Schülerwettbewerben insbesondere in der Begabtenförderung zusammengefasst.

Enrichment und Differenzierung. Die Aufgaben von akademischen Schülerwettbewerben sind meist so gestellt, dass sie curricular Bekanntes neu verknüpfen und akzentuieren, die Aufgaben aber mit den Kompetenzen der SuS lösbar sind (Fauser et al., 2007). Durch diese herausfordernden Aufgaben können Wettbewerbe nach Ozturk and Debelak (2008b) im Unterricht daher als Mittel zur Differenzierung gesehen werden. So bieten Wettbewerbe hauptsächlich eine Anreicherung der Lernumgebung (Enrichment) für SuS, die curriculare Aufgaben schnell und sicher lösen (Bicknell, 2008; Ozturk & Debelak, 2008b; Peters & Sieve, 2013). Die Aufgaben von Wettbewerben sind in der Regel wenig vorstrukturiert und bieten damit insbesondere Raum für kreative Lösungen (Fauser et al., 2007). Dennoch müssen nach Ozturk und Debelak (2008a) in vielen Wettbewerben eigene Gedanken und Ideen (schriftlich) unter Berücksichtigung der Konventionen des betreffenden (Fach)Bereichs ausgedrückt werden. Dadurch lernen Teilnehmerinnen und Teilnehmer die Konventionen eines (Fach-)Bereichs kennen (Peters & Sieve, 2013). Häufig werden Wettbewerbe durch Experten als Mentoren oder Juroren begleitet, was den Teilnehmerinnen und Teilnehmern zum einen reale Einblicke in die Domäne ermöglicht und zum anderen Feedbackmöglichkeiten eröffnet (vgl. Ozturk & Debelak, 2008b; Peters & Sieve, 2013). Nach Fauser und Kollegen (2007) sowie Oswald und Kollegen (2005) sind Wettbewerbe damit ein wichtiger Baustein einer breitaufgestellten Förderung von SuS mit hohem Potential (Begabtenförderung).

Motivatoren. Selbstverständlich können Preise, Anerkennung oder Prestige Anreize für die Teilnahme an einem Wettbewerb darstellen. Nach Oswald und Kollegen (2005) geben die Teilnehmerinnen und Teilnehmer aber an, dass diese extrinsischen Motivatoren nicht ausschlaggebend für die Teilnahme an Wettbewerben seien. Viel mehr scheinen Interesse am Fach bzw. der Domäne oder das Ausloten der eigenen Begabungen bis hin zu Erfahrungen eigener Kompetenz und Eigenverantwortung ausschlaggebender für eine Wettbewerbsteilnahme zu sein (Oswald et al., 2005). So werden die Aufgaben nach Oswald und Kollegen (2005) in der Regel von erfolgreichen SuS im Wettbewerb, aber auch von (Fach-)Lehrern als Herausforderung für die Teilnehmerinnen und Teilnehmer wahrgenommen, die im normalen Schulalltag vermisst wurde.

Zusammenarbeit mit Peers. Viele Wettbewerbe (oder deren Vorbereitungskurse) bieten außerdem den Vorteil, dass Peers mit vergleichbaren Interessen und Leistungsstärken getroffen oder kennen gelernt werden können (vgl. Fauser et al., 2007; Oswald et al., 2005). Herausfordernde Lernsituationen, gepaart mit dem Zusammenarbeiten mit Gleichgesinnten, werden von ehemals erfolgreichen Teilnehmerinnen und Teilnehmern der Olympiaden positiv bewertet.

Vor allem hinsichtlich der Entwicklung des Selbstwertgefühls sowie Fähigkeiten zur Teamarbeit und zur Projektarbeit werden mit steigendem zeitlichen Abstand zum Wettbewerb als zunehmend gewinnbringend beurteilt (Oswald et al., 2005). Bicknell (2008), Fauser und Kollegen (2007) sowie Ozturk und Debelak (2008b) erwarten positive Einflüsse auf die persönliche Entwicklung Begabter durch eine Wettbewerbsteilnahme. Beispielsweise nehmen sie an, dass die Zusammenarbeit in homogenen Gruppen Ausdauer und Beharrlichkeit bei der Aufgabenbearbeitung fördern kann (Fauser et al., 2007). So soll die Teilnahme nicht nur für Sieger von Wettbewerben, sondern für alle Teilnehmerinnen und Teilnehmer gewinnbringend sein (Fauser et al., 2007).

Kompetitive Umwelt. Begabte Teilnehmerinnen und Teilnehmer von Wettbewerben streben zwar nach persönlicher Leistung und der Demonstration der eigenen Fähigkeiten, zusätzlich bewegen sie sich im Wettbewerbssetting aber in einer kompetitiven Umwelt (Bicknell, 2008). Nach Ozturk und Debelak (2008b) ist das Lernen des Umgangs mit Sieg und Niederlage besonders für Begabte ein wichtiger Faktor im Hinblick auf das spätere Leben in einer Gesellschaft, in der insbesondere (hohe) Leistungen anerkannt werden. Trotzdem bieten Wettbewerbe den Teilnehmerinnen und Teilnehmern eine geschützte Zone für das Bearbeiten herausfordernder Aufgaben, da Misserfolg oder schlechte Leistungen keine negativen Konsequenzen haben (Fauser et al., 2007; Ozturk & Debelak, 2008b). So kann die Enttäuschung über schlechtes Abschneiden beispielweise auch Ansporn für kommende Herausforderungen (z.B. erneute Wettbewerbsteilnahme) sein oder dazu führen, dass SuS die eigenen Fähigkeiten realistischer einschätzen (vgl. Fauser et al., 2007; Ozturk & Debelak, 2008a). Dennoch stehen Wettbewerbe immer wieder in der Kritik, SuS unter Druck zu setzen und bloße Leistungsvergleiche anzustellen. Laut Fauser und Kollegen (2007) kann diese Kritik entkräftet werden, wenn ein Wettbewerb hohen pädagogischen Ansprüchen genügt, wie beispielsweise (i) einem offenen, kostenlosen sowie freiwilligen Zugang, (ii) der Anerkennung guter Leistungen oder (iii) einer (pädagogischen) Begleitung der Wettbewerbsteilnahme. Außerdem sollten gute Wettbewerbe auf die Förderung intrinsischer Motivation (wie z.B. Spaß beim Lösen der Aufgaben) ausgerichtet sein (Ozturk & Debelak, 2008b).

Identifikation und Diagnose. Nach Callahan, Husaker, Adams, Moor, und Bland (1995) sowie Fauser und Kollegen (2007) können Wettbewerbe ein Hilfsmittel zur Identifikation Begabter auf Grundlage gezeigter Leistungen in Wettbewerben sein. Sowohl in den USA als auch in Deutschland zeigten Befragungen ehemaliger erfolgreicher Teilnehmerinnen und Teilnehmer an den akademischen Olympiaden sehr gute Schulnoten und (gemessen an der Anzahl wissenschaftlicher Publikationen und abgeschlossener Promotionen) überdurchschnittliche wissenschaftliche Leistungen (Campbell & Verna 2010; Campbell & Walberg, 2010; Lengfelder & Heller, 2002). Damit scheint eine erfolgreiche Wettbewerbsteilnahme mit späteren Spitzenleistungen in Studium und Beruf zusammenzuhängen (vgl. Fauser et al., 2007; Oswald et al., 2005).

Das Beispiel Mathematik-Olympiade[8]

Nach Olson (2005) gehören die akademischen Olympiaden zu den schwierigsten Schülerwettbewerben. Diese Wettbewerbe werden in den Bereichen Biologie, Chemie, Physik und Mathematik auf nationaler und internationaler Ebene angeboten. Die Mathematik-Olympiade ist ein bundesweiter Wettbewerb, der für Schülerinnen und Schüler (SuS) der Klassen 3 bis 12 mit unterschiedlichen alters- und entwicklungsgerechten Schwierigkeitsgraden angeboten wird. SuS mit Spitzenleistungen im Bundesvergleich qualifizieren sich für eine Auswahlrunde zur Internationalen Mathematik-Olympiade und bekommen die Chance, sich auch im internationalen Vergleich zu messen. Der Wettbewerb steht unter der Schirmherrschaft des Bundespräsidenten und wird jährlich im Herbst/Winter (September bis Februar) durch den Verein Mathematik-Olympiaden e.V. veranstaltet.

Für Grundschülerinnen und Grundschüler wird der Wettbewerb in Deutschland seit 2005 als nationaler Wettbewerb auf Landesebene angeboten. Im Grundformat besteht der Wettbewerb aus drei Runden: einer ersten breit angelegten Hausaufgabenrunde, gefolgt von zwei Klausurrunden mit regionaler und landesweiter Ausrichtung. Der Schwierigkeitsgrad der Aufgaben steigt dabei von Runde zu Runde an, thematische Ähnlichkeiten sind häufig vorhanden (Mathematikolympiaden e.V., 2013). Die Wettbewerbsaufgaben stellen dabei insbesondere für Begabte eine Herausforderung dar, denn sie sind komplex konstruiert und Begründungen für notierte Lösungen werden eingefordert. Die Aufgaben erfordern nur wenige über das Curriculum hinausgehende Kompetenzen, sie verknüpfen bekannte Sachverhalte neu und bieten somit auch im Rahmen des Wettbewerbs Möglichkeiten neuer mathematischer Entdeckungen.

Analysiert man die Aufgaben der Mathematik-Olympiade in der Grundschule von 2005 bis 2013, lassen sich acht Aufgabentypen identifizieren. Diese unterscheiden sich in den zur Bearbeitung benötigten, mathematischen Fertigkeiten, Kompetenzen und Strategien. Zum erfolgreichen Bearbeiten aller Aufgabentypen wird ein hohes Maß an mathematischer Sensibilität und Kreativität benötigt. In Tabelle 5.1 sind die Aufgabentypen der Mathematik-Olympiade

[8] Eine weiterführende fachunabhängige Übersicht über empfehlenswerte qualitativ hochwertige Schülerwettbewerbe wurde durch die Kultusministerkonferenz (2009) herausgegeben. Das BMBF fördert deutschlandweit derzeit über 20 Wettbewerbe (vgl. Bundesministerium für Bildung und Forschung, 2015).

in der Grundschule mit kurzer Charakterisierung dargestellt. Alle Aufgaben zwischen 2005 und 2013 können folgendem Schema zugeordnet werden (modifiziert nach Rebholz, 2013).

Tabelle 1.

Aufgabentypen der Mathematik-Olympiade in der Grundschule

Aufgabentyp	Charakterisierung
Logische Schlüsse ziehen	- Komplexe Informationen auf Textbasis verarbeiten - Aufgaben sind durch Strukturieren und Organisieren der Informationsfülle lösbar
Gleichungsbasierte Aufgaben	- Aufgaben basieren mathematisch auf einfachen Gleichungen - Herausforderung: Konzept von Gleichungen in Grundschule noch unbekannt
Platzhalter	- Lösen und Entwerfen von Kryptogrammen - Symbole repräsentieren Zahlen
Würfel und Würfelnetze	- Würfel als dreidimensionales Objekt - Baustein für größerer Objekte - Zusammenhang zwei- und dreidimensionaler Objekte
Geometrie in der Ebene	- Geometrische Objekte zerlegen - Kleinere Objekte in größeren Objekten finden
Muster und Strukturen geometrisch	- Geometrische Muster und Strukturen fortsetzen - Wechsel der Repräsentation zwischen Objekt und Zahl
Geschicktes Rechnen	- Strukturen von Rechnungen erkennen und anwenden - Eigenschaften natürlicher Zahlen
Kombinatorik	- Kombinationsmöglichkeiten finden und konkretisieren

Kursprogramm „Fit für die Mathematik-Olympiade“

Im Folgenden wird ein begleitendes pädagogisches Angebot für mathematisch besonders begabte und hochbegabte Grundschulkinder vorgestellt. Das Angebot bietet sowohl die Gelegenheit zur Vorbereitung auf die Mathematik-Olympiade, als auch die Möglichkeit zum Mathematiktreiben mit Peers sowie die Förderung der Mathematikkompetenz.

Zielgruppe: Mathematisch begabte Grundschulkinder

Mathematisch begabte Grundschülerinnen und Grundschüler sind fasziniert von Mathematik, haben ein besonderes Gefühl für Zahlen und deren Zuordnungen, zeigen eine Begeisterung für geometrische Muster und Zahlenrätsel, sie „tun“ gerne Mathematik (vgl. Heinrich, 2010; Käpnick, 1998, 2013; Kießwetter, 2013). Mathematisch begabte Kinder haben im mathematischen Bereich einen Entwicklungsvorsprung gegenüber Gleichaltrigen und können in der Regel curriculare Mathematik-Aufgaben spielend lösen. Nach Käpnick (1998) und Kießwetter (2013) kann dieser Vorsprung in der Fähigkeit mathematische Probleme zu lösen unter anderem auf eine hohe mathematische Sensibilität und Kreativität – die sich in selbstständigem Erkennen mathematischer Probleme ausdrückt – zurückgeführt werden. Mathematisch Begabte können aber auch auf höherem Niveau mathematisch arbeiten als Gleichaltrige. Dieser Erfolg beim Lösen anspruchsvoller mathematischer Probleme kann bei mathematisch begabten Grundschulkindern auf eine überdurchschnittlich ausgeprägte Fähigkeit im *Erkennen und Nutzen von mathematischen Mustern und Strukturen* zurückgeführt werden. Denn die Fähigkeit zum Erkennen und Nutzen mathematischer Muster und Strukturen wird für den Löseprozess von Mathematikaufgaben als grundlegend gesehen (Aßmus, 2010; Bardy, 2013; Devlin, 2002; Kießwetter, 2013; Nolte, 2013b). Das Erkennen von Gesetzmäßigkeiten (Muster) und deren Zusammenhänge (Strukturen) in mathematischen Problemen erweist sich während des Lösens mathematischer Probleme als großer Vorteil. So kann das Erkennen mathematischer Muster und Strukturen beispielsweise für das Zusammenfassen von Einzel- zu Sammelinformationen genutzt werden. Dieses *(i) Bilden von Superzeichen* auf Grundlage erkannter mathematischer Muster und Strukturen (Kießwetter, 2013), kann nach Nolte (2013b) zu einer Komplexitätsreduktion eines mathematischen Problems führen. Dadurch wird die Verlinkung verschiedener Sachverhalte erleichtert und die mathematisch Begabten können die Struktur eines mathematischen Problems auf noch höherem Niveau erfassen und tiefer in das Problemfeld einer Aufgabe eindringen (vgl. Fritzlar, 2010; Kießwetter, 2013). Des Weiteren wird das Erkennen mathematischer Muster und Strukturen in einer *(ii) Flexibilität und Reversibilität von Gedanken-*

gängen (vgl. Aßmus, 2010, 2013; Fritzlar, 2010; Kießwetter, 2013), einem *(iii) bereichsspezifischen, abstrakten, strukturierten und logischen Denken* (vgl. Bardy, 2013; Devlin, 2002; Wittmann, 2005, July) und dem (unbewussten) *(iv) Wechsel von Repräsentationsebenen und –formen* (vg) angewandt. Erkannte mathematische Muster und Strukturen können außerdem im *(v) räumlichen Vorstellungsvermögen* genutzt werden (vgl. Bardy, 2013; Käpnick, 1998) und nach Nolte (2013a) außerdem zur Änderung der Betrachtungsweisen eines mathematischen Problems beitragen. Häufig sind Grundschülerinnen und Grundschülern mathematische Sachverhalte oder Strategien zum Lösen ähnlicher Probleme bereits bekannt. Durch die erkannten Muster und Strukturen können *(vi) Analogien gebildet und ein Transfer* ermöglicht werden (vgl. Aßmus, 2013, Bardy, 2013; Käpnick, 1998; Selter, 2011).

Fundamental, um mathematische Muster und Strukturen zu erkennen, ist zum einen die Fähigkeit, Informationen zu strukturieren zum anderen die Fähigkeit, gegebene Materialien zu organisieren (vgl. Aßmus, 2013; Bardy, 2013; Fritzlar, 2013; Käpnick, 1998; Kießwetter, 2013; Selter, 2011). Neben den bereits beschriebenen Fähigkeiten zeigen mathematisch Begabte nach Käpnick (1998) noch *(vii) unterstützende Persönlichkeitseigenschaften* wie beispielsweise Anstrengungsbereitschaft, Leistungsmotivation, Freude am Problemlösen oder Beharrlichkeit für das Lösen mathematischer Probleme.

Nach qualitativen Beobachtungen von Deal und Wismer (2010) sowie McAllister und Plourde (2008) können sich mathematische Fähigkeiten asynchron entwickeln. Die überdurchschnittliche Entwicklung einer oder mehrerer der zuvor beschriebenen mathematischen Fähigkeiten kann mit einer (unter-)durchschnittlichen Entwicklung anderer Aspekte mathematischer Fähigkeiten verbunden sein. So zeigen mathematisch Begabte zwar vielfältige Stärken beim Lösen mathematischer Probleme, es können in Bezug auf erfolgreiches Mathematiktreiben und Problemlösen aber auch Schwächen beobachtet werden (Bardy & Hrzán, 2010; Bauersfeld, 2013; Bezold, 2012; Käpnick, 1998; Rotigel & Fello, 2004). Beispielsweise scheinen mathematisch Begabte häufig unscharf im Bilden und Begründen von Hypothesen (Bezold, 2012) oder im vollständigen oder strukturierten Notieren eines Lösungswegs zu sein (Bardy & Hrzán, 2010; Rotigel & Fello, 2004). Sie zeigen Schwierigkeiten bei der Versprachlichung erkannter mathematischer Muster und Strukturen (Käpnick, 1998) oder Fehler beim Übertragen von Lösungsstrategien auf Aufgaben mit veränderten Akzentuierungen der mathematischen Muster und Strukturen (Aßmus, 2010; 2013). Diese Schwächen können in der Arbeit mit mathematisch Begabten eine große Herausforderung darstellen. Vor allem, da die auf den ersten Blick homogene Gruppe in der Entwicklung der verschiedenen mathematischen Fähigkeiten sehr heterogen sein kann.

Kurskonzept

Das Kurskonzept zu „Fit für die Mathematik-Olympiade“ fokussiert die Stärken und Schwächen mathematisch begabter Grundschulkinder. So werden die zuvor beschriebenen überdurchschnittlichen Fähigkeiten als Potenzial genutzt; einerseits um Schwierigkeiten und Schwächen auszubalancieren, andererseits um die Wahrscheinlichkeit für das Ausschöpfen und Weiterentwickeln des Potenzials zu erhöhen. Der Kurs umfasst zehn Doppelstunden, wovon in acht Einheiten jeweils ein Modul bearbeitet wird und zwei Doppelstunden zur Teilnahme an der Mathematik-Olympiade eingeplant sind. Um den Teilnehmerinnen und Teilnehmern erfolgreiches Abschneiden bei der Mathematik-Olympiade zu ermöglichen, basieren die im Kurs bearbeiteten Probleme auf den Ansprüchen der Aufgaben früherer Mathematik-Olympiaden. Die Aufgaben basieren auf den bereits genannten Typen (Tabelle 5.1) und fordern für die erfolgreiche Bearbeitung ein breites Repertoire mathematischer Fähigkeiten. Da sich die Gewichtung der zum Lösen benötigten mathematischen Fähigkeiten (siehe 5.3.1) von Aufgabe zu Aufgabe unterscheiden, wird neben einer Vorbereitung auf die Mathematik-Olympiade eine ganzheitliche Förderung der Mathematikkompetenzen erreicht.

Durch den Einsatz prozessorientierter offener Aufgaben wird das Entdecken von mathematischen Mustern und Strukturen auf verschiedenen (Repräsentations-)Ebenen (z.B. figural oder arithmetisch) ermöglicht. Durch den Aufbau der einzelnen Kursstunden sowie durch die Arbeit in Kleingruppen müssen Lösungsideen und Hypothesen in Worte gefasst werden. Das Notieren gefundener Lösungen als fester Bestandteil einer jeden Kurssitzung soll die Teilnehmerinnen und Teilnehmer optimal auf die Wettbewerbsteilnahme vorbereiten.

Offene Aufgaben. Um den Prozesscharakter der Mathematik zu betonen, sind alle Kurseinheiten charakterisiert durch das Bearbeiten von möglichst offenen Problemstellungen. Diese erlauben durch die offenen Fragestellungen Lösungsansätze und –möglichkeiten auf verschiedenen mathematischen Niveaus und ermöglichen dadurch automatisch eine Differenzierung. Dies bedeutet, dass die Aufgaben je nach Entwicklung der mathematischen Fähigkeiten auf unterschiedlichen Ebenen gelöst werden können, zum Beispiel Lösen durch Abzählen oder durch Verwendung erster algebraischer Kenntnisse. Damit wird die Heterogenität der Teilnehmerinnen und Teilnehmer als Chance genutzt. Nach Nolte (2013b) zeigen mathematisch Begabte eine Vorliebe für reizvolle Probleme. Sie sollten daher an Aufgaben arbeiten, die inhaltlich bekannten (curricularen) (Schul-)Stoff neu verknüpfen, bereichern und vertiefen (Förster & Grohmann, 2013; Käpnick, 2010; Nolte, 2013a). Dadurch soll eine herausfordernde aber nicht überfordernde Situation für die Kinder geschaffen werden (Bardy & Hrzán, 2010). Die Problemlöseaufgaben bieten – wie Bardy und Hrzán (2010) sowie Fritzlar (2013) empfehlen –

Gelegenheiten zu produktiver Eigentätigkeit, eigenen kreativen und phantasievolle Entdeckungen und zur Erweiterung der eigenen heuristischen Problemlösestrategien. Damit knüpfen die Problemlöseaufgaben an die verschiedenen Facetten des Nutzens mathematischer Muster und Strukturen wie beispielsweise der Analogiebildung und den Transfer oder den Wechsel von Repräsentationsebenen an.

Die Aufgaben für den Kurs „Fit für die Mathematik-Olympiade" wurden alle eigens für den Kurs entwickelt und auf Basis zahlreicher Kursdurchführungen optimiert. Sie bieten den mathematisch Begabten die Möglichkeit, Fragestellungen mit variabler Tiefe der mathematischen Muster und Strukturen zu bearbeiten. Somit lassen die Aufgaben Raum für individuelle Lösungswege und verschiedene Lösungsansätze. Den Teilnehmerinnen und Teilnehmern wird dadurch eine aktiv-forschende eigenproduktive Tätigkeit ermöglicht (Förster & Grohmann, 2013; Nolte, 2013b), in deren Verlauf mathematische Muster, Strukturen und regelhafte Zusammenhänge von bekannten schulischen Inhalten erkannt und übertragen, Vermutungen entwickelt und mit eigenen Lösungsstrategien verknüpft werden können (Förster & Grohmann, 2013; Käpnick, 2010; Nolte, 2013b; Rosebrock, 2013; Walther, 2011).

Methodisch-didaktische Umsetzung. Methodisch orientiert sich der Aufbau der einzelnen Kurseinheiten am 3-Phasen-Unterrichtsmodell nach Bezold (2012), das sich in eine Ich-, eine Du- und eine Wir-Phase gliedert und angelehnt ist an Gallin and Ruf (1995; 1999). Durch dieses Unterrichtsmodell werden die prozessbezogenen Tätigkeiten des Mathematiktreibens wie Problemlösen, Kommunizieren, Argumentieren, Modellieren und Darstellen gefördert (Walther, 2011). Jede inhaltliche Doppelstunde folgt dabei schematisch dem gleichen Ablauf (vgl. Abb. 2.1):

Spielerischer Einstieg	Theorie-einheit	Ich-Phase	Du-Phase	Du/Wir-Phase	Wir-Phase	Zusammen-fassung	Spielerischer Abschluss

Abbildung 1: Schematischer Aufbau einer Kurssitzung (nicht maßstabsgetreu)

Um eine vertrauensvolle Atmosphäre bei gemeinsamen Interessen der Teilnehmerinnen und Teilnehmer zu schaffen, wird der Einstieg und auch der Abschluss jeder Kurseinheit dazu genutzt, die lose Gruppe bekannt zu machen und mit kleinen *mathematischen Spielen* eine vertrauensvolle Atmosphäre zu schaffen (Nolte & Pamperien, 2013). Eine *Theorieeinheit* wird dazu genutzt, Vorkenntnisse der Teilnehmer durch das Wiederholen spezieller Inhalte auf einen vergleichbaren Stand zu bringen und verschiedene Lösungsstrategien kennen zu lernen. Der Hauptteil eines jeden Moduls ist aufgebaut nach dem Phasenmodell (vgl. Abb. 2.2), das die Teilnehmer u.a. zur Kommunikation über Mathematik und zum Nutzen mathematischer

Begründungen für gefundene Lösungen anregen soll.

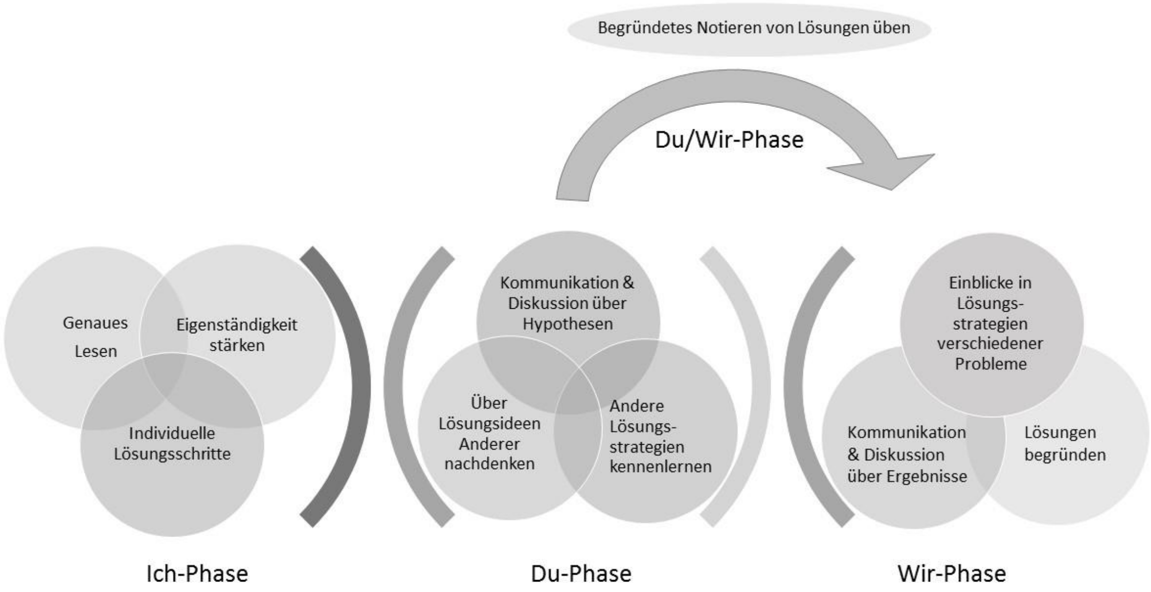


Abbildung 2: Das Phasenmodell der Hauptphase jedes Moduls

In der *Ich-Phase* (links visualisiert) erhalten zwei bis maximal drei Teilnehmerinnen und Teilnehmer des Kurses das gleiche Arbeitsblatt mit dem gleichen mathematischen Problem. Um selbstständiges Arbeiten und die Entwicklung eigener Lösungsideen auch im Hinblick auf die Teilnahme an einem Individualwettbewerb zu fördern, bearbeiten die Teilnehmerinnen und Teilnehmern das Problem einige Zeit alleine. Hier können die eigenen Fähigkeiten im Erkennen und Nutzen mathematischer Muster und Strukturen, eine neue Selbstständigkeit sowie individuelle Stärken entdeckt und genutzt werden (vgl. Bezold, 2012; Walther, 2011).

Die Ich-Phase geht fließend in die *Du-Phase* (mittig in Abb. 2) über. Die Lösung des Problems wird nun in der Kleingruppe gesucht. Einerseits müssen damit die eigenen Lösungsansätze aus der Ich-Phase begründet und nachvollziehbar dargestellt sowie die eigenen Gedanken versprachlicht werden. Auf der anderen Seite müssen sich die Kinder in die Gedanken eines Partners hinein denken, um diese zu verstehen, was häufig zu einem kritischen Hinterfragen der eigenen Ideen führt. Dadurch müssen die Teilnehmerinnen und Teilnehmern gebildete Hypothesen sowie generierte Lösungsansätze und –ideen aus der Ich-Phase verständlich verbalisieren. Die Du-Phase bietet den Vorteil, dass die prozessbezogenen Tätigkeiten – hauptsächlich das Bilden und Suchen von Hypothesen und Begründungen – mit Gleichaltringen gemeinsam durch Einbringen individueller Stärken und Schwächen im kooperativen Setting geübt werden können. Außerdem können beim kooperativen Arbeiten das bewegliche Denken und die Ideenfülle eines jeden Individuums zur Geltung kommen und auch zurückhaltende Teilnehmerinnen und Teilnehmern werden zum Verbalisieren erkannter mathematischer Muster und Strukturen animiert. Die Kleingruppenarbeit bietet den Vorteil, dass eigene Ideen in geschützter Atmosphäre im Kreise Gleichgesinnter eingebracht werden können. So kann das Gefühl der sozialen Eingebundenheit erlebt werden. In Summe trägt das Arbeiten in der Kleingruppe neben einer Entwicklung der allgemeinen mathematischen Kompetenzen (insbesondere das Problemlösen bei Notwendigkeit des Kommunizierens und Argumentierens) auch zur Verbalisierung der eigenen Gedanken und zur Stärkung der Sozialkompetenz bei (Bardy & Hrzán, 2010; Bezold, 2012; Walther, 2011).

Nach erfolgreichem Bearbeiten der Problemaufgabe schließt sich an die Du-Phase eine Vorbereitungsphase auf die Wir-Phase an. Die Teilnehmerinnen und Teilnehmern werden in dieser *Du/Wir*-Phase (siehe Pfeil in Abb. 2) angeleitet, ihre Lösung für den Austausch mit anderen Kursteilnehmerinnen und -teilnehmern zu verschriftlichen. Dies stellt im Hinblick auf die Mathematik-Olympiade einen wichtigen Baustein dar, denn eine genaue, sorgfältige, exakte und begründete Notation wird damit geübt.

Während die vorangegangenen Phasen fließend ineinander übergehen und dem individuellen Bearbeitungstempo der Kleingruppen angepasst sind, nimmt die *Wir-Phase* die letzten 15-20 Minuten, vor einem spielerischen Abschluss, ein. Die Kleingruppen präsentieren sich in dieser Phase gegenseitig ihre Aufgaben und Lösungen. Dabei müssen bei der Präsentation der eigenen Ergebnisse die individuellen Gedanken nochmals begründet und die Lösungsschritte verständlich dargestellt werden. Dies soll erneut die Kommunikations- und Argumentationskompetenz stärken. Bei den Präsentationen anderer Gruppen müssen die Gedanken, Hypothesen, Lösungswege und -schritte anderer verstanden werden. Dabei können verschiedene Denkweisen und Problemlösestrategien sowie Hilfsmittel zur Bearbeitung von mathematischen Problemen kennengelernt werden.

Ist eine Kleingruppe sehr schnell in der Bearbeitung ihres Problems, so sind weitere mathematische Probleme verfügbar, die die Kinder in diesem Fall bearbeiten können. Die Erweiterungsaufgaben sind orientiert an den mathematischen Inhalten der Module. Sie können entweder ergänzend oder wiederholend eingesetzt werden.

Überprüfung der Effektivität des beschriebenen Programms

Der Kurs „Fit für die Mathematik-Olympiade“ wurde im Rahmen des Projekts Formative Evaluation der Hector-Kinderakademien (Golle, Herbein, Hasselhorn, & Trautwein, 2017) entwickelt und in einer ersten empirischen Studie (Schuljahr 2014/15) an sechs Hector-Kinderakademien durchgeführt. Um die Effektivität des Trainings zu überprüfen, wurde ein Kontrollgruppendesign mit Messwiederholung gewählt. Insgesamt nahmen 201 Kinder an der Untersuchung teil, 50 Kinder in der Trainingsgruppe und 151 Kinder in der Kontrollgruppe. Die Kontrollgruppe bestand aus SuS, die nicht für die Hector-Kinderakademien nominiert wurden. Die beiden Gruppen setzten sich zu vergleichbaren Teilen aus Dritt- und Viertklässlern zusammen. Die Erhebung der Daten erfolgte zu Beginn und zum Ende eines Schulhalbjahres, dies entsprach auch dem Beginn und dem Ende des Trainings.

Zu beiden Messzeitpunkten wurden die Mathematik-Kompetenz (DEMAT 2+, Krajewski, Liehm, & Schneider, 2004; DEMAT 3+, Roick, Gölitz, & Hasselhorn, 2004; DEMAT 4, Gölitz, Roick, & Hasselhorn, 2006), das mathematische Interesse, das Selbstkonzept in Mathematik sowie die figuralen sowie kristallinen kognitiven Fähigkeiten via BEFKI-short (Schroeders, Schipolowski, Zettler, Golle, & Wilhelm, 2016) erfasst. Zusätzlich wurden die Leistungsdaten während der Mathematik-Olympiade erhoben. Die Daten wurden mit Hilfe multipler linearer Regressionen ausgewertet. Um die Kurseffekte für mögliche Eingangsunterschiede zwischen den beiden Gruppen kontrollieren zu können, wurden die zum ersten Messzeitpunkt erhobenen Variablen in allen Analysemodellen berücksichtigt (Alter, Geschlecht, Mathematik-Kompetenz, kognitive Fähigkeiten, mathematisches Interesse und Selbstkonzept). Sowohl für die Dritt- als auch für die Viertklässler zeigte sich unter Kontrolle der Eingangsunterschiede ein signifikanter Interventionseffekt auf die Mathematik-Kompetenz und die Leistung in der Mathematik-Olympiade (Näheres siehe Kapitel 3).

Da die Ergebnisse darauf hinweisen, dass die entwickelte Intervention wirksam ist, wurde der Kurs in einer zweiten Studie (Schuljahr 2015/16) mit mehreren Kursleitern an zehn Hector-Kinderakademien erneut evaluiert. Im Vergleich zur ersten Studie sollte die Effektivität des Kurses auch bei einer größeren Anzahl von Kursleitern und mathematisch begabten und interessierten Kindern untersucht werden. In dieser Studie verwendeten wir daher ein so genanntes Warte-Kontrollgruppen-Design mit randomisierter Gruppenzuweisung. Sowohl in einer Prä- als auch in einer Posttestung wurden auch in dieser Studie kognitive Fähigkeiten, Mathematik-Kompetenz sowie mathematisches Selbstkonzept und Interesse in Mathematik erfasst. Ergänzend wurden die Leistungsdaten der Mathematik-Olympiade und die Fähigkeit zum Erkennen von Mustern und Strukturen mittels Matrizentests erhoben. Erste Ergebnisse dieser Untersuchung deuten auf positive Effekte des Kurses auf das Erkennen von Mustern und Strukturen sowie die Leistung in der Mathematik-Olympiade hin. Auf rein arithmetische Fähigkeiten scheint der Kurs keinen Einfluss zu haben. Damit deuten die Ergebnisse darauf hin, dass der Kurs weniger die inhaltsbezogenen mathematischen Kompetenzen, wie beispielsweise Rechnen, sondern eher die prozessbezogenen Kompetenzen, wie beispielsweise mathematisches Argumentieren, zu fördert (Näheres siehe Kapitel 4).

Ausblick: Schülerwettbewerbe in sich ändernden Schulumwelten

Basierend auf Aufgaben, die in der Regel von erfahrenen Experten eines Fach(bereich)s erarbeitet wurden, können SuS im Rahmen von Wettbewerben Einblicke in Themengebiete bekommen, die im normalen Curriculum fehlen oder nur kurz thematisiert werden. SuS können durch intensive Auseinandersetzung mit bestimmten Themen/-gebieten zu kleinen Experten für einen freiwillig gewählten (Fach-)bereich werden. Unter anderem berichten Oswald and colleagues (2005) von ehemaligen Wettbewerbsteilnehmerinnen und -teilnehmern, die sich mit steigendem Abstand zum Wettbewerb merklich für den Fachbereich des Wettbewerbs interessierten und einen Beeinflussung der Berufswahl durch den Wettbewerb angeben. Genau diese Rolle einer Wettbewerbsteilnahme erhoffen sich auch viele Lehrer/innen für ihre SuS und so sind häufig (Fach-)Lehrer/innen die Initiatoren, Motivatoren und Begleiter eines Wettbewerbs (vgl. Oswald et. al, 2005). Lehrkräfte bevorzugen dabei nach Bicknell (2008) und Oswald and colleagues (2005) Team-Wettbewerbe, da das Arbeiten im Team in vielen Disziplinen alltäglich ist und im Wettbewerb erlernt werden kann. Vor allem durch das gemeinsame Arbeiten an einer Herausforderung können alle Teilnehmerinnen und Teilnehmer in ihrer persönlichen Entwicklung und ihrem Verständnis für forschendes Lernen profitieren (Bicknell, 2008; Oswald et al., 2005).

Häufig werden Wettbewerbe vor allem in der Begabtenförderung – wie in Abschnitt 5.1 beschrieben – als Enrichment eingesetzt (Bicknell, 2008; Holling et al., 2009; Oswald et al., 2005). Basierend auf Aufgaben, die seitens der Wettbewerbsorganisatoren auf Grundlage jahrelanger Expertise erarbeitet wurden, bieten Schülerwettbewerbe Möglichkeiten, Begabten heraufordernde Aufgaben zukommen zu lassen. Ozturk und Debelak (2008a, 2008b) schlagen aber außerdem vor, Schülerwettbewerbe als Mittel zur Differenzierung auch im normalen Unterricht einzusetzen. So könnten Wettbewerbe einen Beitrag zu einer begabungsfreundlichen, differenzierenden Lernkultur innerhalb eines Schulbetriebs leisten.

Dies würde die Möglichkeit eröffnen, vielen SuS Einblicke in Curriculum-ergänzenden

Themenbereiche zu geben (vgl. Peters & Sieve, 2013). Für eine breite Partizipation an Schülerwettbewerben als außer und innerunterrichtliche Angebote bräuchte es allerding eine breite Anerkennung von akademischen Schülerwettbewerben in Schulen. In diesem Zusammenhang legt die Schulpolitik zwar indirekt die Grundlage für die Implementierung eines Wettbewerbs (Oswald et al., 2005). Ein konkreter Wettbewerb muss aber hauptsächlich aber auch von (Fach-)Lehrerinnen akzeptiert werden (Bicknell, 2008).

Eine besondere Rolle könnte hier die Ganztagsschule spielen. Denn vor allem im Wandel zur Ganztagsschule könnten Schülerwettbewerbe als leistungsdifferenziertes Angebot noch systematischer etabliert werden. Eine Einbettung eines Schülerwettbewerbs in den Ganztagsbetrieb – idealerweise in Kombination mit einem begleitenden pädagogischen Angebot – ermöglicht vielen SuS die Teilnahme an Wettbewerben. Zusätzlich können Vorbereitungskurse – wie das vorgestellte Programm „Fit für die Mathematik-Olympiade" – Bestandteil des Ganztagsangebots werden und so die fördernde Wirkung von Wettbewerben noch verstärken (Fauser et al., 2007; Oswald et al., 2005; Ozturk, & Debelak, 2008a). So können sich die verschiedenen Funktionen von Schülerwettbewerben – Enrichment und Differenzierung, Zusammenarbeit mit Peers, kompetitive Umwelt, Motivatoren, Identifikation und Diagnose – ergänzen und zu einer individuellen Förderung von SuS beitragen.

References

Aßmus, D. (2010). Merkmale und Besonderheiten mathematisch potentiell begabter Zweitklässler: Ergebnisse einer empirischen Untersuchung. [Characteristics and specifics of prospective mathematically gifted secound graders: Results of an empricial study]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder. Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 59–69). Berlin, Münster: Lit.

Aßmus, D. (2013). Fähigkeiten im Umkehren von Gedankengängen bei potentiell mathematisch begabten Grundschulkindern [Skills for reverse thoughts in prospective mathematically gifted elementary school students]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 45–66). Kronach / Berlin: Mildenberger.

Bardy, P. (2013). *Mathematisch begabte Grundschulkinder: Diagnostik und Förderung*. *Mathematik Primar- und Sekundarstufe I + II*. Berlin, Heidelberg: Springer Spektrum.

Bardy, P., & Hrzán, J. (2010). *Aufgaben für kleine Mathematiker: Mit ausführlichen Lösungen und didaktischen Hinweisen* [Tasks for litte mathematicans. Including detailed solutions and methodically hints] (3. Auflage). *Aulis Schatztruhe für die Grundschule*. Köln: Aulis Verlag.

Bauersfeld, H. (2013). Die Bielefelder Förderansätze. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 17–26). Offenburg: Mildenberger.

Bezold, A. (2012). Förderung von Argumentationskompetenzen auf der Grundlage von Forscheraufgaben. [Fostering the process of justification using research tasks.]. *mathematica didactica*. (35), 73–103.

Bicknell, B. (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom*, *13*(4), 16–20. Retrieved from http://files.eric.ed.gov/fulltext/EJ824763.pdf

Campbell, J. R., & Verna, M. A. (2010). Academic Competitions Serve the US National Interests. *AERA Online Submission*. Retrieved from http://files.eric.ed.gov/fulltext/ED509402.pdf

Campbell, J. R., & Walberg, H. J. (2010). Olympiad Studies: Competitions Provide Alternatives to Developing Talents That Serve National Interests. *Roeper Review*, *33*(1), 8–17. https://doi.org/10.1080/02783193.2011.530202

Deal, L. J., & Wismer, M. G. (2010). NCTM Principles and Standards for Mathematically Talented Students. *Gifted Child Today*, *33*(3), 55–65. https://doi.org/10.1177/107621751003300313

Devlin, K. (2002). *Muster der Mathematik: Ordnungsgesetze des Geistes und der Natur* [Patterns of Mathematics. Laws of arrangements for spirit and nature] (4. Auflage). Heidelberg: Spektrum.

Fauser, P., Messner, R., Beutel, W., & Tetzlaff, S. (2007). *Fordern und fördern: Was Schülerwettbewerbe leisten* [Fosteting and challenging. What academic competitions provide]: Ed. Körber-Stiftung.

Förster, F., & Grohmann, W. (2013). Geöffnete Aufgabensequenzen zur Begabtenförderung im Mathematikunterricht [Open taks to promote gifted students in mathematics education]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 111–125). Kronach / Berlin: Mildenberger.

Fritzlar, T. (2010). Begabung und Expertise. Eine mathematikdidaktiche Perspektive [Giftedness and expertise. The viewpoint of mathematics didactics]. *mathematica didactica*, *33*, 113–140. Retrieved from URL http://mathdid.ph-freiburg.de/documents/md_2010/md_2010_Fritzlar_Begabung.pdf

Fritzlar, T. (2013). Mathematische Begabungen im Grundschulalter. Ein Überblich zu aktuellen mathematikdidaktischen Forschungsarbeiten [Mathematically giftedness in elementary school age. A review of reseach in didactis of mathematics]. *mathematica didactica*, *36*, 5–27.

Gallin, P., & Ruf, U. (1995). *Sprache und Mathematik: 1.-3. Schuljahr: Ich mache das so! Wie machst du es? Das machen wir ab* [Language and mathematics: 1st to 3rd year of schooling: I do it like this! How did you do it? We will do it!]: Interkantonale Lehrmittelzentrale.

Gölitz, D., Roick, T., & Hasselhorn, M. (2006). *DEMAT 4: Deutscher Mathematiktest für vierte Klassen* [German mathematics test for fourth graders]: Hogrefe.

Golle, J., Herbein, E., Hasselhorn, M., & Trautwein, U. (2017). Begabungs- und Talentförderung in der Grundschule durch Enrichment: Das Beispiel Hector-Kinderakademien [Promoting gifted and talented elementary school students via enrichment: Using the example Hector Children‘s Academy]. In Trautwein, U. & Hasselhorn, M. (Ed.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente.* (pp. 177–196). Göttingen: Hogrefe.

Heinrich, F. (2010). Defizitäre Verhaltensweisen beim Bearbeiten mathematischer Probleme [Deficit behaviors when solving mathematical problems]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder. Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 22–33). Berlin, Münster: Lit.

Holling, H., Preckel, F., Vock, M., Roßbach, H.-G., Baudson, T. G., & Kuger, S. (2009). *Begabte Kinder finden und fördern.: Ein Ratgeber für Eltern, Erzieherinnen und Erzieher, Lehrerinnen und Lehrer* [Finding and promoting gifted children. A guidebook for parents, kindergarden teachers and teachers.]. Bonn: Bundesministerium für Bildung und Forschung (BMBF).

Käpnick, F. (1998). *Mathematisch begabte Kinder: Modelle, empirische Studien und Förderungsprojekte für das Grundschulalter* [Mathematically gifted students: Modells, empirical studies and enrichment for elementary school students]: Lang.

Käpnick, F. (2010). „Mathe für kleine Asse“ - Das Münsteraner Konzept zur Förderung mathematisch begabter Kinder. [”Math for young whiz” - The Münster concept for the promotion of mathematically gifted children.]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder. Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 138–150). Berlin, Münster: Lit.

Käpnick, F. (2013). Intuition - ein häufiges Phänomen beim Problemlösen mathematisch begabter Grundschulkinder. [Intuition - A common phenomen in the problem solving of mathematically gifted elementary school students]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 77–93). Kronach / Berlin: Mildenberger.

Kießwetter, K. (2013). Können auch Grundschüler schon im eigentlichen Sinne mathematisch agieren? [Can elementary school students do mathematics?]. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 128–153). Offenburg: Mildenberger.

Krajewski, K., Liehm, S., & Schneider, W. (2004). *DEMAT 2+: Deutscher Mathematiktest für zweite Klassen* [German mathematics tests for second graders]. Weinheim: Beltz.

Lengfelder, A., & Heller, K. A. (2002). German Olympiad studies: Findings from a retrospective evaluation and from in-depth interviews: Where have all the gifted females gone. *Journal of Research in Education*, *12*(1), 86–92. Retrieved from http://www.olympiadprojects.com/v2/pubs_web/ch4_SS.pdf

McAllister, B. A., & Plourde, L. A. (2008). Enrichment Curriculum: Essential for mathematically gifted students. *Education*, *129*(1), 40–49. Retrieved from http://web.a.ebscohost.com/ehost/pdfviewer/pdfviewer?sid=2c0b2b92-484e-4295-8ceb-ae855b7aceed%40sessionmgr4009&vid=2&hid=4214

Nolte, M. (2013a). „Du Papa, die interessieren sich für das was ich denke!“ - Zur Arbeit mit mathematisch besonders begabten Grundschulkindern. [”Daddy, they are interested in what I think!” - About the work with prospective mathematically gifted elementary school students]. In T. Trautmann & W. Manke (Eds.), *Begabung, Individuum, Gesellschaft. Begabtenförderung als pädagogische und gesellschaftliche Herausforderung* (pp. 128–143). Weinheim: Beltz Juventa.

Nolte, M. (2013b). Zum Erkennen und Nutzen von Mustern und Strukturen in Problemlöseprozessen [Recognizing and Using pattern and sturctures while problem solving]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 11–24). Kronach / Berlin: Mildenberger.

Nolte, M., & Pamperien, K. (2013). Besondere mathematische Begabung im Grundschulalter - ein Forschungs- und Förderprojekt [Mathematically giftedness in elementary school age - a project of reseach and fostering]. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 70–72). Offenburg: Mildenberger.

Olson, S. (2005). *Count down: Six kids vie for glory at the world's toughest math competition*. Boston: Houghton Mifflin. Retrieved from http://www.loc.gov/catdir/samples/hm051/2003056897.html

Oswald, F., Hanisch, G., & Hager, G. (2005). *Wettbewerbe und“ Olympiaden“: Impulse zur (Selbst)-Identifikation von Begabungen* [Academic competitions and Olympiads. Looking for talents]: LIT-Verlag.

Ozturk, M. A., & Debelak, C. (2008a). Academic competitions as tools for differentiation in middle school. *Gifted Child Today*, *31*(3), 47–53. Retrieved from http://files.eric.ed.gov/fulltext/EJ803366.pdf

Ozturk, M. A., & Debelak, C. (2008b). Affective Benefits from Academic Competitions for Middle School Gifted Students. *Gifted Child Today*, *31*(2), 48–53. https://doi.org/10.4219/gct-2008-758

Peters, H., & Sieve, B. (2013). Fordern und Fördern mit Wettbewerben - Schülerwettbewerbe in den Naturwissenschaften mit Bezug zur Chemie. [Challenge and foster with competition - academic competitions in science related to chemistry]. *Naturwissenschaften im Unterricht – Chemie*, *24*(136), 2–9.

Rebholz, F. (2013). *Entwicklung und außerschulische Förderung mathematischer Fähigkeiten besonders begabter und hochbegabter Grundschulkinder im Rahmen der Hector-Kinderakademie* [Development and extracurricular promotion of mathematical skills of gifted elementary school children in the context of the Hector Children's Academy Program] (Unveröffentlichte Wissenschaftliche Arbeit für die Zulassung zur Wissenschaftlichen Prüfung für das Lehramt am Gymnasium). Eberhard Karls Universität, Tübingen.

Roick, T., Gölitz, D., & Hasselhorn, M. (2004). *Deutscher Mathematiktest für dritte Klassen (DEMAT 3+)* [German mathematics tests for third graders]. Göttingen: Beltz Test.

Rosebrock, S. (2013). Kreatives Arbeiten mit Zerlegungen [Creative work with decompositions]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 168–181). Kronach / Berlin: Mildenberger.

Rotigel, J. V., & Fello, S. (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today*, *27*(4), 46–51.

Ruf, U., & Gallin, P. (1999). *Dialogisches Lernen in Sprache und Mathematik* [Dialogic learning in language and mathematics]: Kallmeyer.

Schroeders, U., Schipolowski, S., Zettler, I., Golle, J., & Wilhelm, O. (2016). Do the smart get smarter? Development of fluid and crystallized intelligence in 3rd grade. *Intelligence*, *59*, 84–95. https://doi.org/10.1016/j.intell.2016.08.003

Selter, C. (2011). „Ich mark Mate" - Leitideen und Beispiele für interessenförderlichen Unterricht [Ideas and examples for education that fosters interest in mathematics]. In R. Demuth, G. Walther, & M. Prenzel (Eds.), *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* (pp. 131–139). Seelze: Friedrich.

Walther, G. (2011). Die Entwicklung allgemeiner mathematischer Kompetenz fördern. [Foster the development of general mathematical competences]. In R. Demuth, G. Walther, & M. Prenzel (Eds.), *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* (pp. 15–23). Seelze: Friedrich.

Wittmann, C. (2005, July). *Mathematics as the science of patterns–a guideline for developing mathematics education from early childhood to adulthood*. Plenary Lecture at International

Colloquium'Mathematical learning from Early Childhood to Adulthood, Belgium, Mons. Retrieved from http://mathinfo.unistra.fr/fileadmin/upload/IREM/Publications/Annales_didactique/vol_11_et_suppl/adsc11supplweb_wittmaneng.pdf

3

Getting Fit for the Mathematical Olympiad: Positive Effects on Achievement and Motivation?

Rebholz, F., Golle, J., Tibus, M., Herbein, E., Moeller, K., & Trautwein, U. (2017). *Getting Fit for the Mathematical Olympiad: Positive Effects on Achievement and Motivation?* Manuscript under perparation.

This research was supported by a grant from the Hector Foundation II to Ulrich Trautwein. Franziska Rebholz is a doctoral student at the LEAD Graduate School & Research Network [GSC 1028], which is funded by the Excellence Initiative of the German federal and state governments.

Abstract

There are numerous academic competitions (e.g., “Academic Olympiads”) and corresponding trainings around the world that are believed to foster students’ domain-specific achievement and motivation. Although academic competitions are ideal settings in which to study learning processes and outcomes, more empirical studies on the effectiveness of academic competitions and their trainings are needed to determine whether the trainings work and for whom they work. Therefore, we developed and evaluated a math training for preparing third- and fourth-grade students for the German Mathematical Olympiad. In ten 90-min sessions, the training was aimed at fostering process-based mathematical competences (e.g., problem solving). Its effectiveness was evaluated in a quasi-experimental pre- and posttest design (N = 201 students). Results indicated positive training effects on mathematical achievement, positive effects on fourth graders’ task-specific interest in mathematics, and differential effects for math self-concept. Thus, the role of social comparison processes in such trainings for elementary school students is discussed.

Keywords: mathematics competition, enrichment, elementary school children, math training, challenging tasks, domain-specific self-concept

Getting Fit for the Mathematical Olympiad: Positive Effects on Achievement and Motivation?

Every year an increasing number of students around the world participate in regional, national, and international academic competitions in different domains such as mathematics, sciences, history, or languages (Campbell et al., 2000; Forrester, 2010). The underlying assumption behind such academic competitions is that they foster students' competence and motivation (Campbell & Walberg, 2010; Lengfelder & Heller, 2002; Riley & Karnes, 1998). However, the competitive environments of academic competitions (Bicknell, 2008; Wirt, 2011) can also have negative effects on student motivation (e.g., decreasing self-concept; Marsh & Parker, 1984) and can trigger stress or self-doubt (Clinkenbeard, 1989). To prevent negative and to emphasize positive effects of academic competitions, some authors have suggested that trainings can prepare students for such academic competitions (e.g., Cropper, 1998; Ozturk & Debelak, 2008a). Although, from a theoretical point of view, positive influences of such trainings on students' competence and motivation seem plausible, there have been—to the best of our knowledge—no empirical studies that have systematically investigated this assumption.

Therefore, we developed a training to prepare elementary school students for the German Mathematical Olympiad, a national math competition. We evaluated the effects of the training on a sample of third- and fourth-grade students. Before describing the details of the training, we will briefly review the literature on influences of academic competitions and trainings on students' competence and motivation.

Influences of Academic Competitions on Students' Competence and Motivation

Academic competitions provide learning opportunities that are neglected in regular curricula. Therefore, academic competitions offer challenging tasks for targeting students who belong to the group of high achievers who are already able to successfully solve curriculum-based tasks (Bicknell, 2008; Ozturk & Debelak, 2008a, 2008b; Riley & Karnes, 1998). As such, the increase in learning opportunities is supposed to positively influence students' competence in the same domain (cf. Diezmann & Watters, 2001). In fact, there is some—albeit rather fragmentary—empirical support for this assumption. For instance, using retrospective interviews of successful participants of academic competitions, Campbell and Walberg (2010) reported that a very high percentage (52%) of such successful participants of academic Olympiads in the US achieved a PhD degree later in their academic career. In addition, in a German sample, Lengfelder and Heller (2002) reported that over 70% of successful participants of academic competitions were in the top 10 of students in their last year of the German Gymnasium (highest academic track in Germany).

When it comes to the effects of participating in academic competitions on motivation, theoretical predictions are somewhat more complex, and there is a clear need for more studies. Commonly, academic competitions are seen as an opportunity for students—especially high-achieving students—to demonstrate their skills and to experience challenges beyond the requirements of the standard curriculum (Ozturk & Debelak, 2008a, 2008b; Wirt, 2011). Such competitions and being chosen by teachers or parents to participate in such an academic competition might reinforce or further increase students' competence beliefs (see Dai & Rinn, 2008). Hence, it is commonly assumed that motivation for the respective subject domain should increase after a student participates in a competition (cf. Forrester, 2010; Wirt, 2011). However, from a theoretical point of view, academic competitions may also have negative effects. First, students may experience failure when confronted with a challenging task, and failure is known to affect academic self-concept (e.g., Trautwein & Möller, 2016). Second, academic competitions make participants compete with other high-achieving students, a situation they may never have encountered in their regular classes (Ozturk & Debelak, 2008b). More specifically, participation in a competition may change the "frame of reference" for students' self-evaluation (i.e., self-concept; see Shavelson et al., 1976). This has the potential to positively affect students' self-concept (see Dai & Rinn, 2008) when students are aware of the fact that it was their high achievement that allowed them to become a member of the selected group (also see Trautwein, Köller, Lüdtke, & Baumert, 2005). However, frame-of-reference effects might also negatively impact student motivation. In the literature, this effect is known as the Big-Fish-Little-Pond effect (BFLPE; Marsh, 1987; Marsh & Parker, 1984): Of two students with the same individual ability, academic self-concept will likely be higher in the student who is placed in a low-achieving class compared with the student who is placed in a high-achieving class. In other words, high-achieving classmates lead to negative social comparison processes that are typically stronger than potentially positive effects of being part of a selected group of students (see Marsh et al., 2008). In fact, negative frame-of-reference effects have been documented for academic self-concept but also for student interest, joy, and attainment (e.g., Marsh et al., 2008; Trautwein et al., 2006). Looking at expectancy-value theory of achievement motivation (e.g., Eccles, 1983), the latter factors are summarized into value beliefs; besides competence beliefs (i.e., self-concept), value beliefs (i.e., interest, attainment value, utility value and cost) comprise the second factor that drives academic motivation according to this theory. However, although there is a broad literature on the BFLPE and its impact on self-concept and value beliefs, we do not know of any study that has explored the BFLPE in the context of academic competitions.

The Role of Pedagogical Trainings in Fostering Competence and Motivation

To bolster the assumed positive effects of academic competitions on achievement and to prevent potentially negative effects of failure/unfavorable social comparison processes in academic competitions on students' motivation, careful preparation and an introduction to an academic competition may be useful (Cropper, 1998; Ozturk & Debelak, 2008a). In fact, offering a training for a specific academic competition as part of an enrichment program is common practice (for the Academic Olympiads, see e.g., Urhahne, Ho, Parchmann, & Nick, 2012). Assuming that such trainings increase the number of learning opportunities in the respective domain even more than mere participation in an academic competition, any positive effects of academic competitions on domain-specific competences can be expected to be intensified.

Usually, students are selected to participate in such a training on the basis of teacher nominations or by successfully passing previous selection rounds (see the qualification process for the International Olympiads; for an overview, see e.g., http://olympiads.win.tue.nl/). Thus, comparable to academic competitions, the average ability level of students in such trainings is usually higher than the class average as well (cf. Bicknell, 2008; Ozturk & Debelak, 2008b; Riley & Karnes, 1998). Therefore, considering the BFLPE described above, the question that arises is how participating in a training for an academic competition influences students' motivation: Is there a risk that student motivation will decrease when they experience a learning environment characterized by higher academic achievement? As such, one of the explicit aims of any such training should be to prevent negative effects by stabilizing students' motivation (i.e., self-concept and value beliefs), for example, by implementing an individualized teacher frame of reference instead of highlighting social comparisons (see Lüdtke, Köller, Marsh, & Trautwein, 2005).

The Present Study

For a number of reasons, trainings for academic competitions are an ideal setting in which to study learning processes and outcomes. For instance, there is typically a clearly defined achievement outcome (usually performance in the respective competition). Given an appropriate design, this allows researchers to study the effects of various predictor variables on this specific outcome. A similarly interesting question is the evidence (or lack thereof) of participation in academic competition trainings on broader achievement outcomes. More precisely, does participating in a training for the Mathematical Olympiad show transfer effects to classroom-based assessments of (mathematical) competence? In addition, training for an academic competition comes with a change in learning environment (e.g., group membership). This means that potential effects of variations in the learning setting on various processes and outcomes can be investigated. Hence, academic competitions with corresponding trainings provide the perfect opportunity for studying effects of specific learning environments (e.g., a specific competition or training). Thus, this provides the opportunity to contribute to the literature more generally by testing theoretical predictions about specific characteristics of the learning environment.

All the more surprising is the lack of studies that have employed the academic competition setting. Whereas some studies have examined expectations, experiences, or educational pathways of successful participants of an academic competition (e.g., Campbell & Walberg, 2010; Lengfelder & Heller, 2002; Wirt, 2011), effects of academic competitions on students' domain-specific competence and motivation have largely been neglected so far. Moreover, to the best of our knowledge, no studies have analyzed the theoretically plausible effects of academic competitions in combination with a corresponding pedagogical training.

Hence, the present study is the first study we know of to systematically examine the effects of a training for a mathematics competition on (a) performance in the competition, (b) mathematical competences in general, and (c) students' motivation (i.e., self-concept and value beliefs).

The pedagogical training

We developed and evaluated a coherent mathematical training specifically targeting the abilities and challenges of high-achieving elementary school children who were participating in the Mathematical Olympiad. Content and didactical approaches of the training were based on theory as well as empirical findings in mathematically gifted education because high-achieving students should find it easy to solve curricular-based tasks (for more information, see e.g., Diezmann & Watters, 2001; Koshy et al., 2009; Leikin, 2010; McAllister & Plourde, 2008; Rotigel & Fello, 2004). Based on this literature, the focus of the training was on solving challenging mathematical problems. The implemented problems were not challenging because the contents were difficult (e.g., calculations did not involve numbers greater than 100). The problems were challenging because they were comprised of open problems that allowed different solution strategies and required problem solving, modeling, and reasoning. More specifically, the contents of the training were oriented toward the previous requirements of the (German) Mathematical Olympiad for elementary school students and involved, for example, logic problems, cryptograms, cubes, combinatorics, equation-based tasks, sequences (implemented in terms of towers), or riddles. All tasks included in the training were custom made and allowed for several possible solutions (an example task used in the training is shown in Figure 1). Thus, original tasks from previous Mathematical Olympiads were not implemented in the training.

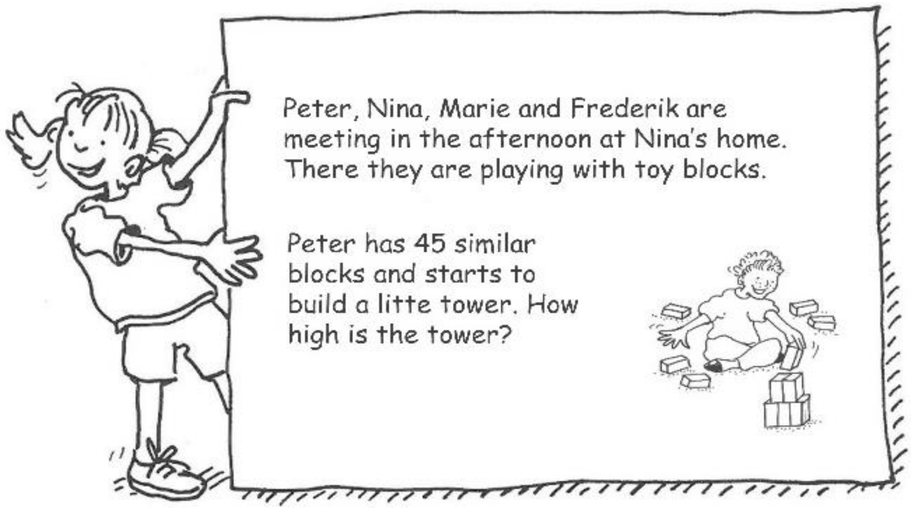


Figure 1. Typical task from the corresponding training program (translated version).

In contrast to the typically competitive setting of academic competitions, the training was based on cooperative learning to enhance motivation for mathematics and to antagonize the competitive setting (e.g., Johnson, 1990; Johnson & Johnson, 1994). Following a specific didactic-methodological model (illustrated in Figure 2, based on Bezold, 2012), students had to (a) attempt an individual solution, (b) discuss and explain these individual solutions in teams (ideally dyads), and (c) justify and discuss their solutions to a challenging mathematical problem. Toward the end of each session, they also had to (d) present the mathematical problem and the solutions produced thus far to other students who were not members of their team.

The aims of components (a) to (d) were to get students accustomed to solving challenging tasks and to foster students' process-based mathematical competences (i.e., [mathematical] problem solving, modelling, and communicating about mathematics/arguing). Overall, the core of the training was comprised of the dyadic problem solving of challenging tasks that considered the requirements of the Mathematical Olympiad. Across the whole training, teachers created an atmosphere in which mistakes were treated as opportunities for learning and for providing individual feedback (using an individualized frame of reference).

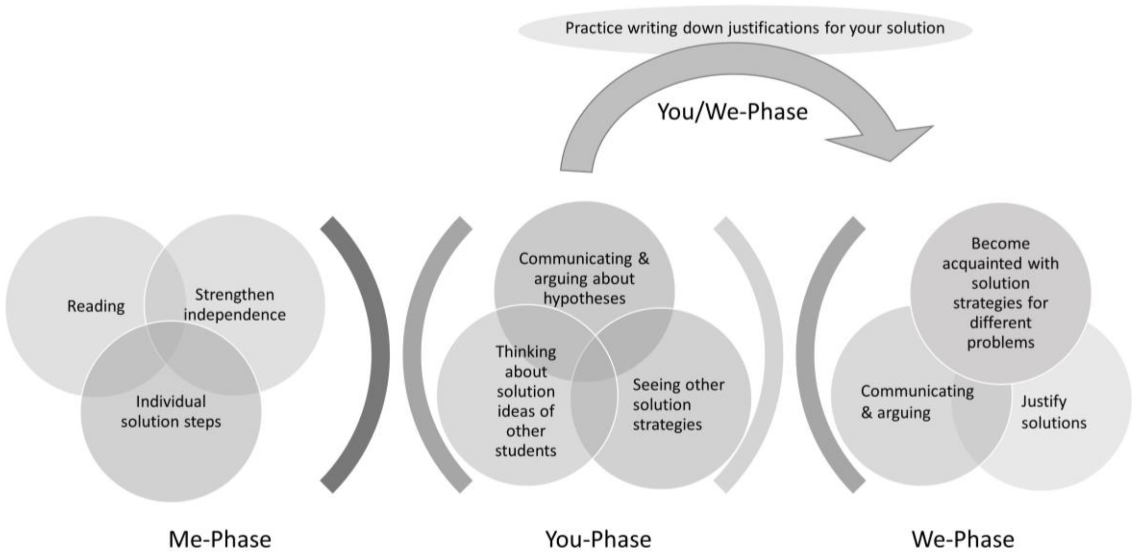


Figure 2. Schematic illustration of the didactic-methodological model used in the training (translated version from Rebholz & Golle, 2017, see Chapter 2).

Design and hypotheses

To evaluate the effectiveness of the training that was designed to prepare students for the Mathematical Olympiad, we used a quasi-experimental pre- and posttest design in a natural setting (Shadish et al., 2002). In addition to a group of children who attended the training, we implemented a control group consisting of students who did not attend the training but participated in the Mathematical Olympiad as well. We posed the following three hypotheses: First, we expected that children participating in the training would perform better in the respective academic competition (Mathematical Olympiad) as they were trained to solve challenging problems that contain contents typically used in the Mathematical Olympiad. Second, we hypothesized that children participating in the training would also improve mathematical competences that are not directly linked to the Mathematical Olympiad more strongly than the children in the control group, which would be indicative of transfer effects. Third, given that the training was offered to third and fourth graders who were taught together in one training group, we expected different effects of the training on motivational constructs (i.e., math self-concept and value beliefs) for the two age groups. On the one hand, students were nominated for the training. Thus, positive influences on motivational constructs could be expected. But, on the other hand, based on findings on reference group effects as described in the BFLPE literature (e.g., Marsh, 1987), different social comparison processes might be plausible for the two age groups. Whereas third graders would be placed in a learning environment with many possibilities for upward comparisons (i.e., with competent fourth graders), fourth graders would be more likely to have opportunities to make downward comparisons (i.e., with third graders) in the training—but perhaps also upward comparisons (i.e., with high-achieving fourth graders). Thus, we expected differential effects on students' self-concept depending on their grade level. Similar effects were expected for students' math value beliefs (Trautwein et al., 2006).

Method

The Training

The training was part of an enrichment program in the German state of Baden-Württemberg (Hector Children's Academy Program; HCAP) that is tailored to the 10% most gifted, talented, interested, and motivated elementary school children (for more information about the HCAP, see Rothenbusch et al., 2016).

The training included 10 modules, each taught in a 90-min session. The training was designed for small groups of six to 10 third and fourth graders. Six local sites of the HCAP participated in this study. The training was taught by different persons (mostly mathematics teachers) at each of these sites. To ensure treatment fidelity, instructors attended a 2-hr qualification session taught by the developer of the training and were given a scripted manual and master copies of all teaching materials (an example task used in the training is presented in Figure 1).

Sample

Data were collected from 201 elementary school children (58% male, age: M = 9.01, SD =.43) in Grade 3 (N = 110, 63 male) or Grade 4 (N = 91, 54 male). The training group comprised 50 children in Grades 3 and 4 [Grade 3: N = 26 (15 male); Grade 4: N = 24 (18 male)] who attended "Getting fit for the Mathematical Olympiad." Children in the training group were from different classes and schools and had been nominated for the extracurricular enrichment program by their teachers. They voluntarily participated in the training. Children in the control group were from 14 classes from six different elementary schools (six fourth-grade classes and eight third-grade classes) from schools that hosted a Hector Children's Academy. Children in the control group attended only standard curricular mathematics classes and did not participate in the training.

Design and Procedure

The training was offered at six local sites of the HCAP. The six courses were attended by five to 11 children and took place over a 5-month period (14 sessions from November 2014 to March 2015). The pre- and posttesting of the training group was integrated into the first two and the last two training sessions separately for each local site. Data were collected from the children in the control group during regular classes at comparable time points (see Figure 3).

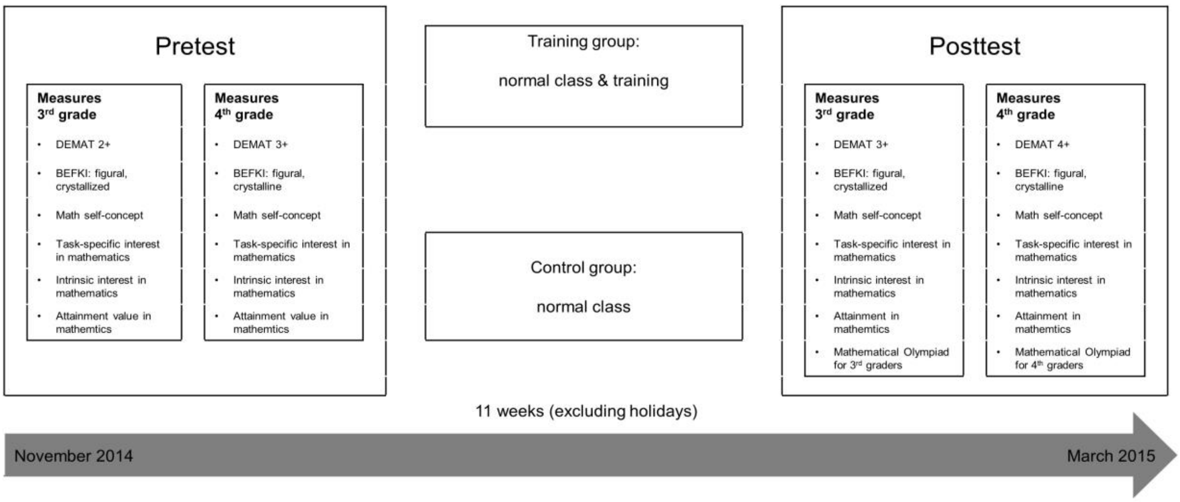


Figure 3. Quasi-experimental pre- and posttest design of the study and the implemented measures.

Trained research assistants and scientists administered the questionnaires and tests. The study was approved by the local ethics committee and local school authorities. Furthermore, parents provided written informed consent for their children's participation prior to the study. All participants took part in the 54th Mathematical Olympiad—the training group as part of the training and the control group during regular math classes.

Measures

Domain-specific achievement outcomes

All scales and corresponding descriptive statistics (including reliabilities) are displayed in Tables 1 and 2. An overview of the measures that were used is provided in Figure 4. We assessed *Performance in the Academic Competition* using the third (most difficult) level of the German *Mathematical Olympiad* for elementary school students (for more information about the competition, see Campbell & Walberg, 2010; Olson, 2005; www.imo-official.org, www.moems.org). The tasks in this academic competition were grade-level-specific and included (complex) word problems (an example task is shown in Figure 4).

> Four children have the same birthday. Anton, Benita, Carla, and Dean are turning 10, 12, 14 and 16 years old, respectively, in 2016.
>
> a) What will be the sum of the kids' ages in 2020?
> b) In which year will the kids' ages sum to 100?
> c) Could the kids' ages ever sum to 163? Justify!

Figure 4. Translated version of a typical task from the Mathematical Olympiad (task no. 550331, 54th Mathematical Olympiad 2015/16, third grade).

Tasks and scoring guidelines were provided by the German Mathematical Olympiad Association. Newly developed tasks were used for the Mathematical Olympiad every year, and these tasks were released by the Mathematical Olympiad Society after the competition. Hence, the tasks were not known to us when we developed the training. Tasks were scored according to the provided scoring guidelines. The dependent variables were z-standardized sum scores for each grade level.

We assessed mathematical competence at pre- and posttest[9] with the *German Mathematics Tests for Second, Third,* and *Fourth Grades* (i.e., pretest: DEMAT 2+, Krajewski et al., 2004, α_{T1} = .92; DEMAT 3+, Roick et al., 2004, α_{T1} = .91; posttest: DEMAT 3+, Roick et al., 2004, α_{T2} = .82; DEMAT 4, Gölitz et al., 2006, α_{T2}= .89). These instruments closely reflect the (German) core standards for the respective grade levels. Hence, different versions of the tests were necessary for the different grades at pre- and posttest. For all measures of mathematical competence, the dependent variable was the obtained sum score, z-standardized by grade level.

Motivational constructs

Competence beliefs were measured with a *math self-concept* scale (six items, e.g., "I'm good at everything that has to do with mathematics," α_{T1} = .87, α_{T2} = .92). Personal value beliefs were assessed with an *intrinsic interest in mathematics* scale (six items, e.g., "I enjoy everything that has to do with math," α_{T1} = .81, α_{T2} = .95) and an *attainment value* scale (three items, e.g., "Everything that has to do with math is important to me," α_{T1} = .70, α_{T2} = .69). In addition, a more activity-oriented form of interest was measured with a scale that asked for *task-specific interest* (three items, e.g., "I like to solve riddles and puzzles in computing magazines and booklets," α_{T1} = .77, α_{T2} = .88). All scales were based on previous instruments (Arens, Trautwein, & Hasselhorn, 2011; Bos, Buddeberg, & Lankes, 2005; Gaspard et al.,

[9] The pretest is abbreviated as T1 and the posttest as T2 in the following.

2015; Ramm, Adamsen, & Neubrand, 2006; Snow, 2011), and the respective items had to be answered on a 4-point scale (ranging from 1 = *not true* to 4 = *exactly*, visually represented with increasing symbol size of stars). Mean values for math self-concept and intrinsic and task-specific interest in mathematics as well as attainment value in mathematics were z-standardized at grade level and used as dependent variables (for more details, see Tables 1 and 2).

Covariates

General cognitive abilities were assessed with the short version of *Berlin Test of Fluid and Crystallized Intelligence* (*BEFKI*; Schroeders et al., 2016). This test included two subscales: figural (Versions A and B) and crystallized cognitive skills. The figural subscale consisted of 16 figural items in which sequences had to be continued twice ($\alpha_{T1, A}$ = .65, $\alpha_{T1, B}$ = .71 / $\alpha_{T2, A}$ = .68, $\alpha_{T2, B}$ = .78). The crystallized subscale required students to indicate the correct answer (out of four alternatives) to 16 questions about general knowledge, for example, “What’s Google?” (α_{T1} = .73, α_{T2} = .73). Again, dependent variables for figural and crystallized cognitive skills were z-standardized sum scores by grade level.

Statistical Analyses

All analyses were run separately by grade level. Group differences at pretest were analyzed by computing *t* tests for independent samples in R (R Core Team, 2015). The effectiveness of the training was evaluated with multiple linear regression analyses separated by grade level using the R package *lavaan* (R Core Team, 2015; Rosseel, 2012). All analyses used the robust maximum likelihood estimator that corrects standard errors for the non-normality of the variables (Rosseel, 2012). Predictors in our regression models were participation in the training or the control group (0 = *control*, 1 = *training*). The dependent variables consisted of performance in the Mathematical Olympiad (third level), mathematical competence, math self-concept, and value beliefs in mathematics. According to the standardization of the dependent variables, the multiple regression coefficient of the group variable indicated the standardized difference between the training and control groups at posttest while pretest performance in mathematical competence, (fluid and crystallized) intelligence, math self-concept, value beliefs in mathematics, and age were controlled for. We controlled for pretest performance on these variables to account for initial differences between the training and control groups. Differential effects between third and fourth graders for each dependent variable were analyzed by testing the estimated differences between the training effects of the two grades against zero.

Missing Data

In our study, the percentage of missing values amounted to 5% at pretest (30% of the missing data occurred in the training group and 70% in the control group) and 12% at posttest (52% of the missing data occurred in the training group and 48% in the control group). There were several reasons for missing data: 10% of the students in the training group left the training because (a) the 14-week program was too long or (b) student-teacher interactions were problematic. One student did not attend the pre- or posttest session but attended all other sessions and participated in the Mathematical Olympiad. In the control group, some children (a) moved away (n = 2), (b) changed schools or classes (n = 5), or (c) did not want to participate in the Mathematical Olympiad (n = 10). The other missing values occurred due to illness. For the implemented measures, the missing values reached a maximum of 18.0% in the training group, and ranged from 1.9% to 22.5% in the control group. When analyzing the treatment effects, we used the full information maximum likelihood approach implemented in R to deal with missing values (Enders, 2010; Graham, 2009; R Core Team, 2015; Rosseel, 2012).

Results

Descriptive Statistics

In a first step, we evaluated differences between the training and control groups at pretest separately for third and fourth graders (see Tables 1 and 2). As to be expected by the way the groups were chosen—the training participants were specifically nominated for the training because of their performance—there were significant differences between the two groups indicated by a 95% CI of Hedges' g that did not include zero. On average, children in the training group showed higher mathematical competence, higher levels of math self-concept, and higher levels of value beliefs in math (for Hedges' g, see Table 3); they were also younger than the children in the control group (see Table 1)[10]. However, inspection of the distribution of scores indicated a considerable overlap between the groups (see Figures 5 and 6).

[10]Due to these differences, we computed additional analyses for which we excluded all control group participants who performed worse than the participants in the training in pretest (N = 95). Also, we excluded all training group participants who were missing at pretest (N = 9). For this smaller sample (N = 97), we observed the same pattern of results (see the Appendix). Thus, we conducted the analyses with the whole sample.

Table 1

Descriptive Statistics for Age, Mathematical Competence, and Cognitive Skills: Means, Standard Deviations, and Internal Consistencies

Construct				Pretest				Posttest			
				N	*M*	*SD*	α	*N*	*M*	*SD*	α
Age		Grade 3	TG	24	8.66	0.52		$t(95) = 2.59, p = .011$			
			CG	73	8.40	0.38					
		Grade 4	TG	23	9.46	0.35		$t(78) = 7.92, p = .059$			
			CG	57	9.66	0.43					
Mathematical competence → DEMAT	2+	Grade 3	TG	26	30.36	7.36	.92				
			CG	78	22.43	7.90					
	3+	Grade 3	TG					17	19.59	4.17	.82
			CG					80	14.66	5.18	
		Grade 4	TG	24	20.08	10.17	.91				
			CG	63	25.35	6.00					
	4	Grade 4	TG					17	24.47	6.19	.89
			CG					61	15.46	7.25	
Cognitive skills BEFKI-short	Figural (A / B)	Grade 3	TG	24	7.17	2.62	.65 / .71	19	8.84	2.27	.68 / .78
			CG	77	6.99	2.58		79	7.27	2.83	
		Grade 4	TG	23	9.52	4.01		17	10.71	3.69	
			CG	62	7.71	3.00		59	8.31	3.24	
	Crystallized	Grade 3	TG	25	10.08	3.33	.73	19	11.89	2.56	.73
			CG	79	9.39	3.09		80	10.35	3.33	
		Grade 4	TG	23	12.18	1.99		17	13.18	2.27	
			CG	64	9.83	3.61		59	10.66	2.93	

Note. *N* = Number of valid answers given by participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha. Measurement points: Pretest = November 2014, Posttest = March 2015. TG = Training group, CG = control group. *t* tests for independent samples were computed to test for significant differences between the TG and the CG at pretest. Two-tailed significance levels are reported.

Table 2

Descriptive Statistics for Motivational Factors: Means, Standard Deviations, and Internal Consistencies

Construct				Pretest				Posttest			
				N	*M*	*SD*	α	*N*	*M*	*SD*	α
Math self-concept		Grade 3	TG	24	3.74	0.35	.87	18	3.67	0.73	.92
			CG	67	3.26	0.71		60	3.41	0.72	
		Grade 4	TG	23	3.69	0.43		17	3.83	0.32	
			CG	63	3.25	0.73		59	3.18	0.79	
Value beliefs in mathematics	Task-specific interest	Grade 3	TG	24	3.58	0.56	.77	18	3.48	0.92	.88
			CG	61	3.18	0.79		50	3.21	0.81	
		Grade 4	TG	23	3.61	0.51		17	3.69	0.46	
			CG	58	3.00	0.83		59	2.92	0.94	
	Intrinsic interest	Grade 3	TG	24	3.77	0.56	.81	18	3.66	0.77	.95
			CG	66	3.25	0.77		58	3.20	0.90	
		Grade 4	TG	23	3.70	0.39		17	3.70	0.72	
			CG	63	2.98	0.90		59	2.88	1.00	
	Attainment value	Grade 3	TG	24	3.81	0.54	.70	18	3.72	0.73	.69
			CG	64	3.44	0.73		55	3.42	0.70	
		Grade 4	TG	23	3.80	0.40		17	3.82	0.41	
			CG	63	3.31	0.75		59	3.41	0.70	

Note. *N* = Number of valid answers given by participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha. Measurement points: Pretest = November 2014, Posttest = March 2015. TG = Training group, CG = control group. *t* tests for independent samples were computed to test for significant differences between the TG and the CG at pretest. Two-tailed significance levels are reported.

Table 3

Baseline Equivalence between the Training and Control Groups at Pretest

	3rd grade students		4th grade students	
	g	95% CI	*g*	95% CI
Mathematical competence	-1.09	[-1.57, -0.60]	-1.02	[-1.57, -0.48]
Math self-concept	-0.75	[-1.24, -0.26]	-0.65	[-1.15, -0.15]
Intrinsic interest in math	-0.72	[-1.21, -0.23]	-0.89	[-1.40, -0.38]
Task-specific interest in math	-0.54	[-1.23, -0.05]	-0.80	[-1.31, -0.29]
Attainment value in math	-0.54	[-1.03, -0.06]	-0.72	[-1.22, -0.22]
Figural cognitive abilities	-0.07	[-0.54, 0.40]	-0.54	[-1.04, -0.05]
Crystallized cogitive abilities	-0.23	[-0.70, 0.24]	-0.71	[-1.21, -0.21]

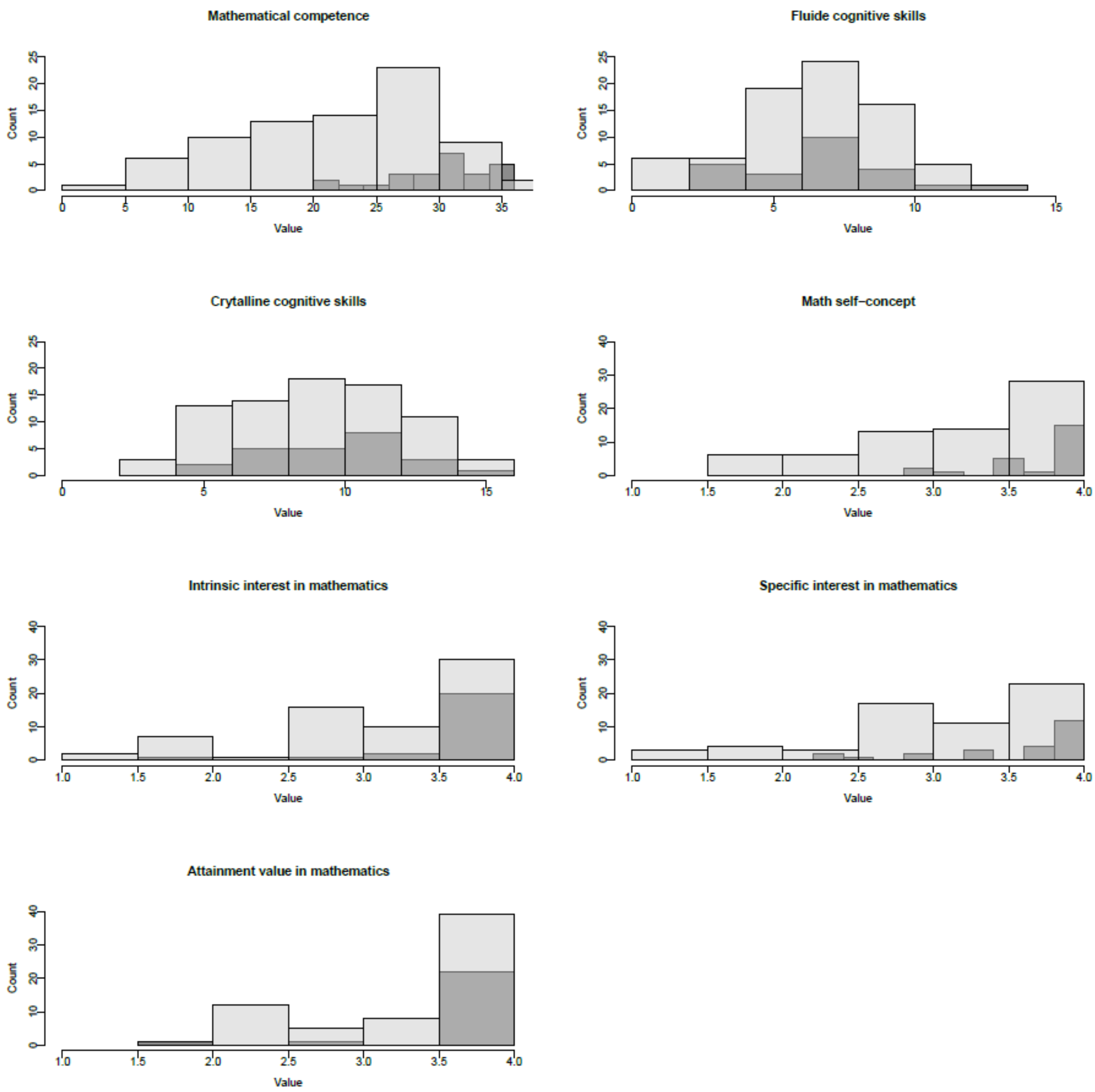


Figure 5. Histograms for pretest values for the third-grade students. The distribution for the training group is colored dark grey, and the control group is light grey.

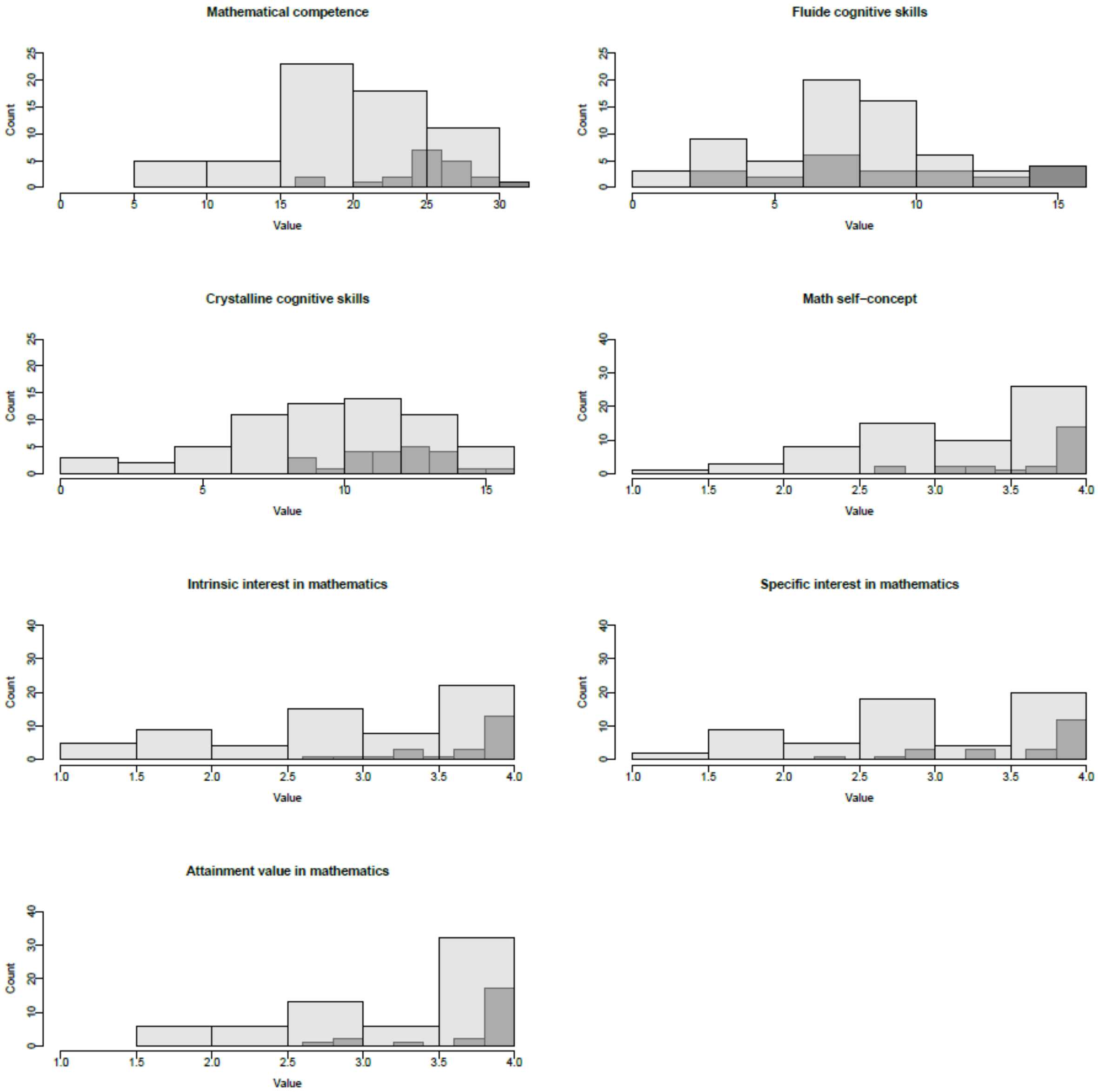


Figure 6. Histograms for pretest values for the fourth-grade students. The distribution for the training group is colored dark grey, and the control group is light grey.

Effects on Domain-Specific Achievement Outcomes

Overall, controlling for pretest values on mathematical competence, fluid and crystallized intelligence, math self-concept, value beliefs in mathematics, and age, the training group showed significantly higher *Performance in the Mathematical Olympiad* ($B_{3rd\ grade} = 0.72$, $p = .034$; $B_{4th\ grade} = 0.58$, $p = .018$) than children who did not attend the training (statistical details are presented in Table 4). Thus, students participating in the training performed between a half and three quarters of standard deviation better in the Mathematical Olympiad than students who did not attend the training when the pretest values were controlled for. There were no differential effects for third vs. fourth graders as indicated by the nonsignificant difference between the grade-specific treatment effects (Δ = 0.14, p = .748).

Regarding the transfer effects on *Mathematical Competence*, the results indicated that overall, the children showed significantly higher mathematical competence after completing the training ($B_{3rd\ grade}$ = 0.71, p = .008; $B_{4th\ grade}$ = 0.55, p = .001) compared with the children in the control group. Thus, students attending the training scored higher on competence than students who did not attend the training after pretest values were controlled for (see Table 4). Again, there were no differential effects for the two grade levels (Δ = 0.16, p = .626).

Effects on Motivational Outcomes

We also analyzed the research questions concerning changes in math self-concept and value beliefs with multiple linear regression. Again, analyses for all dependent variables (i.e., math self-concept, intrinsic and special interest, attainment value) were controlled for the pretest values on mathematical competence, math self-concept, value beliefs in mathematics, fluid and crystallized general cognitive abilities, and age (see Tables 4 and 5).

For both third and fourth graders, we observed no significant treatment effect ($B_{3rd\ grade}$ = -0.27, p = .181; $B_{4th\ grade}$= 0.28, p = .086) for self-concept. However, reflecting the different signs for the treatment effects in the group of third and fourth graders, there was a significant difference between these effects (Δ = 0.55, p = .033). Thus, as expected, differential effect for the two grade levels was observed for math self-concept.

We observed a positive effect on task-specific interest in mathematics only for the fourth graders ($B_{3rd\ grade}$ = 0.05, p = .840, $B_{4th\ grade}$ = 0.59, p = .012) for those children completing the training compared with the control group (for more details, see Table 5). Also, there were no significant treatment effects on value beliefs in mathematics and no differential effects ($\Delta_{intrinsic\ interest}$ = 0.15, p = .656; $\Delta_{attainment\ value}$ = 0.35, p = .295; $\Delta_{task\text{-}specific\ interest}$ = 0.54, p = .104) for value beliefs in mathematics.

Table 4

Effects of the Training Predicting Performance in the Mathematical Olympiad, Mathematical Competence, and Math Self-Concept

	Mathematical Olympiad 3rd level						Mathematical competence						Math self-concept					
	3rd grade			4th grade			3rd grade			4th grade			3rd grade			4th grade		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	1.13	2.29	.622	0.19	1.97	.922	-0.13	0.11	.246	-0.12	0.08	.133	0.08	0.09	.381	-0.03	0.08	.683
Training	**0.72**	**0.34**	**.034**	**0.58**	**0.25**	**.018**	**0.71**	**0.26**	**.008**	**0.55**	**0.16**	**.001**	-0.27	0.20	.181	0.28	0.17	.086
Mathematical competence[a]	0.14	0.11	.184	**0.26**	**0.09**	**.003**	0.14	0.14	.307	**0.60**	**0.08**	**< .001**	**0.29**	**0.14**	**.037**	0.05	0.09	.579
Figural cognitive skills[a]	**0.23**	**0.11**	**.029**	**0.34**	**0.11**	**.001**	0.09	0.10	.354	0.14	0.08	.078	-0.05	0.11	.627	0.06	0.07	.372
Crystallized cognitive skills[a]	**0.21**	**0.09**	**.023**	0.09	0.09	.363	0.05	0.12	.672	-0.11	0.09	.217	0.02	0.11	.828	-0.11	0.08	.188
Math self-concept[a]	0.00	0.15	.938	0.03	0.08	.741	0.19	0.19	.298	0.09	0.08	.270	-0.17	0.19	.383	**0.47**	**0.11**	**< .001**
Intrinsic interest in math[a]	-0.17	0.15	.278	0.08	0.12	.502	0.07	0.21	.720	0.02	0.10	.872	**0.59**	**0.24**	**.013**	-0.08	0.15	.592
Attainment value in math[a]	0.09	0.14	.550	0.01	0.10	.985	-0.20	0.16	.208	0.01	0.08	.878	0.34	0.28	.215	0.12	0.12	.321
Task-specific interest in math[a]	-0.03	0.13	.822	-0.06	0.07	.401	-0.03	0.13	.815	**0.17**	**0.07**	**.011**	-0.28	0.14	.053	**0.35**	**0.10**	**.001**
Age[a]	-0.15	0.27	.582	-0.04	0.20	.854	0.07	0.10	.483	**0.18**	**0.07**	**.008**	-.016	0.10	.101	-0.06	0.07	.432
R^2		.334			.562			.283			.738			.489			.666	

Note. Dependent variables were standardized by grade level prior to analysis. [a]Variables were standardized by grade level prior to analysis. Training was dummy-coded 0 = control group, 1 = training group. Two-tailed significance levels are reported.

Table 5

Effects of the Training Predicting Mathematical Value Beliefs: Intrinsic and Task-Specific Interest and Attainment Value

	Value beliefs in mathematics																	
	Intrinsic interest						Attainment value						Task-specific interest					
	3rd grade			4th grade			3rd grade			4th grade			3rd grade			4th grade		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	-0.02	0.11	.854	-0.06	0.11	.551	-0.03	0.11	.798	-0.09	0.11	.406	0.00	0.12	.999	-0.10	0.13	.462
Training	0.28	0.22	.196	0.43	0.26	.092	0.12	0.23	.594	0.47	0.24	.053	0.05	0.24	.840	**0.59**	**0.24**	**.012**
Mathematical competence[a]	0.17	0.14	.226	-0.12	0.09	.174	0.17	0.17	.318	**-0.27**	**0.10**	**.006**	**0.31**	**0.16**	**.045**	-0.06	0.11	.560
Figural cognitive skills[a]	0.11	0.15	.483	0.02	0.09	.792	-0.00	0.15	.997	0.06	0.11	.602	- 0.22	0.14	.109	0.05	0.11	.668
Crystallized cognitive skills[a]	-0.09	0.12	.435	**-0.17**	**0.08**	**.031**	0.03	0.12	.787	-0.10	0.11	.388	- 0.15	0.12	.214	**-0.22**	**0.09**	**.011**
Math self-con-cept[a]	**-0.57**	**0.24**	**.017**	0.15	0.14	.283	**-0.46**	**0.22**	**.036**	0.22	0.16	.184	- 0.33	0.23	.149	0.16	0.15	.271
Intrinsic interest in math[a]	**0.61**	**0.22**	**.006**	0.10	0.19	.612	**0.54**	**0.23**	**.019**	-0.16	0.20	.421	0.22	0.28	.425	-0.07	0.20	.730
Attainment value in math[a]	0.40	0.25	.112	0.13	0.12	.276	0.34	0.30	.257	**0.48**	**0.13**	**< .001**	0.29	0.31	.350	0.06	0.12	.602
Specific interest in math[a]	-0.12	0.17	.486	**0.42**	**0.12**	**< .001**	-0.08	0.14	.540	0.22	0.13	.102	0.25	0.19	.178	**0.46**	**0.15**	**.002**
Age[a]	0.07	0.12	.583	-0.03	0.09	.729	-0.09	0.13	.462	0.08	0.07	.298	0.17	0.13	.164	-0.01	0.10	.907
R^2		.413			.501			.330			.422			.358			.413	

Note. Dependent variables were standardized by grade level prior to analysis. [a]Variables were standardized by grade level prior to analysis. Participation in training was dummy-coded 0 = control group, 1 = training group. Two-tailed significance levels are reported.

Discussion

In the present study, we investigated the effectiveness of a training that prepared third and fourth graders for the Mathematical Olympiad. In line with our expectations, our data indicated positive effects of the training on achievement outcomes (i.e., performance in the Mathematical Olympiad and in a test of mathematical competence) as well as differential effects on math self-concept between the two grade levels. Further, for the fourth graders only, there was a positive effect on task-specific interest. Contrary to our expectations, there were no differences between third and fourth graders in effects on their value beliefs in mathematics. Of note, the training and control groups differed significantly on their pretest values, so we controlled for all pretest variables in all of our multiple linear regression models.

The positive effects of the training on performance in the Mathematical Olympiad largely confirmed our expectations and previous considerations on the supporting effects of such trainings regarding performance in the respective competition (e.g., Ozturk & Debelak, 2008a, 2008b). At first glance, the significantly better performance in the Mathematical Olympiad observed for students participating in the training is not surprising. However, it should be noted that the training was not based exactly on the requirements of the respective Mathematical Olympiad, but was rather based on information that was released after previous Mathematical Olympiads. Moreover, students did not solve the original tasks from these former Mathematical Olympiads in the training but were instructed more broadly with respect to mathematical problem solving and reasoning. This indicates that a general focus on reasoning and problem solving seems to be transferable to new contents.

These transfer effects for solving new mathematical problems were observed even for general mathematical competence, indicated by significant positive training effects on mathematical competence. Because we assessed mathematical competence by administering a standardized achievement test based on the German education standards for elementary school children, the contents of the test differed considerably from the training contents. This suggests that the observed effects of the training on students' mathematical competence were actually transfer effects from the training to elementary school students' more general mathematical competence assessed by the standardized curriculum-based test. Thus, the more intense focus on challenging tasks in the training seems to positively affect students' mathematical competence. This finding is in line with the idea that it is possible to positively influence students by challenging them if they are already able to solve curriculum-based tasks (see e.g., Diezmann & Watters, 2001). In addition, the idea of challenging students with complex tasks in cooperative settings seems to be a good method for fostering students who already show high domain-specific competence.

For motivational constructs (i.e., self-concept and value beliefs), as expected, we observed a differential effect for math self-concept for the third and fourth graders who participated in the training. More specifically, the statistically significant difference in the development was driven by a (descriptively) negative development of self-concept in third-grade students' math self-concept and an increase in self-concept for fourth graders. The findings supports the notion that social comparison processes are involved in the development of math self-concept in elementary school students. In the experimental groups, these social comparison processes are likely to be more favorable for fourth graders compared to third graders—downwards vs. upwards comparisons (see 1.3.2)—which would explain the differential effect.

We did not observe a parallel differential effect on value beliefs in mathematics. Although—descriptively—the effects were stronger for the fourth graders for all three value constructs, the differences between third and fourth graders did not reach statistical significance. Furthermore, all six regression coefficients (testing the effects of the training on intrinsic interest, task-specific interest and attainment value in the group of third and fourth graders) showed a positive sign. This might be indicative of training effects on value beliefs that are different from the processes that affect self-concept. For instance, the positive experiences of learning more about mathematics and of working in teams (even if the partner seemed more competent) might have stabilized students' value beliefs. These considerations are supported by the positive effect on fourth graders' interest in solving riddles or the like (i.e., task-specific interest). However, further research is required to answer this question.

Taken together, the present study indicates that it is possible to improve mathematical achievement and to affect motivational factors by a training that is geared toward an academic competition. The results of our study corroborate the notion that academic competitions are a promising setting for studying learning processes. In particular, training students for an academic competition contributes to the development of their competence and motivation.

References

Arens, A. K., Trautwein, U., & Hasselhorn, M. (2011). Erfassung des Selbstkonzepts im mittleren Kindesalter: Validierung einer deutschen Version des SDQ I 1Dieser Beitrag wurde unter der geschäftsführenden Herausgeberschaft von Jens Möller angenommen [Self-Concept Measurement with Preadolescent Children: Validation of a German Version of the SDQ I]. *Zeitschrift für Pädagogische Psychologie*, *25*(2), 131–144. https://doi.org/10.1024/1010-0652/a000030

Bezold, A. (2012). Förderung von Argumentationskompetenzen auf der Grundlage von Forscheraufgaben. [Fostering the process of justification using research tasks.]. *mathematica didactica*. (35), 73–103.

Bicknell, B. (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom*, *13*(4), 16–20. Retrieved from http://files.eric.ed.gov/fulltext/EJ824763.pdf

Bos, W., Buddeberg, I., & Lankes, E.-M. (Eds.). (2005). *IGLU: Skalenhandbuch zur Dokumentation der Erhebungsinstrumente* [IGLU: Handbook of scales for documentation of assessment instruments]. Münster: Waxmann. Retrieved from http://www.dandelon.com/intelligentSEARCH.nsf/alldocs/8144B6DCC0DA65A7C125732B004F4B84/

Campbell, J. R., Wagner, H., & Walberg, H. J. (2000). Academic competitions and programs designed to challenge the exceptionally talented. *International handbook of giftedness and talent*, *2*. Retrieved from http://www.olympiadprojects.com/v2/pubs_web/Hdbk.pdf

Campbell, J. R., & Walberg, H. J. (2010). Olympiad Studies: Competitions Provide Alternatives to Developing Talents That Serve National Interests. *Roeper Review*, *33*(1), 8–17. https://doi.org/10.1080/02783193.2011.530202

Clinkenbeard, P. R. (1989). The Motivation to Win Negative Aspects of Success at Competition. *Journal for the Education of the Gifted*, *12*(4), 293–305. https://doi.org/10.1177/016235328901200405

Cropper, C. (1998). Is Competition an Effective Classroom Tool for the Gifted Student? *Gifted Child Today*, *21*(3), 28–31. https://doi.org/10.1177/107621759802100309

Dai, D. Y., & Rinn, A. N. (2008). The Big-Fish-Little-Pond Effect: What Do We Know and Where Do We Go from Here? *Educational Psychology Review*, *20*(3), 283–317. https://doi.org/10.1007/s10648-008-9071-x

Diezmann, C. M., & Watters, J. J. (2001). The Collaboration of Mathematically Gifted Students on Challenging Tasks. *Journal for the Education of the Gifted*, *25*(1), 7–31. https://doi.org/10.1177/016235320102500102

Eccles, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., & Midgley, C. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 97–132). San Francisco: W. H. Freeman.

Enders, C. K. (2010). *Applied missing data analysis.* New York, NY: The Guilford Press.

Forrester, J. H. (2010). *Competitive science events: Gender, interest, science self-efficacy, and academic major choice.* (Dissertation). North Carolina State University. Retrieved from https://repository.lib.ncsu.edu/bitstream/handle/1840.16/6073/etd.pdf?sequence=1&isAllowed=y

Gaspard, H., Dicke, A.-L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B. (2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology*, *107*(3), 663–677. https://doi.org/10.1037/edu0000003

Gölitz, D., Roick, T., & Hasselhorn, M. (2006). *DEMAT 4: Deutscher Mathematiktest für vierte Klassen* [German mathematics test for fourth graders]: Hogrefe.

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. *Annual review of psychology*, *60*, 549–576. https://doi.org/10.1146/annurev.psych.58.110405.085530

IBM Corp. Released. (2013). IBM SPSS Statistics for Windows. Armonk, NY.

Johnson, D. W., & Johnson, R. T. (1994). *Learning together and alone. Cooperative, competitive, and individualistic learning*: ERIC.

Johnson, R. T. (1990). Supporting gifted students' acquisition of relevant knowledge for solving math problems. *Early Child Development and Care*, *63*(1), 37–45. https://doi.org/10.1080/0300443900630106

Koshy, V., Ernest, P., & Casey, R. (2009). Mathematically gifted and talented learners: Theory and practice. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 213–228. https://doi.org/10.1080/00207390802566907

Krajewski, K., Liehm, S., & Schneider, W. (2004). *DEMAT 2+: Deutscher Mathematiktest für zweite Klassen* [German mathematics tests for second graders]. Weinheim: Beltz.

Leikin, R. (2010). Teaching the Mathematically Gifted. *Gifted Education International*, *27*(2), 161–175. https://doi.org/10.1177/026142941002700206

Lengfelder, A., & Heller, K. A. (2002). German Olympiad studies: Findings from a retrospective evaluation and from in-depth interviews: Where have all the gifted females gone. *Journal of Research in Education*, *12*(1), 86–92. Retrieved from http://www.olympiadprojects.com/v2/pubs_web/ch4_SS.pdf

Lüdtke, O., Köller, O., Marsh, H. W., & Trautwein, U. (2005). Teacher frame of reference and the big-fish–little-pond effect. *Contemporary Educational Psychology*, *30*(3), 263–285. https://doi.org/10.1016/j.cedpsych.2004.10.002

Marsh, H. W. (1987). The big-fish-little-pond effect on academic self-concept. *Journal of Educational Psychology*, *79*(3), 280–295. https://doi.org/10.1037/0022-0663.79.3.280

Marsh, H. W., & Parker, J. W. (1984). Determinants of student self-concept: Is it better to be a relatively large fish in a small pond even if you don’t learn to swim as well? *Journal of Personality and Social Psychology*, *47*(1), 213–231. https://doi.org/10.1037/0022-3514.47.1.213

Marsh, H. W., Seaton, M., Trautwein, U., Lüdtke, O., Hau, K. T., O’Mara, A. J., & Craven, R. G. (2008). The Big-fish–little-pond-effect Stands Up to Critical Scrutiny: Implications for Theory, Methodology, and Future Research. *Educational Psychology Review*, *20*(3), 319–350. https://doi.org/10.1007/s10648-008-9075-6

McAllister, B. A., & Plourde, L. A. (2008). Enrichment Curriculum: Essential for mathematically gifted students. *Education*, *129*(1), 40–49. Retrieved from http://web.a.ebscohost.com/ehost/pdfviewer/pdfviewer?sid=2c0b2b92-484e-4295-8ceb-ae855b7aceed%40sessionmgr4009&vid=2&hid=4214

Olson, S. (2005). *Count down: Six kids vie for glory at the world’s toughest math competition*. Boston: Houghton Mifflin. Retrieved from http://www.loc.gov/catdir/samples/hm051/2003056897.html

Ozturk, M. A., & Debelak, C. (2008a). Academic competitions as tools for differentiation in middle school. *Gifted Child Today*, *31*(3), 47–53. Retrieved from http://files.eric.ed.gov/fulltext/EJ803366.pdf

Ozturk, M. A., & Debelak, C. (2008b). Affective Benefits from Academic Competitions for Middle School Gifted Students. *Gifted Child Today*, *31*(2), 48–53. https://doi.org/10.4219/gct-2008-758

R Core Team. (2015). R. Vienna, Austria: the R foundation for Statistical Computing. Retrieved from www.R-project.org

Ramm, G. C., Adamsen, C., & Neubrand, M. (Eds.). (2006). *PISA 2003: Dokumentation der Erhebungsinstrumente* [PISA 2003: Documentation of assessment instruments]. Münster: Waxmann.

Rebholz, F., & Golle, J. (2017). Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade [Fostering mathematical skills in elementary school – the role of academic competions using the example of the Mathematical Olympiad]. In Trautwein, U. & Hasselhorn, M. (Ed.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente.* (pp. 213–228). Göttingen: Hogrefe.

Riley, T. L., & Karnes, F. A. (1998). Mathematics+ competitions= a winning formula! *Gifted Child Today*, *21*(4), 42.

Roick, T., Gölitz, D., & Hasselhorn, M. (2004). *Deutscher Mathematiktest für dritte Klassen (DEMAT 3+)* [German mathematics tests for third graders]. Göttingen: Beltz Test.

Rosseel, Y. (2012). *lavaan: An R package for structural equation modeling and more Version 0.4-9 (BETA)*: Ghent University.

Rothenbusch, S., Zettler, I., Voss, T., Lösch, T., & Trautwein, U. (2016). Exploring reference group effects on teachers' nominations of gifted students. *Journal of Educational Psychology*, *108*(6), 883–897. https://doi.org/10.1037/edu0000085

Rotigel, J. V., & Fello, S. (2016). Mathematically Gifted Students: How Can We Meet Their Needs? *Gifted Child Today*, *27*(4), 46–51. https://doi.org/10.4219/gct-2004-150

Schroeders, U., Schipolowski, S., Zettler, I., Golle, J., & Wilhelm, O. (2016). Do the smart get smarter? Development of fluid and crystallized intelligence in 3rd grade. *Intelligence*, *59*, 84–95. https://doi.org/10.1016/j.intell.2016.08.003

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*: Wadsworth Cengage learning.

Shavelson, R. J., Hubner, J. J., & Stanton, G. C. (1976). Self-concept: Validation of construct interpretations. *Review of Educational Research*, *46*(3), 407–441.

Snow, G. M. (2011). *Development of a Math Interest Inventory to Identify Gifted Students from Underrepresented and Diverse Populations* (Masters Theses & Specialist Projects). Western Kentucky University.

Trautwein, U., Köller, O., Lüdtke, O., & Baumert, J. (2005). Student tracking and the powerful effects of opt-in courses on self-concept: Reflected-glory effects do exist after all. In H. W. Marsh, R. G. Craven, & D. M. McInerney (Eds.), *New frontiers for self research* (pp. 307–327). Greenwich: IAP.

Trautwein, U., Lüdtke, O., Marsh, H. W., Köller, O., & Baumert, J. (2006). Tracking, grading, and student motivation: Using group composition and status to predict self-concept and interest in ninth-grade mathematics. *Journal of Educational Psychology*, *98*(4), 788–806. https://doi.org/10.1037/0022-0663.98.4.788

Trautwein, U., & Möller, J. (2016). Self-Concept: Determinants and Consequences of Academic Self-Concept in School Contexts. In A. A. Lipnevich, F. Preckel, & R. D. Roberts (Eds.), *The Springer Series on Human Exceptionality. Psychosocial Skills and School Systems in the 21st Century* (pp. 187–214). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-319-28606-8_8

Urhahne, D., Ho, L. H., Parchmann, I., & Nick, S. (2012). Attempting to predict success in the qualifying round of the International Chemistry Olympiad. *High Ability Studies*, *23*(2), 167–182. https://doi.org/10.1080/13598139.2012.738324

Wirt, J. L. (2011). *An analysis of Science Olympiad participants' perceptions regarding their experience with the science and engineering academic competition.* (Dissertation). Seton Hall University. Retrieved from http://scholarship.shu.edu/cgi/viewcontent.cgi?article=1014&context=dissertations

Appendix A

A: Analyses for the adjusted sample

We excluded all participants (*N* = 95) from the control group who had worse pretest values on mathematical competence, fluid and crystallized cognitive skills, math self-concept, and value beliefs in mathematics than the worst-performing participant in the training group. Also, we excluded all training group participants who were missing at pretest (*N* = 9). Tables A.1 and A.2 present the descriptive statistics for this adjusted sample. Histograms reflecting the distribution of scores between the training and control groups are presented in Figures A.1 and A.2.

Again, all analyses were run separately for each grade level. The effectiveness of the training was evaluated with multiple linear regression analyses separated by grade level using the R package *lavaan* (R Core Team, 2015; Rosseel, 2012). All analyses used the robust maximum likelihood estimator, which corrects standard errors for the non-normality of the variables (Rosseel, 2012). Predictors in our regression models were participation in the training or the control group (0 = *control*, 1 = *training*), pretest performance in mathematical competence, (fluid and crystallized) intelligence, math self-concept, value beliefs in mathematics, and age. According to the standardization of the dependent variables, the multiple regression coefficient for the group variable indicates the standardized difference between the training and control groups at posttest while pretest performance in mathematical competence, (fluid and crystallized) intelligence, math self-concept, value beliefs in mathematics, and age are controlled for. For each dependent variable, we analyzed differential effects between third and fourth graders by testing the estimated differences between the training effects of the two grades against zero. The results for the adjusted sample are presented in Tables A.3 and A.4. As in the analyses reported in the main text, there were significant effects on students' performance in the Mathematical Olympiad and mathematical competence. Furthermore, there were no differential domain-specific achievement effects for third versus fourth graders as indicated by the nonsignificant difference between the grade-specific treatment effects ($\Delta_{Mathematical\ Olympaid}$ = 0.57, p = .322; $\Delta_{Mathematical\ Competence}$ = -0.36, p = .349). However, the differential effects for third versus fourth graders on math self-concept remained stable ($\Delta_{Math\ Self\text{-}Concept}$ =-0.68, p = .018), and there were no differential effects for value beliefs in mathematics ($\Delta_{intrinsic\ interest}$ = -0.51, p = .172; $\Delta_{attainment\ value}$ = -0.55, p = .127; $\Delta_{task\text{-}specific\ interest}$ =-0.61, p = .099). Again, for task-specific interest, there was a significant positive effect for the fourth graders.

Table A.1

Descriptive Statistics for Age, Mathematical Competence, and Cognitive Skills: Means and Standard Deviations

Construct		Grade level		Pretest			Posttest		
				N	*M*	*SD*	*N*	*M*	*SD*
Age		3rd	TG	23	8.42	0.53	*t*(45) = 2.22,		
			CG	24	8.07	0.34	*p* = .032		
		4th	TG	18	9.43	0.35	*t*(42) =1.48,		
			CG	26	9.63	0.49	*p* = .146		
Mathematical competence → DEMAT	2+	3rd	TG	23	30.57	3.86			
			CG	27	27.20	2.75			
	3+	3rd	TG				16	19.50	4.29
			CG				20	15.92	4.57
		4th	TG	18	25.61	3.57			
			CG	29	23.24	3.77			
	4	4th	TG				11	25.50	7.13
			CG				22	19.36	6.22
Cognitive skills BEFKI-short	Figural (A / B)	3rd	TG	23	7.30	2.58	16	9.18	2.16
			CG	27	8.11	1.53	22	8.76	1.94
		4th	TG	18	10.06	4.21	11	11.54	3.60
			CG	29	9.31	2.65	21	9.29	2.93
	Crystallized	3rd	TG	23	10.04	2.75	16	11.59	2.48
			CG	27	10.81	3.00	22	12.31	2.45
		4th	TG	18	12.61	1.97	11	13.84	1.91
			CG	29	10.79	3.03	21	11.29	2.93

Note. *N* = Number of valid answers from participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha. Measurement time points: Pretest = November 2014, Posttest = March 2015. TG = Training group, CG = control group. *t* tests for independent samples were computed to test for significant differences between the TG and the CG at pretest. Two-tailed significance levels are reported.

Table A.2

Descriptive Statistics for the Motivational Factors: Means and Standard Deviations

Construct		Grade level		Pretest			Posttest		
				N	*M*	*SD*	*N*	*M*	*SD*
Math self-concept		3rd	TG	23	3.75	0.35	16	3.69	0.75
			CG	27	3.65	0.41	17	3.85	0.28
		4th	TG	18	3.81	0.30	13	3.95	0.18
			CG	29	3.71	0.37	28	3.60	0.47
Value beliefs in mathematics	Intrinsic interest	3rd	TG	23	3.76	0.57	16	3.68	0.80
			CG	27	3.60	0.45	17	3.56	0.75
		4th	TG	18	3.75	0.41	13	3.78	0.79
			CG	29	3.78	0.64	28	3.30	0.82
	Attainment value	3rd	TG	23	3.80	0.55	16	3.75	0.76
			CG	27	3.83	0.27	17	3.65	054
		4th	TG	18	3.83	0.35	13	3.90	0.37
			CG	29	3.68	0.55	28	3.61	0.54
	Specific interest	3rd	TG	23	3.56	0.57	16	3.56	0.84
			CG	27	3.47	0.52	17	3.57	0.54
		4th	TG	18	3.70	0.47	13	3.82	0.26
			CG	29	3.38	0.62	28	3.31	0.89

Note. *N* = Number of valid answers from participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha. Measurement time points: Pretest = November 2014, Posttest = March 2015. TG = Training group, CG = control group. *t* tests for independent samples were computed to test for significant differences between the TG and the CG at pretest. Two-tailed significance levels are reported.

Table A.3

Effects of the Training Predicting the Outcome for the Adjusted Sample

	Mathematical Olympiad 3rd level						Mathematical competence						Math self-concept					
	3rd grade			4th grade			3rd grade			4th grade			3rd grade			4th grade		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	1.57	3.18	.622	-1.58	2.38	.505	-0.18	0.17	.275	-0.22	0.17	.213	0.11	0.13	.367	0.14	0.14	.340
Training	**1.14**	**0.49**	**.021**	0.56	0.30	.059	0.24	0.32	.455	**0.59**	**0.22**	**.007**	-0.26	0.17	.142	**0.42**	**0.23**	**.065**
Mathematical competence[a]	-0.29	0.48	.548	0.35	0.19	.057	**0.92**	**0.30**	**.002**	**0.77**	**0.20**	**< .001**	-0.16	0.21	.455	0.13	0.16	.410
Figural cognitive skills[a]	0.22	0.18	.224	**0.53**	**0.17**	**.002**	-0.05	0.11	.632	-0.02	0.12	.876	0.02	0.05	.664	0.01	0.10	.892
Crystallized cog-nitive skills[a]	**0.32**	**0.13**	**.017**	0.24	0.15	.119	0.07	0.10	.491	-0.09	0.14	.539	-0.02	0.04	.633	-0.03	0.08	.708
Math self-concept[a]	0.07	0.29	.799	-0.24	0.32	.459	-0.08	0.25	.749	0.19	0.25	.465	**0.29**	**0.15**	**.049**	0.35	0.18	.052
Intrinsic interest in math[a]	-0.35	0.30	.238	-0.22	0.24	.364	0.03	0.30	.932	-0.11	0.24	.646	**0.57**	**0.15**	**< .001**	-0.34	0.23	.147
Attainment value in math[a]	-0.17	0.25	.497	-0.16	0.18	.368	-0.33	0.24	.163	0.19	0.18	.292	**0.39**	**0.15**	**.010**	0.12	0.14	.384
Specific interest in math[a]	0.11	0.27	.681	0.07	0.15	.664	0.26	0.19	.182	0.20	0.14	.140	-0.05	0.09	.618	0.21	0.13	.103
Age[a]	-0.19	0.37	.614	0.16	0.25	.511	0.13	0.09	.155	0.17	0.09	.057	-0.10	0.08	.169	0.02	0.07	.795
R^2		.336			.588			.406			.646			.838			.320	

Note. Dependent variables were standardized for each grade level prior to analysis. Training was dummy-coded 0 = control group, 1 = training group. Two-tailed significance levels are reported. [a]Variables were standardized for each grade level prior to analysis.

Table A.4

Effects of the Training Predicting the Outcome for the Adjusted Sample

	Value Beliefs in Mathematics																	
	Intrinsic Interest						Attainment Value						Specific Interest					
	3rd grade			4th grade			3rd grade			4th grade			3rd grade			4th grade		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	-0.03	0.14	.836	0.23	0.17	.166	-0.18	0.13	.180	-0.07	0.21	.759	0.14	0.14	.318	0.13	0.20	.523
Training	0.03	0.16	.830	0.54	0.34	.108	0.10	0.18	.574	0.65	0.31	.036	0.08	0.17	.650	**0.68**	**0.32**	**.035**
Mathematical competence[a]	0.05	0.23	.815	0.09	0.22	.660	-0.06	0.21	.784	0.11	0.21	.612	-0.09	0.21	.653	0.11	0.24	.648
Figural cognitive skills[a]	-0.03	0.16	.848	-0.09	0.14	.500	-0.13	0.09	.134	-0.08	0.13	.573	-0.12	0.12	.283	-0.11	0.14	.415
Crystallized cognitive skills[a]	-0.17	0.08	.041	-0.04	0.13	.747	-0.06	0.06	.342	-0.11	0.11	.311	-0.07	0.06	.202	-0.25	0.13	.061
Math self-concept[a]	-0.03	0.27	.920	-0.17	0.27	.544	-0.02	0.20	.930	0.26	0.29	.369	0.14	0.17	.394	0.20	0.34	.564
Intrinsic interest in math[a]	**0.67**	**0.28**	**.019**	-0.21	0.28	.575	**0.85**	**0.25**	**.001**	-0.68	0.35	.053	0.20	0.18	.246	-0.24	0.39	.530
Attainment value in math[a]	0.48	0.18	.007	0.33	0.31	.286	0.34	0.18	.062	**0.62**	**0.24**	**.010**	**0.43**	**0.15**	**.005**	0.09	0.17	.588
Specific interest in math[a]	-0.02	0.13	.874	0.09	0.22	.674	0.11	0.14	.408	0.16	0.17	.354	**0.57**	**0.15**	**< .001**	0.27	0.24	.259
Age[a]	0.01	0.06	.867	-0.03	0.12	.816	0.06	0.05	.239	0.09	0.09	.298	0.08	0.05	.148	0.04	0.14	.777
R^2		.694			.167			.804			.339			.748			.227	

Note. Dependent variables were standardized for each grade level prior to analysis. Participation in the training was dummy-coded 0 = control group, 1 = training group. Two-tailed significance levels are reported. [a]Variables were standardized for each grade level prior to analysis.

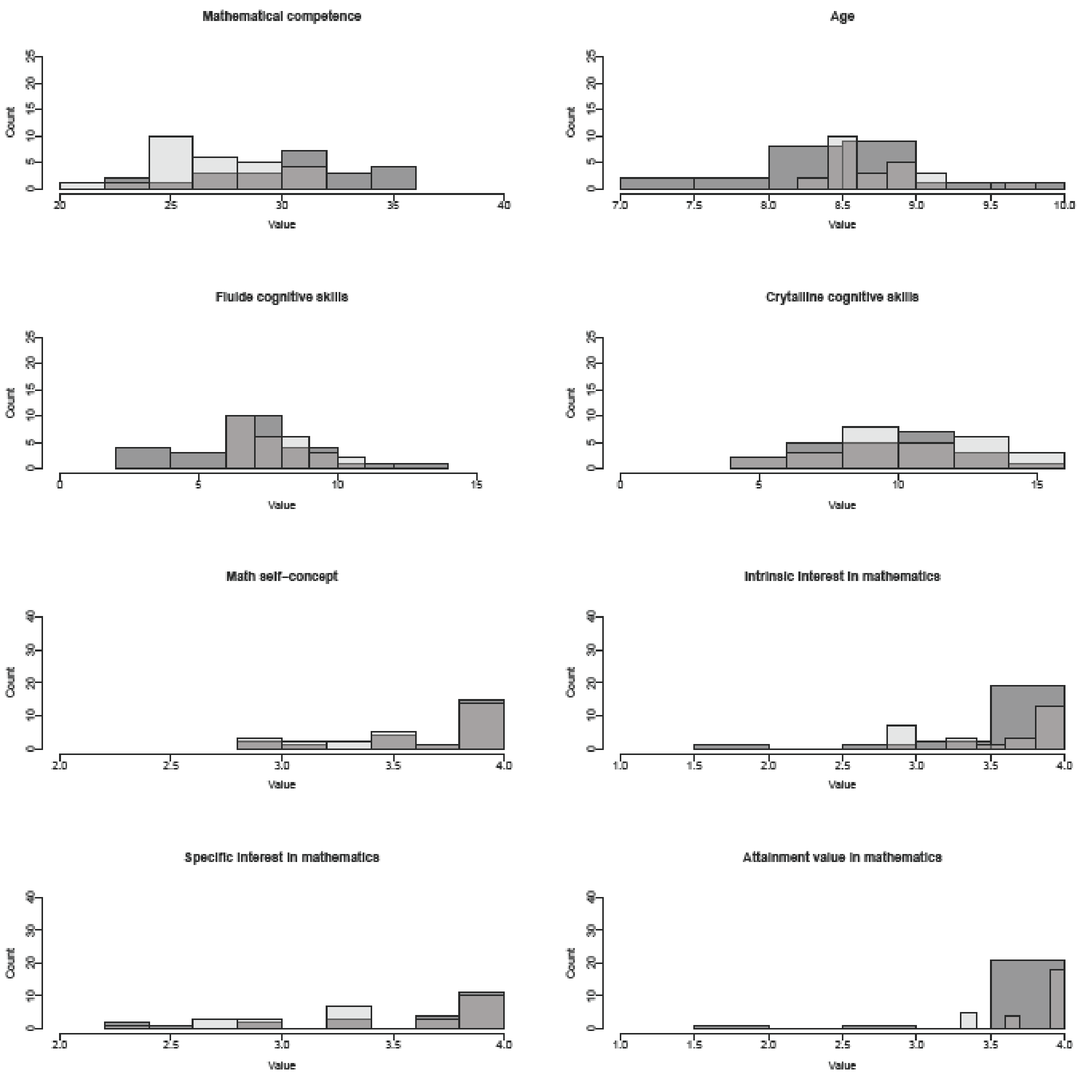


Figure A.1. Histograms for pretest values for the adjusted third-grade sample. The distribution for the training group is colored dark grey, and the control group is light grey.

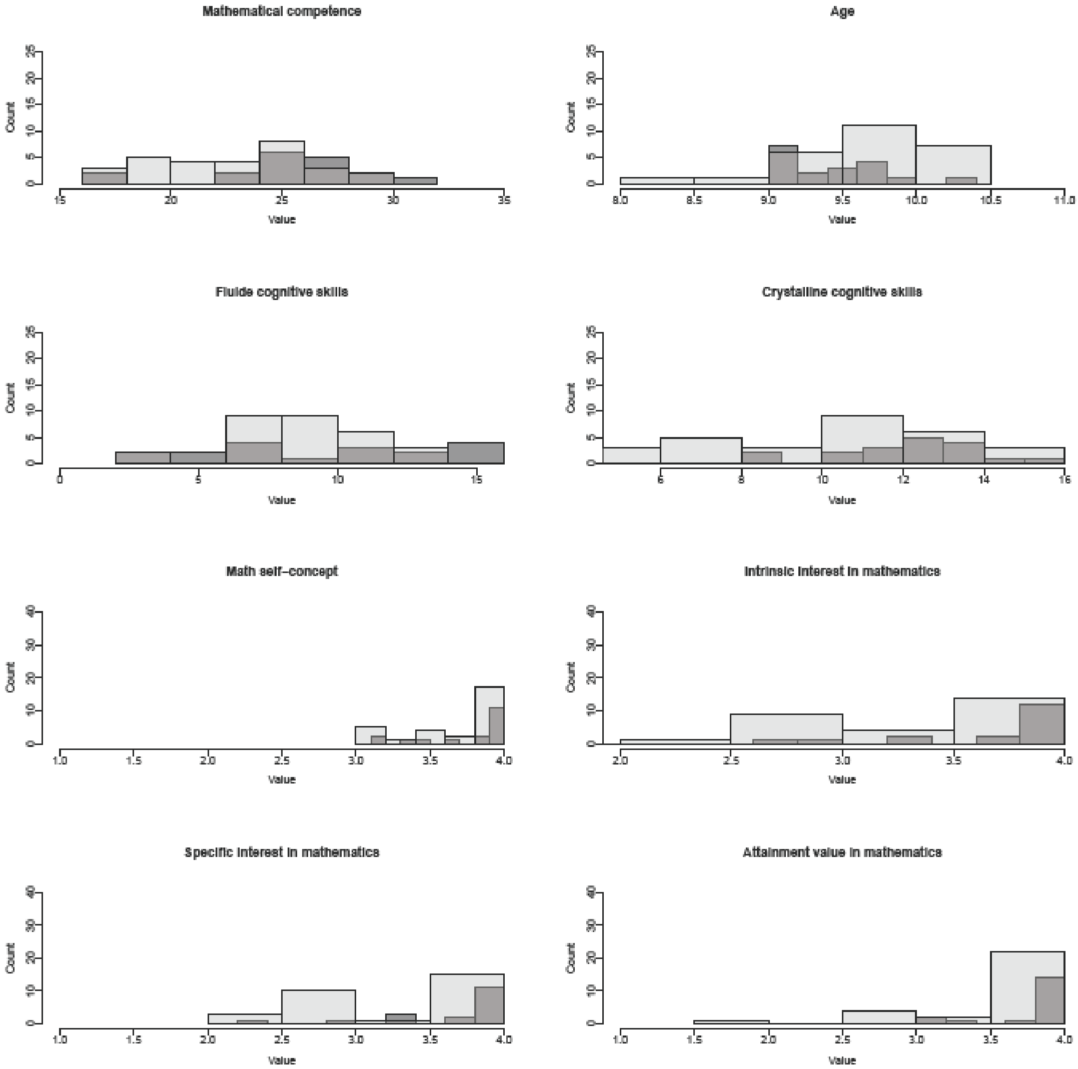


Figure A.2. Histograms for pretest values for the adjusted fourth-grade sample. The distribution for the training group is colored dark grey, and the control group is light grey.

4

Training Process-Based Mathematical Competences – Exploring Effects on Domain-Specific Factors and Domain-General Cognitive Abilities

Rebholz, F., Golle, J., Roesch, S., Trautwein, U., & Moeller, K. (2017). Training Process-Based Mathematical Competences – Exploring Effects on Domain-Specific Factors and Domain-General Cognitive Abilities. Manuscript under perparation.

This research was supported by a grant from the Hector Foundation II to Ulrich Trautwein. Franziska Rebholz is a doctoral student at the LEAD Graduate School & Research Network [GSC 1028], which is funded by the Excellence Initiative of the German federal and state governments.

Abstract

In both numerical cognition research and mathematics education, mathematical achievement is assumed to be driven by domain-specific (i.e., domain-specific numerical abilities and content-based competences) and domain-general factors (i.e., domain-general cognitive abilities and process-based competences). We developed a training for elementary school children that focused on enhancing process-based competences and investigated its effects on domain-specific and domain-general factors. Results of a randomized controlled field trial with 97 children (M_{age} = 8.79 years, 68% male) indicated significant training effects on process-based competences but also transfer effects on domain-general abilities. Furthermore, we observed differential effects on domain-specific factors with girls benefitting more. On the basis of these results, we discuss relations between the different constructs considered in numerical cognition research and mathematics education.

Keywords: domain-specific numerical abilities, domain-general cognitive abilities, numerical cognition, mathematics education, process-based competences

Training Process-Based Mathematical Competences – Exploring Effects on Domain-Specific Factors and Domain-General Cognitive Abilities

Numerical and mathematical abilities are seen as key competences in digital knowledge societies. They are relevant not only in school but also for vocational success and for managing everyday life (e.g., Butterworth et al., 2011; NCTM, 2000; OECD, 2014). As reported by teachers and documented by large-scale assessment studies such as PISA, individual differences in mathematical achievement are considerable (e.g., Klieme, Jude, Baumert, & Prenzel, 2010). Research in both numerical cognition and mathematics education has been conducted to explain these differences.

Numerical cognition research focuses on the cognitive development of numerical abilities. The assumption is that domain-general cognitive abilities (e.g., general cognitive ability) as well as number-specific abilities (e.g., understanding number magnitude) influence mathematical achievement (e.g., Alcock et al., 2016; Fuchs et al., 2010; Passolunghi & Lanfranchi, 2012; Sella et al., 2016; Sullivan et al., 2016; Thompson et al., 2013; Träff, 2013). In comparison with numerical cognition research, mathematics education contrasts content-based mathematical competences (i.e., the actual contents taught in school such as numbers and operations) with broader and more general process-based competences (e.g., [mathematical] problem solving, reasoning, and modeling competences). These are assumed to be necessary to successfully solve mathematical problems (e.g., *Principles and Standards for School Mathematics* in the USA, NCTM, NCTM, 2000; *Education Standards in Mathematics* in Germany, KMK, 2004; Klieme et al., 2003).

Thus, in numerical cognition research as well as in mathematics education, a differentiation is made between rather specific (i.e., content-based competences and domain-specific abilities) and more general factors (i.e., process-based competences and domain-general abilities) that are relevant for dealing with mathematical problems. It seems that both disciplines assume that a common domain-specific basis (i.e., numerical abilities or content-specific competences) underlies mathematical achievement. However, they differ in how they conceptualize more general skills. In numerical cognition research, domain-general skills involve very complex, general cognitive processes that are relevant across several domains and subjects. By contrast, process-based competences in mathematics education refer more closely to the domain of mathematics.

In this study, we aimed to build a bridge between the conceptualizations of numerical cognition and mathematics education research. Thus, we evaluated whether an intervention specifically developed to increase process-based mathematical competences would have differential effects on not only process-based competencies but also on domain-specific factors and more general cognitive skills.

Domain-Specific and Domain-General Abilities in Numerical Cognition Research

In numerical cognition research, numerical abilities are not viewed as a unitary construct but reflect a conglomerate of different domain-specific abilities (e.g., number magnitude, arithmetic facts) and procedures (e.g., basic arithmetic operations; e.g., Dowker, 2005). The vast majority of models of children's numerical development propose a more or less hierarchical development of basic numerical skills and suggest that later arithmetical abilities build on them (e.g., Krajewski & Schneider, 2009a, 2009b; Siegler & Booth, 2004; von Aster & Shalev, 2007). It is important to mention that influences of domain-general cognitive abilities are commonly not considered or are underspecified in these models (e.g., the influence of working memory in the model by von Aster & Shalev, 2007). This is rather surprising given that the most influential model of adult numerical cognition suggested by Dehaene and colleagues (e.g., Dehaene & Cohen, 1995; Dehaene, Piazza, Pinel, & Cohen, 2003; see also Klein et al., 2016 Klein et al., 2016) explicitly proposes the involvement of domain-general abilities such as executive control and working memory in particular when it comes to more complex mathematical problems. Consequently, in most studies in which children's numerical development was evaluated, researchers have attempted to statistically control for the influences of domain-general abilities (e.g., intelligence; e.g., Geary & Moore, 2016; Moeller et al., 2011) and did not further investigate their actual impact on numerical development.

Only recently has more research been devoted to the influence of domain-general cognitive abilities (e.g., general cognitive abilities, working memory, general processing speed) on numerical development in general and academic achievement in mathematics in particular (Alcock et al., 2016; Passolunghi & Lanfranchi, 2012; Thompson et al., 2013; Träff, 2013). For example, Fuchs and colleagues (2010) observed that performance in different topics of school mathematics was differentially predicted by domain-specific numerical factors link performance on the Number Set Test (for more information about the test, see Geary et al., 2009) and domain-general cognitive abilities such as nonverbal problem solving. In this longitudinal study, the authors assessed domain-specific basic numerical and domain-general cognitive abilities to predict performance in curricular tasks (i.e., procedural calculations and mathematical word problems). Amongst other findings, results indicated that domain-general cognitive abilities—amongst others, visual-spatial working memory and executive function—reliably predicted performance in solving mathematical word problems but not in basic arithmetic. By contrast, domain-specific numerical abilities (i.e., number sets, symbolic mental number line) were associated with performance in both tasks (Fuchs et al., 2010).

In another study, Passolunghi and Lufranchi (2012) found a positive effect of domain-general abilities such as working memory and processing speed on domain-specific numerical abilities assessed by an early numeracy test focusing on basic numerical tasks such as magnitude comparison, seriation, use of number words, and so forth (for more information about the test, see van de Rijt, van Luit, & Pennings, 2016). Furthermore, they also found positive effects of both domain-general (i.e., working memory, processing speed) and domain-specific numerical abilities (i.e., magnitude comparison, classification, general understanding of numbers) on later mathematics achievement.

In sum, this evidence suggests that numerical cognition is influenced by both domain-general cognitive and domain-specific numerical abilities. It is interesting that this differentiation made in numerical cognition research resembles the conceptualizations in mathematics education that underlie the development of mathematics curricula and education plans.

Process- and Content-Based Competences in Mathematics Education

The majority of current national curricula emphasize some kind of competence scheme (Klieme et al., 2003; Niss & Højgaard, 2011). In mathematics, for example, educational standards in the US, Germany, and Canada differentiate process-based from content-based competences. On one side, content-based competences embrace specific mathematical content such as numbers and operations, measurements, data analysis, or geometry. For every content-based competence, subtopics such as, for instance, basic arithmetic operations, are elaborated and listed, and achievement goals for different grade levels are formulated (KMK, 2004; NCTM, 2000). On the other side, process-based competences involve a broader, more general part of mathematics including strategies and methods. Process-based competences are necessary for dealing with mathematical problems beyond their actual content. These competencies incorporate problem solving, mathematical reasoning and proofs, communicating mathematically, mathematical modeling, representing mathematics, and building connections between mathematical topics and everyday life (NCTM, 2000; KMK, 2004). Whereas content-based competences are supposed to be learned rather explicitly, process-based competences are conveyed more implicitly through the acquisition and application of content-based mathematical knowledge (NCTM, 2000).

It is important to mention that mathematics education suggests that both process- and content-based competences are necessary for mathematical achievement (Winkelmann & Robitzsch, 2009). Thus, it may be plausible to assume that improvements in mathematical achievement come not only from training number-specific contents but might also depend on existing process-based competences. For instance, solving mathematical word problems strongly relies on both process-based and content-based abilities. For instance, Cummins, Kintsch, Reusser, and Weimer (1988) observed that an arithmetic problem was accurately solved by most first graders when it was presented in a numerical format (e.g., 3 + 5 = ?, reflecting a more content-based format). However, success rates decreased to less than one third of first graders when the very same problem was presented in word format (e.g., "Mary has 3 marbles. John has 5 marbles. How many marbles do they have altogether?" which reflects a more process-based format). This indicates that word problems seem to be much more difficult than problems presented in a numerical format. This seems obvious because, when solving word problems, children are not only required to understand the situation by creating a situational model (Stern, 1992), but they also need to transfer the situational model into a mathematical model (i.e., an arithmetic task). Process-based competences such as mathematical problem solving and modeling are required to make this transition from a situational to a mathematical model. Subsequently, content-based competences need to be applied to solve the derived arithmetic problem. Thus, well-developed process-based competences facilitate the creation of situational mental models of the problem and the transferring of the word problem into an arithmetic problem. Content-based competences such as numbers and operations (i.e., basic arithmetic) are then necessary to actually solve the arithmetic problem. Therefore, more specific content-based and rather general process-based competences interact and facilitate mathematical achievement when they are combined (KMK, 2004; NCTM, 2000).

However, tasks drawing primarily on content-based competences are usually rather basic and normally do not demand highly developed process-based competences. By contrast, more complex tasks usually demand process-based competences in addition to content-based competences. Although there are empirical studies that have evaluated the differentiation between content- and process-based competences (Winkelmann & Robitzsch, 2009), there are—to the best of our knowledge—no studies that have investigated the interplay between content- and process-based competences.

Relations between the Two Classifications

Regardless of whether one's point of view on mathematical achievement comes from numerical cognition research or mathematics education, mathematical achievement is thought to be influenced by both domain-specific (i.e., domain-specific numerical abilities or content-based competences) and more superordinate factors (i.e., domain-general abilities or process-based competences).

Fuchs and colleagues (2010) found that domain-general abilities influenced achievement only on complex mathematical problems (i.e., word problems). By contrast, domain-specific abilities were found to be associated with achievement on basic math problems (i.e., procedural calculation) but also to a lesser degree on complex mathematical problems. In line with these findings, Träff (2013) observed that domain-specific numerical abilities (e.g., subitizing and dot counting) predicted children's performance both in arithmetic fact retrieval (i.e., content-based competences) and in mathematical-word-problem solving (requiring more process-based competences). In addition, he also observed that domain-general cognitive abilities (e.g., general fluid intelligence, working memory) predicted performance in word-problem solving (mostly process-based) as well as in calculations (mostly content-based but more complex than arithmetic fact retrieval). These findings were corroborated by a recent study by Sullivan and colleagues (2016), who observed that domain-general factors (e.g., general fluid intelligence and working memory) seemed to better predict differences in mathematical achievement than domain-specific numerical factors did (i.e., Approximate Number System and dot estimation).

According to the literature, domain-specific numerical abilities predict performance in calculations as well as in mathematical word problems. However, as described above, to come up with a final solution for a mathematical word problem, content-based competences (i.e., calculations) need to be applied as well. Thus, to solve mathematical problems that are presented in a rather process-based format, abilities in basic arithmetic operations are also necessary. Looking more closely at basic arithmetic operations, for example, it seems obvious that domain-specific abilities are part of content-based competences. Therefore, we view both domain-specific numerical abilities and content-based competences as domain-specific factors. Furthermore, the contribution of domain-general abilities increases as the complexity of mathematical problems increases, and this holds true for process-based competences as outlined above. Although process-based mathematical competences and domain-general abilities function at different levels (i.e., process-based competences come into play in domain-specific contexts, whereas domain-general abilities arise in several domains), it might be reasonable to propose that domain-specific factors and process-based competences as well as domain-specific factors and domain-general abilities interact in a similar manner.

Research Questions

In this study, we aimed to experimentally examine the associations between domain-specific factors, process-based competences, and domain-general cognitive abilities. In particular, we evaluated whether a training that specifically focused on process-based competences would lead to differential effects on the domain-specific factors and domain-general cognitive abilities that are necessary for successfully dealing with mathematical problems. On the basis of recent research (Bezold, 2012; Demuth, Walther, & Prenzel, 2011; Fuchs et al., 2010; Selter, 2011), we expected that a training of process-based competences should primarily increase students' performance in process-based competences. Furthermore, we argue that—given the above-described interdependency between process- and content-based competences—a process-based training should also have an impact on domain-specific factors. According to the finding that solving complex mathematical problems is more strongly related to process-based than to content-based competences (Fuchs et al., 2010; Sullivan et al., 2016; Träff, 2013), we expected that performance in tasks that require only basic arithmetic operations—presented in a content-based format—would be less influenced by the training than performance in more complex tasks that also require some process-based competences. In a final step, we explored whether enhancing the process-based competences that drive performance in more complex tasks would also have an influence on domain-general cognitive abilities that are thought to have an increasing influence to more complex problems (see Fuchs et al., 2010; Krajewski & Schneider, 2009a, 2009b; Sullivan et al., 2016; Träff, 2013).

Method

This study was part of the Hector Children's Academy Program (HCAP) in the German state of Baden-Württemberg. In this extracurricular enrichment program, 65 local sites offer enrichment courses for the upper 10% of the most talented, interested, and motivated elementary school children. These children are recruited from all elementary schools in the respective area and are nominated by their teachers (for more information, see Rothenbusch et al., 2016).

The intervention

The training "Getting Fit for the Mathematical Olympiad" was designed for small groups of six to 10 students and included eight modules, each planned for a 90-min session. The modules embraced different topics: (a) geometrics (i.e., cubes, tessellations), (b) algebra (i.e., equation-based tasks, cryptograms), (c) numbers and operations (i.e., magic triangles and squares), (d) and patterns and structures (i.e., combinatorics, logic puzzles). The training included rather complex tasks for fostering process-based competences such as mathematical reasoning and problem solving, whereas the number-specific contents did not go beyond elementary school mathematics.

The training framework was based on research on mathematics education for gifted students and was designed to apply cooperative learning methods (for more information, see e.g., Deal & Wismer, 2010; Diezmann & Watters, 2001; Johnson, 1983; Johnson, 1990; Johnson, Johnson, & Stanne, 2000; McAllister & Plourde, 2008; Rotigel & Fello, 2004). We decided to choose high-achieving students as we supposed these children could easily solve curricular-based tasks and would thus have well-developed basic content-based and process-based competences (see e.g., Koshy et al., 2009; Rotigel & Fello, 2004).

Each module presented the same six core components in the same order: (a) introduction, (b) theoretical input/exercise, (c) individual phase, (d) dyadic phase, (e) discussion phase, and (f) presentation. The beginning of each module consisted of a mathematical game as a playful introduction to increase students' motivation. Afterwards, theoretical input and exercise(s) were presented to prepare students for the contents of the module and to indicate possible solution strategies. The main part comprised an individual problem-solving phase in which students worked on possible solution strategies by themselves, followed by a dyadic solving phase in which they worked with a randomly assigned partner. In the dyadic phase, students had to communicate their individual ideas about solving the problem and discuss different solution steps. The next component required the dyads to prepare a structured transcript to clearly verbalize the arguments behind their solution steps. The end of each module consisted of presenting the mathematical problem and its solution to the other students, followed by a final discussion of the arguments. The aims of components (c) to (f) were to foster *problem solving*, to develop ideas to find justifications (e.g., exploring relationships, looking for patterns and structures), to *communicate* mathematical ideas, and to *argue* about mathematical content to conduct preliminary proofs (Bezold, 2012; Demuth et al., 2011).

We next describe the flow of an exemplary module to illustrate the training approach. In the cryptogram module, the session began with the mathematical game *Mental Arithmetic Wizard*. To play this game, students sat in a circle and counted in turn. However, instead of saying numbers that contained a 3 or a multiple of 3, students had to clap their hands. In addition, students had to stomp the ground for every number that contains a 5 or was a multiple of 5. Thereby, children had to recognize patterns, for instance, the respective multiplication tables had to be remembered flexibly. Subsequently, a problem-oriented discussion in class about the definitions of figures and the differences between numbers was initiated to prepare students to solve the cryptograms involving digits. Afterwards, some exercises to indicate possible applications were conducted, such as discussing how many digits one would have to write to note all numbers from 1 to 10.

In the main part of this module, students worked on mathematical problems dealing with cryptograms (for an example, see Figure 1A). First, students tried to solve a cryptogram individually. Each student had the opportunity to work on the problem, look for well-founded relations, and evaluate individual solutions. Following the individual phase, students worked in dyads to solve and discuss the same cryptogram before they were asked to write down their common solution and corresponding justifications. Based on this, the cryptogram and its solutions had to be presented to all fellow students. The training session ended with another mathematical game. In *Find the Calculation*, one student considered a multiplication problem, and the others had to guess what the problem was. Therefore, possible solutions and multiplicands had to be verbalized under the condition that the person considering the problem was allowed to answer with only “higher,” “lower,” “yes,” or “no.” Besides multiplication tables, this game allowed the children to discuss the associative law of multiplication in a playful way (for more about the training, see Rebholz & Golle, 2017, Chapter 2).

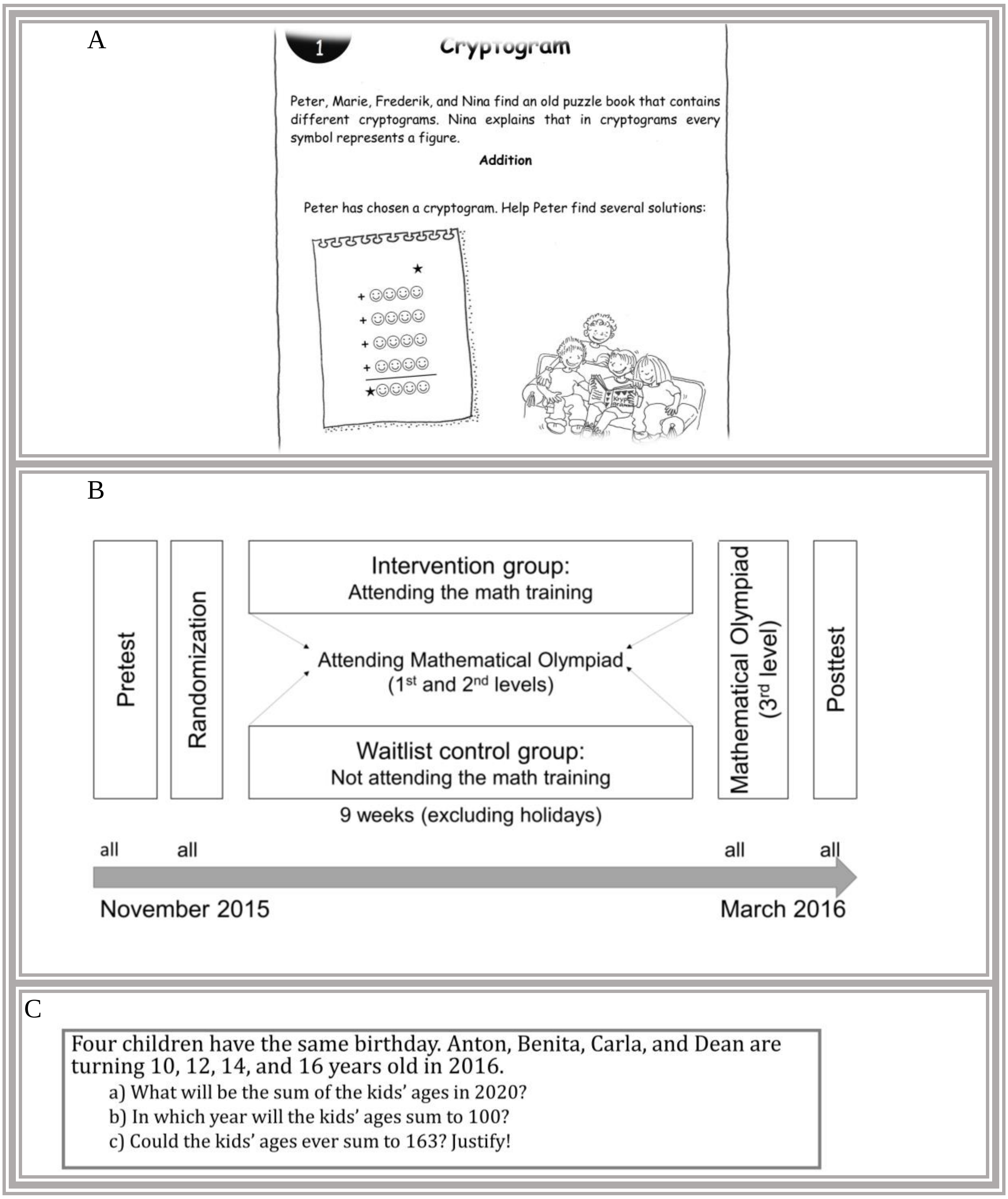


Figure 1. (A) One task from the module cryptograms (as a typical task from the training program). (B) Experimental design of the present study. (C) Translated version of a typical task from the Mathematical Olympiad (task no. 550331, 54th Mathematical Olympiad 2015/16, third grade).

Participants and Procedure

Data were collected from 10 different voluntarily participating local sites of the HCAP. The training was conducted in small groups of five to 10 children. Ten volunteer instructors (50% male; age: *M* = 46.30, *SD* = 16.56) taught the training (for further characteristic of the instructors, see Table 1). Overall, 97 third- and fourth-grade elementary school children took part in the study (68% male; age: *M* = 8.79, *SD* = 0.69). We obtained written informed consent from parents and course instructors prior to the study.

Table 1

Characteristics of Instructors

Characteristic	*N*	*M*	*SD*	*Min*	*Max*
Age [years]	10	46.70	15.56	26	67
Experience with the HCAP [number of courses]	10	6.60	8.06	0	25
Teaching experience [years]	10	19.00	17.21	1	40
Experiences with mathematics [years]	10	24.67	20.62	4	55

To evaluate the effectiveness of the training, we used a multisite randomized controlled repeated-measures field trial (Friedman et al., 2010). A total of 52 children (age *M* = 8.70, *SD* = 0.59; 62% male) were randomly assigned to the mathematics training and 45 children (age *M* = 8.91, *SD* = 0.78; 76% male) to the waitlist control group (for more details, see Figure 2). An independent person performed the randomization. To ensure treatment fidelity, the instructors took a half-day course taught by the developer of the intervention and were given a scripted manual and master copies of all teaching materials. Pretest and posttest measurements for both groups were embedded in the first and last course sessions, respectively, and took about 90 min each (including a 5-min break). All measures were tested via paper-pencil tests and administered by trained research assistants who were blind to the group allocation of the participants. The control group had the opportunity to attend the training after the posttest (see Figure 1B) but attended only regular mathematics classes in school while the intervention group completed the training. The local ethics committee approved the study.

Measures

Domain-specific factors

All scales and their corresponding descriptive statistics as well as their pretest and post-test correlations are displayed in Tables 2, 3, and 4. To assess skills in *basic arithmetic operations,* we used four subscales from a German standardized arithmetic test (HRT; Haffner, Baro, Parzer, & Resch, 2005: (a) addition (1- to 3-digit numbers; e.g., 1 + 6 or 26 + 13), (b) subtraction (1-to-3-digit numbers; e.g., 4 - 1 or 23 - 6), (c) multiplication (1- and 2-digit numbers; e.g., 3 * 1 or 11 * 2), and (d) division (1- and 2-digit numbers; e.g., 6 ÷ 2 or 72 ÷ 8). Each subscale consisted of 40 items with increasing numerical values. Students were asked to solve as many items as possible within a time limit of 2 min on each scale.

To assess *complex content-based mathematical competences*, we used another three subscales from the HRT (Haffner et al., 2005): (a) magnitude comparison (children had to fill in the correct sign to describe the respective relation, <, >, or =; e.g., 11 _ 12, 99 _ 200 - 100), (b) problem completion (children had to fill in the missing number to correctly solve the problem; e.g., 6 + _ = 7, 13 – 12 = 9 -_), and (c) number sequences (children had to continue a given sequence, e.g., 5, 1, 6, 2, 7, 3, …). The first two subscales consisted of 40 items administered with a time limit of 2 min each scale. The last subscale consisted of 20 items with a time limit of 3 min for the whole scale. Students had to solve as many items as possible within the respective time limit.

Domain-specific factors were assessed at pretest and posttest using the HRT. We used sum scores of correctly solved items from the HRT subscales in the statistical analyses.

Process-based mathematical competences

To assess the process-based competences—mathematical *problem solving* and *(mathematical) reasoning*—with regard to word problems, we used participants' performance in a three-level mathematical competition for elementary school students (i.e., the 55th Mathematical Olympiad). The tasks in the Mathematical Olympiad are complex and require students to justify their solutions (for an example, see Figure 1C). Thus, although content-based competences are necessary to calculate the final solutions to the problems, process-based competences are needed first to grasp and model the problems. The German Mathematical Olympiad Association constructed all of the items. Thus, the problems were not known to the developers of the training, the instructors and the students before the training began and thus were not used to develop the intervention. Moreover, the intervention did not exactly mirror the requirements of the 55th Mathematical Olympiad from 2015; rather, the tasks in the intervention were based on tasks from previous Mathematical Olympiads from 2005 to 2013.

Performance in the Mathematical Olympiad (reflected by sum scores) was assessed only on the posttest at the third, most difficult level. Due to the fact that no task was used at more than one level, it was not possible to implement a pre-posttest design to measure process-based competences in particular.

Domain-general cognitive abilities

We assessed *domain-general cognitive abilities* at both occasions with the fluid intelligence subscale from a German intelligence test (BEFKI-short; Schroeders et al., 2016) consisting of 16 items (time limit 15 min) with two-step figural seriations. Moreover, at posttest, we also administered two subscales (i.e., matrices; 15 items, time limit 3 min) and (one-step) figural seriation (15 items; time limit 4 min) from the Culture Fair Test 20-R (Weiß, Albinus, & Arzt, 2006). Finally, crystallized intelligence was also assessed with the second part of BEFKI-short (Schroeders et al., 2016) on the pretest with 16 items (time limit 8 min). This test was used to control for potential baseline differences between the training and control groups. For these measures, sum scores of correctly solved items were used in further analyses (see Table 3 for examples).

Additional psychological measures

Motivational factors have been found to be related to mathematical competences in general (e.g., Musu-Gillette et al., 2015; Wigfield & Eccles, 2000). However, all children who participated in the current study wanted to attend the math training. Thus, we did not expect systematic differences on motivational factors between the training and control groups. However, to be able to control for motivational factors, we assessed them at pretest and posttest. Using the scales developed by Gaspard and colleagues (2015), we assessed *mathematics self-concept* (four items) and *(intrinsic) interest in mathematics* (six items) with measures that were adapted for this age group. Response scales ranged from 1 (*not true*) to 4 (*exactly*; e.g., “I’m good at everything that has to do with mathematics”). In addition, we assessed *need for cognition* (six items) with Baudson, Strobel, and Preckel’s (2012) instrument. The response scale ranged from 1 (*not true*) to 5 (*exactly*; e.g., “I like solving tricky tasks”). Mean scores were used in further analyses.

Statistical analyses

We tested baseline differences for statistical significance to exclude systematic group differences before the training had begun (t tests and a chi-square test). There were no significant group differences at pretest, neither for the distribution of gender ($\chi 2$ =1.58, df = 1, p = .210) nor for the cognitive or motivational variables (for more details, see Tables 2, 3 and 4). We evaluated the effectiveness of the training with multiple linear regression analyses using the R package lavaan (R Core Team, 2015; Rosseel, 2012). All variables were z-standardized prior to the analyses except for the categorical variables gender (0 = girls, 1 = boys) and group membership (0 = control, 1 = intervention).

We computed two types of multiple linear regression models separately for each dependent variable. In the first models, we evaluated overall differences between the training and control groups and controlled for gender and pretest performance. We included gender in this step because there were fewer girls than boys in our total sample. We included pretest performance as a predictor to increase power and minimize standard errors. Because the dependent variables were z-standardized, the multiple regression coefficient of the group variable indicated the standardized difference between the training and control groups at posttest while gender and pretest performance were controlled for.

As there is an ongoing debate on the role of gender differences in mathematics performance (e.g., Hyde et al., 1990; Liu et al., 2008; Liu & Wilson, 2009), we also conducted an analysis with a second type of model in which we added the Group Membership x Gender interaction to the first models. In this second set of models, the multiple regression coefficient of the group variable indicated the standardized difference between the training and control group at posttest while pretest performance was controlled for girls only (as they had the dummy code of 0). Here, the coefficient for the interaction term represents the difference in the treatment effect for girls and boys, again while pretest performance was controlled for.

In all analyses, we used the robust maximum likelihood estimator, which corrects the standard errors for the non-normality of the variables. Missing values occurred in both groups (for more details, see Figure 2 or Tables 2, 3 and 4). We used the full information maximum likelihood approach to deal with missing values (Enders, 2010; Graham, 2009; R Core Team, 2015; Rosseel, 2012).

Table 2

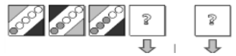
Descriptive Statistics for the Mathematical Measures: Means, Standard Deviations, Internal Consistencies, Number of Items, and Examples

Construct		Pretest					Posttest						Number of items and example	
		N	*M*	*SD*	*p*	α	*N*	*M*	*SD*	α	*p*	$r_{\text{pre-posttest}}$		
Age	IG	47	8.73	0.54	.158		49	9.09	0.57					
	CG	40	8.86	0.84			38	9.09	0.80					
Basic arithmetic operations	IG	46	101.91	17.23	.217	.85	47	106.39	16.43	0.85	.722	$r(75) = .83$, $p < .001$	160	Addition, subtraction, multiplication, division
	CG	41	97.46	16.11			38	105.08	17.77					
Complex content-based competences	IG	47	58.65	11.26	.098	.56	49	62.71	10.93	0.56	.413	$r(76) = .77$, $p < .001$	100	Magnitude comparison, problem completion, number sequences
	CG	41	54.66	11.07			38	60.72	11.72					
Performance in Mathematical Olympiad	IG						46	12.47	7.12	0.47	.003			See Figure 1C
	CG						38	8.40	5.46					

Note. *N* = Number of valid answers from participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha (calculated with SPSS 22, IBM Corp. Released, 2013). Measurement points: Pretest = November 2015, Posttest = March 2016. IG = Intervention group, CG = control group. *t* tests for independent samples (R Core Team, 2015) were computed to test for significant differences between the IG and the CG. Chi-square test was calculated with R. Two-tailed significance levels are reported.

Table 3

Descriptive Statistics for Cognitive Abilities: Means, Standard Deviations, Internal Consistencies, Number of Items, and Examples

Construct		Pretest					Posttest						
		N	*M*	*SD*	*p*	α	*N*	*M*	*SD*	α	*p*	$r_{\text{pre-posttest}}$	Number of items and example
Figural cognitive skills													
→ BEFKI-short	IG	47	9.23	2.42	.915	.61	49	9.48	2.53	0.56	.958	$r(75) = .39$, $p < .001$	16 
	CG	40	8.83	3.08			38	9.39	2.44				
→ Culture fair test 20-R	IG						48	22.10	2.73	0.86	.027		30 
	CG						38	20.66	3.27				
Crystallized intelligence → BEFKI-short	IG	47	10.40	2.40	.426	.54							16 What's google?
	CG	42	10.10	2.82									

Note. *N* = Number of valid answers from participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha (calculated with SPSS 22; IBM Corp. Released, 2013). Measurement points: Pretest = November 2015, Posttest = March 2016. IG = Intervention group, CG = control group. *t* tests for independent samples (R Core Team, 2015) were computed to test for significant differences between the IG and the CG. Chi-square test was calculated with R. Two-tailed significance levels are reported.

Table 4

Descriptive Statistics for the Motivational Covariates: Means, Standard Deviations, Internal Consistencies, Number of Items, and Examples

Construct		Pretest					Posttest				$r_{pre\text{-}posttest}$	Number of items and example/subscales	
		N	*M*	*SD*	*p*	α	*N*	*M*	*SD*	α			
Interest in mathematics	IG	47	4.52	0.54	.604	.85	45	4.24	0.69	.94	$r(74) = .58$, $p < .001$	6	I like everything that has to do with mathematics.
	CG	41	4.45	0.75			38	4.37	0.78				
Need for cognition	IG	47	3.17	0.57	.520	.84	46	3.17	0.57	.85	$r(75) = .53$, $p < .001$	6	I like solving tricky tasks.
	CG	42	3.08	0.75			37	3.09	0.63				
Mathematics self-concept	IG	47	4.47	0.61	.988	.90	44	4.13	0.72	.91	$r(73) = .35$, $p = .002$	4	I'm good at everything that has to do with math.
	CG	41	4.48	0.73			38	4.23	0.73				

Note. *N* = Number of valid answers from participating children, *M* = mean, *SD* = standard deviation, α = Cronbach's alpha (calculated with SPSS 22; IBM Corp. Released, 2013). Measurement points: Pretest = November 2015, Posttest = March 2016. IG = Intervention group, CG = control group. *t* tests for independent samples (R Core Team, 2015) were computed to test for significant differences between the IG and the CG at pretest. Two-tailed significance levels are reported.

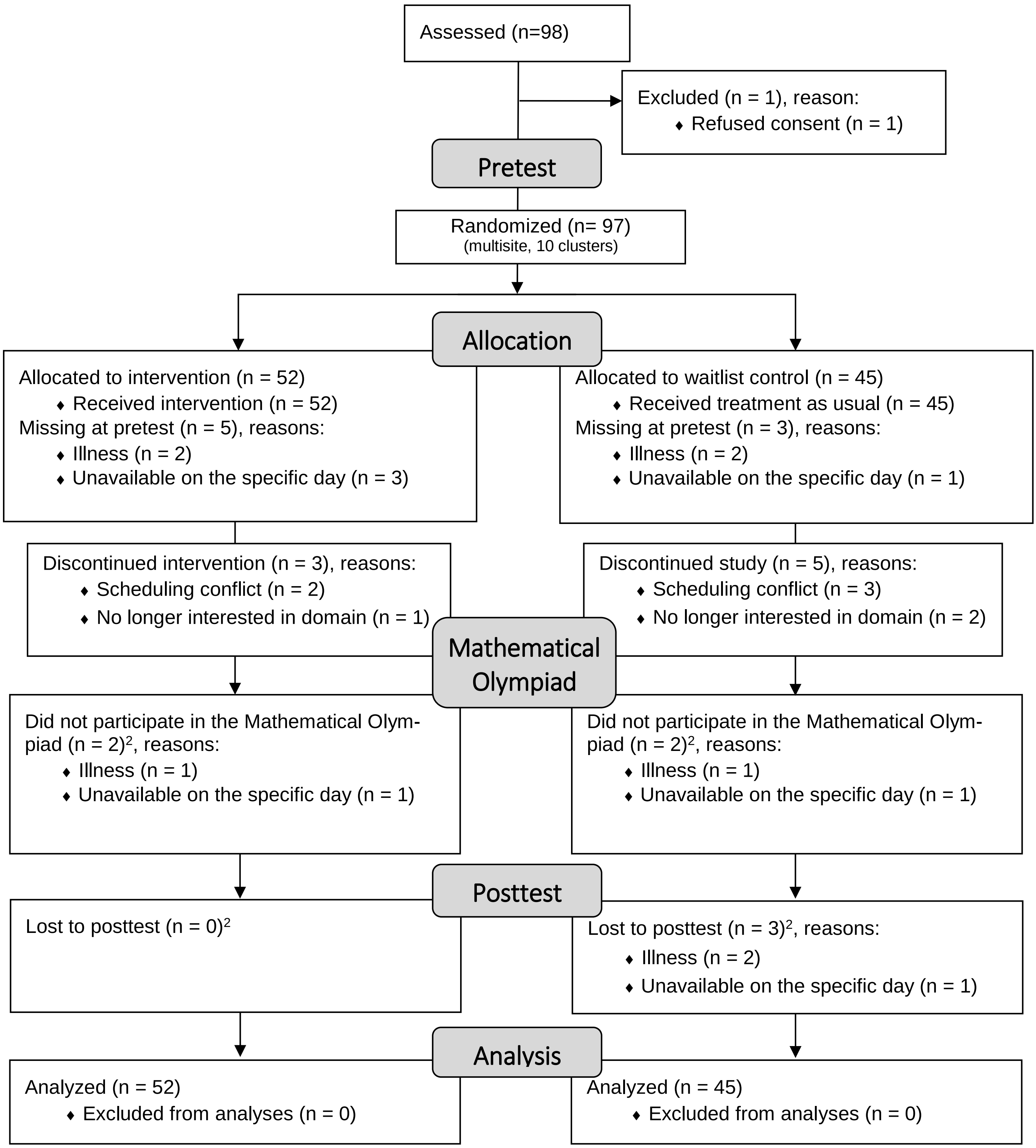


Figure 2. Flow chart of the study (based on the CONSORT Flow diagram, 2010). The missing values from different measurement occasions (pretest, Mathematical Olympiad, and posttest) cannot be ascribed to the same participants. 2Excluding participants who discontinued.

Results

Domain-specific factors

For *basic arithmetic operations*, there was no significant intervention effect ($B_{\text{Intervention}}$ = -0.01, p = .751, see Table 5). Nevertheless, the interaction between gender and group membership was significant ($B_{\text{Intervention×Gender}}$ = -0.45, p = .034, see Table 6), indicating that the intervention effect was more pronounced for girls than for boys, even though the actual simple effect of the training was not significant for girls or boys (B_{Girls} = 0.26, p = .058, B_{Boys} = -0.19, p = .252). Even though the children in the control and intervention groups did not differ significantly in their skills on basic arithmetic operations after the training, girls benefitted significantly more from the training.

Similarly, the analyses indicated no significant training effect for *complex content-based competences* ($B_{\text{Intervention}}$ = -0.11, p = .465). But again, this effect was qualified by gender as indicated by the significant interaction term ($B_{\text{Intervention×Gender}}$ = -0.78, p = .003). An inspection of the beta weight (see Table 6) revealed that the training effect was significantly more pronounced for girls in comparison with boys, with a significant training effect for girls (B_{Girls} = .52, p = .010, B_{Boys} = -.26, p = .149). Therefore, comparable to the situation for *basic arithmetic operations*, girls seemed to specifically benefit from the intervention with regard to their *complex content-based competences* (see Tables 5 and 6).

Process-based mathematical competences

The regression results revealed a significant training effect on performance in the *Mathematical Olympiad*, $B_{\text{Intervention}}$ = 0.63, p = .002. The nonsignificant interaction term ($B_{\text{Intervention×Gender}}$ = -0.71, p = .074, Table 6) indicated that the effect was not further qualified by gender. This finding suggests that the process-based competences of the children who completed the intervention significantly improved in comparison with the children in the control group (see Table 5).

Domain-general cognitive abilities

Finally, the analyses indicated that the two groups differed significantly in their *general cognitive abilities* at the end of the training as assessed by the CFT 20-R, $B_{\text{Intervention}}$ = 0.49, p = .012. This effect was also not qualified by gender, $B_{\text{Intervention×Gender}}$ = -0.26, p = .541. However, this finding was not substantiated by the results for fluid general cognitive abilities assessed by BEFKI-short ($B_{\text{Intervention}}$ = 0.08, p = .682; $B_{\text{Intervention×Gender}}$ = 0.34, p = .417). Thus, for general cognitive abilities, we observed a positive trend, indicated by a significantly positive intervention effect for CFT 20-R scores (see Tables 5 and 6).

Table 5

Effects of the Intervention Predicting the Outcome (Average Causal Effects)

	Basic arithmetic operations[a]			Complex content-based competences[a]			Performance in Mathematical Olympiad[a,b]			Figural cognitive skills (BEFKI-short)[a,c]			Figural cognitive skills (CFT 20R)[a,c]		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	0.02	0.11	.851	-0.11	0.15	.465	**-0.43**	0.19	**.024**	-0.25	0.21	.239	-0.20	0.21	.343
Intervention	-0.01	0.15	.751	-0.01	0.15	.975	**0.63**	0.20	**.002**	0.08	0.20	.682	**0.49**	**0.20**	**.012**
Gender	-0.07	0.12	.581	0.13	0.16	.402	0.12	0.221	.553	0.30	0.20	.142	-0.13	0.20	.524
Pretest[a]	**0.86**	**0.06**	**< .001**	**0.76**	**0.09**	**< .001**	**0.27**	**0.11**	**.013**	**0.38**	**0.12**	**.001**	**0.41**	**0.08**	**< .001**
R^2		.695			.575			.177			.168			.233	

Note. Dependent variables were standardized prior to the analyses. Intervention was dummy-coded 0 = control group, 1 = intervention. Gender was dummy-coded 0 = girls, 1 = boys. Two-tailed significance levels are reported. [a]Variables were standardized prior to the analyses. [b]The pretest variable was the performance in Level 1 of the competition standardized prior to the analyses. [c]Pretest variable was the cognitive figural skills (BEFKI-short) standardized prior to the analyses.

Table 6

Differential Effects of the Intervention Predicting the Outcome (Effects Separated by Gender)

	Basic arithmetic operations[a]			Complex content-based competences[a]			Performance in Mathematical Olympiad[a,b]			Figural cognitive skills (BEFKI-short)[a,c]			Figural cognitive skills (CFT 20R)[a,c]		
	B	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*	*B*	*SE*	*p*
Intercept	-0.17	0.10	.068	**-0.45**	**0.14**	**.001**	**-0.76**	0.18	< **.001**	-0.08	0.28	.771	-0.32	0.30	.281
Intervention	0.26	0.14	.058	**0.52**	**0.20**	**.010**	**1.12**	0.29	< **.001**	-0.15	0.34	.647	0.67	0.35	.059
Gender	0.20	0.16	.206	**0.60**	0.19	**.002**	**0.57**	**0.24**	**.019**	0.08	0.33	.802	0.03	0.35	.930
Intervention x Gender	**-0.45**	**0.21**	**.034**	**-0.78**	**0.26**	**.003**	-0.71	0.40	.074	0.34	0.41	.417	-0.26	0.42	.541
Pretest[a]	**0.88**	**0.06**	**< .001**	**0.78**	**0.08**	< **.001**	**0.30**	**0.11**	**.006**	**0.37**	**0.12**	**.002**	**0.41**	**0.08**	< **.001**
R^2		.705			.610			.195		.171				.233	

Note. Dependent variables were standardized prior to the analyses. Intervention was dummy-coded 0 = control group, 1 = intervention. Gender was dummy-coded 0 = girls, 1 = boys. Two-tailed significance levels are reported. [a]Variables were standardized prior to the analyses. [b]Pretest variable was the performance in Level 1 of the competition standardized prior to the analyses. [c]Pretest variable was the cognitive figural skills (BEFKI-short) standardized prior to the analyses.

Discussion

In this study, we aimed to evaluate whether a training of process-based competences (i.e., *problem solving*, *communicating*, *arguing*) for elementary school children would differentially improve their process-based competences as well as their domain-specific factors (i.e., basic arithmetic operations and complex content-based competences) and domain-general cognitive abilities.

Overall, the results indicated that the training was successful in fostering process-based competences—as reflected by participants' performance in the Mathematical Olympiad—but also general cognitive abilities—reflected by their performance on the CFT 20-R. However, the latter finding was not substantiated by the results of the BEFKI-short and thus should be treated with some caution. As different mathematical tasks were used in the training and the Mathematical Olympiad, this indicates that the process-based mathematical competences acquired by attending the training seemed to be transferable to new mathematical problems. Especially when conceiving of mathematics as the science of patterns (e.g., Devlin, 1996), it seems plausible that a training of process-based competences such as problem solving and the recognition of patterns should increase performance in solving new word problems as used in the Mathematical Olympiad.

Furthermore, there were no significant training effects on domain-specific factors. These missing effects might have resulted from the sample recruited for the training. The training was developed for elementary school students who are very good at mathematics (average math grade: M = 1.35, SD = 0.48 on a scale ranging from 1 to 6 with 1 representing the best grade). However, solving the tasks in the training (which focused primarily on process-based mathematical competences) required only very basic calculations (i.e., numerical content in numbers ranging from 1 to 10). This might not have been challenging for this group of children because more complex content (e.g., multiplication with results larger than 100) may be necessary to challenge such children (e.g., Käpnick, 2014).

Considering the nonsignificant treatment effects on basic arithmetic but significant effects on domain-general abilities and process-based competences, the results are in line with previous findings in numerical cognition research. For instance, with respect to domain-general cognitive abilities (e.g., intelligence, working memory), Fuchs and colleagues (2010) and Träff (2013) found that the domain-general cognitive abilities predicted performance in rather complex mathematical tasks (i.e., word problems) but not in basic tasks (e.g., subitizing, basic arithmetic).

Likewise, at least some domain-specific numerical abilities (e.g., understanding magnitudes or the place-value-system) may form the basis for developing content-based mathematical skills (e.g., algebraic skills or the ability to deal with measurements), which tend to be more complex and applied. These ideas regarding the interrelation between the two conceptualizations seem reasonable but need further research to substantiate them. Even though there is increasing research interest in numerical cognition research on influences of domain-general cognitive abilities on children's numerical development, future research needs to investigate the interplay between domain-specific factors (i.e., domain-specific numerical abilities and content-based competences) and broader constructs (i.e., domain-general cognitive abilities and process-based mathematical competences) in more detail. These connections need to be specified in order to inform mathematical instruction.

Regarding the similarity between the conceptualizations of numerical cognition research as well as mathematics education, we already pointed out that both differentiate between domain-specific factors (e.g., number magnitudes, arithmetical operations) and the skills that are additionally necessary to successfully deal with mathematical problems (e.g., problem solving or general cognitive ability). Actually, one might even go so far as to argue that process-based competences reflect an application of domain-general cognitive abilities (e.g., executive functioning or working memory) in a mathematical context and on mathematical problems.

As there is evidence for gender differences in mathematical achievement (e.g., Hyde et al., 1990; Liu et al., 2008; Liu & Wilson, 2009), we included gender and the interaction between gender and group membership as predictors in the regression models. It is interesting that the training effects on process-based competences and domain-general abilities did not differ between boys and girls. This suggests that transfer mechanisms catalyzing performance on these constructs may be comparable for girls and boys. However, effects on basic and especially complex domain-specific factors were more pronounced and even significant for girls only. However, it is important to note that there were significant gender differences in domain-specific factors at pretest—basic arithmetic operations: $t(85) = 2.47$, $p = .016$; complex content-based competences: $t(86) = 2.37$, $p = .020$—which might explain why girls benefitted more from the training. Because girls' performance was poorer than boys' performance on the pretest, this left more room for improvements for girls.

The significant effect on girls' complex content-based competences may suggest that the training had specific effects on more complex mathematical tasks that require superordinate skills. This idea is in line with our expectation that our training of process-based competences may be beneficial for factors that go beyond standard procedures in executing basic arithmetic operations but require some more advanced process-based skills (e.g., to flexibly model arithmetic problems as in the problem completion task or the recognition of patterns in the number sequences task). This provides a nice illustration that, as the complexity of mathematical problems increases, it might not be exclusively domain-specific mathematical factors that drive performance, but rather, process-based competences and domain-general cognitive abilities may come into play.

Another explanation for the different results for boys and girls might be that girls benefitted more from the core components of the intervention, in particular from communicating about mathematics with other children and the intensive transcription involved in justifying solutions (parts of cooperative learning; e.g., Johnson et al., 2000)—but specifically so for domain-specific factors. These transfer effects of our training might be due to the fact that girls tend to prefer cooperative learning methods more than boys (e.g., Johnson & Engelhard, 1992). As a consequence, they might have profited from the training on a broader scale than boys. It is interesting, however, that Lee (1995) found that girls' and boys' reports of their experiences in cooperative learning depended on group composition. It might thus be interesting to explore whether interactions between group members in cooperative learning situations depend on gender. But, otherwise, the results for gender differences did not occur for the broader constructs of process-based and domain-general cognitive abilities.

Last but not least, there are some points that should be noted when interpreting the results of this study. Unfortunately, for some outcome measures, the reliability coefficients that we found were not so good (e.g., complex content-based competences [HRT] or figural cognitive skills [BEFKI-short]; see Tables 2 and 3). However, considering that we reported retest reliabilities (see Tables 2 and 3) and that all measures were assessed in group settings, these reliabilities are acceptable. Furthermore, to assess students' process-based competences, we used only tasks from the German Mathematical Olympiad for elementary school students. As this academic competition seems to be one of the most challenging ones (Olson, 2005; www.imo-official.org), it might be fruitful to include a standardized measure of process-based competences (e.g., less complex word problems) at pretest and posttest. Moreover, as the data were collected only at the beginning and end of the intervention, no conclusions can be drawn about the effectiveness of single elements of the training. It might be desirable to include intermediate surveys in future research to identify effects of training components or components that work better than others in fostering students' content-based or process-based competences.

Taken together, our results indicate that a training that focused on enhancing process-based competences had differential effects on process-based competences, domain-general abilities, and domain-specific factors. In particular, our study revealed differential effects of the training of process-based mathematical competences on these competences but also on domain-general cognitive abilities. In addition, we observed beneficial effects on complex content-based competences for girls only, possibly indicating that girls profited more from the collaborative nature of the intervention. In sum, this study provided initial empirical support for conceptual similarities in numerical cognition as well as mathematics education research regarding factors that contribute to children's numerical development (i.e., domain-specific numerical and domain-general cognitive abilities vs. content-based and process-based mathematical competences, respectively). Even the results of our training study seem to corroborate the notion of a conceptual similarity between the contributions made to mathematical achievement by domain-specific numerical abilities and content-based competences as well as process-based competences and domain-general cognitive abilities. The present study is thus a first step toward a closer integration of the literature on numerical cognition research and mathematics education. Despite all the benefits of interdisciplinary research for the two research communities, such research is especially beneficial for those who need to acquire mathematical skills: the children.

References

Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., Smedt, B. de, Gilmore, C.,. . . Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. *Journal of Numerical Cognition*, *2*(1), 20–41. https://doi.org/10.5964/jnc.v2i1.10

Baudson, T. G., Strobel, A., & Preckel, F. (2012). Validierung einer neuen Need for Cognition (NFC)-Skala für Grundschülerinnen und Grundschüler: Struktur, Messinvarianz und Zusammenhänge mit Intelligenz- und Leistungsmaßen [Validation of a new Need for Cognition (NFC)-Scale for elementary school students: Structure, measurement invariance and correlations with intelligence and performance]. In R. Riemann (Ed.), *48. Kongress der Deutschen Gesellschaft für Psychologie* (p. 59). Lengerich: Pabst Science Publishers.

Bezold, A. (2012). Förderung von Argumentationskompetenzen auf der Grundlage von Forscheraufgaben. [Fostering the process of justification using research tasks.]. *mathematica didactica*. (35), 73–103.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science (New York, N.Y.)*, *332*(6033), 1049–1053. https://doi.org/10.1126/science.1201536

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive psychology*, *20*(4), 405–438. https://doi.org/10.1016/0010-0285(88)90011-4

Deal, L. J., & Wismer, M. G. (2010). NCTM Principles and Standards for Mathematically Talented Students. *Gifted Child Today*, *33*(3), 55–65. https://doi.org/10.1177/107621751003300313

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical cognition*, *1*(1), 83–120.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive neuropsychology*, *20*(3), 487–506. https://doi.org/10.1080/02643290244000239

Demuth, R., Walther, G., & Prenzel, M. (Eds.). (2011). *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* [Developing education with SINUS: 10 Models for mathematics and science in elementary schools]. Seelze: Friedrich.

Devlin, K. (1996). *Mathematics: The science of patterns: the search for order in life, mind and the universe* (2nd print., paperback ed.). New York: Scientific American Library.

Diezmann, C. M., & Watters, J. J. (2001). The Collaboration of Mathematically Gifted Students on Challenging Tasks. *Journal for the Education of the Gifted*, *25*(1), 7–31. https://doi.org/10.1177/016235320102500102

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuro-science and education*. New York, NY: Psychology Press.

Enders, C. K. (2010). *Applied missing data analysis*. New York, NY: The Guilford Press.

Friedman, L. M., Furberg, C., & DeMets, D. L. (2010). *Fundamentals of clinical trials* (4th). New York, NY: Springer.

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M.,. . . Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental psychology*, *46*(6), 1731–1746. https://doi.org/10.1037/a0020662

Gaspard, H., Dicke, A.-L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B. (2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology*, *107*(3), 663–677. https://doi.org/10.1037/edu0000003

Geary, D. C., & Moore, A. M. (2016). Cognitive and brain systems underlying early mathe-matical development. *Progress in brain research*, *227*, 75–103. https://doi.org/10.1016/bs.pbr.2016.03.008

Geary, D. C., Bailey, D. H., & Hoard, M. K. (2009). Predicting mathematical achievement and mathematical learning disability with a simple screening tool: The number sets test. *Journal of Psychoeducational Assessment*, *27*(3), 265–279.

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. *Annual review of psychology*, *60*, 549–576. https://doi.org/10.1146/annurev.psych.58.110405.085530

Haffner, J., Baro, K., Parzer, P., & Resch, F. (2005). *Heidelberger Rechentest (HRT 1-4)* [Hei-delberg Arithemtic Test]. Göttingen: Hogrefe.

Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender differences in mathematics perfor-mance: A meta-analysis. *Psychological Bulletin*, *107*(2), 139–155. https://doi.org/10.1037/0033-2909.107.2.139

IBM Corp. Released. (2013). IBM SPSS Statistics for Windows. Armonk, NY.

Johnson, C., & Engelhard, G. (1992). Gender, academic achievement, and preferences for co-operative, competitive, and individualistic learning among African-American adolescents.

The Journal of psychology, *126*(4), 385–392. https://doi.org/10.1080/00223980.1992.10543371

Johnson, D. W., Johnson, R. T., & Stanne, M. B. (2000). *Cooperative Learning Methods: A Meta-Analysis.* University of Minnesota. Retrieved from https://pdfs.semanticscholar.org/93e9/97fd0e883cf7cceb3b1b612096c27aa40f90.pdf

Johnson, M. L. (1983). Identifying and teaching mathematically gifted elementary school children. *Arithmetic Teacher*, *30*(5), 25–26.

Johnson, R. T. (1990). Supporting gifted students' acquisition of relevant knowledge for solving math problems. *Early Child Development and Care*, *63*(1), 37–45. https://doi.org/10.1080/0300443900630106

Käpnick, F. (2014). *Mathematiklernen in der Grundschule* [Learning mathematics in elementary schools]. Berlin, Heidelberg: Springer.

Klein, E., Suchan, J., Moeller, K., Karnath, H.-O., Knops, A., Wood, G.,. . . Willmes, K. (2016). Considering structural connectivity in the triple code model of numerical cognition: Differential connectivity for magnitude processing and arithmetic facts. *Brain structure & function*, *221*(2), 979–995. https://doi.org/10.1007/s00429-014-0951-1

Klieme, E., Jude, N., Baumert, J., & Prenzel, M. (2010). PISA 2000–2009: Bilanz der Veränderungen im Schulsystem [PISA 2000-2009: A review of changes in the schhol system]. In E. Klieme, C. Artelt, J. Hartig, N. Jude, O. KÃ¶ller, M. Prenzel, & S. Wolfgang (Eds.), *PISA 2009. Bilanz nach einem Jahrzehnt* (pp. 277–300). Waxmann Verlag.

Klieme, E., Avenarius, H., Blum, W., Döbrich, P., Gruber, H., Prenzel, M.,. . . Tenorth, H.-E. (2003). *Zur Entwicklung nationaler Bildungsstandards* [For the development of national educatioanl standards].

Koshy, V., Ernest, P., & Casey, R. (2009). Mathematically gifted and talented learners: Theory and practice. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 213–228. https://doi.org/10.1080/00207390802566907

Krajewski, K., & Schneider, W. (2009a). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*(6), 513–526. https://doi.org/10.1016/j.learninstruc.2008.10.002

Krajewski, K., & Schneider, W. (2009b). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of experimental child psychology*, *103*(4), 516–531. https://doi.org/10.1016/j.jecp.2009.03.009

Kultusministerkonferenz (Ed.). (2004). *Bildungsstandards im Fach Mathematik für den Primarbereich.* [Decisions of the Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany. Scholastic standards for mathematics for elementary schools]. Neuwied: Luchterhand.

Lee, M. (1995). Gender, Group Composition, and Peer Interaction in Computer-Based Cooperative Learning. *Journal of Educational Computing Research*, *9*(4), 549–577. https://doi.org/10.2190/VMV1-JCVV-D9GA-GN88

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child development*, *81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Liu, O. L., & Wilson, M. (2009). Gender Differences in Large-Scale Math Assessments: PISA Trend 2000 and 2003. *Applied Measurement in Education*, *22*(2), 164–184. https://doi.org/10.1080/08957340902754635

Liu, O. L., Wilson, M., & Paek, I. (2008). A multidimensional Rasch analysis of gender differences in PISA mathematics. *Journal of applied measurement*, *9*(1), 18. Retrieved from https://gse.berkeley.edu/sites/default/files/users/mark-wilson/Wilson8.pdf

McAllister, B. A., & Plourde, L. A. (2008). Enrichment Curriculum: Essential for mathematically gifted students. *Education*, *129*(1), 40–49. Retrieved from http://web.a.ebscohost.com/ehost/pdfviewer/pdfviewer?sid=2c0b2b92-484e-4295-8ceb-ae855b7aceed%40sessionmgr4009&vid=2&hid=4214

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H.-C. (2011). Early place-value understanding as a precursor for later arithmetic performance—a longitudinal study on numerical development. *Research in developmental disabilities*, *32*(5), 1837–1851. https://doi.org/10.1016/j.ridd.2011.03.012

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*(4), 343–370. https://doi.org/10.1080/13803611.2015.1057161

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*: National Council of Teachers of.

Niss, M., & Højgaard, T. (2011). *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde. Retrieved from IMFUFA, Roskilde university website: http://milne.ruc.dk/imfufatekster/pdf/485web_b.pdf

Olson, S. (2005). *Count down: Six kids vie for glory at the world's toughest math competition*. Boston: Houghton Mifflin. Retrieved from http://www.loc.gov/catdir/samples/hm051/2003056897.html

Organisation for Economic Co-operation and Development. (2014). *PISA 2012 Ergebnisse: Was Schülerinnen und Schüler wissen und können - Schulleistungen in Lesekompetenz, Mathematik und Naturwissenschaften* [PISA 2012 results: What students know and can do - students performance in reading, mathematics and science]. Bielefeld: Bertelsmann.

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *The British journal of educational psychology*, *82*(Pt 1), 42–63. https://doi.org/10.1111/j.2044-8279.2011.02039.x

R Core Team. (2015). R. Vienna, Austria: the R foundation for Statistical Computing. Retrieved from www.R-project.org

Rebholz, F., & Golle, J. (2017). Rebholz, F. & Golle, J. (2017). Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade. [Fostering mathematical skills in elementary school – die role of academic competions using the example of the Mathematical Olympiad.]. In U. Trautwein & M. Hasselhorn (Eds.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente (PSYNDEXalert)* (pp. 213–228). Göttingen: Hogrefe.

Rosseel, Y. (2012). *lavaan: An R package for structural equation modeling and more Version 0.4-9 (BETA)*: Ghent University.

Rothenbusch, S., Zettler, I., Voss, T., Lösch, T., & Trautwein, U. (2016). Exploring reference group effects on teachers' nominations of gifted students. *Journal of Educational Psychology*, *108*(6), 883–897. https://doi.org/10.1037/edu0000085

Rotigel, J. V., & Fello, S. (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today*, *27*(4), 46–51.

Schroeders, U., Schipolowski, S., Zettler, I., Golle, J., & Wilhelm, O. (2016). Do the smart get smarter? Development of fluid and crystallized intelligence in 3rd grade. *Intelligence*, *59*, 84–95. https://doi.org/10.1016/j.intell.2016.08.003

Sella, F., Sader, E., Lolliot, S., & Cohen Kadosh, R. (2016). Basic and advanced numerical performances relate to mathematical expertise but are fully mediated by visuospatial skills. *Journal of experimental psychology. Learning, memory, and cognition*, *42*(9), 1458–1472. https://doi.org/10.1037/xlm0000249

Selter, C. (2011). „Ich mark Mate" - Leitideen und Beispiele für interessenförderlichen Unterricht [Ideas and examples for education that fosters interest in mathematics]. In R. Demuth, G. Walther, & M. Prenzel (Eds.), *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* (pp. 131–139). Seelze: Friedrich.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child development*, *75*(2), 428–444.

Stern, E. (1992). Warum werden Kapitänsaufgaben „gelöst"? [Why specific tasks get „solved"?]. *Der Mathematikunterricht*, *38*(5), 7–29.

Sullivan, J., Frank, M. C., & Barner, D. (2016). Intensive math training does not affect approximate number acuity: Evidence from a three-year longitudinal curriculum intervention. *Journal of Numerical Cognition*, *2*(2), 57–76. https://doi.org/10.5964/jnc.v2i2.19

Träff, U. (2013). The contribution of general cognitive abilities and number abilities to different aspects of mathematics in children. *Journal of experimental child psychology*, *116*(2), 139–156. https://doi.org/10.1016/j.jecp.2013.04.007

van de Rijt, B.A.M., van Luit, J.E.H., & Pennings, A. H. (2016). The Construction of the Utrecht Early Mathematical Competence Scales. *Educational and Psychological Measurement*, *59*(2), 289–309. https://doi.org/10.1177/00131644995920006

von Aster, M. G. von, & Shalev, R. S. (2007). Number development and developmental dyscalculia. *Developmental medicine and child neurology*, *49*(11), 868–873. https://doi.org/10.1111/j.1469-8749.2007.00868.x

Weiß, R., Albinus, B., & Arzt, D. (2006). *Grundintelligenztest Skala 2-Revision (CFT 20-R)* [Intelligence Test Scale 2-Revision (CFT 20-R)]: Hogrefe.

Wigfield, & Eccles. (2000). Expectancy-Value Theory of Achievement Motivation. *Contemporary Educational Psychology*, *25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Winkelmann, H., & Robitzsch, A. (2009). Modelle mathematischer Kompetenzen: Empirische Befunde zur Dimensionalität [Models of mathematical competences.]. *D. Granzer, O. Köller, A. Bremerich-Vos, M. van den Heuvel-Panhuizen, K. Reiss & G. Walther (Hg.), Bildungsstandards Deutsch und Mathematik. Leistungsmessung in der Grundschule*, 169–196.

5

General Discussion

5 General Discussion

Sophisticated mathematical competences are assumed to enable students to cope with educational and vocational requirements as well as the demands of everyday life (e.g., Grønmo et al., 2015; Murnane et al., 1995; NCTM, 2000; OECD, 2014). One way to challenge students who are already able to solve mathematical problems and tasks in elementary school is to encourage them to participate in academic competitions. Academic competitions are assumed to enhance students' domain-specific competence and motivation by providing the opportunity to work on problems in a domain of interest (e.g., Forrester, 2010; Oswald et al., 2005).

The present dissertation explored such considerations by asking about the appropriateness of academic competitions and corresponding trainings in enriching learning environments in mathematics. Therefore, in a first step, the concept of mathematical competences was reviewed, factors that were found to influence the acquisition of mathematical competences were summarized, and the characteristics and needs of mathematically gifted students were derived (Chapter 1). In a second step, the role of academic competitions in promoting gifted students was reviewed and—using the example of the Mathematical Olympiad for elementary school children—a corresponding training was delineated. Two effectiveness studies (cf. Herbein, 2016) examined the effects of the training on the motivation to do mathematics (i.e., self-concept and value beliefs), mathematical competences, and performance in the competition (see Chapters 3 and 4) by evaluating a mathematical training that was developed to prepare students for the requirements of a particular competition (i.e., Mathematical Olympiad). In the following, these effects are discussed in general, and several limitations and strengths are pointed out. To conclude, implications for educational practice and further research are mentioned.

5.1 Discussion of General Findings

To challenge students who are already able to solve curriculum-based tasks and problems, academic competitions are one possible approach for enriching learning environments (Bicknell, 2008; Cropper, 1998; Ozturk & Debelak, 2008b; Peters & Sieve, 2013). Around the world, there are numerous national and international academic competitions. Besides providing learning environments in which gifted students can develop their skills, academic competitions are supposed to fulfill several roles: (a) enrichment and differentiation, (b) fostering the ability to work in teams with peers, (c) motivating students to pursue the respective domain, (d) identifying the tough students, and (e) providing competitive environments (see Chapter 2). One of the most prominent and challenging types of academic competition are the academic Olympiads (Olson, 2005). Using the example of the (German) Mathematical Olympiad for elementary school students, the requirements with regard to contents (i.e., different types of typical tasks) and in terms of successful participation (e.g., the necessity of solutions and justifications) of this particular competition are described in detail (see Chapter 2). Indeed, mathematical competitions in general and the Mathematical Olympiad in particular provide challenging tasks. However, in line with a socio-cognitive-constructivist understanding of learning, if students are going to benefit from participating in the competition, they should be able to handle the requirements of the competition (e.g., Kießwetter, 2013). Otherwise, the intended positive effects may turn negative: Not only might students learn nothing, but their motivation to do mathematics in the future could disappear (see Chapter 1.5.2. and Chapter 2).

To give students the opportunity to be prepared to participate in the Mathematical Olympiad, a training based on the strengths and weaknesses of mathematically gifted students was developed. In the training "Getting fit for the Mathematical Olympiad," (a) cooperative learning[11] was chosen to simulate the competitive setting of the academic competition and to enhance students' motivation to do mathematics along with their mathematical competences (e.g., Johnson & Johnson, 1990, 1994). Further core components besides a specific cooperative method in which students first worked alone to solve a challenging mathematical problem, then talked about their approaches in cooperative teams that, again, presented their solution to the mathematical problem to other students who solved another problem were implemented. (b) Mathematical games were included at the beginning and end of each module to also enhance

[11] Cooperative learning is characterized by students who work together in positive interdependence (each member is important) to achieve shared learning goals with individual accountability (all members make their own contributions to the group's success according to their strengths). Thereby, students actively promote each other's learning (see Johnson & Johnson, 1990).

motivation and to implement playful competitive situations with regard to the competition (see e.g., Randel, Morris, Wetzel, & Whitehill, 1992). The challenging mathematical problems were (c) open tasks that enabled students to apply several approaches and (d) a structured notation of approaches and solutions that were inherent parts of the training (cf. context-specific writing; Seo, 2015). The challenging open tasks implemented in the training were based on the requirements of former Mathematical Olympiads but were reformulated to become open tasks (see Chapter 2) and to match the pedagogical framework (i.e., four fictive kids [Peter, Nina, Marie, and Frederick] presenting or having to solve the mathematical problems, see Rebholz, 2013; cf. Demuth et al., 2011). Thereby, especially process-based mathematical competences (i.e., [mathematical] problem solving, communication, as well as arguing and justifications) were assumed to be triggered (see Chapters 2, 3, and 4) as there is a belief that students develop an understanding of mathematics when they actively “describe their strategies in detail” (Franke et al., 2007, p. 229).

In the following, the findings of two empirical studies that evaluated the effectiveness of “Getting fit for the Mathematical Olympiad” are discussed with a focus on how the training influenced cognitive (e.g., competences) and noncognitive factors (e.g., math self-concept) as well as the gender differences indicated by the second Study (Chapter 4).

5.1.1 Effects of the training on motivational factors

In line with other studies that have examined motivational factors for elementary school students (e.g., Ehm, 2014; Selter et al., 2016), both participants who were nominated for the training (see Chapters 3 [Study 1] and 4 [Study 2]) and students who were not nominated for the training (see Chapter 3) reported relatively high motivation (i.e., a noncognitive factor) for mathematics as indicated by high mean values of math self-concept and value beliefs (i.e., intrinsic interest in Studies 1 and 2, and also attainment value and task-specific interest in Study 1). As expected, students who participated in the mathematical training showed higher mathematical competences than students who were not nominated to participate in the training (see the descriptive results of Study 1 in Chapter 3). In line with other enrichment measures (e.g., Zeidner & Schleyer, 1999), the ability level of the students in the training was actually higher than the participants had been accustomed to encountering in their regular classes. Against the background of how social comparison processes help form people’s domain-specific motivation (i.e., the big-fish-little-pond effect, BFLPE; e.g., Marsh, 1987; Marsh & Parker, 1984), it was comforting to find that no direct effects on students’ math self-concept, intrinsic interest, and attainment value were indicated by the results of Study 1 (see Chapter 3). Nevertheless, differential effects on students' math self-concept for third and fourth graders were identified. This hints at different social comparison processes for students in the different grade levels, such that third graders (who were in class with higher achieving fourth graders) tended to experience negative development in their math self-concept, opposite the experience of fourth-grade students (who were in class with lower achieving third graders; i.e., who experienced positive development in their self-concept; see Chapter 3). Unfortunately, for practical reasons—participants in the control group of the randomized controlled field trail in Study 2 did not have the opportunity to participate in the academic competition in a prepared manner until 1 year later—mostly third graders participated in Study 2. Thus, social comparison processes between the two grade levels could not be examined.

Third and fourth graders in the training not only worked together in cooperative teams to solve challenging tasks, but they also worked on the same tasks. Indeed, the contents of the tasks did not go further than the German educational standards for second graders, and the tasks were open so that several solution approaches could be applied (see Chapter 2). However, fourth graders experienced 1 more year of formal learning in mathematics in school and, thus, their mathematical competences were very likely to be more sophisticated in comparison with the third graders. Thus, fourth graders probably experienced the challenging tasks implemented in the training as less challenging than the third graders. Plus, being aware that the students participating in the training were considered the "clever kids" may have boosted fourth graders evaluations of their own competences (see Basking-in-reflected-glory-effect; e.g., Marsh et al., 2000). This assumption was corroborated by the positive treatment effect on students' task-specific interest where significant positive effects were indicated by the results of the study only for the fourth graders. Overall, the missing treatment effects on value beliefs and math self-concept could actually be interpreted positively because this indicates that the core components of the training were successful in arousing both the competitive environment of the academic competition and negative social comparison processes.

5.1.2 Effects of the training on cognitive factors

In both the quasi-experimental study (see Chapter 3) and the study based on a randomized controlled field trial (see Chapter 4), positive influences on performance in the Mathematical Olympiad were indicated for students who participated in the training. These students achieved higher scores in the tasks used in the competition in comparison with students who did not participate in the training, when differences on the pretest and gender were controlled for. More precisely, in Study 1 (see Chapter 3), third and fourth graders who participated in the training scored nearly three fourths (third graders) respectively over half (fourth graders) a standard deviation higher than students who did not participate. As the tasks of the Mathematical Olympiad require process-based competences (i.e., problem solving and arguing), one could even go so far to interpret this positive effect on the competition's tasks as an increase in students' process-based competences. Even if the training and the control group showed no pretest differences as in Study 2 (see Chapter 4), students who participated in the training scored nearly two thirds standard deviation higher in the last and most difficult level of the Mathematical Olympiad[12] in comparison with the control group students. Therewith, it looks like the training preparation, which was based on the requirements of previous versions of the competition, increased the likelihood of successful participation. But, of course, these findings are not surprising when considering that the core components of the training—especially getting used to solving challenging tasks and becoming more familiar with formulating hypotheses and solutions—were implemented in the training to prepare students for the Mathematical Olympiad. Considering that the Mathematical Olympiad tasks that were used as outcome measures were not known before the training was developed and that the mathematical problems implemented in the training were reformulated, one could even speculate that the results represent some kind of transfer effect. Students were able to demonstrate the mathematical competences they acquired while solving the tasks in the training when they encountered new mathematical problems. Nevertheless, the findings are in line with expectations (i.e., to become more successful participants) connected to the implementation of such trainings (see e.g., Ozturk & Debelak, 2008a, 2008b; Petersen & Wulff, 2017).

When looking at treatment effects with respect to the results of the studies related to mathematical competences, the findings were more ambiguous. In Study 1 (see Chapter 3), results indicated that the students who participated in the training showed higher general mathematical competences in comparison with the children who did not participate. Mathematical competences on the pretest and motivational factors (i.e., self-concept, value beliefs) were controlled for as there were significant differences on the pretest for the variables that have been shown to influence later mathematical competences (see e.g., Bailey, Siegler et al., 2014; Duncan et al., 2007; Eccles et al., 1983; Marsh et al., 2005; Murayama et al., 2013; Watts et al., 2015). More precisely, third and fourth grade students who participated in the training scored nearly three fourths of a standard deviation respectively more than half a standard deviation

[12] Differences between the training and the control group in their performance in the Mathematical Olympiad were not observed when also considering the lowest level as an outcome measure (Level 1: $t(62.91) = -0.62$, $p = .537$).

better on a standardized test—which was oriented on the curriculum of the respective grade—for assessing mathematical competence in comparison with students who did not participate in the training when prior mathematical competences and motivation to do mathematics before the training started were controlled for (see Chapter 3). Indeed the measure for assessing mathematical competences had quite a good reliability, but the extent to which the test was oriented on the curriculum of the respective grade was revealed to be problematic. For example, some contents (e.g., written division) were already taught in some classes but not in others (c.f. Gölitz et al., 2006; Krajewski et al., 2004; Roick et al., 2004). Thus, although all Study 2 participants (see Chapter 4) were nominated to participate in the training, another standardized test for assessing mathematical competences was implemented in the second study.

Against the expectations for transfer effects from the training on general mathematical competences raised by the first study, these results were not corroborated by the results of the second study in a one-to-one fashion: Neither basic arithmetic operations nor complex content-based competences—therefore, the content-based competences that are closer to the process-based competences than basic arithmetic operations—seemed to be influenced by the training. Nevertheless, the results indicated that students who participated in the training showed higher figural cognitive skills after the training when figural cognitive skills on the pretest were controlled for (see Chapter 4). Considering that figural cognitive skills are a domain-general cognitive ability, this could be interpreted as a hint that students who participated in the training may have profited from the training on a more general level than only by receiving an increase in their domain-specific abilities.

Therewith, overall, the two studies indicated that the training "Getting fit for the Mathematical Olympiad" was successful in promoting students' process-based mathematical competences that are conveyed by their performance in the tasks of the Mathematical Olympiad. And further, transfer effects to general mathematical competences (Study 1) and domain-general cognitive abilities (Study 2) seem likely.

Differential effects of the training on gender

In line with studies reporting gender differences in the mathematical competences of girls and boys (e.g., Hyde et al., 1990; Hyde, 2016, see Chapter 1.1.3), descriptive results from Study 2 indicated differences on the pretest between boys' and girls' mathematical competences in favor of boys (see Chapter 4). Looking for differential effects of the training for boys and girls, the results of Study 2 indicated that girls who participated in the training showed a greater increase in their content-based mathematical competences in comparison with boys, indicated by the significant regression coefficient for the interaction term. Thus, although no overall treatment effect on content-based competences (i.e., basic arithmetic operations and complex content-based operations) was observed, girls who participated in the training even showed higher complex content-based competences in comparison with girls who did not participate. Regarding the effect on process-based mathematical competences (i.e., performance in the Mathematical Olympiad), no differential effect was observed as the regression coefficient for the interaction was not significant. Overall, these results may indicate that boys and girls had different experiences while they were in the cooperative learning situation (e.g., Johnson & Engelhard, 1992; Lee, 1995). Perhaps some characteristics that are usually attributed to girls (e.g., behaving in a more adaptive fashion and being more willing to persist in learning situations; Steinmayr & Spinath, 2008) increased the success with which cooperative learning was able to support the development of cognitive factors (i.e., content- and process-based competences). But, perhaps the preexisting differences in girls' and boys' mathematical competences can explain the differential effects (Wendt, Steinmayr et al., 2016). Going further, one might even speculate that the tasks implemented in the training were not challenging enough for the students who showed more sophisticated mathematical competences before the training started (i.e., boys).

5.2 Limitations and Strengths

Several strengths but also limitations must be considered when interpreting the results of the studies and the present dissertation. In the following, first, the strengths of the present dissertation are summarized. Second, the limitations, especially with regard to the interpretation of the treatment effects, are presented.

5.2.1 Strengths and limitations of the effectiveness studies

First, compared with other approaches that are used in education to foster students' abilities (e.g., mathematical competences), the assumed effectiveness of the training "Getting fit for the Mathematical Olympiad" was examined with empirical studies as part of this dissertation. Even the training—which was developed on the basis of the literature—was evaluated and put into practice (see e.g., Herbein, 2016; Humphrey et al., 2016). A second strength of the present dissertation is its interdisciplinarily: Both for the development of the training and for the studies that were included, findings from different research traditions (education science, mathematical education, psychology) were combined, and different topics were considered (e.g., concept of competences, mathematical competences, numerical cognition, motivation research, mathematical giftedness). For example, Study 1 examined not only the effectiveness of the training regarding success in the Mathematical Olympiad and on mathematical competences but also social comparison processes in the training (see Chapter 3). In Study 2, the cognitive processes were investigated in detail by looking at whether the training, which was targeted toward process-based mathematical competences, could influence content-based mathematical competences and domain-general cognitive abilities (see Chapter 4).

Although one could argue that the quasi-experimental design of the first study (see Chapter 3) was weak because the two groups (children in the training vs. in the control group) showed significant differences on the pretest. But, these differences were controlled for in the multiple regression analysis that was computed to analyze the data. And further, a second study with a stronger design was included in the present dissertation: To examine the effectiveness of the training in Study 2, a randomized controlled field trial (RCFT) was used. Such RCFTs are considered the "best approach for demonstrating the effectiveness of a novel educational intervention" (Torgerson & Torgerson, 2013, p. 2). Using an RCFT, "differences in outcomes can be attributed to the presence or absence of the intervention, rather than to some other factor" (Twone & Hilton, 2004, p. 3). Therewith, the effects of the training reported in Study 2 could be attributed to the training rather than to other factors that have been shown to influence the acquisition of mathematical competences (see Chapters 1.2, 1.3, and 1.4).

In both empirical studies, some students did not—for several reasons (see Chapters 3 and 4)—participate in each testing occasion. But, in the multiple linear regression models—implemented with the R package *lavaan*—the full information maximum likelihood approach was used to deal with these missing values (Enders, 2010; Graham, 2009; Rosseel, 2012). Further, all analyses were conducted with the *robust maximum likelihood estimator,* which corrects the standard errors for the non-normality of the variables.

Besides these strengths, there were some limitations that have to be considered with regard to studies: First, the training was conducted only within the framework of the Hector Children's Academy Program (HCAP). Thus, the two effectiveness studies were based on quite a specific sample—students who were nominated by their former teacher to participate in the extracurricular enrichment program targeting the upper 10% of the most gifted, talented, interested, and creative students (see Herbein, 2016; Rothenbusch et al., 2016; Schiefer, 2017). As academic competitions are in general developed for students who are already able to solve curriculum-based tasks and need extracurricular challenges to nurture their potential, this sample was chosen for the present dissertation. Nevertheless, the specific sample limits the generalizability of the findings. Hence, students participating the HCAP in general and the mathematical training in particular cannot be classified as (mathematically) gifted (or talented) with regard to models defining (mathematical) giftedness (e.g., showing high domain-general cognitive abilities such as intelligence, see Chapter 1.5). However, children participating in the HCAP, for example, tend to have families with a higher social background and show higher domain-general cognitive skills (e.g., intelligence) than the societal mean as well as higher competences (e.g., mathematical competences) than their classmates (see Rothenbusch et al., 2016). Social background (e.g., Klibanoff et al., 2006; Sirin, 2005, see also Chapter 1.4), domain-general cognitive skills (e.g., Deary et al., 2007; Spinath et al., 2010, see also Chapter 1.3.1), and prior competences (e.g., Georges et al., 2017; Schneider et al., 2016 see also Chapter 1.2) have all been shown to influence the acquisition of mathematical competences in previous studies. Thus, it is not possible to directly transfer the results to another group of children, and the question of whether specifically the training "Getting fit for the Mathematical Olympiad" or more generally preparation for an academic competition would also be effective for students other than the students nominated for the Hector Children's Academy Program remains unanswered.

In the first study, six different course instructors taught the training; in the second study, there were 10 instructors. To ensure that the training was conducted as intended, these instructors participated in a half day seminar and were given a scripted manual that included master copies of all materials but also schedules and background information (Rebholz, Golle, Oschatz, & Trautwein, 2017). In fact, both studies revealed the effectiveness of the training (see Chapters 2, 3, and 4). On the positive side, this in a sign of that the program is effective when put into practice, or in other words, when implementing the intervention under real-world conditions (cf. Humphrey et al., 2016). On the negative side, it is impossible to know for certain whether the course instructors did what they were supposed to do. The studies that were conducted revealed only that the training was effective on average for the seven (Study 1) or 10 (Study 2) course instructors who participated. To ensure that the training was taught as intended, some researchers have recommended that implementation fidelity (i.e., the degree to which the program or intervention was implemented as intended; see e.g., Carroll et al., 2007; Humphrey et al., 2016) should be assessed. For future studies that will examine the effectiveness of educational interventions such as the training "Getting fit for the Mathematical Olympiad," questionnaires or observations should be included to ensure that the program is taught as intended.

5.2.2 Processes that may influence the effects of the training

Many domain-general and domain-specific factors have been shown to influence the development of mathematical competences (see e.g., Schneider et al., 2016). For example, several studies have indicated that students' mathematical competences are strongly based on prior mathematical competences (e.g., Bailey, Siegler et al., 2014; Cerda et al., 2015; Duncan et al., 2007; Watts et al., 2015). But mathematical competences are supposed to depend on a large number of various different complex subcompetences and processes and many subskills and subprocesses, such as logical inference, memorization of calculation procedures, and working memory (Thompson et al., 2013). Although previous mathematical competences were controlled for when determining the effects of the training in both studies (see Chapters 2, 3, and 4), it is still possible to argue that the mathematical competences of the students who showed higher mathematical competences before the training started (e.g., the training group in Study 1) increased as a result of their higher mathematical potential and not as a consequence of the training. However, the results of Study 2 argue against this: First, Study 2 was conducted as a randomized controlled field trial as all participants in this sample were nominated for the program by their former teacher, and so there were no mean differences on the pretest between the training and control groups (see Chapter 4). Indeed, effects of the training were not observed for students' competences in basic arithmetic operations or for the complex content-based competences. But, effects of the training were observed for the girls who showed lower mathematical competences at the beginning of the training than the boys. This finding speaks in favor of processes that depend less on previous mathematical competences, especially because effects on broader process-based mathematical competences (assessed by performance in the Mathematical Olympiad) were significant for both boys and girls (see Chapter 4).

Also examining the effectiveness of a mathematical intervention, Watts and colleagues (2017) reported that the treatment had effects on state (time varying) factors but no effects on trait factors (characteristics that exceed a stable influence on students' mathematical competences). Transferring their argumentation that trait mathematics rather is somewhat general to academic domains (Watts et al., 2017) into the classification of domain-specific and domain-general factors, the positive effect on domain-general cognitive abilities (see Chapter 4) is all the more surprising. Considering that this effect is not an artefact, the training might serve to facilitate students' further acquisition of mathematical competences as, for example, Watts and colleagues (2017) have argued that mathematical competences are influenced by rather stable (domain-general) characteristics more than time-varying (domain-specific) characteristics from the previous time point.

One further limitation that can occur when interpreting treatment effects on mathematical competences are the differences in motivational factors of the participants of the training in comparison with the control group because, in Study 1, significant differences were reported between the training and control groups. In Study 2, differences in motivation (especially interest) could be assumed only because the treatment group had the privilege of being able to attend the training before the control group. In general, differences in motivational factors have been shown to influence the acquisition of later mathematical competences (see e.g., Helmke, 1998; Marsh & Craven, 2006). Some authors have attributed this to the ability to deal with more challenging mathematical contents by showing more engagement and persistence (see e.g., Hidi & Harackiewicz, 2000), which again can lead to more effective learning and solution strategies (see e.g., Kriegbaum et al., 2015; Middleton & Spanias, 1999). Transferring these considerations to the training, which was the basis of the present dissertation, one could argue that these mechanisms were not observed and were not even observable at pretest and that students in the training, because they were catalyzed by higher motivation, showed higher mathematical competences and therefore also better performance in the Mathematical Olympiad.

Last but not least, when interpreting the effectiveness of the training, it is important to consider that the learning environment provided in the training consisted of several core components. All these core components—cooperative learning, mathematical games, challenging open tasks, and a structured notation of approaches and solutions—were aimed at fostering students' mathematical competences and motivation for mathematics and counterbalancing social comparison processes and especially the competitive environment. For all core components that were implemented, there are studies and considerations that corroborate these positive effects. For example, cooperative learning is deemed an effective teaching method for fostering students' outcomes such as competences and motivation (see e.g., Johnson et al., 2000; Slavin, 1983a). In their meta-analysis, Johnson and colleagues (2000) reported significant positive effects of cooperative learning for students' achievement in comparison with individualistic or competitive learning. A more precise study that examined the effects of a special form of cooperative learning conducted with 12th graders in physics using a quasi-experimental design indicated positive effects on students' self-reported cognitive activation, intrinsic motivation, and interest in physics (Hänze & Berger, 2007). However, cooperative learning was not revealed to be effective per se (e.g., Slavin, 1983a, 1983b). For instance, in a study by Battistich, Solomin, and Delucchi (1993), the results indicated that the positive influence of cooperative learning on students' competences and motivation depended on the quality of the interactions in the groups. In their study, the authors assessed students' competences (i.e., achievement), motivation (e.g., intrinsic motivation), and the processes in small groups that were supposed to work cooperatively (Battistich et al., 1993). In connection to the training, it was not possible to ensure that all groups in the training solved their mathematical problems cooperatively. As working alone, working together, writing solutions down, presenting them to other students, as well as listening to the problems and solutions of other groups (i.e., the specific didactic-methodological model) were inherent parts of the schedule of each module, the likelihood that the procedure would be successful was quite high. Nevertheless, the quality of students' interactions was not controlled for. Similar considerations are also necessary with respect to the other core components. Using the example of cooperative learning demonstrated that the mechanisms that are responsible for the positive effects of the training cannot be explained from within the framework of the present dissertation.

5.3 Implications for Educational Practice and Further Research

The present dissertation was aimed at answering the question of whether, on the one hand, academic competitions are an appropriate approach for fostering gifted students by arguing the merits of a training program that can be implemented to prepare students for a specific competition. On the other hand, the present dissertation contributed to the question of how mathematical competences and success in an academic math competition can be fostered. However, some questions remain open in the framework of the present dissertation. Thus, considerations for further educational research are summarized in the following. Further, based on the results of the present dissertation, some implications for educational practice can be derived.

5.3.1 Future educational research

As explained in Chapter 5.2.2, the question of which core component was the main cause of the effectiveness of the training remained unanswered in the present dissertation. Thus, for further research, first, a consideration of treatment fidelity might contribute to the understanding of the effectiveness of the individual core component. Knowing which component had been implemented in which of the training groups might provide hints about the more and less effective core components. Further, differential effects of the training should be investigated in more detail: Who and what makes the training effective? Therefore, also domain-general and domain-specific cognitive abilities as well as the noncognitive factors should be assessed before and after the training. Thus, further studies that consider all such factors as well as treatment fidelity are necessary for investigating the effectiveness of the training. Ideally, another randomized controlled field trial with a waitlist control group should be conducted with more than the 10 courses. Based on the findings of Bailey and colleagues (2016), whose study indicated that fadeout after a successful intervention was caused by preexisting differences, on the one hand, studies should also investigate for whom “Getting fit for the Mathematical Olympiad” is (most) effective in terms of mathematical competences and success in the Mathematical Olympiad. But also, social comparison processes between students in the training should be investigated in this RCFT. On the other hand, long-term effects of the training sound like a fruitful question. Perhaps the effects of the extracurricular enrichment (i.e., Getting fit for the Mathematical Olympiad) disappear as students are no longer challenged enough (cf. Bailey et al., 2016).

Thereby, gender differences should also be counted as the differential effects of “Getting fit for the Mathematical Olympiad” for boys and girls indicated by the results of the second study provide room for more speculation about gender differences in mathematics. In a larger sample, it would be promising to look at whether boys and girls already show characteristic strengths (e.g., boys are better problem solvers than girls; cf. Brehl et al., 2012; Wendt, Steinmayr et al., 2016) even for the specific sample of students nominated for extracurricular enrichment.

Based on hints for effective core components from the study described above, the effectiveness of single core components could be examined by manipulating only a single element at a time between groups. For example, the effects of the mathematical games—that are thought to be effective at enhancing students' motivation and for preparing students for the competitive setting of the competition—can be investigated by having some groups playing the games as intended and others not.

Academic competitions

Regarding academic competitions, different views must be considered in further research. First, the appropriateness of academic competitions for fostering domain-specific competence and motivation should be investigated empirically. At the moment, the effectiveness of academic competitions has been explored only through retrospective studies that have surveyed previously successful participants by looking at their vocational success and asking them what benefits they attribute to the academic competition (e.g., Campbell & Walberg, 2010; Fauser et al., 2007; Lengfelder & Heller, 2002; Oswald et al., 2005; Wirt, 2011). However, questioning only the successful participants offers a nonrepresentative and very selective sample. Thus, studies in which only the participation in an academic competition is manipulated would be necessary to substantiate the theoretically plausible positive influence of academic competitions. In the long run, it would also be fruitful to examine whether the assumed effects of an academic competition on competences and motivation again influence vocational success. The first results regarding this question were reported by Forrester (2010) who conducted semistructured interviews and asked the participants of an academic competition in science about why they decided to participate. Amongst others, her results indicated that students attributed their increased interest in science to the competition. She concluded that academic competitions have the potential to influence academic choices (i.e., choosing to be a science major) and can pique students' interest in the domain of the competition (Forrester, 2010).

Further, there is the question of which factors drive successful participation in an optional extracurricular enrichment program such as academic competitions: Are these factors comparable to the factors that have been examined with respect to their influence on academic achievement in general? In studies by Urhahne and colleagues (2012) as well as Stang, Urhahne, Nick, and Parchmann (2014), success in academic science competitions was mostly predicted by previous knowledge, motivational factors (i.e. competence and value beliefs), expected success, and relative costs. Both studies that were intended to predict successful performance based on the expectancy-value-model (EVT; e.g., Eccels et al., 1983, 2010; Wigfield, 1994; Wigfield & Eccels, 2000) of achievement motivation were based on adolescent samples. Therewith, the results of the studies by Urhahne and colleagues (2012) as well as Strang an colleagues (2014) are not perfectly in line with the EVT according to which only motivational factors (see Chapter 1.4.1) directly influence later achievement/performance/competences (Wigfield & Eccles, 2000). Further studies are necessary to determine whether factors that influence success in an academic competition differ from the factors that influence general academic achievement.

In the area of sports, training for a competition is a common approach. To underpin the assumption that this approach can also be transferred to the field of education, the present dissertation indicated that preparation for an academic math competition could also be a useful measure in terms of successful participation. Further, there were also hints that such a measure could be successful in fostering students' domain-specific competence (i.e., mathematical competences). But, to make a more general statement, further trainings for other math competitions or even competitions in other fields are necessary. However, this leads to the problem of whether, in this case, the trainings are actually comparable, for example, if they trigger different contents or use different methods.

Mathematical competences

In mathematics education, there is quite a broad consensus that mathematical competences can be differentiated into content-based and process-based parts (e.g., NCTM, 2000; Stanat et al., 2012; Winkelmann & Robitzsch, 2009). But, the results of the second study indicated that the training that targeted process-based mathematical competences had more of an influence on domain-general cognitive abilities than on content-based competences (i.e., domain-specific factors). Thus, the interplay of domain-general and domain-specific cognitive abilities as well as process- and content-based mathematical competences should be investigated on the basis of the models of numerical cognition described in Chapters 1.2.3 and 1.2.4 (see also, e.g., Dehaene, 2011; Krajewski & Schneider, 2009a, 2009b; von Aster & Shalev, 2007). In particular—as mathematical competences are a hierarchical construct in which later mathematical competences are based on prior mathematical competences—questions regarding the processes and mechanisms that enable elementary school students to acquire mathematical competences remain open in this dissertation.

5.3.2 Implications for educational practice

First—as indicated by the positive effects of the training on students' performance in the academic competition—preparing students for an academic competition has positive influences on students' competences and success in the competition (see Chapters 2, 3, and 4). Even in such trainings, it does seem to be enough to provide challenging problems that are based on the requirements of the competition. It is not necessary to work with the original tasks to increase the likelihood of successful participation. Going further, concentrating on process-based competences and the necessary solution strategies (e.g., a systematic approach, see e.g., Demuth et al., 2011; Käpnick, 1998) allows for transfers to other new challenging problems as well. In detail, a combination of cooperative learning, mathematical games, challenging open tasks, and structured notation has revealed positive effects in enhancing students' mathematical competences and their likelihood to successfully participate in the Mathematical Olympiad.

Second, the effectiveness of the training that is aimed at fostering process-based competences and mathematical competences in general underpin the importance of such broader process-based competences. Providing challenges to all students may contribute to enhancing the sophisticated mathematical competences of all students as suggested by Franke and colleagues (2007):

> Within the field of mathematics education, researchers seem to agree in principle that classrooms that support mathematical proficiency would be places where students are encouraged to be curious about mathematical ideas, where they can develop their mathematical intuition and analytic capabilities, where they can learn to talk about and with mathematical expertise. (Franke et al., 2007, p. 229)

Therewith, third, indicted by the effects of the training "Getting fit for the Mathematical Olympiad," continuous promotion in a weekly mathematical training can contribute to the promotion of mathematical competences. The training incorporated in the present dissertation is one of (at the moment) five so-called Hector Core Courses, which all show positive effects on students' development (see e.g., Herbein, 2016; Schiefer, 2017). Overall, this corroborates the implementation of regular extracurricular enrichment measures as done in the Hector Children's Academy Program.

Last but not least, whether mathematical competences are operationalized by differentiating between content- and process-based mathematical competences or whether both domain-specific and domain-general abilities are considered to drive mathematical competences, mathematical competences are supposed to be a complex multidimensional construct. However, a holistic approach to the concept of competences as assumed by social and educational science is not considered in the operationalization of competences when assessing students' abilities. But, considering all factors that have been shown to influence or to be correlated with mathematical competences (cf. Chapter 1), the results of this dissertation corroborate such a holistic approach.

6. References

Abernathy, T. V., & Vineyard, R. N. (2001). Academic competitions in science: What are the rewards for students? *The Clearing House*, *74*(5), 269–276. Retrieved from http://www.jstor.org/stable/30189679

Abramson, L. Y., Seligman, M. E., & Teasdale, J. D. (1978). Learned helplessness in humans: Critique and reformulation. *Journal of abnormal psychology*, *87*(1), 49.

Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., de Smedt, B., Gilmore, C.,. . . Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. *Journal of Numerical Cognition*, *2*(1), 20–41. https://doi.org/10.5964/jnc.v2i1.10

Aljughaiman, A. M., & Ayoub, A. E. A. (2012). The Effect of an Enrichment Program on Developing Analytical, Creative, and Practical Abilities of Elementary Gifted Students. *Journal for the Education of the Gifted*, *35*(2), 153–174. https://doi.org/10.1177/0162353212440616

Arens, A. K., Trautwein, U., & Hasselhorn, M. (2011). Erfassung des Selbstkonzepts im mittleren Kindesalter: Validierung einer deutschen Version des SDQ I 1Dieser Beitrag wurde unter der geschäftsführenden Herausgeberschaft von Jens Möller angenommen [Self-Concept Measurement with Preadolescent Children: Validation of a German Version of the SDQ I]. *Zeitschrift für Pädagogische Psychologie*, *25*(2), 131–144. https://doi.org/10.1024/1010-0652/a000030

Arens, A. K., Yeung, A. S., Craven, R. G., & Hasselhorn, M. (2011). The twofold multidimensionality of academic self-concept: Domain specificity and separation between competence and affect components. *Journal of Educational Psychology*, *103*(4), 970.

Arvey, R. D., Bouchard, T. J., Carroll, J. B., Cattell, R. B., Cohen, D. B., Dawis, R. V., & Willerman, L. (1994). Mainstream science on intelligence. *Wall Street Journal*, *13*, A18.

Aßmus, D. (2010). Merkmale und Besonderheiten mathematisch potentiell begabter Zweitklässler: Ergebnisse einer empirischen Untersuchung. [Characteristics and specifics of prospective mathematically gifted secound graders: Results of an emprical study]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder*. *Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 59–69). Berlin, Münster: Lit.

Aßmus, D. (2013). Fähigkeiten im Umkehren von Gedankengängen bei potentiell mathematisch begabten Grundschulkindern [Skills for reverse thoughts in prospective mathematically gifted elementary school students]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen* *mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 45–66). Kronach / Berlin: Mildenberger.

Atkinson, J. W. (1957). Motivational determinants of risk-taking behavior. *Psychological review*, *64*(6p1), 359.

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental Dynamics of Math Performance From Preschool to Grade 2. *Journal of Educational Psychology*, *96*(4), 699–713. https://doi.org/10.1037/0022-0663.96.4.699

Baddeley, A. D. (1986). *Working memory*. Oxford: Clarendon Press.

Baddeley, A. D., & Hitch, G. (1974). Working Memory. In A. D. Baddeley & G. Hitch (Eds.), *Psychology of Learning and Motivation. Psychology of Learning and Motivation* (Vol. 8, pp. 47–89). Elsevier. https://doi.org/10.1016/S0079-7421(08)60452-1

Bailey, D. H., Nguyen, T., Jenkins, J. M., Domina, T., Clements, D. H., & Sarama, J. S. (2016). Fadeout in an early mathematics intervention: Constraining content or preexisting differences? *Developmental psychology*, *52*(9), 1457–1469. https://doi.org/10.1037/dev0000188

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental science*, *17*(5), 775–785.

Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2014). State and trait effects on individual differences in children's mathematical development. *Psychological science*, *25*(11), 2017–2026. https://doi.org/10.1177/0956797614547539

Baltes, P. B., Staudinger, U. M., & Lindenberger, U. (1999). Lifespan psychology: Theory and application to intellectual functioning. *Annual review of psychology*, *50*, 471–507. https://doi.org/10.1146/annurev.psych.50.1.471

Bandura, A., & Jourden, F. J. (1991). Self-regulatory mechanisms governing the impact of social comparison on complex decision making. *Journal of Personality and Social Psychology*, *60*(6), 941.

Bardy, P. (2013). *Mathematisch begabte Grundschulkinder: Diagnostik und Förderung*. *Mathematik Primar- und Sekundarstufe I + II*. Berlin, Heidelberg: Springer Spektrum.

Bardy, P., & Hrzán, J. (2010). *Aufgaben für kleine Mathematiker: Mit ausführlichen Lösungen und didaktischen Hinweisen* [Tasks for litte mathematicans. Including detailed solutions and methodically hints] (3. Auflage). *Aulis Schatztruhe für die Grundschule*. Köln: Aulis Verlag.

Battistich, V., Solomin, D., & Delucchi, K. (1993). Interaction Processes and Student Outcomes in Cooperative Learning Groups. *The Elementary School Journal*, *94*(1), 19–32.

Baudson, T. G., Strobel, A., & Preckel, F. (2012). Validierung einer neuen Need for Cognition (NFC)-Skala für Grundschülerinnen und Grundschüler: Struktur, Messinvarianz und Zusammenhänge mit Intelligenz- und Leistungsmaßen [Validation of a new Need for Cognition (NFC)-Scale for elementary school students: Structure, measurement invariance and correlations with intelligence and performance]. In R. Riemann (Ed.), *48. Kongress der Deutschen Gesellschaft für Psychologie* (p. 59). Lengerich: Pabst Science Publishers.

Bauersfeld, H. (2013). Die Bielefelder Förderansätze. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 17–26). Offenburg: Mildenberger.

Baumert, J., Kunter, M., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (2013). Professional Competence of Teachers, Cognitivel Activating Instruction, and the Development of Students' Mathematical Literacy (COACTIV): A Reaearch Program. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers. Results from the COACTIV project* (pp. 1–24). Springer Science & Business Media.

Bayrhuber, M., Leuders, T., Bruder, R., & Wirtz, M. (2010). Erfassung und Modellierung mathematischer Kompetenz: Aufdeckung kognitiver Strukturen anhand des Wechsels von Darstellungs-und Repräsentationsform [Assessing and modeling mathematical competens]. *Zeitschrift für Pädagogik, Beiheft*, *56*, 28–39.

Beck, E., Guldimann, T., & Zutavern, M. (1991). Eigenstandig lernende Schulerinnen und Schuler: Bericht uber ein empirisches Forschungsprojekt [Autonomous Learners. An Empirical Research Project]. *Zeitschrift für Pädagogik*, *37*(5), 735–768.

Benbow, C. P. (1988). Sex differences in mathematical reasoning ability in intellectually talented preadolescents: Their nature, effects, and possible causes. *Behavioral and Brain Sciences*, *11*(2), 169–183.

Bernholt, S., Neumann, K., & Nentwig, P. (2012). Making it Tangible - Specifiying Learning Outcomes in Science Education. In S. Bernholt, K. Neumann, & P. Nentwig (Eds.), *Learning outcomes in science education. Making it tangible* (pp. 13–27). Münster: Waxmann.

Bezold, A. (2012). Förderung von Argumentationskompetenzen auf der Grundlage von Forscheraufgaben. [Fostering the process of justification using research tasks.]. *mathematica didactica*. (35), 73–103.

Bicknell, B. (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom*, *13*(4), 16–20. Retrieved from http://files.eric.ed.gov/fulltext/EJ824763.pdf

Bloom, B. S. (1976). *Human characteristics and school learning*. Columbus: McGraw Hill Higher Education.

Blum, W. (2012). Einführung [Introduction]. In W. Blum, C. Drüke-Noe, R. Hartung, & O. Köller (Eds.), *Bildungsstandards Mathematik: konkret. Sekundarstufe I: Aufgabenbeispiele, Unterrichtsanregungen, Fortbildungsideen; mit CD-ROM* (6th ed., pp. 14–32). Berlin: Cornelsen.

Blum, W., Neubrand, M., Ehmke, T., Senkbeil, M., Jordan, A., Ulfig, F., & Carstensen, C. H. (2004). Mathematische Kompetenz [Mathematical competence]. In PISA-Konsrotium Deutschland (Ed.), *PISA 2003. Der Bildungsstand der Jugendlichen in Deutschland ; Ergebnisse des zweiten internationalen Vergleichs* (pp. 47–92). Münster: Waxmann.

Blums, A., Belsky, J., Grimm, K., & Chen, Z. (2016). Building Links Between Early Socioeconomic Status, Cognitive Ability, and Math and Science Achievement. *Journal of Cognition and Development*, *18*(1), 16–40. https://doi.org/10.1080/15248372.2016.1228652

Böhme, K., & Roppelt, A. (2012). Geschlechtsbezogene Disparitäten [Gender disparity]. In P. Stanat, H. A. Pant, K. Böhme, & D. Richter (Eds.), *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik. Ergebnisse des IQB-Ländervergleichs 2011* (pp. 173–190). Münster: Waxmann.

Bong, M., & Skaalvik, E. M. (2003). Academic self-concept and self-efficacy: How different are they really? *Educational Psychology Review*, *15*(1), 1–40.

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental psychology*, *42*(1), 189.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child development*, *79*(4), 1016–1031.

Bos, W., Lanke, E.-M., Prenzel, M., Schwippert, K., Valtin, R., & Walther, G. (2003). *Erste Ergebnisse aus IGLU: Schülerleistungen am Ende der vierten Jahrgangsstufe im internationalen Vergleich: Zusammenfassung ausgewählter Ergebnisse* [First results form

IGLU: Students performance at the end of 4th grade in international comparison]. Hamburg. Retrieved from http://www.kmk.org/fileadmin/Dateien/pdf/PresseUndAktuelles/2003/iglu_kurz-end.pdf

Bos, W., Wendt, H., Köller, O., & Selter, C. (Eds.). (2012). *TIMSS 2011: Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern im internationalen Vergleich* [TIMSS 2011. Mathematical and science competences of elementary school children in international comparison]. Münster/New York/München/Berlin: Waxmann.

Bos, W. (Ed.). (2008). *TIMSS 2007: Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern in Deutschland im internationalen Vergleich*: Waxmann Verlag.

Bos, W., Buddeberg, I., & Lankes, E.-M. (Eds.). (2005). *IGLU: Skalenhandbuch zur Dokumentation der Erhebungsinstrumente* [IGLU: Handbook of scales for documentation of assessment instruments]. Münster: Waxmann. Retrieved from http://www.dandelon.com/intelligentSEARCH.nsf/alldocs/8144B6DCC0DA65A7C125732B004F4B84/

Bradley, L., & Bryant, P. (1985). *Rhyme and reason in reading and spelling* (Vol. 1). Ann Arbor, MI: University of Michigan Press.

Bransford, J. D., Brown, A., & Cocking, R. (2000). *How people learn: Mind, brain, experience, and school*. Washington, D.C.: National Academy Press.

Bransford, J. D., Zech, L., Schwartz, D., Barron, B., Vye, N., & The Cognition and Technology Group at Vanderbilt. (2012). Fostering Mathematical Thinking in Middle School Students: Lessons From Research. In R. J. Sternberg & T. Ben-Zeev (Eds.), *Studies in Mathematical Thinking and Learning Series. The Nature of Mathematical Thinking* (pp. 203–250). Hoboken: Taylor and Francis.

Brehl, T., Wendt, H., & Bos, W. (2012). Geschlechtsspezifische Unterschiede in mathematischen und naturwissenschaftlichen Kompetenzen [Gender disparity in mathematical and science competences]. In W. Bos, H. Wendt, O. Köller, & C. Selter (Eds.), *TIMSS 2011. Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern im internationalen Vergleich* (pp. 203–230). Münster/New York/München/Berlin: Waxmann.

Bruder, R., Hefendehl-Hebeker, L., Schmidt-Thieme, B., & Weigand, H.-G. (Eds.). (2015). *Handbuch der Mathematikdidaktik* [Handbook of didactis in mathematics]. Berlin: Springer Spektrum. Retrieved from http://dx.doi.org/10.1007/978-3-642-35119-8

Brunner, M., Krauss, S., & Kunter, M. (2008). Gender differences in mathematics: Does the story need to be rewritten? *Intelligence*, *36*(5), 403–421. https://doi.org/10.1016/j.intell.2007.11.002

Brunner, M., Krauss, S., & Martignon, L. (2011). Eine alternative Modellierung von Geschlechtsunterschieden in Mathematik [Alternative Measurement Models to Assess Gender Differences in Mathematics]. *Journal für Mathematik-Didaktik*, *32*(2), 179–204. https://doi.org/10.1007/s13138-011-0026-2

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, *8*(1), 36–41. https://doi.org/10.1111/cdep.12059

Bullock, M., & Ziegler, A. (1997). Entwicklung der Intelligenz und des Denkens: Ergebnisse aus dem SCHOLASTIK-Projekt [Development of intelligence and thinking. Results of SCHOLASTIK-project]. *Entwicklung im Grundschulalter*, *S*, 27–35.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science (New York, N.Y.)*, *332*(6033), 1049–1053. https://doi.org/10.1126/science.1201536

Callahan, C. M., Hunsaker, S. L., Adams, C. M., Moore, S. D., & Bland, L. C. (1995). Instruments Used in the Identification of Gifted and Talented Students.

Campbell, J. R., & Verna, M. A. (2010). Academic Competitions Serve the US National Interests. *AERA Online Submission*. Retrieved from http://files.eric.ed.gov/fulltext/ED509402.pdf

Campbell, J. R., Wagner, H., & Walberg, H. J. (2000). Academic competitions and programs designed to challenge the exceptionally talented. *International handbook of giftedness and talent*, *2*. Retrieved from http://www.olympiadprojects.com/v2/pubs_web/Hdbk.pdf

Campbell, J. R., & Walberg, H. J. (2010). Olympiad Studies: Competitions Provide Alternatives to Developing Talents That Serve National Interests. *Roeper Review*, *33*(1), 8–17. https://doi.org/10.1080/02783193.2011.530202

Carroll, C., Patterson, M., Wood, S., Booth, A., Rick, J., & Balain, S. (2007). A conceptual framework for implementation fidelity. *Implementation science*, *2*(1), 40.

Ceci, S. J. (1991). How much does schooling influence general intelligence and its cognitive components? A reassessment of the evidence. *Developmental psychology*, *27*(5), 703–722. https://doi.org/10.1037/0012-1649.27.5.703

Cerda, G., Pérez, C., Navarro, J. I., Aguilar, M., Casas, J. A., & Aragón, E. (2015). Explanatory model of emotional-cognitive variables in school mathematics performance: A longitudinal study in primary school. *Frontiers in psychology*, *6*, 1363. https://doi.org/10.3389/fpsyg.2015.01363

Chomsky, N. (1968). *Language and mind*. New York: Harcourt, Brace & World.

Cipolotti, L., & Butterworth, B. (1995). Toward a multiroute model of number processing: Impaired number transcoding with preserved calculation skills. *Journal of Experimental Psychology: General*, *124*(4), 375.

Clark, C. A. C., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental psychology*, *46*(5), 1176.

Clements, D. H., & Sarama, J. S. (2007). Early Childhood Mathematics Learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning. A project of the National Council of Teachers of Mathematics* (pp. 461–555). IAP.

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science (New York, N.Y.)*, *333*(6045), 968–970. https://doi.org/10.1126/science.1204537

Clinkenbeard, P. R. (1989). The Motivation to Win Negative Aspects of Success at Competition. *Journal for the Education of the Gifted*, *12*(4), 293–305. https://doi.org/10.1177/016235328901200405

Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: Teaching the craft of reading, writing and mathematics. In L. B. Resnick (Ed.), *Knowing, learning, and instruction. Essays in honor of Robert Glaser* (pp. 453–494). Hillsdale: Lawrence Erlbaum Associates, Inc.

Cropper, C. (1998). Is Competition an Effective Classroom Tool for the Gifted Student? *Gifted Child Today*, *21*(3), 28–31. https://doi.org/10.1177/107621759802100309

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive psychology*, *20*(4), 405–438. https://doi.org/10.1016/0010-0285(88)90011-4

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive psychology*, *20*(4), 405–438.

Dai, D. Y., & Rinn, A. N. (2008). The Big-Fish-Little-Pond Effect: What Do We Know and Where Do We Go from Here? *Educational Psychology Review*, *20*(3), 283–317. https://doi.org/10.1007/s10648-008-9071-x

Deal, L. J., & Wismer, M. G. (2010). NCTM Principles and Standards for Mathematically Talented Students. *Gifted Child Today*, *33*(3), 55–65. https://doi.org/10.1177/107621751003300313

Deary, I. J., Strand, S., Smith, P., & Fernandes, C. (2007). Intelligence and educational achievement. *Intelligence*, *35*(1), 13–21. https://doi.org/10.1016/j.intell.2006.02.001

Deci, E. L., & Ryan, R. M. (1993). Die Selbstbestimmungstheorie der Motivation und ihre Bedeutung für die Pädagogik. *Zeitschrift für Pädagogik*, *39*(2), 223–238.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1), 1–42.

Dehaene, S. (2011). *The number sense: How the mind creates mathematics*: OUP USA.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical cognition*, *1*(1), 83–120.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive neuropsychology*, *20*(3), 487–506. https://doi.org/10.1080/02643290244000239

Demuth, R., Walther, G., & Prenzel, M. (Eds.). (2011). *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule*. [Developing education with SINUS: 10 Models for mathematics and science in elementary schools]. Seelze: Friedrich.

de Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of experimental child psychology*, *103*(2), 186–201.

Devlin, K. (1996). *Mathematics: The science of patterns: the search for order in life, mind and the universe* (2nd print., paperback ed.). New York: Scientific American Library.

Devlin, K. (2002). *Muster der Mathematik: Ordnungsgesetze des Geistes und der Natur* [Patterns of Mathematics. Laws of arrangements for spirit and nature] (4. Auflage). Heidelberg: Spektrum.

Devlin, K. (2003). *The millennium problems: The seven greatest unsolved mathematical puzzles of our time*. New York: Basic Books.

Devlin, K. J. (2004). *Sets, functions, and logic: An introduction to abstract mathematics* (3rd ed.). *Chapman & Hall/CRC mathematics*. Boca Raton, Fla.: Chapman & Hall/CRC.

Diezmann, C. M., & Watters, J. J. (2000). Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today*, *23*(4), 14–52.

Diezmann, C. M., & Watters, J. J. (2001). The Collaboration of Mathematically Gifted Students on Challenging Tasks. *Journal for the Education of the Gifted*, *25*(1), 7–31. https://doi.org/10.1177/016235320102500102

Diezmann, C. M., & Watters, J. J. (2016). Catering for Mathematically Gifted Elementary Students: Learning from Challenging Tasks. *Gifted Child Today*, *23*(4), 14–52. https://doi.org/10.4219/gct-2000-737

Ditton, H., & Krüsken, J. (2009). Denn wer hat, dem wird gegeben werden? Eine Längsschnittstudie zur Entwicklung schulischer Leistungen und den Effekten der sozialen Herkunft in der Grundschulzeit. *Journal for educational research online*, *1*(1), 33–61.

Dörner, D., & Güss, C. D. (2013). PSI: A computational architecture of cognition, motivation, and emotion. *Review of General Psychology*, *17*(3), 297–317.

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. New York, NY: Psychology Press.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P.,. . . Brooks-Gunn, J. (2007). School readiness and later achievement. *Developmental psychology*, *43*(6), 1428.

Eccles, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., & Midgley, C. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 97–132). San Francisco: W. H. Freeman.

Eccles, J., Wigfield, A., Harold, R. D., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child development*, *64*(3), 830–847.

Eccles, J. S., & Wigfield, A. (1995). In the mind of the actor: The structure of adolescents' achievement task values and expectancy-related beliefs. *Personality and social psychology bulletin*, *21*(3), 215–225.

Edwards, E. L., Nichols, E. D., & Sharpe, G. H. (1972). Mathematical competencies and skills essential for enlightened citizens. *The Mathematics Teacher*, *65*(7), 671–677.

Ehm, J.-H. (2014). *Akademisches Selbstkonzept im Grundschulalter. Entwicklungsanalyse dimensionaler Vergleiche und Exploration differenzieller Unterschiede.* [Academic self-concept in elementary school. Developmental analysis of dimensional comparisons and exploration of differences.] (Dissertation). Johann-Wolfgang-Goethe-Universität, Frankfurt am Main. Retrieved from URN: urn:nbn:de:0111-opus-95657

Ehm, J.-H., Nagler, T., Lindberg, S., & Hasselhorn, M. (2014). Dimensionale Vergleichseffekte zwischen Lesen, Rechtschreiben und Rechnen. Eine Erweiterung des I/E-Modells für die Grundschule [Dimensional Comparison Effects Between Reading, Spelling and Math. An Extension of the I/E-Model for Elementary School]. *Zeitschrift für Pädagogische Psychologie*, *28*, 51–56. https://doi.org/10.1024/1010-0652/a000117

Enders, C. K. (2010). *Applied missing data analysis*. New York, NY: The Guilford Press.

Fauser, P., Messner, R., Beutel, W., & Tetzlaff, S. (2007). *Fordern und fördern: Was Schülerwettbewerbe leisten* [Fosteting and challenging. What academic competitions provide]: Ed. Körber-Stiftung.

Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). A longitudinal study of gender differences in young children’s mathematical thinking. *Educational researcher*, *27*(5), 6–11.

Fischer, U., Moeller, K., Bientzle, M., Cress, U., & Nuerk, H.-C. (2011). Sensori-motor spatial training of number magnitude representation. *Psychonomic bulletin & review*, *18*(1), 177–183. https://doi.org/10.3758/s13423-010-0031-3

Fleischer, J., Koeppen, K., Kenk, M., Klieme, E., & Leutner, D. (2013). Kompetenzmodellierung: Struktur, Konzepte und Forschungszugänge des DFG-Schwerpunktprogramms [Modeling of competencies: structure, concepts and research approaches of the DFG priority program]. *Zeitschrift für Erziehungswissenschaft*, *16*(1), 5–22. https://doi.org/10.1007/s11618-013-0379-z

Floyd, R. G., Evans, J. J., & McGrew, K. S. (2003). Relations between measures of Cattell-Horn-Carroll (CHC) cognitive abilities and mathematics achievement across the school-age years. *Psychology in the Schools*, *40*(2), 155–171.

Forrester, J. H. (2010). *Competitive science events: Gender, interest, science self-efficacy, and academic major choice.* (Dissertation). North Carolina State University. Retrieved from https://repository.lib.ncsu.edu/bitstream/handle/1840.16/6073/etd.pdf?sequence=1&isAllowed=y

Förster, F., & Grohmann, W. (2013). Geöffnete Aufgabensequenzen zur Begabtenförderung im Mathematikunterricht [Open taks to promote gifted students in mathematics education].

In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 111–125). Kronach / Berlin: Mildenberger.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics Teaching and Classroom Practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning. A project of the National Council of Teachers of Mathematics* (pp. 225–253). IAP.

Freudenthal, H. (1986). *Didactical phenomenology of mathematical structures*: Springer Science & Business Media.

Friedman, L. M., Furberg, C., & DeMets, D. L. (2010). *Fundamentals of clinical trials* (4th). New York, NY: Springer.

Friso-van den Bos, I., van der Ven, S. H.G., Kroesbergen, E. H., & van Luit, J. E.H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, *10*, 29–44. https://doi.org/10.1016/j.edurev.2013.05.003

Fritzlar, T. (2010). Begabung und Expertise. Eine mathematikdidaktiche Perspektive [Giftedness and expertise. The viewpoint of mathematics didactics]. *mathematica didactica*, *33*, 113–140. Retrieved from URL http://mathdid.ph-freiburg.de/documents/md_2010/md_2010_Fritzlar_Begabung.pdf

Fritzlar, T. (2013). Mathematische Begabungen im Grundschulalter. Ein Überblich zu aktuellen mathematikdidaktischen Forschungsarbeiten [Mathematically giftedness in elementary school age. A review of reseach in didactis of mathematics]. *mathematica didactica*, *36*, 5–27.

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M.,. . . Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental psychology*, *46*(6), 1731–1746. https://doi.org/10.1037/a0020662

Gallin, P., & Ruf, U. (1995). *Sprache und Mathematik: 1.-3. Schuljahr: Ich mache das so! Wie machst du es? Das machen wir ab* [Language and mathematics: 1st to 3rd year of schooling: I do it like this! How did you do it? We will do it!]: Interkantonale Lehrmittelzentrale.

Gaspard, D.-P. H. (2015). *Promoting Value Beliefs in Mathematics. A Multidimensional Perspective and the Role of Gender.* Eberhard Karls Universität Tübingen.

Gaspard, H., Dicke, A.-L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B. (2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology*, *107*(3), 663–677. https://doi.org/10.1037/edu0000003

Geary, D. C., & Moore, A. M. (2016). Cognitive and brain systems underlying early mathematical development. *Progress in brain research*, *227*, 75–103. https://doi.org/10.1016/bs.pbr.2016.03.008

Geary, D. C., Saults, S. J., Liu, F., & Hoard, M. K. (2000). Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. *Journal of experimental child psychology*, *77*(4), 337–353. https://doi.org/10.1006/jecp.2000.2594

Geary, D. C., Bailey, D. H., & Hoard, M. K. (2009). Predicting mathematical achievement and mathematical learning disability with a simple screening tool: The number sets test. *Journal of Psychoeducational Assessment*, *27*(3), 265–279.

Georges, C., Hoffmann, D., & Schiltz, C. (2017). Mathematical abilities in elementary school: Do they relate to number–space associations? *Journal of experimental child psychology*, *161*, 126–147.

Goldstein, D., & Wagner, H. (1993). After school programs, competitions, school olympics, and summer programs. In K. A. Heller (Ed.), *International Handbook of Research and Development of Giftedness and Talent* (pp. 593–604). Oxford: Pergamon.

Gölitz, D., Roick, T., & Hasselhorn, M. (2006). *DEMAT 4: Deutscher Mathematiktest für vierte Klassen* [German mathematics test for fourth graders]: Hogrefe.

Golle, J., Herbein, E., Hasselhorn, M., & Trautwein, U. (2017). Begabungs- und Talentförderung in der Grundschule durch Enrichment: Das Beispiel Hector-Kinderakademien [Promoting gifted and talented elementary school students via enrichment: Using the example Hector Children‘s Academy]. In Trautwein, U. & Hasselhorn, M. (Ed.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente.* (pp. 177–196). Göttingen: Hogrefe.

Gottfredson, L. S., & Deary, I. J. (2004). Intelligence predicts health and longevity, but why? *Current Directions in Psychological Science*, *13*(1), 1–4.

Grabner, R. H., Ansari, D., Reishofer, G., Stern, E., Ebner, F., & Neuper, C. (2007). Individual differences in mathematical competence predict parietal brain activation during mental calculation. *NeuroImage*, *38*(2), 346–356. https://doi.org/10.1016/j.neuroimage.2007.07.041

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. *Annual review of psychology*, *60*, 549–576. https://doi.org/10.1146/annurev.psych.58.110405.085530

Grebe, U. (2013). *Haben Mädchen mehr Mühe mit Mathe?* [Do girls have more trouble with mathematics?]. Hamburg: Diplomica Verlag. Retrieved from http://gbv.eblib.com/patron/FullRecord.aspx?p=1594238

Grønmo, L. S., Lindquist, M., Arora, A., & Mullis, I. V. S. (2015). TIMSS 2015 mathematics framework. *TIMSS*, 11–28.

Guay, F., Marsh, H. W., & Boivin, M. (2003). Academic self-concept and academic achievement: Developmental perspectives on their causal ordering. *Journal of Educational Psychology*, *95*(1), 124–136. https://doi.org/10.1037/0022-0663.95.1.124

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental psychology*, *48*(5), 1229.

Haag, N., & Roppelt, A. (2012). Der Ländervergleich im Fach Mathematik [German National Assessment in mathematics]. In P. Stanat, H. A. Pant, K. Böhme, & D. Richter (Eds.), *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik. Ergebnisse des IQB-Ländervergleichs 2011* (pp. 117–127). Münster: Waxmann.

Haffner, J., Baro, K., Parzer, P., & Resch, F. (2005). *Heidelberger Rechentest (HRT 1-4)* [Heidelberg Arithemtic Test]. Göttingen: Hogrefe.

Hanna, G. (2000). Declining gender differences from FIMS to TIMSS. *Zentralblatt für Didaktik der Mathematik*, *32*(1), 11–17.

Hansford, B. C., & Hattie, J. A. (1982). The relationship between self and achievement/performance measures. *Review of Educational Research*, *52*(1), 123–142.

Hänze, M., & Berger, R. (2007). Cooperative learning, motivational effects, and student characteristics: An experimental study comparing cooperative learning and direct instruction in 12th grade physics classes. *Learning and Instruction*, *17*(1), 29–41. https://doi.org/10.1016/j.learninstruc.2006.11.004

Hartig, J. (Ed.). (2008). *Assessment of competencies in educational contexts* (1. Aufl.). Cambridge, Mass.: Hogrefe. Retrieved from http://lib.myilibrary.com/detail.asp?id=369040

Hartig, J., & Jude, N. (2008). Strukturen sprachlicher Kompetenzen [Structures of language competences]. In E. Klieme, W. Eichler, A. Helmke, R. H. Lehmann, G. Nold, H.-G. Rolff,. . . H. Willenberg (Eds.), *Unterricht und Kompetenzerwerb in Deutsch und Englisch. Ergebnisse der DESI-Studie* (pp. 191–201). Weinheim: Beltz.

Hartig, J., & Klieme, E. (2006). Kompetenz und Kompetenzdiagnostik [Competences and assessing competences]. In K. Schweizer (Ed.), *Leistung und Leistungsdiagnostik. Mit 18 Tabellen* (pp. 127–143). Berlin, Heidelberg: Springer Medizin Verlag Heidelberg.

Hasemann, K., Gasteiger, H., & Padberg, F. (Eds.). (2014). *Anfangsunterricht Mathematik* [Mathematics in elementary school]. Berlin, Heidelberg: Springer Berlin Heidelberg.

Hasselhorn, M., & Gold, A. (2017). *Pädagogische Psychologie: Erfolgreiches Lernen und Lehren* [Educational Psychology] (4., aktualisierte Auflage). *Standards Psychologie*. Stuttgart: Verlag W. Kohlhammer. Retrieved from http://www.kohlhammer.de/wms/instances/KOB/appDE/nav_product.php?product=978-3-17-031976-9;X:MVB

Heinrich, F. (2010). Defizitäre Verhaltensweisen beim Bearbeiten mathematischer Probleme [Deficit behaviors when solving mathematical problems]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder. Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 22–33). Berlin, Münster: Lit.

Heller, K. A. (Ed.). (1993). *International Handbook of Research and Development of Giftedness and Talent*. Oxford: Pergamon.

Heller, K. A., Mönks, F. J., Subotnik, R., & Sternberg, R. J. (Eds.). (2000). *International handbook of giftedness and talent*: Elsevier.

Helmke, A. (1998). Vom Optimisten zum Realisten? Zur Entwicklung des Fähigkeitskonzeptes vom Kindergarten bis zur 6. Klassenstufe. In F. E. Weinert (Ed.), *Entwicklung im Kindesalter* (pp. 115–132). Weinheim: Beltz.

Henningsen, M., & Stein, M. K. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524. https://doi.org/10.2307/749690

Herbein, E. (2016). *Public Speaking Training as an Enrichment Program for Elementary School Children. Conceptualization, Evaluation, and Implementation* (Dissertation). Eberhard Karls Universität, Tübingen.

Hidi, S., & Harackiewicz, J. M. (2000). Motivating the academically unmotivated: A critical issue for the 21st century. *Review of Educational Research*, *70*(2), 151–179.

Höffler, T. N., Bonin, V., & Parchmann, I. (2017). Science vs. Sports: Motivation and Self-concepts of Participants in Different School Competitions. *International Journal of Science and Mathematics Education*, *15*(5), 817–836. https://doi.org/10.1007/s10763-016-9717-y

Holling, H., Preckel, F., Vock, M., Roßbach, H.-G., Baudson, T. G., & Kuger, S. (2009). *Begabte Kinder finden und fördern.: Ein Ratgeber für Eltern, Erzieherinnen und Erzieher, Lehrerinnen und Lehrer* [Finding and promoting gifted children. A guidebook for parents, kindergarden teachers and teachers.]. Bonn: Bundesministerium für Bildung und Forschung (BMBF).

Humphrey, N., Lendrum, A., Ashworth, E., Frearson, K., Buck, R., & Kerr, K. (2016). *Implementation and process evaluation (IPE) for interventions in education settings: A synthesis of the literature.* Retrieved from Retrieved from Education Endowment: https://educationendowmentfoundation.org.uk/public/files/Evaluation/Setting_up_an_Evaluation/IPE_Review_Final.pdf

Hyde, J. S. (2016). Sex and cognition: Gender and cognitive functions. *Current opinion in neurobiology*, *38*, 53–56. https://doi.org/10.1016/j.conb.2016.02.007

Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender differences in mathematics performance: A meta-analysis. *Psychological Bulletin*, *107*(2), 139–155. https://doi.org/10.1037/0033-2909.107.2.139

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., & Williams, C. C. (2008). Gender similarities characterize math performance. *Science (New York, N.Y.)*, *321*(5888), 494–495.

Hyde, J. S. (2005a). The gender similarities hypothesis. *American psychologist*, *60*(6), 581.

Hyde, J. S. (2005b). The gender similarities hypothesis. *American psychologist*, *60*(6), 581.

IBM Corp. Released. (2013). IBM SPSS Statistics for Windows. Armonk, NY.

IQB (= Institut zur Qualitätsentwicklung im Bildungswesen) (ed.). (2008). *Kompetenzstufenmodell zu den Bildungsstandards im Fach Mathematik für den Primarbereich (Jahrgangsstufe 4).* [Competence level model for the educational standards in mathematics for elemetary school]. Retrieved from https://www.iqb.hu-berlin.de/bista/ksm/KSM_GS_Mathemati_3.pdf

Johnson, C., & Engelhard, G. (1992). Gender, academic achievement, and preferences for cooperative, competitive, and individualistic learning among African-American adolescents. *The Journal of psychology*, *126*(4), 385–392. https://doi.org/10.1080/00223980.1992.10543371

Johnson, D. W., & Johnson, R. T. (1990). Using cooperative learning in math. *Cooperative learning in mathematics: A handbook for teachers*, 103–125.

Johnson, D. W., & Johnson, R. T. (1994). *Learning together and alone. Cooperative, competitive, and individualistic learning*: ERIC.

Johnson, D. W., Johnson, R. T., & Stanne, M. B. (2000). *Cooperative Learning Methods: A Meta-Analysis.* University of Minnesota. Retrieved from https://pdfs.semanticscholar.org/93e9/97fd0e883cf7cceb3b1b612096c27aa40f90.pdf

Johnson, M. L. (1983). Identifying and teaching mathematically gifted elementary school children. *Arithmetic Teacher*, *30*(5), 25–26.

Johnson, R. T. (1990). Supporting gifted students' acquisition of relevant knowledge for solving math problems. *Early Child Development and Care*, *63*(1), 37–45. https://doi.org/10.1080/0300443900630106

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental psychology*, *45*(3), 850.

Käpnick, F. (1998). *Mathematisch begabte Kinder: Modelle, empirische Studien und Förderungsprojekte für das Grundschulalter* [Mathematically gifted students: Modells, empirical studies and enrichment for elementary school students]: Lang.

Käpnick, F. (2010). „Mathe für kleine Asse" - Das Münsteraner Konzept zur Förderung mathematisch begabter Kinder. ["Math for young whiz" - The Münster concept for the promotion of mathematically gifted children.]. In M. Fuchs & F. Käpnick (Eds.), *Begabungsforschung: Mathematisch begabte Kinder. Vol. 8. Eine Herausforderung für Schule und Wissenschaft* (2nd ed., pp. 138–150). Berlin, Münster: Lit.

Käpnick, F. (2013). Intuition - ein häufiges Phänomen beim Problemlösen mathematisch begabter Grundschulkinder. [Intuition - A common phenomen in the problem solving of mathematically gifted elementary school students]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 77–93). Kronach / Berlin: Mildenberger.

Käpnick, F. (2014). *Mathematiklernen in der Grundschule* [Learning mathematics in elementary schools]. Berlin, Heidelberg: Springer.

Kastens, C., Gabriel, K., & Lipowsky, F. (2013). Selbstkonzeptentwicklung im Anfangsunterricht. In F. Lipowsky, G. Faust, & C. Kastens (Eds.), *Persönlichkeits-und Lernentwicklung an staatlichen und privaten Grundschulen. Ergebnisse der PERLE-Studie zu den ersten beiden Schuljahren* (pp. 99–128). Waxmann Verlag.

Kell, H. J., Lubinski, D., & Benbow, C. P. (2013). Who rises to the top? Early indicators. *Psychological science*, *24*(5), 648–659.

Kießwetter, K. (2013). Können auch Grundschüler schon im eigentlichen Sinne mathematisch agieren? [Can elementary school students do mathematics?]. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 128–153). Offenburg: Mildenberger.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

Klein, E., Suchan, J., Moeller, K., Karnath, H.-O., Knops, A., Wood, G.,. . . Willmes, K. (2016). Considering structural connectivity in the triple code model of numerical cognition: Differential connectivity for magnitude processing and arithmetic facts. *Brain structure & function*, *221*(2), 979–995. https://doi.org/10.1007/s00429-014-0951-1

Kleinginna, P. R., & Kleinginna, A. M. (1981). A categorized list of emotion definitions, with suggestions for a consensual definition. *Motivation and emotion*, *5*(3), 263–291.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher" math talk.". *Developmental psychology*, *42*(1), 59.

Klieme, E., Jude, N., Baumert, J., & Prenzel, M. (2010). PISA 2000–2009: Bilanz der Veränderungen im Schulsystem [PISA 2000-2009: A review of changes in the schhol system]. In E. Klieme, C. Artelt, J. Hartig, N. Jude, O. KÃ¶ller, M. Prenzel, & S. Wolfgang (Eds.), *PISA 2009. Bilanz nach einem Jahrzehnt* (pp. 277–300). Waxmann Verlag.

Klieme, E. (2004). Was sind Kompetenzen und wie lassen sie sich messen? [What are competences and how can they be assessed]. *Pädagogik*, *6*, 10–13.

Klieme, E. (2006). Empirische Unterrichtsforschung: Aktuelle Entwicklungen, theoretische Grundlagen und fachspezifische Befunde. Einführung in den Thementeil. *Zeitschrift für Pädagogik*, *52*(6), 765–773.

Klieme, E., Artelt, C., Hartig, J., Jude, N., KÃ¶ller, O., Prenzel, M., & Wolfgang, S. (Eds.). (2010). *PISA 2009: Bilanz nach einem Jahrzehnt* [PISA 2009. Resume after one decade]: Waxmann Verlag.

Klieme, E., Avenarius, H., Blum, W., Döbrich, P., Gruber, H., Prenzel, M.,. . . Tenorth, H.-E. (2003). *Zur Entwicklung nationaler Bildungsstandards* [For the development of national educatioanl standards].

Klieme, E., Eichler, W., Helmke, A., Lehmann, R. H., Nold, G., Rolff, H.-G.,. . . Willenberg, H. (Eds.). (2008). *Unterricht und Kompetenzerwerb in Deutsch und Englisch. Ergebnisse der DESI-Studie* [Education and acquiring competences in German and English. Results form the DESI-study]. Weinheim: Beltz.

Klieme, E., & Hartig, J. (2008). Kompetenzkonzepte in den Sozialwissenschaften und im erziehungswissenschaftlichen Diskurs [Concepts of competences in social and edcuational science discourse]. *Kompetenzdiagnostik*, 11–29.

Klieme, E., Hartig, J., & Rauch, D. (2008). The concept of competence in educational contexts. In J. Hartig (Ed.), *Assessment of competencies in educational contexts* (1st ed., pp. 3–22). Cambridge, Mass.: Hogrefe.

Klieme, E., & Leutner, D. (2006). Kompetenzmodelle zur Erfassung individueller Lernergebnisse und zur Bilanzierung von Bildungsprozessen. Beschreibung eines neu eingerichteten Schwerpunktprogramms der DFG [Competence models to assess individual learning and evaluate educational processes. Describing a new research approaches of the DFG priority program]. *Zeitschrift für Pädagogik*, *52*(6), 876–903.

Klieme, E., Neubrand, M., & Lüdtke, O. (2001). Mathematische Grundbildung: Testkonzeption und Ergebnisse [Basic education in mathematics: Concept of the test and results]. In Baumert, J., Klieme, E., Neubrand, M., Prenzel, M., Schiefele, U., Schneider, W. et al. (Ed.), *PISA 2000* (pp. 139–190). Opladen: Leske + Buderich.

Klieme, E., & Rakoczy, K. (2008). Empirische Unterrichtsforschung und Fachdidaktik. Outcome-orientierte Messung und Prozessqualität des Unterrichts. *Zeitschrift für Pädagogik*, *54*(2), 222–237.

Köller, O. (1998). *Zielorientierungen und schulisches Lernen* [Goal-orientation and education in school]: Waxmann Verlag.

Köller, O. (2010). Bildungsstandards [Educational standards]. In R. Tippelt & B. Schmidt (Eds.), *Handbuch Bildungsforschung* (pp. 529–548). Springer.

Köller, O., & Klieme, E. (2000). Geschlechtsdifferenzen in den mathematisch-naturwissenschaftlichen Leistungen [Gender differences in mathematics and science performance]. In J. Baumert, W. Bos, & R. H. Lehmann (Eds.), *Mathematische und physikalische Kompetenzen am Ende der gymnasialen Oberstufe* (pp. 373–404). Berlin, Heidelberg: Springer.

Köller, O., & Parchmann, I. (2012). Competencies: The German Notion of Learning Outcomes. In S. Bernholt, K. Neumann, & P. Nentwig (Eds.), *Learning outcomes in science education. Making it tangible* (pp. 151–171). Münster: Waxmann.

Koshy, V., Ernest, P., & Casey, R. (2009). Mathematically gifted and talented learners: Theory and practice. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 213–228. https://doi.org/10.1080/00207390802566907

Krajewski, K., Liehm, S., & Schneider, W. (2004). *DEMAT 2+: Deutscher Mathematiktest für zweite Klassen* [German mathematics tests for second graders]. Weinheim: Beltz.

Krajewski, K. (2008). Prävention der Rechenschwäche [The early prevention of math problems]. In W. Schneider & M. Hasselhorn (Eds.), *Handbuch der pädagogischen Psychologie* (pp. 360–370). Hogrefe Verlag.

Krajewski, K., & Schneider, W. (2009a). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*(6), 513–526. https://doi.org/10.1016/j.learninstruc.2008.10.002

Krajewski, K., & Schneider, W. (2009b). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of experimental child psychology*, *103*(4), 516–531. https://doi.org/10.1016/j.jecp.2009.03.009

Kriegbaum, K., Jansen, M., & Spinath, B. (2015). Motivation: A predictor of PISA's mathematical competence beyond intelligence and prior test achievement. *Learning and Individual Differences*, *43*, 140–148. https://doi.org/10.1016/j.lindif.2015.08.026

Kriegbaum, K., & Spinath, B. (2016). Explaining Social Disparities in Mathematical Achievement: The Role of Motivation. *European Journal of Personality*, *30*(1), 45–63. https://doi.org/10.1002/per.2042

Kulik, J. A., & Kulik, C.-L. C. (1987). Meta-analytic Findings on Grouping Programs. *Gifted Child Quarterly*, *36*(2), 73–77. https://doi.org/10.1177/001698629203600204

Kultusministerkonferenz. (2004a). *Bildungsstandards der Kultusministerkonferenz: Erläuterungen zur Konzeption und Entwicklung* [Educational Standards of the Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of

Germany. Explanations regarding concepts and development]. Neuwied: Luchterhand. Retrieved from https://www.kmk.org/fileadmin/Dateien/veroeffentlichungen_beschluesse/2004/2004_12_16-Bildungsstandards-Konzeption-Entwicklung.pdf

Kultusministerkonferenz (Ed.). (2004b). *Bildungsstandards im Fach Mathematik für den Primarbereich.* [Decisions of the Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany. Scholastic standards for mathematics for elementary schools]. Neuwied: Luchterhand.

Kunina-Habenicht, O., Rupp, A. A., & Wilhelm, O. (2009). A practical illustration of multidimensional diagnostic skills profiling: Comparing results from confirmatory factor analysis and diagnostic classification models. *Studies in Educational Evaluation*, *35*(2), 64–70.

Kunter, M., & Trautwein, U. (2013). *Psychologie des Unterrichts* [Psychology in education]: UTB.

Kunter, M., & Voss, T. (2011). Das Modell der Unterrichtsqualität in COACTIV: Eine multikriteriale Analyse. In M. Kunter, J. Baumert, & W. Blum (Eds.), *Professionelle Kompetenz von Lehrkräften. Ergebnisse des Forschungsprogramms COACTIV* (pp. 85–113). Weinheim: Waxmann Verlag.

Kunter, M., & Voss, T. (2013). The Model of Instructional Quality in COACTIV: A Multicriteria Analysis. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers. Results from the COACTIV project* (pp. 97–124). Springer Science & Business Media.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition*, *93*(2), 99–125. https://doi.org/10.1016/j.cognition.2003.11.004

Laski, E. V., & Siegler, R. S. (2007). Is 27 a big number? Correlational and causal connections among numerical categorization, number line estimation, and numerical magnitude comparison. *Child development*, *78*(6), 1723–1743.

Leder, G., & Forgasz, H. (2008). Mathematics education: New perspectives on gender. *ZDM*, *40*(4), 513–518. https://doi.org/10.1007/s11858-008-0137-5

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37–46.

Lee, K., Ng, S.-F., Ng, E.-L., & Lim, Z.-Y. (2004). Working memory and literacy as predictors of performance on algebraic word problems. *Journal of experimental child psychology*, *89*(2), 140–158.

Lee, M. (1995). Gender, Group Composition, and Peer Interaction in Computer-Based Cooperative Learning. *Journal of Educational Computing Research*, *9*(4), 549–577. https://doi.org/10.2190/VMV1-JCVV-D9GA-GN88

LeFevre, J.-A. (2016). Numerical cognition: Adding it up. *Canadian journal of experimental psychology = Revue canadienne de psychologie experimentale*, *70*(1), 3–11. https://doi.org/10.1037/cep0000062

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child development*, *81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Leikin, R. (2010). Teaching the Mathematically Gifted. *Gifted Education International*, *27*(2), 161–175. https://doi.org/10.1177/026142941002700206

Lengfelder, A., & Heller, K. A. (2002). German Olympiad studies: Findings from a retrospective evaluation and from in-depth interviews: Where have all the gifted females gone. *Journal of Research in Education*, *12*(1), 86–92. Retrieved from http://www.olympiadprojects.com/v2/pubs_web/ch4_SS.pdf

Leuders, T. (2011). Kompetenzorientierung - eine Chance für die Weiterentwicklung des Mathematikunterrichts? In K. Eilerts, A. H. Hilligus, G. Kaiser, & P. Bender (Eds.), *Kompetenzorientierung in Schule und Lehrerbildung. Paderborner Beiträge zur Unterrichtsforschung und Lehrerbildung. Festschrift für Hans-Dieter Rinkens* (Vol. 15). Münster: LIT-Verlag.

Leuders, T. (2014). Modellierungen mathematischer Kompetenzen – Kriterien für eine Validitätsprüfung aus fachdidaktischer Sicht [Modelling mathematical competences - criteria for examining validity viewed form didactics]. *Journal für Mathematik-Didaktik*, *35*(1), 7–48. https://doi.org/10.1007/s13138-013-0060-3

Leutner, D., Fleischer, J., Wirth, J., Greiff, S., & Funke, J. (2012). Analytische und dynamische Problemlösekompetenz im Lichte internationaler Schulleistungsvergleichsstudien [Analytic and dynamic problem solving competence in international large-scale studies]. *Psychologische Rundschau*, *63*(1), 34–42. https://doi.org/10.1026/0033-3042/a000108

Link, T., Moeller, K., Huber, S., Fischer, U., & Nuerk, H.-C. (2013). Walk the number line – An embodied training of numerical concepts. *Trends in Neuroscience and Education*, *2*(2), 74–84. https://doi.org/10.1016/j.tine.2013.06.005

Link, T., Nuerk, H.-C., & Moeller, K. (2014). On the relation between the mental number line and arithmetic competencies. *The quarterly journal of experimental psychology*, *67*(8), 1597–1613.

Liu, O. L., & Wilson, M. (2009). Gender Differences in Large-Scale Math Assessments: PISA Trend 2000 and 2003. *Applied Measurement in Education*, *22*(2), 164–184. https://doi.org/10.1080/08957340902754635

Liu, O. L., Wilson, M., & Paek, I. (2008). A multidimensional Rasch analysis of gender differences in PISA mathematics. *Journal of applied measurement*, *9*(1), 18. Retrieved from https://gse.berkeley.edu/sites/default/files/users/mark-wilson/Wilson8.pdf

Loos, A., & Ziegler, G. M. (2015). Gesellschaftliche Bedeutung der Mathematik [Social significane of mathematics]. In R. Bruder, L. Hefendehl-Hebeker, B. Schmidt-Thieme, & H.-G. Weigand (Eds.), *Handbuch der Mathematikdidaktik* (pp. 3–18). Berlin: Springer Spektrum.

Lubinski, D., & Benbow, C. P. (2006). Study of Mathematically Precocious Youth After 35 Years: Uncovering Antecedents for the Development of Math-Science Expertise. *Perspectives on psychological science*, *1*(4), 316–345. https://doi.org/10.1111/j.1745-6916.2006.00019.x

Lüdtke, O., Köller, O., Artelt, C., Stanat, P., & Baumert, J. (2002). Eine Überprüfung von Modellen zur Genese akademischer Selbstkonzepte: Ergebnisse aus der PISA-Studie. *Zeitschrift für Pädagogische Psychologie*, *16*(3/4), 151–164. https://doi.org/10.1024//1010-0652.16.34.151

Lüdtke, O., Köller, O., Marsh, H. W., & Trautwein, U. (2005). Teacher frame of reference and the big-fish–little-pond effect. *Contemporary Educational Psychology*, *30*(3), 263–285. https://doi.org/10.1016/j.cedpsych.2004.10.002

Marsh, H. W. (1986). Verbal and math self-concepts: An internal/external frame of reference model. *American Educational Research Journal*, *23*(1), 129–149.

Marsh, H. W. (1987). The big-fish-little-pond effect on academic self-concept. *Journal of Educational Psychology*, *79*(3), 280–295. https://doi.org/10.1037/0022-0663.79.3.280

Marsh, H. W. (1989). Age and sex effects in multiple dimensions of self-concept: Preadolescence to early adulthood. *Journal of Educational Psychology*, *81*(3), 417.

Marsh, H. W. (1990). Causal ordering of academic self-concept and academic achievement: A multiwave, longitudinal panel analysis. *Journal of Educational Psychology*, *82*(4), 646.

Marsh, H. W. (2014). Academic Self-Concept: Theory, Measurement, and Research. In J. Suls (Ed.), *Psychological Perspectives on the Self. The Self in Social Perspective* (4th ed., pp. 59–98). Hoboken: Taylor and Francis.

Marsh, H. W., Byrne, B. M., & Yeung, A. S. (1999). Causal ordering of academic self-concept and achievement: Reanalysis of a pioneering study and. *Educational psychologist*, *34*(3), 155–167.

Marsh, H. W., Chessor, D., Craven, R., & Roche, L. (1995). The effects of gifted and talented programs on academic self-concept: The big fish strikes again. *American Educational Research Journal*, *32*(2), 285–319.

Marsh, H. W., & Craven, R. G. (2006). Reciprocal effects of self-concept and performance from a multidimensional perspective: Beyond seductive pleasure and unidimensional perspectives. *Perspectives on psychological science*, *1*(2), 133–163. https://doi.org/10.1111/j.1745-6916.2006.00010.x

Marsh, H. W., & Hau, K.-T. (2003). Big-Fish—Little-Pond effect on academic self-concept: A cross-cultural (26-country) test of the negative effects of academically selective schools. *American psychologist*, *58*(5), 364–376. https://doi.org/10.1037/0003-066X.58.5.364

Marsh, H. W., Kong, C.-K., & Hau, K.-T. (2000). Longitudinal multilevel models of the big-fish-little-pond effect on academic self-concept: Counterbalancing contrast and reflected-glory effects in Hong Kong schools. *Journal of Personality and Social Psychology*, *78*(2), 337.

Marsh, H. W., & Martin, A. J. (2011). Academic self-concept and academic achievement: Relations and causal ordering. *The British journal of educational psychology*, *81*(Pt 1), 59–77. https://doi.org/10.1348/000709910X503501

Marsh, H. W., & Parker, J. W. (1984). Determinants of student self-concept: Is it better to be a relatively large fish in a small pond even if you don’t learn to swim as well? *Journal of Personality and Social Psychology*, *47*(1), 213–231. https://doi.org/10.1037/0022-3514.47.1.213

Marsh, H. W., Seaton, M., Trautwein, U., Lüdtke, O., Hau, K. T., O’Mara, A. J., & Craven, R. G. (2008). The Big-fish–little-pond-effect Stands Up to Critical Scrutiny: Implications for

Theory, Methodology, and Future Research. *Educational Psychology Review*, *20*(3), 319–350. https://doi.org/10.1007/s10648-008-9075-6

Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child development*, *76*(2), 397–416.

Marsh, H. W., & Yeung, A. S. (1997). Coursework Selection: Relations to Academic Self-Concept and Achievement. *American Educational Research Journal*, *34*(4), 691–720. https://doi.org/10.2307/1163354

McAllister, B. A., & Plourde, L. A. (2008). Enrichment Curriculum: Essential for mathematically gifted students. *Education*, *129*(1), 40–49. Retrieved from http://web.a.ebscohost.com/ehost/pdfviewer/pdfviewer?sid=2c0b2b92-484e-4295-8ceb-ae855b7aceed%40sessionmgr4009&vid=2&hid=4214

Merton, R. K. (1968). The Matthew effect in science. *Science (New York, N.Y.)*, *159*(3810), 56–63.

Meyer, H. (2014). *Didaktische Modelle* [Didactical modells] (11. Auflage). Berlin: Cornelsen Schulverlage. Retrieved from http://gbv.eblib.com/patron/FullRecord.aspx?p=2080721

Meyer, H. (2016). *Was ist guter Unterricht?* [What is good education?] (11. Auflage). Berlin: Cornelsen.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, 65–88.

Ministerium für Kultus, Jugend und Sport Baden-Württemberg. (2016). *Bildungsplan der Grundschule: Mathematik* [Educational standards for elementary school: Mathematics]: Neckar-Verlag GmbH. Retrieved from http://www.bildungsplaene-bw.de/site/bildungsplan/get/documents/lsbw/export-pdf/depot-pdf/ALLG/BP2016BW_ALLG_GS_M.pdf

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H.-C. (2011). Early place-value understanding as a precursor for later arithmetic performance—a longitudinal study on numerical development. *Research in developmental disabilities*, *32*(5), 1837–1851. https://doi.org/10.1016/j.ridd.2011.03.012

Möller, J., & Köller, O. (2001). Dimensional comparisons: An experimental approach to the internal/external frame of reference model. *Journal of Educational Psychology*, *93*(4), 826–835. https://doi.org/10.1037/0022-0663.93.4.826

Möller, J., Pohlmann, B., Köller, O., & Marsh, H. W. (2009). A meta-analytic path analysis of the internal/external frame of reference model of academic achievement and academic self-concept. *Review of Educational Research*, *79*(3), 1129–1167.

Möller, J., Streblow, L., Pohlmann, B., & Köller, O. (2006). An extension to the internal/external frame of reference model to two verbal and numerical domains. *European Journal of Psychology of Education*, *21*(4), 467–487.

Mönks, F. J., & Mason, E. J. (2000). Developmental Psychology and Giftedness: Theories and Research. In K. A. Heller, F. J. Mönks, R. Subotnik, & R. J. Sternberg (Eds.), *International handbook of giftedness and talent* (pp. 141–156). Elsevier.

Murayama, K., Pekrun, R., Lichtenfeld, S., & Vom Hofe, R. (2013). Predicting long-term growth in students' mathematics achievement: The unique contributions of motivation and cognitive strategies. *Child development*, *84*(4), 1475–1490.

Murnane, R. J., Willett, J. B., & Levy, F. (1995). *The growing importance of cognitive skills in wage determination.*

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*(4), 343–370. https://doi.org/10.1080/13803611.2015.1057161

Nagy, G., Garrett, J., Trautwein, U., Cortina, K. S., Baumert, J., & Eccles, J. (2008). Gendered high school course selection as a precursor of gendered occupational careers: The mediating role of self-concept and intrinsic value. In H. M. G. Watt & J. S. Eccles (Eds.), *Gender and occupational outcomes. Longitudinal assessments of individual, social, and cultural influences* (pp. 115–143). American Psychological Association.

Nagy, G., Trautwein, U., Baumert, J., Köller, O., & Garrett, J. (2007). Gender and course selection in upper secondary education: Effects of academic self-concept and intrinsic value. *Educational Research and Evaluation*, *12*(4), 323–345. https://doi.org/10.1080/13803610600765687

Nagy, G., Watt, H. M. G., Eccles, J. S., Trautwein, U., Lüdtke, O., & Baumert, J. (2010). The Development of Students' Mathematics Self-Concept in Relation to Gender: Different Countries, Different Trajectories? *Journal of Research on Adolescence*, *20*(2), 482–506.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*: National Council of Teachers of.

Navarro, J. I., Aguilar, M., Alcalde, C., Ruiz, G., Marchena, E., & Menacho, I. (2011). Inhibitory processes, working memory, phonological awareness, naming speed, and early arithmetic achievement. *The Spanish Journal of Psychology*, *14*(2), 580–588.

Navarro, J. I., Aguilar, M., Marchena, E., Ruiz, G., Menacho, I., & van Luit, J. E. H. (2012). Longitudinal study of low and high achievers in early mathematics. *The British journal of educational psychology*, *82*(Pt 1), 28–41. https://doi.org/10.1111/j.2044-8279.2011.02043.x

Neisser, U., Boodoo, G., Bouchard, T. J., JR., Boykin, A. W., Brody, N., Ceci, S. J.,. . . Urbina, S. (1996). Intelligence: Knowns and unknowns. *American psychologist*, *51*(2), 77–101. https://doi.org/10.1037/0003-066X.51.2.77

Neumann, K., Bernholt, S., & Nentwig, P. (2012). Learning Outcomes in Science Education:: A Synthesis of the International Views in Definingm Assessing and Fostering Science Learning. In S. Bernholt, K. Neumann, & P. Nentwig (Eds.), *Learning outcomes in science education. Making it tangible* (pp. 501–519). Münster: Waxmann.

Niklas, F., & Schneider, W. (2012). Die Anfänge geschlechtsspezifischer Leistungsunterschiede in mathematischen und schriftsprachlichen Kompetenzen [The beginnings of gender differences in mathematical and writing competences]. *Zeitschrift für Entwicklungspsychologie und Pädagogische Psychologie*.

Niss, M., & Højgaard, T. (2011). *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde. Retrieved from IMFUFA, Roskilde university website: http://milne.ruc.dk/imfufatekster/pdf/485web_b.pdf

Noel, M.-P., & Seron, X. (1993). Arabic number reading deficit: A single case study or when 236 is read (2306) and judged superior to 1258. *Cognitive neuropsychology*, *10*(4), 317–339.

Nolte, M. (2013a). „Du Papa, die interessieren sich für das was ich denke!" - Zur Arbeit mit mathematisch besonders begabten Grundschulkindern. ["Daddy, they are interested in what

I think!" - About the work with prospective mathematically gifted elementary school students]. In T. Trautmann & W. Manke (Eds.), *Begabung, Individuum, Gesellschaft. Begabtenförderung als pädagogische und gesellschaftliche Herausforderung* (pp. 128–143). Weinheim: Beltz Juventa.

Nolte, M. (2013b). Zum Erkennen und Nutzen von Mustern und Strukturen in Problemlöseprozessen [Recognizing and Using pattern and sturctures while problem solving]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 11–24). Kronach / Berlin: Mildenberger.

Nolte, M., & Pamperien, K. (2013). Besondere mathematische Begabung im Grundschulalter - ein Forschungs- und Förderprojekt [Mathematically giftedness in elementary school age - a project of reseach and fostering]. In H. Bauersfeld & K. Kießwetter (Eds.), *Wie fördert man mathematisch besonders befähigte Kinder? Ein Buch aus der Praxis für die Praxis* (5th ed., pp. 70–72). Offenburg: Mildenberger.

Oelkers, J., & Reusser, K. (2008). *Expertise: Qualität entwickeln, Standards sichern, mit Differenz umgehen* [Expertise: Developing quality, secure standards and deal with heterogeniety]: BMBF, Referat Bildungsforschung.

Olson, S. (2005). *Count down: Six kids vie for glory at the world's toughest math competition*. Boston: Houghton Mifflin. Retrieved from http://www.loc.gov/catdir/samples/hm051/2003056897.html

Organisation for Economic Co-operation and Development. (1999). *Measuring Student Knowledge and Skills: A New Framework for Assessment.*: OECD Publishing. Retrieved from https://www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/33693997.pdf

Organisation for Economic Co-operation and Development. (2004). *Programme for International Student Assessment: Problem Solving For Tomorrow's World: First Measures Of Cross-curricular Competencies From Pisa 2003*: OECD Publishing. Retrieved from https://www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/34009000.pdf

Organisation for Economic Co-operation and Development. (2007). *PISA 2006, science competencies for tomorrow's world. PISA 2006: v. 1*. Paris: OECD. Retrieved from http://213.253.134.43/oecd/pdfs/browseit/9807011E.PDF

Organisation for Economic Co-operation and Development. (2014). *PISA 2012 Ergebnisse: Was Schülerinnen und Schüler wissen und können-Schülerleistungen in Lesekompetenz, Mathematik und Naturwissenschaften.* [PISA 2012 results: What students know and can do-students performance in reading, mathematics and science]. Bielefeld: Bertelsmann. Retrieved from http://www.oecd-ilibrary.org/docserver/download/9814035e.pdf?expires=1501762988&id=id&accname=guest&checksum=308521B34AEFEA1030FCC07A9CB1CF7C

Organisation for Economic Co-operation and Development (Ed.). (2016). *PISA 2015 Results (Volume I)*: OECD Publishing. Retrieved from http://www.keepeek.com/Digital-Asset-Management/oecd/education/pisa-2015-results-volume-i_9789264266490-en#.WYwnnOlCRPY

Oswald, F., Hanisch, G., & Hager, G. (2005). *Wettbewerbe und" Olympiaden": Impulse zur (Selbst)-Identifikation von Begabungen* [Academic competitions and Olympiads. Looking for talents]: LIT-Verlag.

Ozturk, M. A., & Debelak, C. (2008a). Academic competitions as tools for differentiation in middle school. *Gifted Child Today*, *31*(3), 47–53. Retrieved from http://files.eric.ed.gov/fulltext/EJ803366.pdf

Ozturk, M. A., & Debelak, C. (2008b). Affective Benefits from Academic Competitions for Middle School Gifted Students. *Gifted Child Today*, *31*(2), 48–53. https://doi.org/10.4219/gct-2008-758

Pajares, F., & Schunk, D. H. (2002). Self and self-belief in psychology and education: A historical perspective. *Improving academic achievement: Impact of psychological factors on education*, 3–31.

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *The British journal of educational psychology*, *82*(Pt 1), 42–63. https://doi.org/10.1111/j.2044-8279.2011.02039.x

Peters, H., & Sieve, B. (2013). Fordern und Fördern mit Wettbewerben - Schülerwettbewerbe in den Naturwissenschaften mit Bezug zur Chemie. [Challenge and foster with competition - academic competitions in science related to chemistry]. *Naturwissenschaften im Unterricht – Chemie*, *24*(136), 2–9.

Petersen, S., & Wulff, P. (2017). The German Physics Olympiad—identifying and inspiring talents. *European Journal of Physics*, *38*(3), 34005. https://doi.org/10.1088/1361-6404/aa538f

Pinxten, M., Marsh, H. W., Fraine, B. de, van den Noortgate, W., & van Damme, J. (2014). Enjoying mathematics or feeling competent in mathematics? Reciprocal effects on mathematics achievement and perceived math effort expenditure. *The British journal of educational psychology*, *84*(Pt 1), 152–174. https://doi.org/10.1111/bjep.12028

Poropat, A. E. (2009). A meta-analysis of the five-factor model of personality and academic performance. *Psychological Bulletin*, *135*(2), 322–338. https://doi.org/10.1037/a0014996

Praetorius, A.-K., Kastens, C., Hartig, J., & Lipowsky, F. (2016). Haben Schüler mit optimistischen Selbsteinschätzungen die Nase vorn? [Are Students With Optimistic Self-Concepts One Step Ahead? Relations Between Optimistic, Realistic, and Pessimistic Self-Concepts and the Achievement Development of Primary School Children]. *Zeitschrift für Entwicklungspsychologie und Pädagogische Psychologie*, *48*(1), 14–26. https://doi.org/10.1026/0049-8637/a000140

Primi, R., Ferrão, M. E., & Almeida, L. S. (2010). Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math. *Learning and Individual Differences*, *20*(5), 446–451. https://doi.org/10.1016/j.lindif.2010.05.001

Pyrt, M. C. (2000). Talent development in science and technology. In K. A. Heller, F. J. Mönks, R. Subotnik, & R. J. Sternberg (Eds.), *International handbook of giftedness and talent* (pp. 427–437). Elsevier.

R Core Team. (2015). R. Vienna, Austria: the R foundation for Statistical Computing. Retrieved from www.R-project.org

Ramm, G. C., Adamsen, C., & Neubrand, M. (Eds.). (2006). *PISA 2003: Dokumentation der Erhebungsinstrumente* [PISA 2003: Documentation of assessment instruments]. Münster: Waxmann.

Randel, J. M., Morris, B. A., Wetzel, C. D., & Whitehill, B. V. (1992). The effectiveness of games for educational purposes: A review of recent research. *Simulation & gaming*, *23*(3), 261–276.

Rebholz, F., & Golle, J. (2017). Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade [Fostering mathematical skills in elementary school – the role of academic competions using the example of the Mathematical Olympiad]. In Trautwein, U. & Hasselhorn, M. (Ed.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente.* (pp. 213–228). Göttingen: Hogrefe.

Rebholz, F. (2013). *Entwicklung und außerschulische Förderung mathematischer Fähigkeiten besonders begabter und hochbegabter Grundschulkinder im Rahmen der Hector-Kinderakademie* [Development and extracurricular promotion of mathematical skills of gifted elementary school children in the context of the Hector Children’s Academy Program] (Unveröffentlichte Wissenschaftliche Arbeit für die Zulassung zur Wissenschaftlichen Prüfung für das Lehramt am Gymnasium). Eberhard Karls Universität, Tübingen.

Rebholz, F., Golle, J., Oschatz, K., & Trautwein, U. (2017). *„Fit für die Mathematik-Olympiade“: Ein Trainingsporgramm zur Förderung mathematischer Fähigkeiten besonders begabter und hochbegabter Grundschulkinder*. Under preperation [”Getting fit for the Mathematical Olympiad”- A training program to foster mathematical skills of gifted elementary school children].

Rebholz, F., & Golle, J. (2017). Rebholz, F. & Golle, J. (2017). Förderung mathematischer Fähigkeiten in der Grundschule - Die Rolle von Schülerwettbewerben am Beispiel der Mathematik-Olympiade. [Fostering mathematical skills in elementary school – die role of academic competions using the example of the Mathematical Olympiad.]. In U. Trautwein & M. Hasselhorn (Eds.), *Tests und Trends - Jahrbuch der pädagogisch-psychologischen Diagnostik, Band 15. Begabungen und Talente (PSYNDEXalert)* (pp. 213–228). Göttingen: Hogrefe.

Reis, S. M., & Renzulli, J. S. (2010). Is there still a need for gifted education? An examination of current research. *Learning and Individual Differences*, *20*(4), 308–317. https://doi.org/10.1016/j.lindif.2009.10.012

Reiss, K., Roppelt, A., Haag, N., Pant, H. A., & Köller, O. (2012). Kompetenzstufenmodelle im Fach Mathematik [Competence level models in mathematics]. In P. Stanat, H. A. Pant, K. Böhme, & D. Richter (Eds.), *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik. Ergebnisse des IQB-Ländervergleichs 2011* (pp. 72–84). Münster: Waxmann.

Reiss, K., & Winkelmann, H. (2009). Kompetenzstufenmodelle für das Fach Mathematik im Primarbereich [Competence level models in mathematics for elementary school]. In D. Granzer, O. Köller, A. Bremerich-Voss, M. van den Heuvel-Panhuizen, K. Reiss, & G.

Walther (Eds.), *Bildungsstandards Deutsch und Mathematik. Leistungsmessung in der Grundschule* (pp. 120–141). Weinheim: Beltz.

Richter, D., Engelbert, M., Böhme, K., Haag, N., Hannighofer, J., Reimers, H.,. . . Stanat, P. (2012). Anlage und Durchführung des Ländervergleichs [Establishment and implementation of the German National Assessment]. In P. Stanat, H. A. Pant, K. Böhme, & D. Richter (Eds.), *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik. Ergebnisse des IQB-Ländervergleichs 2011* (pp. 85–102). Münster: Waxmann.

Riley, T. L., & Karnes, F. A. (1998). Mathematics+ competitions= a winning formula! *Gifted Child Today, 21*(4), 42.

Rinn, A. N. (2007). Effects of programmatic selectivity on the academic achievement, academic self-concepts, and aspirations of gifted college students. *Gifted Child Quarterly, 51*(3), 232–245.

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological science*, *24*(7), 1301–1308.

Rittle-Johnson, B., & Siegler, R. S. The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *Studies in developmental psychology. The development of mathematical skills* (pp. 75–110). Hove, England: Psychology Press/Taylor & Francis (UK).

Roberts, R. D., & Lipnevich, A. A. (2012). From general intelligence to multiple intelligences: Meanings, models, and measures. *APA Handbook Educational Psychology, Part 2. Waschington: APA.*

Robins, S., & Mayer, R. E. (1993). Schema training in analogical reasoning. *Journal of Educational Psychology*, *85*(3), 529–538. https://doi.org/10.1037/0022-0663.85.3.529

Roick, T., Gölitz, D., & Hasselhorn, M. (2004). *Deutscher Mathematiktest für dritte Klassen (DEMAT 3+)* [German mathematics tests for third graders]. Göttingen: Beltz Test.

Roppelt, A., & Reiss, K. (2012). Beschreibung der im Fach Mathematik untersuchten Kompetenzen [Describing comeptences in mathematics]. In P. Stanat, H. A. Pant, K. Böhme, & D. Richter (Eds.), *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik. Ergebnisse des IQB-Ländervergleichs 2011* (p. 34). Münster: Waxmann.

Rosebrock, S. (2013). Kreatives Arbeiten mit Zerlegungen [Creative work with decompositions]. In T. Fritzlar & F. Heinrich (Eds.), *Kompetenzen mathematisch begabter Grundschulkinder erkunden und fördern.* (4th ed., pp. 168–181). Kronach / Berlin: Mildenberger.

Rosseel, Y. (2012). *lavaan: An R package for structural equation modeling and more Version 0.4-9 (BETA)*: Ghent University.

Rothenbusch, S., Zettler, I., Voss, T., Lösch, T., & Trautwein, U. (2016). Exploring reference group effects on teachers' nominations of gifted students. *Journal of Educational Psychology*, *108*(6), 883–897. https://doi.org/10.1037/edu0000085

Rotigel, J. V., & Fello, S. (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today*, *27*(4), 46–51.

Rotigel, J. V., & Fello, S. (2016). Mathematically Gifted Students: How Can We Meet Their Needs? *Gifted Child Today*, *27*(4), 46–51. https://doi.org/10.4219/gct-2004-150

Ruf, U., & Gallin, P. (1999). *Dialogisches Lernen in Sprache und Mathematik* [Dialogic learning in language and mathematics]: Kallmeyer.

Sax, L. J., Kanny, M. A., Riggers-Piehl, T. A., Whang, H., & Paulson, L. N. (2015). "But I'm Not Good at Math": The Changing Salience of Mathematical Self-Concept in Shaping Women's and Men's STEM Aspirations. *Research in Higher Education*, *56*(8), 813–842. https://doi.org/10.1007/s11162-015-9375-x

Schecker, H. (2012). Standards, Competencies and Outcomes.: A Critical View. In S. Bernholt, K. Neumann, & P. Nentwig (Eds.), *Learning outcomes in science education. Making it tangible* (pp. 219–234). Münster: Waxmann.

Schenke, K., Rutherford, T., Lam, A. C., & Bailey, D. H. (2016). Construct Confounding Among Predictors of Mathematics Achievement. *AERA Open*, *2*(2), 233285841664893. https://doi.org/10.1177/2332858416648930

Schiefer, J. (2017). *Promoting and measuring elementary school children's understanding of science* (Dissertation). Eberhard Karls Universität, Tübingen. Retrieved from http://nbn-resolving.de/urn:nbn:de:bsz:21-dspace-741679

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental psychology*, *47*(6), 1525.

Schneider, W., Küspert, P., & Krajewski, K. (2016). *Die Entwicklung mathematischer Kompetenzen* [Development of mathematical competences] (2., aktualisierte und erweiterte Auflage). *StandardWissen Lehramt: Vol. 3899*. Paderborn: Ferdinand Schöningh. Retrieved from http://www.utb-studi-e-book.de/9783838546162

Schrader, F.-W., & Helmke, A. (2008). Determinanten der Schulleistung [Determinants of school performance]. In M. K. W. Schweer (Ed.), *Schule und Gesellschaft: Vol. 24. Lehrer-Schüler-Interaktion. Inhaltsfelder, Forschungsperspektiven und methodische Zugänge* (2nd ed., Vol. 2, pp. 285–302). Wiesbaden: VS Verlag für Sozialwissenschaften / GWV Fachverlage GmbH Wiesbaden.

Schroeders, U., Schipolowski, S., Zettler, I., Golle, J., & Wilhelm, O. (2016). Do the smart get smarter? Development of fluid and crystallized intelligence in 3rd grade. *Intelligence*, *59*, 84–95. https://doi.org/10.1016/j.intell.2016.08.003

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. *ZDM*, *49*(3), 307–322. https://doi.org/10.1007/s11858-017-0864-6

Seidel, T., & Shavelson, R. J. (2007). Teaching Effectiveness Research in the Past Decade: The Role of Theory and Research Design in Disentangling Meta-Analysis Results. *Review of Educational Research*, *77*(4), 454–499. https://doi.org/10.3102/0034654307310317

Sella, F., Sader, E., Lolliot, S., & Cohen Kadosh, R. (2016). Basic and advanced numerical performances relate to mathematical expertise but are fully mediated by visuospatial skills. *Journal of experimental psychology. Learning, memory, and cognition*, *42*(9), 1458–1472. https://doi.org/10.1037/xlm0000249

Selter, C. (2011). „Ich mark Mate“ - Leitideen und Beispiele für interessenförderlichen Unterricht [Ideas and examples for education that fosters interest in mathematics]. In R. Demuth, G. Walther, & M. Prenzel (Eds.), *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* (pp. 131–139). Seelze: Friedrich.

Selter, C., Walther, G., Wessel, J., & Wendt, H. (2012). Mathematische Kompetenzen im internationalen Vergleich: Testkonzeption und Ergebnisse [Mathematical competences in international comparsion]. In W. Bos, H. Wendt, O. Köller, & C. Selter (Eds.), *TIMSS 2011. Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern im internationalen Vergleich* (pp. 69–122). Münster/New York/München/Berlin: Waxmann.

Selter, C., Walther, G., Wessel, J., & Wendt, H. (2016). Mathematische Kompetenzen im internationalen Vergleich: Testkonzeption und Ergebnisse [Mathematical competences in international comparsion]. In H. Wendt, W. Bos, C. Selter, O. Köller, K. Schwippert, & D. Kasper (Eds.), *TIMSS 2015 Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern in Deutschland im internationalen Vergleich* (79–204). Waxmann.

Seo, B.-I. (2015). Mathematical Writing: What Is It and How Do We Teach It? *Journal of Humanistic Mathematics*, *5*(2), 133–145. https://doi.org/10.5642/jhummath.201502.12

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*: Wadsworth Cengage learning.

Shavelson, R. J., Hubner, J. J., & Stanton, G. C. (1976). Self-concept: Validation of construct interpretations. *Review of Educational Research*, *46*(3), 407–441.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child development*, *75*(2), 428–444.

Siegler, R. S., & Lortie-Forgues, H. (2014). An Integrative Theory of Numerical Development. *Child Development Perspectives*, *8*(3), 144–150. https://doi.org/10.1111/cdep.12077

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological science*, *14*(3), 237–250.

Simonton, D. K. (2003). Expertise, Competence, and Creative Ability: The Perplexing Complexities. In R. J. Sternberg & E. L. Grigorenko (Eds.), *The Psychology of Abilities, Competencies, and Expertise* (pp. 213–240). Cambridge University Press.

Sirin, S. R. (2005). Socioeconomic status and academic achievement: A meta-analytic review of research. *Review of Educational Research*, *75*(3), 417–453.

Slavin, R. E. (1983a). *Cooperative learning*. New York: Longman.

Slavin, R. E. (1983b). When does cooperative learning increase student achievement? *Psychological Bulletin*, *94*(3), 429–445.

Snow, G. M. (2011). *Development of a Math Interest Inventory to Identify Gifted Students from Underrepresented and Diverse Populations* (Masters Theses & Specialist Projects). Western Kentucky University.

Spinath, B., Freudenthaler, H., & Neubauer, A. C. (2010). Domain-specific school achievement in boys and girls as predicted by intelligence, personality and motivation. *Personality and Individual Differences*, *48*(4), 481–486. https://doi.org/10.1016/j.paid.2009.11.028

Stake, J. E., & Mares, K. R. (2001). Science enrichment programs for gifted high school girls and boys: Predictors of program impact on science confidence and motivation. *Journal of Research in Science Teaching*, *38*(10), 1065–1088. https://doi.org/10.1002/tea.10001

Stanat, P., Pant, H. A., Böhme, K., & Richter, D. (Eds.). (2012). *Kompetenzen von Schülerinnen und Schülern am Ende der vierten Jahrgangsstufe in den Fächern Deutsch und Mathematik: Ergebnisse des IQB-Ländervergleichs 2011* [Competences of elementary school students at the end of 4th grade]. Münster: Waxmann. Retrieved from http://www.content-select.com/index.php?id=bib_view&ean=9783830977773

Stang, J., Urhahne, D., Nick, S., & Parchmann, I. (2014). Wer kommt weiter? Vorhersage der Qualifikation zur Internationalen Biologie- und Chemie-Olympiade auf Grundlage des Leistungsmotivations-Modells von Eccles. *Zeitschrift für Pädagogische Psychologie*, *28*(3), 105–114. https://doi.org/10.1024/1010-0652/a000127

Steenbergen-Hu, S., & Moon, S. M. (2010). The Effects of Acceleration on High-Ability Learners: A Meta-Analysis. *Gifted Child Quarterly*, *55*(1), 39–53. https://doi.org/10.1177/0016986210383155

Steinmayr, R., & Spinath, B. (2008). Sex differences in school achievement: What are the roles of personality and achievement motivation? *European Journal of Personality*, *22*(3), 185–209.

Steinmayr, R., Wirthwein, L., & Schöne, C. (2014). Gender and numerical intelligence: Does motivation matter? *Learning and Individual Differences*, *32*, 140–147. https://doi.org/10.1016/j.lindif.2014.01.001

Stern, E. (1992). Warum werden Kapitänsaufgaben „gelöst“? [Why specific tasks get „solved“?]. *Der Mathematikunterricht*, *38*(5), 7–29.

Stern, E. (1998). *Die Entwicklung des mathematischen Verständnisses im Kindesalter* [Developing mathematical understanding in childhood]: Pabst Lengerich.

Stern, E. (2017). Früh übt sich: Neuere Ergebnisse aus der LOGIK-Studie zum Lösen mathematischer Textaufgaben [Early practice. New results for the LOGIK-study about solving mathematical word problems]. In A. Fritz, S. Schmidt, & G. Ricken (Eds.), *Pädagogik. Handbuch Rechenschwäche. Lernwege, Schwierigkeiten und Hilfen bei Dyskalkulie* (3rd ed., pp. 116–130). Weinheim, Basel: Beltz.

Sternberg, R. J. (2011). From Intelligence to Leadership. *Gifted Child Quarterly*, *55*(4), 309–312. https://doi.org/10.1177/0016986211421872

Sternberg, R. J., & Zhang, L.-f. (1995). What do we mean by giftedness? A pentagonal implicit theory. *Gifted Child Quarterly*, *39*(2), 88–94.

Stubbe, T. C., Schwippert, K., & Wendt, H. (2016). Soziale Disparitäten der Schülerleistungen in Mathematik und Naturwissenschaften [Social Disparencies in Students' Mathematical and Science Competences]. In H. Wendt, W. Bos, C. Selter, O. Köller, K. Schwippert, & D. Kasper (Eds.), *TIMSS 2015 Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern in Deutschland im internationalen Vergleich* (pp. 299–316). Waxmann.

Stumpf, E. (2011). *Begabtenförderung für Gymnasiasten: Längsschnittanalysen zu homogenen Begabtenklassen und Frühstudium* [Promoting gifted students: Analysis of longitudinal a longitudinal study in homogenous gifted classes and early studies]: Lit.

Suárez-Álvarez, J., Fernández-Alonso, R., & Muñiz, J. (2014). Self-concept, motivation, expectations, and socioeconomic level as predictors of academic performance in mathematics. *Learning and Individual Differences*, *30*, 118–123. https://doi.org/10.1016/j.lindif.2013.10.019

Subotnik, R. F., Olszewski-Kubilius, P., & Worrell, F. C. (2011). Rethinking giftedness and gifted education: A proposed direction forward based on psychological science. *Psychological science in the public interest*, *12*(1), 3–54. https://doi.org/10.1177/1529100611418056

Sullivan, J., Frank, M. C., & Barner, D. (2016). Intensive math training does not affect approximate number acuity: Evidence from a three-year longitudinal curriculum intervention. *Journal of Numerical Cognition*, *2*(2), 57–76. https://doi.org/10.5964/jnc.v2i2.19

Szűcs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2014). Cognitive components of a mathematical processing network in 9-year-old children. *Developmental science*, *17*(4), 506–524.

Taub, G. E., Keith, T. Z., Floyd, R. G., & McGrew, K. S. (2008). Effects of general and broad cognitive abilities on mathematics achievement. *School Psychology Quarterly*, *23*(2), 187.

Thompson, J. M., Nuerk, H.-C., Moeller, K., & Kadosh, R. C. (2013). The link between mental rotation ability and basic numerical representations. *Acta psychologica*, *144*(2), 324–331.

Torgerson, C. J., & Torgerson, D. J. (2013). *Randomised trials in education: An introductory handbook*. London: Education Endowment Foundation.

Träff, U. (2013). The contribution of general cognitive abilities and number abilities to different aspects of mathematics in children. *Journal of experimental child psychology*, *116*(2), 139–156. https://doi.org/10.1016/j.jecp.2013.04.007

Trautwein, U., Köller, O., Lüdtke, O., & Baumert, J. (2005). Student tracking and the powerful effects of opt-in courses on self-concept: Reflected-glory effects do exist after all. In H. W. Marsh, R. G. Craven, & D. M. McInerney (Eds.), *New frontiers for self research* (pp. 307–327). Greenwich: IAP.

Trautwein, U., Lüdtke, O., Marsh, H. W., Köller, O., & Baumert, J. (2006). Tracking, grading, and student motivation: Using group composition and status to predict self-concept and interest in ninth-grade mathematics. *Journal of Educational Psychology*, *98*(4), 788–806. https://doi.org/10.1037/0022-0663.98.4.788

Trautwein, U., Marsh, H. W., Nagengast, B., Lüdtke, O., Nagy, G., & Jonkmann, K. (2012). Probing for the multiplicative term in modern expectancy–value theory: A latent interaction modeling study. *Journal of Educational Psychology*, *104*(3), 763–777. https://doi.org/10.1037/a0027470

Trautwein, U., & Möller, J. (2016). Self-Concept: Determinants and Consequences of Academic Self-Concept in School Contexts. In A. A. Lipnevich, F. Preckel, & R. D. Roberts (Eds.), *The Springer Series on Human Exceptionality. Psychosocial Skills and School Systems in the 21st Century* (pp. 187–214). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-319-28606-8_8

Twone, L., & Hilton, M. (Eds.). (2004). *Implementing randomized field trials in education: Report of a workshop.* Committee on Research in Education, National Research Council. National Academy of Science. Washington, D.C.: The National Academies Press.

Urhahne, D., Ho, L. H., Parchmann, I., & Nick, S. (2012). Attempting to predict success in the qualifying round of the International Chemistry Olympiad. *High Ability Studies*, *23*(2), 167–182. https://doi.org/10.1080/13598139.2012.738324

van de Rijt, B.A.M., van Luit, J.E.H., & Pennings, A. H. (2016). The Construction of the Utrecht Early Mathematical Competence Scales. *Educational and Psychological Measurement*, *59*(2), 289–309. https://doi.org/10.1177/0013164499592006

van der Ven, S. H.G., Klaiber, J. D., & van der Maas, H. L.J. (2016). Four and twenty blackbirds: How transcoding ability mediates the relationship between visuospatial working memory and math in a language with inversion. *Educational Psychology*, *37*(4), 487–505. https://doi.org/10.1080/01443410.2016.1150421

van der Ven, S. H.G., van der Maas, H. L.J., Straatemeier, M., & Jansen, B. R.J. (2013). Visuospatial working memory and mathematical ability at different ages throughout primary school. *Learning and Individual Differences*, *27*, 182–192. https://doi.org/10.1016/j.lindif.2013.09.003

Vaughn, V. L., Feldhusen, J. F., & Asher, J. W. (1991). Meta-Analyses and Review of Research on Pull-Out Programs in Gifted Education. *Gifted Child Quarterly*, *35*(2), 92–98. https://doi.org/10.1177/001698629103500208

Vecchione, M., Alessandri, G., & Marsicano, G. (2014). Academic motivation predicts educational attainment: Does gender make a difference? *Learning and Individual Differences*, *32*, 124–131. https://doi.org/10.1016/j.lindif.2014.01.003

Veenman, S., Kenter, B., & Post, K. (2010). Cooperative Learning in Dutch Primary Classrooms. *Educational Studies*, *26*(3), 281–302. https://doi.org/10.1080/03055690050137114

von Aster, M. G., & Shalev, R. S. (2007). Number development and developmental dyscalculia. *Developmental medicine and child neurology*, *49*(11), 868–873. https://doi.org/10.1111/j.1469-8749.2007.00868.x

Voyer, D., & Voyer, S. D. (2014). Gender differences in scholastic achievement: A meta-analysis. *Psychological Bulletin*, *140*(4), 1174.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*(4), 817–835. https://doi.org/10.1037/a0016127

Walther, G. (2011). Die Entwicklung allgemeiner mathematischer Kompetenz fördern. [Foster the development of general mathematical competences]. In R. Demuth, G. Walther, & M. Prenzel (Eds.), *Unterricht entwickeln mit SINUS. 10 Module für den Mathematik- und Sachunterricht in der Grundschule.* (pp. 15–23). Seelze: Friedrich.

Walther, G., Schwippert, K., Lankes, E.-M., & Stubbe, T. C. (2008). Können Mädchen doch rechnen? Vertiefende Analysen zu Geschlechtsdifferenzen im Bereich Mathematik auf Basis der Internationalen Grundschul-Lese-Untersuchung IGLU [Can girls count?]. *Zeitschrift für Erziehungswissenschaft*, *11*(1), 30–46.

Wang, J. J., Halberda, J., & Feigenson, L. (2017). Approximate number sense correlates with math performance in gifted adolescents. *Acta psychologica*, *176*, 78–84. https://doi.org/10.1016/j.actpsy.2017.03.014

Watts, T. W., Clements, D. H., Sarama, J., Wolfe, C. B., Spitler, M. E., & Bailey, D. H. (2017). Does Early Mathematics Intervention Change the Processes Underlying Children's Learning? *Journal of Research on Educational Effectiveness*, *10*(1), 96–115.

Watts, T. W., Duncan, G. J., Chen, M., Claessens, A., Davis-Kean, P. E., Duckworth, K.,. . . Susperreguy, M. I. (2015). The Role of Mediators in the Development of Longitudinal Mathematics Achievement Associations. *Child development*, *86*(6), 1892–1907. https://doi.org/10.1111/cdev.12416

Webb, N. L., Day, R., & Romberg, T. A. (1988). *Evaluation of the use of "Exploring data" and "Exploring probability"*: Madison: Wisconsin Center for Education Research.

Weinert, F. E. (1999). *Konzepte der Kompetenz* [Concept of competences]: Organisation for Economic Co-operation and Development.

Weinert, F. E. (2001a). Concept of competence: A conceptual clarification. In D. S. E. Rychen & L. H. E. Salganik (Eds.), *Defining and selecting key competencies* (pp. 45–66). Seaattle: Hogrefe & Huber.

Weinert, F. E. (Ed.). (2001b). *Leistungsmessungen in Schulen* [Assessing performance in schools]. Weinheim und Basel: Beltz.

Weinert, F. E. (2001c). Vergleichende Leistungsmessung in Schulen - eine umstrittene Selbstverständlichkeit [Comparative performance measurement in schools - a controversial self-evident]. In F. E. Weinert (Ed.), *Leistungsmessungen in Schulen* (pp. 17–32). Weinheim und Basel: Beltz.

Weiß, R., Albinus, B., & Arzt, D. (2006). *Grundintelligenztest Skala 2-Revision (CFT 20-R)* [Intelligence Test Scale 2-Revision (CFT 20-R)]: Hogrefe.

Welsh, J. A., Nix, R. L., Blair, C., Bierman, K. L., & Nelson, K. E. (2010). The development of cognitive skills and gains in academic school readiness for children from low-income families. *Journal of Educational Psychology*, *102*(1), 43.

Wendt, H., Bos, W., Selter, C., Köller, O., Schwippert, K., & Kasper, D. (Eds.). (2016). *TIMSS 2015 Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern in Deutschland im internationalen Vergleich* [Mathematial and science competences of German elementary school students in international comparisons]: Waxmann.

Wendt, H., Steinmayr, R., & Kasper, D. (2016). Geschlechterunterschiede in mathematischen und naturwissenschaftlichen Kompetenzen [Gender differences in mathematical and science competences]. In H. Wendt, W. Bos, C. Selter, O. Köller, K. Schwippert, & D. Kasper (Eds.), *TIMSS 2015 Mathematische und naturwissenschaftliche Kompetenzen von Grundschulkindern in Deutschland im internationalen Vergleich* (pp. 257–297). Waxmann.

Wigfield, & Eccles. (2000). Expectancy-Value Theory of Achievement Motivation. *Contemporary Educational Psychology*, *25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Wigfield, A. (1997). Reading motivation: A domain-specific approach to motivation. *Educational psychologist*, *32*(2), 59–68. https://doi.org/10.1207/s15326985ep3202_1

Wigfield, A., & Eccles, J. S. (1992). The development of achievement task values: A theoretical analysis. *Developmental review*, *12*(3), 265–310. https://doi.org/10.1016/0273-2297(92)90011-P

Wigfield, A., & Eccles, J. S. (1994). Children's competence beliefs, achievement values, and general self-esteem: Change across elementary and middle school. *The Journal of Early Adolescence*, *14*(2), 107–138.

Wigfield, A., Eccles, J. S., Fredricks, J. A., Simpkins, S., Roeser, R. W., & Schiefele, U. (2015). Development of achievement motivation and engagement. In M. E. Lamb, R. M. Lerner, & S. B. Bonner (Eds.), *Handbook of child psychology and developmental science* (pp. 657–702). Wiley.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D., Arbreton, A. J. A., Freedman-Doan, C., & Blumenfeld, P. C. (1997). Change in children's competence beliefs and subjective task values across the elementary school years: A 3-year study. *Journal of Educational Psychology*, *89*(3), 451.

Wigfield, A., & Karpathian, M. (1991). Who am I and what can I do? Children's self-concepts and motivation in achievement situations. *Educational psychologist*, *26*(3-4), 233–261.

Wigfield, A., Tonks, S., & Klauda, S. L. (2009). Expectancy-value theory. *Handbook of motivation at school*, 55–75.

Wilson, M. (1992). Measuring levels of mathematical understanding. *Mathematics assessment and evaluation: Imperatives for mathematics educators*, 213–241.

Winkelmann, H., & van den Heuvel-Panhuizen, M. (2009). Geschlechtsspezifische mathematische Kompetenzen [Gender differences in mathematical competences]. In D. Granzer, O. Köller, A. Bremerich-Voss, M. van den Heuvel-Panhuizen, K. Reiss, & G. Walther (Eds.), *Bildungsstandards Deutsch und Mathematik. Leistungsmessung in der Grundschule* (pp. 142–156). Weinheim: Beltz.

Winkelmann, H., & Robitzsch, A. (2009). Modelle mathematischer Kompetenzen: Empirische Befunde zur Dimensionalität [Models of mathematical competences.]. *D. Granzer, O. Köller, A. Bremerich-Vos, M. van den Heuvel-Panhuizen, K. Reiss & G. Walther (Hg.), Bildungsstandards Deutsch und Mathematik. Leistungsmessung in der Grundschule*, 169–196.

Winkelmann, H., Robitzsch, A., Stanat, P., & Köller, O. (2012). Mathematische Kompetenzen in der Grundschule [Mathematical competences in elementary school]. *Diagnostica*, *58*(1), 15–30. https://doi.org/10.1026/0012-1924/a000061

Winkelmann, H., van den Heuvel-Panhuizen, M., & Robitzsch, A. (2008). Gender differences in the mathematics achievements of German primary school students: Results from a German large-scale study. *ZDM*, *40*(4), 601–616. https://doi.org/10.1007/s11858-008-0124-x

Winter, H. (1995). Mathematikunterricht und Allgemeinbildung [Mathematics and general education]. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, *21*(61), 37–46.

Wirt, J. L. (2011). *An analysis of Science Olympiad participants' perceptions regarding their experience with the science and engineering academic competition.* (Dissertation). Seton Hall University. Retrieved from http://scholarship.shu.edu/cgi/viewcontent.cgi?article=1014&context=dissertations

Wittmann, C. (2005, July). *Mathematics as the science of patterns–a guideline for developing mathematics education from early childhood to adulthood*. Plenary Lecture at International Colloquium 'Mathematical learning from Early Childhood to Adulthood', Belgium, Mons. Retrieved from http://mathinfo.unistra.fr/fileadmin/upload/IREM/Publications/Annales_didactique/vol_11_et_suppl/adsc11supplweb_wittmaneng.pdf

Zeidner, M., & Schleyer, E. J. (1999). The big-fish–little-pond effect for academic self-concept, test anxiety, and school grades in gifted children. *Contemporary Educational Psychology*, *24*(4), 305–329.

Ziegler, A. (2008). *Hochbegabung* [Giftedness]. München: Reinhardt.